Extended stability range of weak grids with Voltage Source Converters through impedance-conditioned grid synchronization

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Abstract

The stability range of power transfer in high impedance, weak, grids with Voltage Source Converters (VSC) can be extended a simple modification of the conventional Synchronous Reference Frame Phase Locked Loop (PLL) used for grid synchronization. By introducing an impedance-conditioning term in the voltage used by the PLL, the VSC control system is virtually synchronized to the voltage at a stronger point in the grid. By synchronization to the voltage behind the corresponding virtual impedance, the proposed approach can be utilized to influence the reactive power flow of the VSC in a way that will inherently enhance the power transfer capability of the high-impedance grid. To verify the impact of the proposed approach, an analytical model of a VSC HVDC terminal is used to calculate the achievable steady-state power transfer capability, as well as the range where small-signal stability is ensured, in a grid with low SCR when applying different levels of virtual impedance in the grid synchronization. The validity of the stability assessment based on the analytical model, and the increased stability range achieved with the proposed impedance-conditioned PLL, are also verified by time-domain simulations in the Matlab/Simulink environment.

1 Introduction

Voltage Source Converters (VSCs) operating in very weak grids with low Short Circuit Ratio (SCR) are known to face stability challenges related to the control system design and tuning, and corresponding interaction with the grid impedance. Since the challenges of weak grids are most commonly encountered for HVDC transmission systems, several studies on the stability of VSC-based HVDC converter stations under such conditions have been published during the last years [1]-[6]. However, similar stability problems can occur for any VSC-based application operating under weak grid conditions, as confirmed by several recent studies on the stability of VSCs for grid integration of renewables or for general purpose operation in weak distribution systems [7]-[10].

From the previous studies of VSC stability limitations under weak grid conditions, the identified instability phenomena have been mainly associated with two issues:

- 1) The reactive power flow in the system has significant impact on the stable operating range of the VSC and the corresponding power transfer capability of the weak grid. Thus, local voltage control and/or reactive power or current injection from the converter has been shown to improve both the steady-state power transfer capability and the large-signal transient stability range of the VSC [1], [3], [11], [12], [13]. The achievable stability range, and the stability improvements that can be achieved by reactive power injection, is also closely related to the X/R ratio or impedance angle of the equivalent grid impedance as seen from the VSC converter [4], [5].
- 2) The tuning of the Phase Locked Loop (PLL) used for grid synchronization can have a significant impact on the dynamic performance and stability range of VSCs operating in weak grids [5], [7], [9], [10], [14]-[16]. In general terms, the possible stability problems related to the PLL are associated with the interaction between the converter operation and the grid impedance, and instability can occur if the local voltage measurements used by the PLL becomes more influenced by the converter itself than by the grid voltage behind the equivalent grid impedance [9], [10], [16]. This effect can to some extent be attenuated by reducing the bandwidth of the PLL, but this will correspondingly limit the dynamic response of the converter control system [4], [5], [7], [15].

To avoid the potential instability due to high influence of the converter operation on the local voltage measurements used for grid synchronization by conventional PLLs, a voltage-sensor-less approach for synchronizing to a remote point in a weak grid was proposed in [17], [18]. By using a Virtual Flux-based estimation, this approach was shown to significantly improve the achievable VSC stability range and the power transfer capability in a weak grid. The stability improvement was attributed partly to the synchronization to a remote and stiff voltage, but also to the corresponding change of reference frame orientation which ensured an appropriate reactive power injection from the VSC. Thus, in case the total

Fig. 1. Overview of Control system for VSC Converter operated in a weak grid

grid impedance was known, the approach even allowed for reaching very close to the theoretical power transfer limit through an ideal reactance, as long as the VSC operation was not reaching over-modulation. A variety of the same approach, which combined the use of a Virtual Impedance with the estimation of the Virtual Flux in the grid from available voltage measurements instead from a voltage-sensor-less estimation, was further proposed in [19]. Later, the use of a Virtual Impedance in the VSC control system has also been proposed in [20], [21]. However, all these approaches require either a dedicated estimation method or an additional impedance-based control loop to be included in the VSC control system.

