DOA DETECTION VERSUS COMBINED SPATIAL TEMPORAL REFERENCE BEAMFORMING

Miguel A. Lagunas, Gregori Vázquez

T.S.C. Department. E.T.S.I. Telecomunicación
Apdo. 30.002, 08080 Barcelona, Spain

Abstract: This paper presents a procedure for CIR beamforming avoiding SVD-like procedures, and derived from the classic CINR solution. The resulting nulling, for impinging interferers, is of the same degree that when using noise subspace procedures. The method is based on the use of a temporal reference, already available in the satellite payload, together with the spatial steered beamvector (the Applebaum solution). The resulting procedure is a valuable alternative to the classic covariance loading methods, since does not requires further matrix inversion, and the noise estimate is derived from the existing hardware. Some results corresponding to a data relay satellite (DRS) scenario are reported.

I. INTRODUCTION

Modern satellite communication systems work with low level desired signals in a highly contaminated scenarios. The heaviest case is given when by using spread spectrum signals, the desired signal impinges the array at CNR (carrier to noise ratios) ranging in the margin of -5dB up to 0dB at the element level. Based in the despreading ratio, given by a channel bandwidth equal to 5 MHz for a symbol rate of 1 Mbaud, adequate signal to noise ratio are obtained in the corresponding receiver. The spreading is achieved by a Gold code used to generate a pseudorandom digital carrier, this digital carrier is multiplied (logic AND) with the binary data; the resulting signal is finally modulated using BPSK.

Because no despreading is available at the satellite, rough BPSK detection has to be used in-board to regenerate the signal before retransmission proceeds. In general, even when despreading is available, all the receiving systems are well behaved under white noise and absence of interferences. Carrier tracking, timing, phase adquisition units, matched

filtering or correlation filter, and ML detector are designed under no colored noise; this means that very low level interferers, at the array output, may degrade very much the overall performance of the relay operation. This somehow reveals up to what degree optimum beamforming needs not to be the objective in the design of adaptive arrays for communications. A better nulling of interferers, at the array level, is much desired that improving the CINR (carrier to interference plus noise ratio). This objective of maximising the CIR (carrier to interference ratio), instead the CINR, is achieved at the expense of a greater noise level at the array output. Nevertheless, the rest of systems, in the receiver, are well protected against white noise, revealing the interest of CIR designs. The above arguments in favour of CIR instead of CINR for adaptive arrays are also reinforced in TV satellite systems due to the sensitivity of video signals to low level interferers.

It is very well known that optimum CIR beamforming requires of the noise level estimation σ^2 , in order to form the corresponding weight vector. The resulting procedure use to be referred as the loading covariance method [3] [4].

This paper will deal with the estimation of the noise level without interruption of the communication link. This is done with already available in-board hardware and without requiring the computation of SVD of the associated covariance matrix. Of course, avoiding SVD for maximum CIR beamformer, and feasible estimates, impose a suboptimum character in the resulting solution. Also section IV reports a new approach for CIR beamformer design which overcomes some ill-conditioned problems associated with negative loading of the covariance matrix.

Finally, it is important to remark that CIR beamformers are very well suited to reinforce source nulling when superresolution procedures are used in DOA estimation. Furthermore, CIR beamforming is very well suited to procedures of DOA estimation based on a set of

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Applebaum solutions [1] as it has been recently reported by M. Amin [2]. The idea reported by M. Amin results in a high resolution procedure, based on existing analog or digital loops; and it is of high interest for existing in-board transponders.

II. A REVIEW OF THE MAXIMUM CIR-BEAMFORMER

In general, all terminal design provides a reference test oscillator that using a directional coupler could be connected simultaneously to all the array elements. This testing signal, that could be subtracted at the array output after the final down-conversion to base-band, allows to the maintenance of adequate desired signal levels, calibration, as well as monitoring of frequency and phase shifts eventually produced by component aging.

