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INTRINSIC SCATTER OF FRC: AN ALTERNATIVE PHILOSOPHY TO ESTIMATE CHARACTERISTIC VALUES

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INTRINSIC SCATTER OF FRC: AN ALTERNATIVE PHILOSOPHY TO ESTIMATE CHARACTERISTIC VALUES

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Abstract: The results from small scale laboratory tests of fibre reinforced concrete (FRC) usually show a high scatter. However, several studies indicate that the real scatter on the post-cracking response of the material reduces considerably with the increase of the size of the elements tested. Such observations highlights a possible contradiction in the design of FRC since the characteristic values estimated from small scale tests might not be representative of large scale structures. This could penalize the material, leading to higher fibre consumption, less competitive solutions and problems in the quality control. The main objective of the present study is to address this fundamental issue. The aim is to evaluate the scatter that is intrinsic to the FRC and how it is affected by the size of the element, the type of concrete, the type and content of fibre. For that, a novel numerical approach is proposed for the simulation of the material and its variability. Then, an extensive parametric study is conducted with more than 35000 models, each one unique in terms of fibre distribution. Based on this analysis, equations are proposed to estimate the intrinsic scatter depending on several parameters. Finally, an alternative formulation is defined to estimate the characteristic value of the FRC considering the real structure in which it will be applied. The results derived from this study represent a contribution towards a more efficient design of structures and the reduction of the non-conformities in the quality control of the FRC.

Keywords: fibre reinforced concrete; scatter; characteristic residual strength; numerical model; parametric study

1 INTRODUCTION

The design of Fibre Reinforced Concrete (FRC) elements has been performed with multi linear stress-strain diagrams that intend to represent the post-cracking response of the material. The definition of these diagrams usually relies on an inverse approach based on the results of small-scale tests, such as the bending test. However, the results obtained in the laboratory are not directly

introduced in the models from the literature. First, they receive a statistic treatment in order to estimate the characteristic value, which has a 95% theoretical probability of being the lowest resistant capacity obtained. This estimation takes into account that the material and the processes are subjected to a certain degree of variability represented either by the standard deviation (SD) or by the coefficient of variation (CV).

Due to its intrinsic properties, the results from small scale laboratory tests of FRC normally show a high scatter. As an example, Figure 1 presents the average scatter in the residual strength measured in the bending test EN 14651 by different authors from the literature for crack mouth opening displacements of 0.5 mm ($f_{R,1}$), 1.5 mm ($f_{R,2}$), 2.5 mm ($f_{R,3}$) and 3.5 mm ($f_{R,4}$) [1-4]. Notice that the CV is on the average 15%, which is considerably higher than the observed in the compressive and the tensile strength tests of concrete. This high scatter leads to characteristic values that are far below the average residual strength measured. In some cases, the reduction is so remarkable that almost no contribution of fibres may be considered in the design.

Several authors indicate that the real scatter on the post-cracking behaviour of FRC reduces considerably with the increase of the cracking surface, which is especially evident when real-scale elements are tested [5-10]. For instance, Di Prisco [5] indicates that the scatter measured in the test of full scale slabs was 2.3%, in contrast with the 18% and the 23% found for different residual strengths measured with the bending test UNI 11039. Moreover, in the test of different panels that with a cracked area considerably bigger than the found in the bending test, Bernard [11] reported a CV ranging from 6% to 9%. The study conducted by de la Fuente [6] found that the CV of the residual load measured in the test of concrete pipes with plastic fibres was between 2.4% and 9.8%, with an average of 7.0%. In a similar study with steel fibres [12], the author found a CV between 3.8% and 9.8% (6.8% on the average).

Such observations highlight a possible contradiction in the design of FRC since the variability and, consequently, the characteristic values from the small scale tests are not representative of the real structure. This might penalize the material, ultimately leading to higher fibre consumption, to less competitive solutions in comparison with traditional reinforced concrete and to problems in the quality control.