To avoid the need for advanced estimation methods or additional impedance-based control loops in the VSC control system, this paper proposes a simple modification to the traditional Synchronous Reference Frame (SRF) PLL by including an impedance-based compensation term in the voltages that are used for grid synchronization. This compensation can be based on the current flowing into the grid, and the implementation requires only simple calculations based on voltage and current measurements to be included in the PLL. Thus, a quasi-stationary synchronization to a remote point in the weak grid can be achieved in a similar way as for the more complicated control strategies discussed in [17]-[19], while achieving the same improvement of stability range and power transfer capability. The achieved stability improvement will be identified from a detailed analytical model of a VSC converter connected to a weak grid, and will be verified by time-domain simulations.

2 Increasing the stability range in weak grids

For presenting the implementation and functionality of the proposed impedance-conditioned PLL, a simple configuration of a VSC HVDC terminal connected to a weak grid will be assumed. The system configuration and assumed control

structure, as well as the details of the proposed PLL implementation will be presented in the following sub-sections.

2.1 Overview of general system configuration

The assumed system configuration is shown in Fig. 1, indicating a VSC HVDC converter terminal connected to a high-impedance grid through a LC filter and a transformer. The converter currents are assumed to be controlled by conventional decoupled PI current controllers in the SRF [22]. To avoid that possible LC oscillations in the filter influences the stability range of the system, an active damping algorithm is included in the studied configuration, providing the stabilizing voltage references v_{AD}^* indicated in the figure, [23], [24]. However, the steady-state power transfer capability of the weak grid is not significantly influenced by the LC transients in the system, so passively damped filters as commonly applied in HVDC applications will yield similar results for the stability limitations of the system.

As shown in Fig. 1, the investigated VSC converter terminal is assumed to have an outer loop PI power controller providing the active, d-axis, current reference. The outer loop PI controller is assumed since it has been demonstrated to be more robust and stable than feed-forward power control based on division with the measured grid voltage amplitude [25]. However, the proposed approach will help to improve the stability range of the system independent of the power control implementation. The limitations on the active power transfer capability of the system will also be similar for cases where the converter is dc-voltage-controlled. Thus, the proposed approach can be directly applied to control schemes with outer loop dc-voltage controllers, or any other active power control strategy, in the same way as the Virtual Flux-based approaches from [17]-[19].

Fig. 2. Details of proposed Phase Locked Loop with virtual impedance conditioning of voltage measurements

2.2 Impedance-conditioned PLL

In the following, a PLL implementation as indicated in the upper part of Fig. 2 is assumed [26]. This PLL structure applies low-pass filters on the estimated d- and q-axis voltage components, while an inverse tangent function is used for identifying the phase angle error e_θ . A PI-controller is then tracking the grid frequency, which is again integrated to obtain the phase angle estimate θ_{PLL} . The inverse tangent function is used in this case to maximize the linear tracking range of the PLL, but the proposed impedance term can be included in any other traditional PLL implementation used for orienting the SRF to the d-axis (or q-axis) of a measured three-phase voltage signal [27].

As mentioned, the proposed impedance-conditioned PLL requires only a simple modification to the traditional SRF PLL. The impedance conditioning is then obtained by subtracting the estimated quasi-stationary voltage drop across a virtual impedance from the voltage measurement available for the converter control system. Thus, the PLL can synchronize to an estimated, "remote" voltage $\tilde{v}_{VI}$, which can be calculated on complex space vector form from (1) by assuming a virtual impedance given in per unit as $z_v = r_v + j\omega_g l_v$.

$$\tilde{v}_{VI} = v_o - r_v \mathbf{i}_o - j \cdot \omega_g l_v \mathbf{i}_o \quad (1)$$

The corresponding implementation of the d- and q-axis voltage drop across the virtual impedance term is shown to the lower right of Fig. 2. The frequency-dependent voltage drop across the virtual inductance is then calculated by using the estimated grid frequency available from the PLL itself.