At the same time, due to the low level of the desired signal which use spread spectrum modulation. The receiving array is pointed by spatial reference using a GSLC; being the pointing direction the aperture broadside ± 0.5 degrees. This means that adding the test signal (with a directional coupler which introduces the necessary attenuation), is like adding a high level signal in the desired direction (i.e. the aperture broadside). Assuming that the resulting array covariance is given by (1)

$$\underline{R} = \alpha_d \underline{S}_d \underline{S}_d^H + \sum_{i=2}^{IS} \alpha_i \underline{S}_i \underline{S}_i^H + \sigma^2 \underline{I} \quad (1)$$

being $(\alpha_d, \underline{S}_d)$ the desired signal level and steering vector respectively, σ^2 the noise level and $(\alpha_i, \underline{S}_i)$ the same couple describing IS interferences in the scenario.

As described before the low level desired signal is almost masked by the test signal; in consequence we may assume that the desired signal level α_d is known, and it is given only by the test signal.

The principal component decomposition of $\underline{R}$ is given by (2); where λ_d are the signal eigenvalues for $q=1$ to IS, and for $q=IS+1$ to Q (the number of antenna elements) the noise eigenvalues.

$$\underline{R} = \sum_{q=1}^Q \lambda_q \underline{U}_q \underline{U}_q^H \quad (2)$$

The optimum CIR beamformer is given by the only-signal based inverse of $\underline{R}$, this signal inverse of the covariance matrix is given, within a constant, by:

$$\Delta \text{CIR} = \sum_{q=0}^{IS} \frac{1}{\lambda_d \cdot \sigma^2} \underline{U}_q \underline{U}_q^H(\underline{S}_d) \quad (3)$$

where $\underline{U}_q(\underline{S}_d)$ is the response of the signal eigenvector $\underline{U}_q$ to the desired direction denoted by the corresponding steering vector $\underline{S}_d$. Note that the above solution for optimum CIR beamforming comes from selecting a beamformer $\underline{A}$ by the new noise eigenvector generated when the desired signal is subtracted from the given covariance matrix (4). Let us to follow this procedure in order to know the relationship between the test signal level and the non-directional noise level.

$$\underline{R}_0 = \underline{R} - \alpha_d \underline{S}_d \underline{S}_d^H \quad (4)$$

The new noise eigenvector requires only of a constant to point the desired direction, with 0dB., and nulling all the interferers signals which remain in $\underline{R}_0$.

As it can be concluded from the above presentation optimum CIR beamformer reduces to a principal component analysis in order to determine accurately the signal subspace and signal eigenvalues of the inverse covariance matrix $\underline{R}^{-1}$. Next section will provide one way to circumvent this problem. At the same time, it will be shown the involved advantages of using the test signal to increase the desired signal level.

III. NOISE LEVEL ESTIMATE FROM TEMPORAL REFERENCE

A tentative to CIR beamforming may claim that it is not necessary to add the test signal in order to measure the noise level for the optimum CIR beamformer. But, whenever the desired signal is present at a very low level, as denoted before, it becomes immersed in the noise subspace making difficult to distinguish the signal subspace from the noise subspace. We are assuming that interferers come from PSK or unmodulated signal pulses coming from ground communication terminal and radar stations. Thus, measuring noise level and signal eigenvectors becomes a difficult task whenever the desired signal impinges the aperture with very low level. In the other hand, it may be argued that whenever the desired signal is low, optimum CIR could be achieved by means the extended noise subspace of $\underline{R}$; we refer to it as extended in the sense that we presume that the desired signal is included on it. From a pure theoretical point of view this is true; but, in practice, some times the desired signal is recognized, and sometimes not, depending on the current scenario. This makes the procedure not robust since the adequate design depends on the levels (of the interferers) which of course we do not have under control.

Based on the above limitations we propose the use of the test signal added to the low desired signal. The effects of the test signal is two-fold: it stabilizes the optimum solution

(3) and provides an easy estimate of the noise level σ^2 . Increasing the desired signal tends to increase all the signal eigenvalues, allowing a clear separation between the two subspaces.