Despite that, few studies have been conducted about the influence of the dimensions of the elements or the fibre content on the scatter of the tests. The key motive is the considerably high and prohibitive number of specimens needed – both small and large scale – in order to obtain a statistically representative estimation of the scatter. In that sense, the advances on the numerical simulation of materials and on the modelling of the pull-out response of FRC open new horizons to estimate the intrinsic variability in the behaviour of the material. Nevertheless, even in this case limitations for the study are evident since the numerical assessment of the scatter would require the

simulation of each fibre from the FRC and the analysis of thousands of models. Although important advances on the discrete simulation of the interaction fibre-matrix were achieved [13-15], the approach generally used leads to a long processing time that render the analysis of several models with different fibre distributions very difficult.

The objective of the present study is to address this fundamental issue. The aim is to evaluate the scatter that is intrinsic to the FRC and how it is affected by the size of the element, the type of concrete, the type and content of fiber. For that, a novel numerical approach is proposed for the simulation of the material and its variability. Subsequently, an extensive parametric study is conducted with more than 35000 models, each one unique in terms of fibre distribution. Based on this analysis, equations are proposed to estimate the intrinsic CV depending on several parameters. Finally, an alternative formulation is defined to estimate the characteristic value of the FRC considering the real structure in which it will be applied.

The results derived from this study represent a contribution towards a more efficient design of structures and the reduction of the non-conformities in the quality control of the FRC. Furthermore, the approach used in this paper might open a new field for the analysis of the mechanical properties of the material.

2 WORK PHILOSOPHY

In any real test, the variation of the surrounding conditions and of the elements characterized produces scatter of the results. In the case of concrete elements, the chief sources of scatter may be classified in the three groups shown in Figure 2. The first source or type of variation is related with the precision of the equipment and of the set-up used in the test. The second type is related with the process of production of the samples. The third type of scatter is the intrinsic to the material as a result of the random distribution of the composite formed by fibres and concrete. It takes into account that, even if the production and the testing procedures were kept the same, the position of the fibres would vary slightly from one specimen to the other. Consequently, small variations on the mechanical behaviour would still occur.

While the first and the second types are highly affected by the personal or the machinery involved with the production and testing, the intrinsic scatter just depend on the probabilistic laws that govern the distribution and orientation of the aggregates and the fibres. This nuance is relevant since the intrinsic type will always be present even though its intensity might vary between real-scale and small-scale specimens tested in the laboratory. If so, a global factor could be proposed to correct the intrinsic scatter of small-scale elements, which would be valid for all applications regardless of the scatter of the production process or the testing procedure.

As mentioned before, the experimental determination of the intrinsic scatter is highly complex. The variations measured in the tests are the consequence of the summed effect of the three sources from Figure 2 and it would be difficult to dissociate the influence of each source. In this context, the best alternative is to estimate the intrinsic scatter numerically. For that, a discrete approach must be applied to model the FRC.

As shown in Figure 3, this implies the use of two modulus of analysis: one for the fibre placement and the other for the structural analysis. In the first modulus, the fibres are distributed following probabilistic laws that govern their position and their orientation. Then, a finite element model (FEM) with this fibre profile and the mechanical behaviour of the specimen is assembled is. This procedure should be repeated a statistically significant number of times with the same input parameters. Each time, a different model is obtained and processed, giving every time slightly different results due to the natural randomness in the position of the fibres. Together, all the results may be used to estimate the intrinsic scatter of the material. The next section presents a detailed description of both moduli used to obtain the intrinsic scatter of FRC.