A vector diagram illustrating the potential impact of the proposed impedance conditioning on the SRF orientation of the PLL is shown in Fig. 3. In this figure, δ_g is the phase

Fig. 3. Vector diagram showing the possible range of SRF orientation for the impedance-conditioned PLL

displacement between the grid voltage v_g and the actual voltage vector v_o measured at the filter capacitors under a certain operating condition, and is therefore indicating the possible range of SRF orientations that can be obtained by the impedance conditioned PLL. The phase angle $\delta\theta_{PLL,0}$ is the steady-state displacement between the grid voltage vector and the orientation of the PLL SRF given by the voltage vector $\tilde{v}_{VI}$, while θ_{PLL} is the continuously increasing, instantaneous phase angle estimated by the PLL and used for the SRF reference frame transformations in the control system. Considering this vector diagram, it should be clear that in case the virtual impedance is set to zero, or if the current flowing into the grid is zero, the impedance conditioned PLL will be synchronized directly to the measured voltage, as any conventional PLL. However, when the system is loaded, the virtual impedance can be selected so that the PLL is

synchronized to the estimated voltage at any electrical distance between the measured voltages and the equivalent grid voltage, as indicated by the range of δ_g shown in the figure. As it will be shown in the following, the stability range under very weak grid conditions can be increased by increasing the virtual impedance towards the total equivalent impedance in the grid. However, operation with full grid impedance compensation can require high converter voltage and will therefore be limited by the available dc voltage, in the same way as discussed in [17], [18].

It should be noted that the equivalent grid impedance as seen from a converter is not always exactly known and might also change with breaker operations in the grid. Thus, accurate and full compensation of the actual grid impedance might be impractical. However, any compensation of known or estimated minimum impedances in the grid will help to improve the stability range without any negative effects on the converter operation, as will be demonstrated in the following. It can also be noted that the proposed approach can be combined with a remote measurement of active and/or reactive power flow to allow for estimating the equivalent grid impedance in a similar way as proposed in [19].

2.3 System modelling for stability identification

To verify the increase of stability range that can be achieved with the proposed approach, an analytical model of the converter and control system configuration from Fig. 1 has been developed. This model is based on the modelling approach from [28], and is described in detail in the Appendix. The applied model allows for simple numerical assessment of the stationary power transfer capability or loadability of the system, and also for small signal stability analysis to verify if identified steady-state operating points are stable.

3 System analysis

In the following, results from analysis of the system from Fig. 1 are presented by applying the model developed in the appendix. These results are used to verify the claimed improvement in stability range and to illustrate how the impedance conditioning is influencing the operating conditions of the VSC terminal.

3.1 Stability limits with conventional synchronization

As a point of reference, the stability limits of the system are first calculated for the case of a conventional PLL, i.e. with zero virtual impedance in the PLL from Fig. 2. The stability limit is first calculated by solving the non-linear system model from (20) for the highest power transfer the results in a numerical solution for the steady-state operating point. Additionally, the highest value of the active power reference that result in a small-signal stable operating point is found from the linearized system model according to (18). The stability limits are calculated for the maximum power transfer capability in both direction, i.e. for inverter operation of the VSC ($p > 0$) and for rectifier operation of the VSC ($p < 0$), and the results are plotted in Fig. 4 as a function of the equivalent

Fig. 4. Stability limits of the investigated system with conventional PLL, as a function of grid impedance ($\varphi_g = 80^\circ$)

grid impedance. For this plot, a grid impedance angle φ_g of 80° is assumed ($\varphi_g = \tan^{-1}(l_g/r_g)$), while l_g and r_g are considered as the equivalent per unit grid inductance and grid resistance, including the series impedance of the transformer, as seen from the filter capacitor indicated in Fig. 1.

From the curves in Fig. 4, it is clearly seen that the power transfer capability of the system is decreasing nonlinearly as the grid impedance is increasing. It is also noticeable that the small-signal stability range is identical to the power transfer capability of the system. Thus, instability is in this case only reached if the power cannot be transferred through the grid impedance and the system reaches voltage collapse. For rectifier operation, the power transfer capability with the assumed control approach and no reactive power support from the converter is falling below 1.0 pu already for a grid impedance exceeding 0.4 pu (i.e. SCR = 2.5). As expected, the power transfer capability is higher for inverter operation, but still the grid impedance must be below about 0.6 pu (corresponding to a SCR of about 1.7) for the VSC to be able to transfer 1.0 pu power. It can also be noted that the plotted curves are closely reassembling an inverse function of the grid impedance, which means that the power transfer capability is almost linearly dependent on the SCR (since $SCR = 1/|z_g|$). Thus, the power transfer capability achieved with VSCs applying the assumed control system configuration is corresponding well with the simplified curves for power transfer limitations developed in [4].