Searching for the new noise eigenvalue of matrix (4), it is interesting the relationship between the eigenvalues of $\underline{R}$ and the corresponding ones to $\underline{R}_0$ [5]. Because matrix $\underline{R}_0$ is obtained from $\underline{R}$ by substrating a dyad belonging to its signal subspace, it is easy to obtain the mentioned relationship between both set of eigenvalues.

The relationship is given by (5).

$$\frac{1}{\alpha_d} = \sum_{q=0}^{IS} \frac{|\omega_q|^2}{\lambda_q - \lambda_q^0} \quad (5)$$

This last expression differs from the one shown in reference [5] in the summation index, which is the number of interferers instead the matrix order. The reason for this is that the noise subspace has null contribution in the summatory, due to the special character of the dyad, which is orthogonal to the mentioned subspace. Note that $|\omega_q|^2$ is given by the components of vector $\underline{\omega}$ shown in (6).

$$\underline{\omega} = \underline{U}^H \underline{S}_d \quad (6.a) \quad \text{or} \quad \omega_q = \underline{U}_q^H \underline{S}_d \quad (6.b)$$

$$\omega_q = 0 \quad \text{for } IS < q < Q \quad (6.c)$$

Note that the noise eigenvalues, as expected, do not change remaining also the same noise eigenvectors whenever $\underline{S}_d$ is at the signal subspace of $\underline{R}$.

Thus the noise level is related with the desired signal level by (7).

$$\frac{1}{\alpha_d} = \sum_{q=0}^{IS} \frac{|\underline{U}_q^H \underline{S}_d|^2}{\lambda_q - \sigma^2} \quad (7)$$

This relationship could be used to compute very precisely the noise level from the signal level. Nevertheless, the high desired signal levels, taking into account that this produces high signal eigenvalues, the approximation shown in (8) can be used to compute and estimate of σ^2 .

$$\frac{1}{\lambda_q - \sigma^2} \approx \frac{1}{\lambda_q} + \frac{\sigma^2}{\lambda_q^2} \quad (8)$$

At the same time, the two summatory that result in (7) after introducing approximation (8) become closed quadratic forms from the inverse covariance matrix.

$$\underline{A} \underline{R} = \underline{S}_d \quad (\text{the Applebaum solution}) \quad (9.a)$$

$$\sum_{q=0}^{IS} \frac{|\underline{U}_q^H \underline{S}_d|^2}{\lambda_q} = \underline{S}_d^H \underline{R}^{-1} \underline{S}_d = \underline{A}^H \underline{S}_d \quad (9.b)$$

and

$$\sum_{q=0}^{IS} \frac{|\underline{U}_q^H \underline{S}_d|^2}{\lambda_q} = \underline{A}^H \underline{A} \quad (9.c)$$

In summary, the noise power estimate is given by (10), where it has been referred (9.b) as the inverse of the maximum likelihood estimate of the desired signal level.

$$\hat{\sigma}^2 = \left(\frac{1}{\alpha_d} - \frac{1}{\alpha_d} \right) (\underline{A}^H \underline{A}) \quad (10)$$

Note that this last relationship has an interesting reading because the noise estimate is given by the product of the noise bandwidth of the optimum beamformer $\underline{A}$ by the difference between the inverse of the desired signal level and the inverse of the corresponding maximum likelihood estimate. With this reading of the above formula it is clear that the noise estimate is always positive due to the fact that the leakage introduced in the estimate will produce a positive difference in the second term of (10).

IV. A NEW APPROACH TO THE OPTIMUM CIR BEAMFORMER

Once the noise power level has been computed, the so-called covariance loading methods, subtract partially or in full the estimated value from the main diagonal of the given covariance matrix, may be applied. Through its inverse the CIR beamformer or improved DOA estimates are obtained.

Following the above procedure the issue of robustness lacks due to the ill-conditioned character of the resulting matrix. By subtracting some percentage of σ^2 the problem disappears, but at the expense of lower optimality in the resulting solution. A better approach which avoids facing ill-conditioned covariance matrixes, without requiring further computations, is given after using the same approximation that motivates the previously reported noise level estimate.