2.1 Modulus 1: Distribution of the fibres

The distribution of fibres may be described in terms of their position and direction. The position of the fibres central point is defined by the coordinates (P_x, P_y, P_z) . Moreover, the unitary vector $(\vec{D}_x, \vec{D}_y, \vec{D}_z)$ gives the orientation of the fibres in the axes X, Y and Z. To determine each of these parameters, a flexible and independent probabilistic law is used. Depending on the input data, the probabilistic law may vary from a uniform distribution to a full normal distribution. For a general axis i , the position P_i is given by (1) in which ϕ is the cumulative normal distribution, V_{P_i} is a purely random variable generated between $V_{P_{i,max}}$ and $V_{P_{i,min}}$, μ_{P_i} is the average position of the fibre, σ_{P_i} is its standard deviation, whereas $P_{i,max}$ and $P_{i,min}$ indicate the limit position of the fibre in the space.

$$P_i = \frac{\phi(V_{P_i}, \mu_{P_i}, \sigma_{P_i}) \cdot (P_{i,max} - P_{i,min})}{\phi(V_{P_{i,max}}, \mu_{P_i}, \sigma_{P_i}) - \phi(V_{P_{i,min}}, \mu_{P_i}, \sigma_{P_i})} + P_{i,min} \quad (1)$$

A similar equation may be used to estimate the direction of the fibres. However, in this case an addition calculation is necessary to obtain the components of the unitary vector on the general axis i , according with (2) and (3). The variables are analogous to the ones described for P_i . Notice that for $V_{Dt_{i,min}} \approx V_{Dt_{i,max}}$ the probabilistic follow a uniform distribution. On the contrary, for $V_{Dt_{i,min}} \ll V_{Dt_{i,max}}$ the distribution follows a Gauss distribution.

$$D_i = \frac{Dt_i}{\sqrt{Dt_x^2 + Dt_y^2 + Dt_z^2}} \quad (2)$$

$$Dt_i = \frac{\phi(V_{Dt_i}, \mu_{Dt_i}, \sigma_{Dt_i}) \cdot (Dt_{i,max} - Dt_{i,min})}{\phi(V_{Dt_{i,max}}, \mu_{Dt_i}, \sigma_{Dt_i}) - \phi(V_{Dt_{i,min}}, \mu_{Dt_i}, \sigma_{Dt_i})} + Dt_{i,min} \quad (3)$$

In the case of FRC, it is also important to consider the presence of elements that might condition the orientation of fibres. A clear example is the walls of the formwork, which clearly limit the possible directions that the fibre might assume. To account for the influence of this wall effect, first the position (P_x, P_y, P_z) is fixed according with (1) for the centre point of the fibre within the sampling space. Then, the unitary vector ($\vec{D_x}, \vec{D_y}, \vec{D_z}$) is generated with (3) and the extremities of the fibres are checked for the walls of the formwork. If any of the extremities is outside the sampling space, the unitary vector ($\vec{D_x}, \vec{D_y}, \vec{D_z}$) is randomly defined and checked again. This procedure is repeated until the whole fibre is inside the sample. Figure 4 shows a typical fibre distribution obtained with the modulus 1 for an approximately isotropic FRC with a content of 30 kg/m³.

A uniform distribution was used to determine the fibre position and orientation in the specimens for the present study. In order to evaluate differences in the behaviour of FRC depending on the level of isotropy of the material, two definitions of the unitary vector were considered. To simulate an quasi-isotropic concrete (IC) that would be obtained if a very dry material was used, the values of $Dt_{x,min}$, $Dt_{y,min}$ and $Dt_{z,min}$ were set to -1 whereas $Dt_{x,max}$, $Dt_{y,max}$ and $Dt_{z,max}$ were set to 1. On the other hand, to simulate a fibre distribution typical of an anisotropic concrete (AC), the values of $Dt_{z,min}$ and $Dt_{z,max}$ were changed to -0.5 and 0.5, respectively. The profile obtained in this case was checked against the experimental results from the literature [3, 16, 17] for fibre distribution and orientation in self-compacting concrete, which should present a high degree of anisotropy. In this study, the axis Z is considered parallel to the casting direction.

2.2 Modulus 2: Definition and processing of FEM

To allow the assessment of the intrinsic scatter, fibres should be represented individually. Other studies from the literature have simulated discretely the behaviour of each fibre in the concrete mass by means of FEM [13, 14, 18]. Nevertheless, the analysis of these FEM are very time consuming what limits the number of cases that may be processed. This clearly restricts its applicability in practice and in studies about the scatter that require the analysis of a great number of models. Notice that in the present work, tens of thousands of models should be processed in order to obtain representative values of scatter for different fibre contents and geometries.