3.2 Stability limits with impedance conditioned PLL

The impact of the proposed approach for impedance conditioning on the power transfer capability of the system is shown in Fig. 5, where the same curves as in Fig. 4 are plotted for increasing levels of virtual impedance included in the PLL from Fig. 2. For simplicity the phase angle of the virtual impedance is kept equal to the grid impedance angle, and the share of compensation is increased by 10 % for each curve shown in the figure. From these curves, it is clear that the power transfer capability can be significantly increased by

Fig. 5. Stability limits of the investigated system with increasing level of impedance compensation ($\phi_g = 80^\circ$)

Fig. 7. Stability limits as a function of the impedance compensation level for different grid impedance ($\phi_g = 80^\circ$)

applying the proposed impedance conditioning approach. Thus, with a virtual impedance corresponding to a sufficient share of the actual grid impedance, it is even possible to achieve 1.0 pu power transfer in both directions with a SCR of 1.0, i.e. with 1.0 pu grid impedance. To further illustrate this, the power transfer capability as a function of the impedance compensation share is plotted in Fig. 7 for a range of grid impedances from 0.1 pu to 1.0 pu. These curves show how the effect of the virtual impedance becomes more and more important as the grid impedance is increasing (i.e. as the SCR is reduced). The curves also show that with full impedance compensation, a wide range of power transfer capability can be achieved for inverter operation of the VSC. As expected, the power transfer capability is always lower for rectifier operation of the VSC, due to the voltage drop across the equivalent grid resistance.

It should be considered that full impedance compensation in the PLL might lead to requirements for higher currents and voltages from the VSC. Thus, even if a wide range of power transfer capability can be achieved with increasing the virtual impedance, the operating conditions of the VSC should be

Fig. 6. Stability limits as a function of the impedance compensation level for a system with SCR = 1 ($\phi_g = 80^\circ$)

taken into account when selecting a suitable virtual impedance. To investigate these issues, the power transfer capability limit and the corresponding voltage and current amplitudes at the filter capacitors of the VSC are plotted in Fig. 6 as a function of the impedance compensation share for the case of 1.0 pu grid impedance (SCR = 1.0) with $\phi_g = 80^\circ$. These curves show that the converter voltage and currents can increase excessively for more than about 50% compensation since this implies significant reactive power injection from the converter. However, for inverter operation of the VSC, a virtual impedance share in the range of 30–40 % of the actual equivalent grid impedance is enough to ensure a power transfer capability of 1.0 pu in this case, with a filter voltage around 1.0 pu and a current amplitude of about 1.2 pu. Thus, the converter must be slightly over-rated in current capability to be able to achieve 1.0 pu power transfer capability in this case. For rectifier operation, a higher share of virtual impedance, with a correspondingly higher injection of reactive power from the converter, is required to be able to reach 1.0 pu power transfer capability with an SCR of 1.0, due to the effect of the resistive voltage drop across the equivalent grid resistance. However, when increasing the virtual impedance towards 70 % of the grid impedance stable operation can still be ensured on the cost of about 50 % over-rating in current, which results in about 1.2 pu voltage due to

the reactive power injection required to ensure the power transfer capability. Although it might not be realistic to design a VSC for such operating conditions, this proves that the proposed impedance conditioning in the grid synchronization makes it possible to keep the system stable beyond the lower acceptable SCR limits of about 1.3 found in [15]. From practical considerations, a simple selection of virtual impedance corresponding to the known minimum impedances in the system, like for instance the equivalent series impedance of the transformer and the lines in the radial part of the transmission system, can be appropriate. This will ensure a significantly extended stability range to avoid stability problems under normal operating conditions.

It should also be noted that the investigated control structure with a fixed reactive q-axis current reference has been shown to be small-signal stable within the full range of power transfer capability, both with the conventional grid synchronization and with any level of virtual impedance included in the proposed PLL implementation. This is similar to the findings by trial-and-error simulation for the case of Virtual Flux-based grid synchronization in [17]–[19], and indicates that the quasi-stationary approximation of the applied virtual impedance terms do not have any significant impact on the small-signal stability of the system. However, when adding outer loop reactive power controllers or ac voltage controllers, as in the control structure used for the investigations in [15], the system can reach small-signal instability before reaching the steady-state power transfer limitations. This issue is left for further investigations.