Since optimum CIR beamforming requires the computation of (3), reproduced here as (11.a) for the presentation, and the Applebaum solution is given by (11.b),

$$\underline{A} \text{CIR} = \sum_{q=0}^{IS} \frac{1}{\lambda_q - \sigma^2} \underline{U}_q \underline{U}_q^H \underline{S}_d \quad (11.a)$$

$$\underline{A} = \sum_{q=0}^{IS} \frac{1}{\lambda_q} \underline{U}_q \underline{U}_q^H \underline{S}_d \quad (11.b)$$

we may use again the approximation (8). Doing in this way, the following relationship results :

$$\hat{\underline{A}} \text{CIR} = \underline{A} + \sigma^2 \underline{R}^{-1} \underline{A} \quad (12)$$

Also, in terms of the covariance inverse and the desired source steering vector, the suboptimum CIR beamformer is shown in (13).

$$\hat{A}_{CIR} = (\underline{I} + \sigma^2 \underline{R}^{-1}) \underline{R}^{-1} \underline{S}_d \quad (13)$$

In order to normalise the beampattern (0dB at the desired signal) the above weighthvector is divided by

$$\underline{S}_d^H \underline{R}^{-1} \underline{S}_d + \sigma^2 \underline{S}_d^H \underline{R}^{-2} \underline{S}_d = 1/\alpha_d \quad (14)$$

In summary, the resulting CIR beamforming is given by (15).

$$\hat{A}_{CIR} = \alpha_d (\underline{A} + \sigma^2 \underline{R}^{-1} \underline{A}) \quad (15)$$

Note that the procedure provided herein do not requires the inverse of the near singular covariance matrix which results in classical covariance loading methods. At the same time, the SVD of the covariance matrix has been changed by the computation of two quadratic forms, which have been also set as a function of the Applebaum beamformer, in order to estimate the noise level.

V PERFORMANCE ANALYSIS

The aperture tested was a linear equally spaced array, half wavelength interelement separation, with 8 elements. The scenario was formed by the desired signal at -5 db. of CNR (carrier to noise ratio) spread spectrum modulated arriving from the aperture broadside. The interferers were two; one located at -10 degrees and 3dB. of CNR being frequency modulated (FM), second one was PSK-4 (four level phase keying modulation) with four samples per symbol, located at 35 degrees and 3 dB. of CNR. All the angles of arrival are given from the aperture broadside. The test signal available was an unmodulated carrier at 10 dB. of CNR. The covariance matrix was formed after 4000 snapshots. The procedures are: the Capon beamformer (CINR design), the optimum SVD-CIR design (as it is shown in (11.a)), the loading covariance method (LOA), and, the procedure reported in this work and summarized in (15) and denoted by MIO in the table.

Test signal level 10 dB.

Interference				
location	CINR	SVD	LOA	MIO
(FM) -10°	-33dB	-47dB.	-46dB.	-51dB.
(PSK-4) -35°	-42dB.	-66 dB.	-62dB.	-100dB.

Test signal level..... 5 dB.

Interference

location	CINR	SVD	LOA	MIO
(FM) -5 °	-17dB.	-65dB	-32dB	-40dB.
(PSK-4) -30°	-36dB	-53dB	-44dB	-78dB

Table II Nulling performance for the same scenario and aperture of the CINR, SVD based, Loading covariance and this paper procedure.

From this table it can be concluded, regardless numerical errors, the herein reported procedure produces, most of the cases the same performance that the optimum SVD CIR beamformer. The noise estimate was the same for the loading covariance method and the paper's method. The table also shows that the covariance loading method, due to the ill-conditioned character of the corrected matrix does not null the interferers with the same accuracy that the reported method. It is important to remark that the adequate design of the CIR beamformer resides in the null positioning. This is not evident from a visual analysis of the corresponding beampatterns.

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