FOCUSING WIDEBAND ARRAYS FROM 2-D SPECTRAL ESTIMATES

M. A. Lagunas, G. Vázquez

Universitat Politècnica Catalunya, Apt. 30002, Barcelona, Spain

Abstract

This paper describes the scope of focusing at the spectral level, showing up to what degree the time and the spatial support of the aperture represents a fundamental bound in performance. Finally some solutions coming from the spectral estimation domain are envisaged.

1. Spectral estimation and focusing.

Focusing a wideband source requires to average over a given frequency band the received energy in the aperture over constant DOAs. The procedure is adequately explained as a 2-D power level estimation problem. Let the signal source at the sensor level given by

$$x_{sq}(t) = \frac{1}{2\pi} \int X_s(w) e^{j(wt - \vec{k}_s \cdot \vec{r}_q)} dw \quad (1)$$

being s and q the source and sensor index respectively. The global signal is obtained after adding over index s all the sources impinging the array from the aperture environment.

$$x_q(t) = \sum_{s=1}^S x_{sq}(t) + n_q(t) \quad (2)$$

We claim that focusing (i. e. determining the procedure to obtain

arriving energy from a given $\vec{k}$) has to be derived from the two-dimensional spectral estimate that corresponds to the data periodogram $P(w, \Omega)$ given in (3).

$$P(w, \Omega) = \left| \frac{1}{Q} \sum_{q=0}^Q \left(\int x_q(t) e^{-j\omega t} dt \right) e^{-jq\Omega \rho_0} \right|^2 \quad (3)$$

Before we continue, it is interesting to note that, as concern with spectral estimation purposes, the signal periodogram as well as its corresponding autocorrelation function provides the adequate starting point in order to derive any kind of spectral estimates enhancing special features.

Thus, the mentioned periodogram in the noise free case and a single source reduces to (4).

$$P_s(w, \Omega) = X(w) \sum_{q=0}^Q e^{j\vec{k}_s \cdot \vec{r}_q} e^{-jq\Omega \rho_0} \quad (4)$$

The problem arises from the complex expression of the dot product of the wave vector with the sensor location vector. This expression, shown in (5) where θ is the azimuth angle and ϕ is the elevation angle, reveals that, even if a given Ω_s the corresponding periodogram is maximum, finding the

corresponding angles of arrival of the source (θ_s, φ_s) may be a difficult task.

$$R_s r_q = \frac{w d_q}{c} \{ \cos \varphi_q \cos \varphi_s + \sin \varphi_q \sin \varphi_s \cos(\theta_s - \theta_q) \} \quad (5)$$

One way out to overcome this problem is to proceed to a direct exploration in accordance with (5). That means that the second FFT should be done with the following frequency exploration

$$q \Omega \varphi_0 = \{ \cos \varphi_q \cos \varphi + \sin \varphi_q \sin \varphi \cos(\theta - \theta_q) \} \quad (6)$$

$$\text{where } \varphi = s/S \quad s = -S, S$$

$$\text{and } \theta = r/R \quad r = -R, R$$

This procedure achieves the first step of focussing which is just to convert the 2-D representation in an energy versus combined frequency DOA representation.

$$P(\omega, \Omega) \rightarrow P(\omega, \varphi, \theta) \quad (7)$$

The second and final step, to get the focused image, is to average over the array response frequency band.

$$P_f(\varphi, \theta) = \int_w P(\omega, \varphi, \theta) d\omega \quad (8)$$

At this point focusing is ended and, as claimed before, the adequate output from any procedure to implement is a plot of energy versus DOA parameters only.

As a consequence note that when doing focusing two different

procedures of spectral estimation have to be used. First a low bias and high statistical stability procedure is required in order to get the multidimensional energy estimate $P(\omega, \varphi, \theta)$. Due to the desired features, an analysis of the currently reported methods, reduces our choice to either a periodogram or a power maximum likelihood estimate. It may be stated that the above choice is dictated by the importance of providing adequate levels of energy impinging the array. Resolution is not the main issue at this step. That is not longer the case for the second step. The procedures to apply over $P_f(\varphi, \theta)$ need resolution, always trading with adequate confidence levels, in order to determine as accurate as possible the source angles of arrival. It should be noted that $P_f(\varphi, \theta)$ has to be viewed as a periodogram estimate, thus its inverse Fourier Transform will define the correlation mask over which high resolution procedures should be applied.

2. Some fundamental limitations.

One of the nice advantages of looking focusing at the spectral level is to find out clearly which limitations and associated bounds in performance we are going to face in the procedure. These bounds are found in focusing at the covariance level [4] but the mathematical framework around it does not allow an easy understanding of the phenomena. In [1] the authors reported an explanation for these limitations in the case of a linearly

equally spaced array, here a generalized point of view is reported.