3.3 Verification of calculated stability limits

To verify that the stability limits calculated in the previous sections are accurately representing the investigated system, they have been verified by time-domain simulations in the Matlab/Simulink/SimPowerSystems environment. In this case, an average model of the VSC is used for the simulations, although it can also be verified that the results will not be significantly influenced by the use of a switching model of the converter. A set of simulation results obtained with 1.0 total grid impedance and $\phi_g = 80^\circ$ are shown in Fig. 8, where the reference case of a conventional PLL is compared to the case with 50 % virtual impedance included in the PLL.

Fig. 8 a) shows a set of simulation results for inverter operation of the VSC, when the power reference is increased in steps from 0 to 1.0 pu, with smaller steps around the calculated stability limit with the conventional PLL. At simulation time $t = 4.0$ s, the power reference is stepped from 0.650 pu to 0.675 pu, and it is clear from the blue dashed curve in the figure that the system with the conventional PLL becomes unstable at that power level as predicted by the results from Fig. 4. With the 50 % impedance compensation, the system is stable for the same conditions, and can also reach the full 1.0 pu power transfer as expected from the curves in Fig. 7 and Fig. 6.

Fig. 8. Time domain simulations for verifying the stability limits calculated from the system model without impedance conditioning and with 50 % virtual impedance

In Fig. 8 b), a similar simulation is repeated for the case of rectifier operation of the VSC. In this case, it is clearly seen that the system with the conventional PLL becomes unstable when the power reference is stepped from -0.450 pu to -0.475 pu, corresponding to the stability limit shown in Fig. 4, while the system with the 50 % impedance compensation is still stable. However when the power reference is stepped from -0.650 pu to -0.675 , also the case with 50 % impedance compensation reaches its power transfer capability and the system collapses, as predicted by the curves in Fig. 7 and Fig. 6. This effectively verifies that the model presented in the appendix can be used to accurately assess the stability and power transfer capability of the investigated system

4 Conclusion

A simple method for extending the achievable range of power transfer capability in weak grids with Voltage Source Converters (VSCs) has been proposed. The extended stability range is achieved by introducing a virtual impedance-based term in the voltage used by the PLL for grid synchronization. This virtual impedance can be utilized to synchronize the

VSC control system to a virtual remote point in the grid, where the estimated voltage will be less influenced by the converter operation. The synchronization to such a remote point in the grid is also influencing the reference frame orientation of the VSC control system and can by that be utilized to inherently provide load-dependent reactive power support from the VSC. This can be considered to have a similar effect as inserting a virtual series capacitor in a weak, inductive, transmission line through the VSC control system, and will increase the power transfer capability correspondingly. The extended stability range that can be achieved with the proposed method has been verified by using a nonlinear analytical model for calculating the steady-state power transfer capability with various grid impedance values and various levels of virtual impedance. The small-signal stability of the obtained operating points has also been confirmed, and the validity of the calculated stability limits has been verified by time-domain simulations.

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Fig. 9. Implementation of active damping of LC oscillations

Appendix

A. Non-linear system model

In the following, a model of the entire system from Fig. 1 will be developed on state-space form, resulting in a non-linear state space model. This model can be used as basis for calculating steady-state operating points and thus the steady-state stability limits of the system, and can also be linearized to result in a small-signal state-space model for small-signal stability studies. The model development is based on [28], but is adapted to the particular control system and synchronization method under study.

A.1 Electrical system model

The electrical system from Fig. 1 can be modelled on state-space form in a synchronously rotating dq reference frame as given by (2), where all bold symbols represent space vectors written on the form given by (3).

$$\begin{aligned} \frac{d\mathbf{i}_{cv}}{dt} &= \frac{\omega_b}{l_f} \mathbf{v}_{cv} - \frac{\omega_b}{l_f} \mathbf{v}_o - \left(\frac{r_{lf} \omega_b}{l_f} + j \cdot \omega_g \omega_b \right) \mathbf{i}_{cv} \\ \frac{d\mathbf{v}_o}{dt} &= \frac{\omega_b}{c_f} \mathbf{i}_{cv} - \frac{\omega_b}{c_f} \mathbf{i}_g - j \cdot \omega_g \omega_b \cdot \mathbf{v}_o \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{d\mathbf{i}_o}{dt} &= \frac{\omega_b}{l_g} \mathbf{v}_o - \frac{\omega_b}{l_g} \mathbf{v}_g - \left(\frac{r_g \omega_b}{l_g} + j \cdot \omega_g \omega_b \right) \mathbf{i}_o \\ \mathbf{x} &= x_d + j \cdot x_q \end{aligned} \quad (3)$$