Therefore, a simplified approach is proposed considering that in the cases analysed only the fibres crossing the cracking plane contribute to the post-cracking response and that the position of the cracking plane in the specimens is approximately known in advance. Rather than representing the entire fibre length, only the points at which they cross the cracking plane is represented. This point receives interface properties with a stress-displacement diagram that equals the pull-out response obtained for the orientation and the embedded length of each fibre. This strategy of simulation reduces considerably the time required for the analysis.

2.2.1 Pull-out response of a single fibre

Several models are available to predict the pull-out response of a single fibre [19-23]. In the present study, the models proposed by Laranjeira for straight [24] and for hooked [25] fibres was used since they include a straightforward formulation and an extensive database of input parameters is available. The application of these models require the execution of at least one pull-out test of a single fibre aligned with the load direction in order to obtain the key points of the load-crack opening curve.

With that, it is possible to estimate the load-crack opening curve theoretically for different fibre embedded lengths and inclinations with the cracked plane. It is important to remark that both models were extensively validated with the results from Leung [26], Robins [27] and Van Gysel [28]. The input parameters used for this study were the described by Laranjeira [23-25].

The straight and the hooked fibres modelled presented a length of 60 mm and an aspect ratio of 80, with a tensile strength, an elastic modulus and an ultimate strain of 1050 MPa, 200 GPa and 35%, respectively. The ultimate tangential debonding stress was estimated as 51% of the tensile strength of concrete. The friction coefficient of 0.6 between the steel fibre and the concrete was assumed after debonding occurs. Figure 5 presents the pull-out key points based on the experimental results considered in the work of Laranjeira [23-25] for a single fibre embedded 30 mm in the concrete matrix and aligned with the load.

Concretes with average compressive strengths of 30 MPa and of 50 MPa were considered. The other properties of the material such as elastic modulus and tensile strength were calculated according with the formulations proposed in the Eurocode 2.

2.2.2 Definition and processing of FEM

The pull-out curve estimated depending on the inclination and the embedded length for each fibre that cross the cracked plane is then introduced in the FEM, which is processed to obtain the structural behaviour of the specimen. All specimens are simulated under an isostatic configuration with two supports that extend along the width of the element. In the opposite face, a constant displacement rate is applied to the centreline between the supports. The numerical simulation of the

specimen under this configuration does not require the use of sophisticated 3D FEM with solid elements. Accurate results may be obtained with plane stress models, which were used in the present study after being properly validated with 3D models. All the FEM analyses were performed with the program DIANA 9.2.

In total, two types of models are analyzed. The first of them considers a specimen with the dimension used in the bending test specified in the EN 14651, which includes the notch. The second model represents the situation of real-scale elements. To allow a better comparison with the results, the geometry of the specimens obeys the same proportions as the bending test, except for the simulation of the notch that is not included in the real structure.

The concrete is represented in all models with triangular elements that behave as a nonlinear material with a parabolic diagram under compression and a linear in tension. Interface elements were placed at the centre of the specimen where the crack should appear. The interface presents a high stiffness and a traction cut-off behaviour with softening in tension. Furthermore, 2-points interface element with fictitious area were placed between the surfaces that form the cracking plane in order to simulate the pull-out response of each fibre crossing this plane. It is important to remark that this model is adequate for the simulation of elements that present a preferential cracking plane. If considerable hardening should occur, additional interface elements have to be placed at the specimen to account for the fact that cracking extends to a wider length.

The size and the total number of elements used in the different FEM were defined after a study on the influence of the mesh on the results obtained. The mesh was defined to assure a high refinement close to the interface where the points representing each fibre are placed. In this zone, the division of finite elements mesh is approximately 1.2 mm. Figure 6 shows the comparison between the results from the FEM and the experimental results from Blanco [29]. Both of them were obtained for the bending test of FRC specimens with 20 kg/m³ of steel fibre of the type considered in the present study.

