Normalized parameters for Y-branch optical waveguides

Jaume Recolons, Lluís Torner, and Fernando Canal

Department of Signal Theory and Communications, Polytechnic University of Catalonia, Apdo. 30002, 08080 Barcelona, Spain

Received October 24, 1990

Y-branch optical waveguides are analyzed in terms of normalized parameters by means of the usual modal analysis. With these parameters the number of variables of the problem is reduced, and universal plots for the mode conversion are obtained. Our results show that the normalized parameters are useful for the design of branching waveguides more tolerant of fabrication errors.

Nonparallel waveguides are used in a large number of electro-optic switches and modulators, such as directional couplers and Mach-Zehnder and crossing channel switches.¹ It has been shown theoretically² that mode conversion in the converging or diverging regions caused by the asymmetries introduced in the fabrication process results in a degradation of the extinction ratio of the device. Usually such mode-conversion effects are studied by means of a numerical modal analysis in which the nonparallel waveguide sections are replaced by five-layer waveguide steps.²⁻⁴ The object of the current study is to reduce the number of the variables involved in that treatment by introducing normalized parameters for branching waveguides.⁵ This will enable us to obtain universal results for the mode-converted power and to establish rules of similarity among such waveguides.

Our first step is to introduce normalized parameters for the five-layer waveguide resulting from the superposition of two three-layer planar waveguides, as shown in Fig. 1, where the refractive indices n_2 and n_4 correspond to the guiding layers. In a similar way as for three-layer waveguides,⁶ we define the normalized effective index for the eigenmodes:

$$b = (N^2 - n_1^2)/(n_4^2 - n_5^2), \quad (1)$$

where N is the effective index and the normalized thicknesses for the guiding layers are

$$V_2 = kd_2(n_2^2 - n_1^2)^{1/2},$$

$$V_4 = kd_4(n_4^2 - n_5^2)^{1/2}, \quad (2)$$

where $k = 2\pi/\lambda$, λ being the free-space wavelength. We define as well the normalized separation between the guiding layers and three asymmetry coefficients as follows:

$$V_3 = \frac{2s}{d_2 + d_4}, \quad (3)$$

$$a = (n_1^2 - n_5^2)/(n_4^2 - n_5^2),$$

$$g = (n_3^2 - n_1^2)/(n_4^2 - n_5^2),$$

$$p = (n_4^2 - n_2^2)/(n_4^2 - n_5^2). \quad (4)$$

Thus a gives the external asymmetry between the

surrounding layers and p gives the internal asymmetry between the two guiding layers. This choice is somewhat arbitrary, but it suits our purposes. It can be seen straightforwardly that the eigenvalue equation for TE modes can be expressed entirely in terms of these normalized parameters. Therefore the solution for the normalized effective index b will be a function of the other six parameters.

We will limit ourselves to the case $n_1 = n_3 = n_5 \equiv n_0$, where n_0 is the substrate index, which is the most common case in electro-optic switches. With this assumption we obtain $a = g = 0$, so the only variables that we have to deal with are V_2 , V_3 , V_4 , and p ; b is a function of them. In this case the normalized parameter p offers an intuitive measure of the electromagnetic asymmetry of the structure, while its geometrical symmetry is related to V_2 and V_4 . Moreover, we will study only the case $n_0 < n_2 \leq n_4$, so that $p, b, \in [0, 1]$, since the opposite situation can be reduced to this one.

The optical Y branch was first studied by Burns and Milton³ and Yajima⁴ by means of a five-layer model and a step approximation (see Fig. 1). They showed

Fig. 1. (a) Five-layer model for two three-layer parallel waveguides. (b) Step approximation for two separating waveguides.

that the input modes of the five-layer waveguide transform adiabatically into the modes of the output waveguides as they travel along the separating region. During this evolution, mode conversion occurs. This process can be expressed by the following equations:

$$A_{j,m+1} = \sum_{l=0}^1 C_{lj} A_{l,m} \exp[i(\beta_{j,m+1} - \beta_{l,m})z_m], \quad j = 0, 1, \quad (5)$$

which indicate the mode conversion at each step. $A_{l,m}$ and $\beta_{l,m}$ are, respectively, the complex amplitude and the propagation constant of mode l at step m , and z_m is the position of the step. C_{lj} is the coupling coefficient between modes l and j and is given by

$$C_{lj} = F_{lj} \frac{I_{lj}}{\sqrt{I_{ll}I_{jj}}}, \quad (6)$$

where I_{lj} is the overlap integral between the corresponding mode field distribution evaluated at steps m and $m+1$, respectively, and

$$F_{lj} = \frac{2(\beta_{l,m}\beta_{j,m+1})^{1/2}}{\beta_{l,m} + \beta_{l,m+1}} \frac{\beta_{j,m} + \beta_{l,m+1}}{\beta_{j,m} + \beta_{j,m+1}}. \quad (7)$$

Since I_{lj} depends only on the transversal field distribution of modes l and j , it can be expressed entirely in terms of the normalized parameters given above. Moreover, if we expand the prefactor F_{lj} in powers of $\delta n/n_0$, where $\delta n = n_4 - n_0$, we obtain

$$F_{lj} \simeq 1 + \frac{1}{4}[(b_{l,m} - b_{j,m})(b_{l,m+1} - b_{j,m+1}) - \frac{1}{2}(b_{l,m} - b_{j,m+1})^2] \left(\frac{\delta n}{n_0}\right)^2. \quad (8)$$

Then, provided that $\delta n/n_0 \ll 1$ and the expression enclosed in brackets takes on values between zero and unity, F_{lj} will be a slowly varying function of the waveguide parameters, so C_{lj} will depend basically on the normalized parameters.