Considering that the modulation index $\mathbf{m}$ used for the PWM operation of the VSC is calculated as indicated in Fig. 1 from a division of the voltage reference from the control system by the actual dc voltage v_{dc} , the converter output voltage $\mathbf{v}_{cv}$ can be assumed to be equal to the reference voltage $\mathbf{v}_{cv}^*$, as expressed by (4). As this effectively decouples the ac side dynamics from the dc voltage of the VSC, any dynamics of the dc side are not included in the modelling.

$$\mathbf{m} = \frac{\mathbf{v}_{cv}^*}{v_{DC}}, \quad \mathbf{v}_{cv} = \mathbf{m} \cdot v_{DC} \rightarrow \mathbf{v}_{cv} \approx \mathbf{v}_{cv}^* \quad (4)$$

A.2 Model of inner loop controllers

From the control system block diagram in Fig. 1, the voltage reference for the converter can be expressed by (5). In this equation, the integrator states of the PI current controllers are defined by (6), while the stabilizing voltage reference from the active damping algorithm is given by (7). The applied scheme for active damping is shown in Fig. 9, and the corresponding states of the low-pass filter used to extract the oscillating components are defined by (8).

$$\mathbf{v}_{cv}^* = k_{pc} (\mathbf{i}_{cv}^* - \mathbf{i}_{cv}) + k_{ic} \cdot \gamma + j \cdot l_1 \cdot \omega_{VSM} \cdot \mathbf{i}_{cv} + \mathbf{v}_o - \mathbf{v}_{dq,AD}^* \quad (5)$$

$$\frac{d\mathbf{i}}{dt} = \mathbf{i}_{cv}^* - \mathbf{i}_{cv} \quad (6)$$

$$\mathbf{v}_{AD}^* = k_{AD} (\mathbf{v}_o - \boldsymbol{\phi}) \quad (7)$$

$$\frac{d\boldsymbol{\phi}}{dt} = \omega_{AD} \cdot \mathbf{v}_o - \omega_{AD} \cdot \boldsymbol{\phi} \quad (8)$$

A.3 Model of outer loop power controller

The active power flow from the VSC into the grid is defined by (9). A low pass filter with an internal state variable defined by (10) is applied on the actual power flow before it is used as the feedback signal for the outer loop power controller indicated in Fig. 1. The active, d-axis current reference resulting from the PI power controller is given by (11), where the internal state of the PI controller is defined by (12).

$$p_o = \text{Re}(\mathbf{v}_o \cdot \tilde{\mathbf{i}}_o) = v_{o,d} \cdot i_{o,d} + v_{o,q} \cdot i_{o,q} \quad (9)$$

$$\frac{dp_m}{dt} = \omega_{LP,p} \cdot p_o - \omega_{LP,p} \cdot p_m \quad (10)$$

$$i_{cv,d}^* = k_{pp} (p^* - p_m) + k_{ip} \kappa_p \quad (11)$$

$$\frac{d\kappa_p}{dt} = p^* - p_m \quad (12)$$

A.4 Model of PLL with impedance conditioning

The proposed PLL with impedance conditioning can be modelled in the same way as the conventional PLL, as further described in [28]. Thus, the states of the low-pass filters used in the PLL is defined by (13), with the only difference from the conventional PLL being that the voltage $\tilde{\mathbf{v}}_{VT}$ calculated from the virtual impedance according to (1) is used as an input to the low-pass filter instead of the measured filter voltage $\mathbf{v}_o$.

$$\frac{d\mathbf{v}_{PLL}}{dt} = -\omega_{LP,PLL} \cdot \mathbf{v}_{PLL} + \omega_{LP,PLL} \cdot \tilde{\mathbf{v}}_{VT} \quad (13)$$

The integrator state of the PI-controller used for tracking the grid frequency deviation is then defined by (14). Thus, the deviation of the PLL frequency from the grid frequency can be expressed by (15), and the phase angle deviation between the grid voltage vector and the orientation of the PLL can be defined by (16)

$$\frac{d\varepsilon_{PLL}}{dt} = \arctan\left(\frac{v_{PLL,q}}{v_{PLL,d}}\right) \quad (14)$$

$$\delta\omega_{PLL} = k_{p,PLL} \cdot \arctan\left(\frac{v_{PLL,q}}{v_{PLL,d}}\right) + k_{i,PLL} \cdot \varepsilon_{PLL} \quad (15)$$

$$\frac{d\delta\theta_{PLL}}{dt} = \delta\omega_{PLL} \cdot \omega_b \quad (16)$$