Let us assume that we have infinite resolution at the time and angle level (i. e. $x_{SQ}(t)$ has infinite spatial support and duration); in this case a wideband source will produce a set of Dirac functions multiplied by the energy associated to the source at each frequency.

$$\sum_{i=1}^I \delta(\omega - \omega_i) \delta(\theta, \varphi - \theta_s, \varphi_s) P(\varphi_s, \theta_s) \quad (9)$$

To simplify our derivation, let us assume all the energy levels produced by the source are the same namely $P(\varphi_s, \theta_s)$. The focused energy will be:

$$I_0 P(\varphi_s, \theta_s) \delta(\theta, \varphi - \theta_s, \varphi_s) \quad (10)$$

Note that as expected from a coherent processing the gain of focussing is $10 \cdot \log$ of the number of averages. The situation changes dramatically when, depending on the array geometry, we convert Ω in φ and θ with a finite data support. In this case a window $W(\cdot)$ replaces the Dirac function; in addition, mapping Ω in φ and θ causes that the width of $W(\cdot)$ depends on the frequency such that (9) is replaced by

$$\sum_{i=1}^I \delta(\omega - \omega_i) W(\theta, \varphi, \theta_s, \varphi_s, \omega) P(\varphi_s, \theta_s) \quad (11)$$

To be more precise the width of the angular window decreases with the frequency in accordance with the

reduced size of the array aperture at low frequencies. As a consequence the focused plot will be (12).

$$P(\varphi_s, \theta_s) \sum_{i=1}^I W(\theta, \varphi, \theta_s, \varphi_s, \omega_i) \quad (12)$$

The sum shown in (12) yet preserving the $10 \cdot \log$ of the number of averages gain for $\theta = \theta_s$ and $\varphi = \varphi_s$, produces an unexpected spread window which reduces the performance of any high resolution procedure prepared to detect deterministic sinusoids in noise as it is the case of mostly of the so called high resolution methods.

To be more precise just consider the case of an array solving only azimuthal angles. The maxima in the two dimensional periodogram will be at the spectral frequency Ω that produces the best fit to the spatial frequency produced by a source impinging the array with an angle φ . This best fit is given by:

$$\sum_{q=0}^{Q-1} \left(q\Omega - \frac{d_q \omega}{c} \cos(\varphi - \varphi_q) \right)^2 \Big|_{\min} \quad (13)$$

where an arbitrary constant (the array phase reference) can be subtracted to each term in the summation. From (13) it is clear that the maxima in W will follow a linear relationship with w .

Figure 1. Spatial window along line pattern of focusing.

Thus, looking at figure 1, it is evident the lines of exploration, yet preserving the line of maxima produced by a given source, never will focus or add coherently the corresponding spread produced by the window associated to the finite size of the aperture. Returning to the effects of unaligned spread, adding along the lines of exploration is like superimposing different sinc functions of different duration.

Figure 2. Spatial windows of different width added during focusing.

As a consequence, when applying high resolution methods over the focused energy versus DOA (see 8), the unaligned windows produce an extra sidelobe level (α in figure 2) that

avoids the expected improvement of performance after coherent averaging along the source frequency band.

It is important to note that even when the source is at the broadside the mentioned drawback exists. Only for relative small bandwidths this phenomena will not be important and the gain derived from focusing will be in correspondence with a perfect coherent average.

At the same time, note that focusing at the 2-D spectral level, referred sometimes as resampling [3] or interpolated arrays [2] for the case of linearly equally spaced arrays, also suffers of this problem. In any case the non-uniform spread is a deterministic phenomena and, in this sense, do not destroy the spatial distribution of background noise as non-unitary transforms do in the case of focussing at the covariance level [4].

Some opportunities in order to reduce the stated problem can be envisaged and they will be described in the next section.

3. Separate spectral estimation.

In accordance with what has been stated in the previous section, it seems clear that the use of frequency dependent (or independent in the ideal case) procedures, to obtain a two-dimensional spectral estimate, will be preferred since they will reduce the bound imposed by the

constant bandwidth analysis imposed by the Fourier Transform.

One way of getting more quality, or well conditioned spread, is just to use a 2-D power maximum likelihood estimate. Regardless the spread will be constant, the low sidelobe level of the MLM estimate will perform better than the 2-D periodogram. This has been checked out by the authors in simulated scenarios.

Another possibility, more risky than the periodogram or MLM, consists on the use of autoregressive or SVD based methods to obtain the 2-D spectral estimate. Of course, we have to keep in mind that 2-D versions of these procedures are highly time consuming. An alternative is to use them just in the second step of a two step or a separate 2-D spectral estimate. At this point note that phase has to be preserved during the first step, which forces the Fourier Transform the first procedure. After, and along index q , autoregressive or SVD methods may be used. It is important to note that going in this way the procedure is going to face a tradeoff between bias and resolution. Resolution is required in order to remove the window spread effect; but, at the same time low bias is highly required in order to average the adequate values along the preselected pattern in the variable w (see (8)).

4. References.

[1] M. A. Lagunas, G. Vázquez, J. F. Sitjar. " Specific 2-D spectral

estimation for wideband beamforming". Proceedings ICASSP-90 paper EG.21, pp 2707-2710, Albuquerque, USA.

[2] G. Bienvenu et al. "Coherent wideband high resolution processing for a linear array". Proceedings ICASSP-89 paper US.2, pp 2799-2802, Glasgow, Scotland.

[3] J. Krolik, D. Swingler. "Focused wideband array processing by spatial resampling". ASSP Transactions, vol 38, nº 2, pp 356-359, February 1990.

[4] H. Wang, M. Kaveth. "Coherent signal Subspace processing for the detection and estimation of angles of arrival of multiple wide-band sources". ASSP Transactions vol 33, pp 823-831, August 1985.