In fact, our whole problem is z dependent, so we have an additional dimension. This means that we need to add a new parameter in order to normalize the exponential terms of Eq. (5). We proceed as follows. By expanding $(\beta_{j,m+1} - \beta_{l,m})z_m$ in powers of $\delta n/n_0$ and keeping only first-order terms, we obtain

$$(\beta_{j,m+1} - \beta_{l,m})z_m \simeq (b_{j,m+1} - b_{l,m}) \left(\frac{\delta n}{2n_0}\right)^{1/2} \frac{V_{3,m}}{2\alpha} \times \left(\frac{V_2}{\sqrt{1-p}} + V_4\right), \quad (9)$$

where $V_{3,m} \approx 2\alpha z_m/(d_2 + d_4)$, α being the half-branching angle. Therefore we can define a normalized branching angle as follows:

$$\frac{1}{\theta} \equiv \frac{1}{2\alpha} \left(\frac{\delta n}{2n_0}\right)^{1/2}, \quad (10)$$

so we have

$$(\beta_{j,m+1} - \beta_{l,m})z_m \simeq (b_{j,m+1} - b_{l,m}) \times \frac{V_{3,m}}{\theta} \left(\frac{V_2}{\sqrt{1-p}} + V_4\right). \quad (11)$$

The normalized branching angle admits of an interesting interpretation since it makes clear that the effect of changing the direction of propagation in an optical waveguide increases as the ratio $\delta n/n_0$ decreases. As a matter of fact, for given values of p , V_2 , and V_4 , each value of the normalized branching angle sets up an equivalence class formed by branching waveguides with different refractive indices and branching angle but with equal θ .

By applying Eq. (5) in a recurrent way we obtain the final converted mode amplitudes. Since the calculation is carried up from $V_{3,m} = 0$ to $V_{3,m} \rightarrow \infty$, the final results will not depend on this variable. Thus we have $A_{j,\infty} = A_{j,\infty}(V_2, V_4, p, \theta)$.

Using the normalized five-layer model, we have studied the effect of the asymmetries in a branching waveguide like that of Fig. 1, assuming that the TE_0 mode with a unity power is launched in branch A and that the modes propagate toward branches B and C. We have used in our calculations discrete steps with $\Delta V_3 = V_{3,m+1} - V_{3,m} = 0.002$. In Fig. 2 we show the behavior of the total converted power from TE_0 to TE_1 modes of the structure as a function of the normalized branching angle θ for the case $V_2 = V_4 = 3$, taking p as a parameter. As expected, the amount of converted mode power increases with the branching angle until it reaches a saturation level.⁴ In this case the exponential terms in Eq. (5) tend to 1, so Eq. (5) reduces to

$$A_{j,m+1} \simeq C_{lj}A_{l,m} + C_{jj}A_{j,m}, \quad l \neq j = 0, 1. \quad (12)$$

Therefore in the saturation region a further increase in θ results in small variations of the converted mode power [i.e., $A_{j,\infty} \simeq A_{j,\infty}(V_2, V_4, p)$]. Nevertheless, we must keep in mind that we have neglected radiated modes and the effect of wave-front tilt at the output branches, so the last assertion holds only within the range of applicability of the staircase approach. We can also see that the saturation level for the converted

Fig. 2. Converted power from TE_0 to TE_1 modes of the Y branch at the end of the separating region (P_1 as a function of the normalized branching angle θ for the case $V_2 = V_4 = 3$).

Fig. 3. Saturation values for the output power at branch B (P_B) as a function of the asymmetry measure p for the case $V_4 = 3$.

mode power is reached more quickly and has a greater value when the symmetry between the output branches increases.

In Fig. 3 we have plotted the saturation value for the output power at the end of branch B as a function of the asymmetry measure p , taking $V_4 = 3$ and V_2 as a parameter. This plot shows that the output power is equally divided between both output branches for the case of a totally symmetric Y branch ($V_2 = V_4$ and $p = 1$). This situation can be reproduced for a nonsymmetric one by compensating for the geometrical asymmetry by means of an electromagnetic asymmetry.⁷ On the other hand, any Y branch with $2 < V_2 < 4$ will exhibit a behavior placed within the limits shown in Fig. 3. This means that in the neighborhood of the value $p \sim 0.2$ the saturated output power is, for all practical purposes, not affected by a change in V_2 . Also, one can note that the behavior of the Y branch with $V_2 = 2$ is less sensitive to variations of the parameter p than to other plotted Y branches. All this suggests that, since there are many different Y branches that act as 3-dB couplers, we can choose the refractive indices and widths of the layers in order to achieve the greatest degree of tolerance in the fabrication process.

In conclusion, normalized parameters for separating waveguides have been introduced by means of the usual modal analysis. Their range of applicability is determined by the condition $\delta n/n_0 \ll 1$ (see also Ref. 5 for some numerical calculations), which is a loose restriction in the domain of integrated optics since the index changes produced by the usual fabrication techniques are small. We have proved that, under this assumption, the behavior of the Y branch can be explained as a function of four parameters, namely, the normalized waveguide thicknesses of both output branches, the normalized branching angle, and the asymmetry parameter p . This enables us to reduce the number of variables with which we must deal and to obtain universal plots that establish rules of similarity among different Y branches. The plots obtained suggest that this can be useful for the design of Y branches that are less sensitive to fabrication tolerances. Moreover, the normalized parameters reported here provide a powerful tool for the study of the effects of the asymmetries in electro-optic switches.⁸ Research in this direction is now in progress.

This study has been partially supported by grant PB-87-0798-C0302 from the Comisión Interministerial de Ciencia y Tecnología of the Spanish Government. We also acknowledge the support of the Comissió Interdepartamental de Recerca i Tecnologia of the Catalan Government.

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