The resulting PLL phase angle determines the orientation of the SRF used for the implementation of the VSC control system. The grid voltage can then be expressed in the corresponding SRF dq frame as given by (17)

$$\mathbf{v}_g = \hat{\mathbf{v}}_g e^{-j\delta\theta_{PLL}} \quad (17)$$

A.5 Resulting non-linear state-space model

A model of the overall system from Fig. 1 can be achieved by replacing the algebraic equations from the previous subsections into the differential equations. This results in a non-linear state-space model of the system as given by (20) on the next page.

A.6 Steady-state system model

A steady-state system model can be directly achieved from (20) by setting all derivative terms to zero. The model is then reduced to a set of linear and non-linear algebraic equations that can be solved for the steady state operating conditions of the system states as a function of the reference and input signals. This is the model used to calculate the steady-state power transfer limitations presented in section 3.1 and 3.2.

B. Linearized small-signal state space model

To verify the small-signal stability of the system in any operating point that can be calculated from the non-linear model, a small signal state-space model can be easily obtained by linearizing (20). This will result in a linearized small-signal model on the form given by (18), with the state vector $\mathbf{x}$ and the input vector $\mathbf{u}$ defined by (19). This is the model that is used to verify the small signal stability limit indicated in Fig. 4 and any other operating condition shown in the subsequent figures. The resulting model can also be used to analyse the dynamic response of the system for small deviations around any operating point, as for instance the dynamics observed when the system is settling around a new operating point as a consequence of the power reference step simulated in Fig. 8.

$$\Delta\dot{\mathbf{x}} = \mathbf{A} \cdot \Delta\mathbf{x} + \mathbf{B} \cdot \Delta\mathbf{u} \quad (18)$$

$$\mathbf{x} = \begin{bmatrix} v_{o,d} & v_{o,q} & i_{l,d} & i_{l,q} & \gamma_d & \gamma_q & i_{o,d} & i_{o,q} & \phi_d & \phi_q \\ v_{PLL,d} & v_{PLL,q} & \varepsilon_{PLL} & \delta\theta_{PLL} & p_m & \kappa_p \end{bmatrix}^T \quad (19)$$

$$\mathbf{u} = \begin{bmatrix} i_{l,q}^* & p^* & \hat{v}_g & \omega_g \end{bmatrix}^T$$

C. Parameters used for system analysis

The following electrical parameters and controller settings have been used for all analysis of the system presented in the paper. The grid impedance and the virtual impedance are not listed in the table, as their values are varied for the different investigations.

Parameter	Value	Parameter	Value
Rated voltage $V_{S,LL,RMS}$	220 kV	Filter inductance L_f	0.08 pu
Rated power S_b	1200 MVA	Filter resistance r_{lf}	0.003 pu
Rated angular frequency ω_b	$2\pi \cdot 50$ Hz	Filter capacitance C_f	0.074 pu
Current controller gains, k_{pc}, k_{ic}	1.27, 14.25	Grid voltage $\hat{v}_g$	1.0 pu
Power controller gains, k_{pp}, k_{ip}	0.10, 50.0	PLL Low pass filter, $\omega_{LP,PLL}$	200 rad/s
Power measurement filter, T_{fp}	5 ms	PLL gains, $k_{p,PLL}, k_{i,PLL}$	0.05, 2.53

Table I Parameters of investigated system configuration

$$\begin{aligned}
\frac{dv_{o,d}}{dt} &= \omega_b \omega_g v_{o,q} + \frac{\omega_b}{c_f} i_{cv,d} - \frac{\omega_b}{c_f} i_{o,d} \\
\frac{dv_{o,q}}{dt} &= -\omega_b \omega_g v_{o,d} + \frac{\omega_b}{c_f} i_{cv,q} - \frac{\omega_b}{c_f} i_{o,q} \\
\frac{di_{cv,d}}{dt} &= -\frac{\omega_b k_{AD}}{l_f} v_{o,d} - \frac{\omega_b (k_{pc} + r_f)}{l_f} i_{cv,d} + \frac{\omega_b k_{ic}}{l_f} \gamma_d + \frac{\omega_b k_{AD}}{l_f} \varphi_d - \frac{\omega_b k_{pc} k_{pp}}{l_f} p_m + \frac{\omega_b k_{pc} k_{ip}}{l_f} \kappa_m \\
&\quad - \omega_b k_{i,PLL} i_{cv,q} \mathcal{E}_{PLL} + \omega_b k_{p,PLL} i_{cv,q} \tan^{-1} \left(\frac{v_{PLL,q}}{v_{PLL,d}} \right) + \frac{\omega_b k_{pc} k_{pp}}{l_f} p^* \\
\frac{di_{cv,q}}{dt} &= -\frac{\omega_b k_{AD}}{l_f} v_{o,q} - \frac{\omega_b (k_{pc} + r_f)}{l_f} i_{cv,q} + \frac{\omega_b k_{ic}}{l_f} \gamma_q + \frac{\omega_b k_{AD}}{l_f} \varphi_d \\
&\quad + \omega_b k_{i,PLL} i_{cv,d} \mathcal{E}_{PLL} + \omega_b k_{p,PLL} i_{cv,d} \tan^{-1} \left(\frac{v_{PLL,q}}{v_{PLL,d}} \right) + \frac{\omega_b k_{pc}}{l_f} i_{cv,q}^* \\
\frac{d\gamma_d}{dt} &= -i_{cv,d} - k_{pp} p_m + k_{ip} \kappa - k_{pp} p^* \\
\frac{d\gamma_q}{dt} &= -i_{cv,q} + i_{cv,q}^* \\
\frac{di_{o,d}}{dt} &= \frac{\omega_b}{l_g} v_{o,d} - \frac{\omega_b r_g}{l_g} i_{o,d} + \omega_b \omega_g i_{o,q} - \frac{\omega_b \hat{v}_g \cos(\delta\theta_{PLL})}{l_g} \\
\frac{di_{o,q}}{dt} &= \frac{\omega_b}{l_g} v_{o,q} - \omega_b \omega_g i_{o,d} - \frac{\omega_b r_g}{l_g} i_{o,q} + \frac{\omega_b \hat{v}_g \sin(\delta\theta_{PLL})}{l_g} \\
\frac{d\varphi_d}{dt} &= \omega_{AD} v_{o,d} - \omega_{AD} \varphi_d \\
\frac{d\varphi_q}{dt} &= \omega_{AD} v_{o,q} - \omega_{AD} \varphi_q \\
\frac{dv_{PLL,d}}{dt} &= \omega_{LP,PLL} v_{o,d} - \omega_{LP,PLL} r i_{o,d} - \omega_{LP,PLL} v_{PLL,d} + \omega_{LP,PLL} \omega_g l i_{o,q} + \omega_{LP,PLL} k_{i,PLL} l i_{o,q} \mathcal{E}_{PLL} + \omega_{LP,PLL} k_{p,PLL} l i_{o,q} \tan^{-1} \left(\frac{v_{PLL,q}}{v_{PLL,d}} \right) \\
\frac{dv_{PLL,q}}{dt} &= \omega_{LP,PLL} v_{o,q} - \omega_{LP,PLL} r i_{o,q} - \omega_{LP,PLL} v_{PLL,q} - \omega_{LP,PLL} \omega_g l i_{o,d} - \omega_{LP,PLL} k_{i,PLL} l i_{o,d} \mathcal{E}_{PLL} - \omega_{LP,PLL} k_{p,PLL} l i_{o,d} \tan^{-1} \left(\frac{v_{PLL,q}}{v_{PLL,d}} \right) \\
\frac{d\mathcal{E}_{PLL}}{dt} &= \tan^{-1} \left(\frac{v_{PLL,q}}{v_{PLL,d}} \right) \\
\frac{d\delta\theta_{PLL}}{dt} &= \omega_b k_{p,PLL} \tan^{-1} \left(\frac{v_{PLL,q}}{v_{PLL,d}} \right) + \omega_b k_{i,PLL} \mathcal{E}_{PLL} \\
\frac{dp_m}{dt} &= -\omega_{LP,p} p_m + \omega_{LP,p} v_{o,d} i_{o,d} + \omega_{LP,p} v_{o,q} i_{o,q} \\
\frac{d\kappa_p}{dt} &= -p_m + p^*
\end{aligned} \tag{20}$$