The limit of proportionality is almost the same in both curves, although a slight difference appears just after cracking. Such difference may be attributed to the fact that a more abrupt release of energy may occur in the real experiment when the specimen cracks. This produces a steeper drop in the load applied. Despite that, a good fit is obtained for the residual branch of the curve, which is the most relevant part for this study. This result indicates that the model is capable of reproducing the mechanical behaviour observed during the bending test.

3 PARAMETRIC STUDY

Even if the probabilistic law used to generate the FEM were always the same, every model generated would be slightly different in terms of the fibre arrangement and, consequently, of the structural response. To estimate the intrinsic scatter among these results, several of models were processed with the same probabilistic input data.

Moreover, to compare the scatter of specimens with different characteristics, many series of models were analysed through a parametric study. The main variables considered were the height (125, 150, 200 and 300 mm) of the specimen, the width (150, 500, 1000, 2000 mm) of the specimen, the type of concrete (IC and AC), the compressive strength of concrete (30 MPa and 50 MPa), the type of fibre (hooked and straight) and the content of fibre (30, 45 and 60 kg/m³). The possible combination of these variables provided 145 series.

The minimum number of models processed within each series to obtain a statistically representative intrinsic scatter varied depending on the dimensions and on the fibre content. For instance, in the simulation of the bending test with 30 kg of hooked fibres, 1000 models were analysed in order to calculate a single value of scatter. On the contrary, bigger specimens with higher fibre contents required between 75 and 100 models each. More than 35000 finite element models were analysed throughout the study. In each model, a curve Load-CMOD was obtained.

In order to achieve a fair and direct comparison between the scatter for models with different cracked section and concrete compressive strengths, the CV is used. This dimensionless coefficient provides information about the variation in relation to the average results, thus yielding a normalised value. The same evaluation would not be possible if the analysis was based on SD since its order of magnitude depends on the characteristics of each model, thus hindering the comparison of the scatter of models with different sizes.

4 RESULTS AND ANALYSIS

The results from the parametric study are analysed in terms of the probabilistic variability of the fibre arrangement in the cracked section. This study provides valuable information towards explaining the following analysis about the scatter on the mechanical behaviour of the material. The same information could hardly be obtained through experimental means, since a prohibitive number of tests with small and large specimens would be required to achieve representative results.

4.1 Scatter on fibre distribution

The scatter on fibre distribution is only assessed in the cracked surface since these are the fibres responsible for the post-cracking response. For the series of models generated, the intrinsic CV (CV_I) for the number of fibres crossing this plane, for the shortest embedded length and for the angle between the fibre and a line perpendicular to the cracked plane were assessed. All these parameters should influence the final mechanical behaviour of the elements, as reported in experimental and numerical studies from the literature [24, 25, 30].

Figure 7 depicts the CV_I estimated depending on the width of the cracked section with height equal to 125 mm and 300 mm, considering an IC and fibres contents of 30 kg/m³ and 60 kg/m³. Similar curves were obtained for models with other geometries and fibre contents. The results show that the CV_I for the number of fibres crossing the cracked section, the embedded length and the angle are similar. It is clear that the increase on the width leads to a considerable reduction on the CV_I . The same trend is observed if the curves corresponding to the widths are compared. Again, the increase in the dimension produces a reduction of the CV_I .

This outcome is related with the likelihood of obtaining big variations depending on the total number of fibres that cross the cracked section. Small cracked sections should present a reduced number of fibres whose distribution follow the probabilistic law. Consequently, the influence of a single fibre on the total scatter estimated is much higher than in bigger sections, which should present more fibres. In this last case, the total scatter is less influenced by a single fibre since more unities are present to mitigate variations.

Figure 8.a and b depict the CV_I of the number of fibres, the embedded length and the angle depending on the fibre content for IC and for AC, respectively. The figure only includes the results for an element with 125 mm of height and 150 mm of width. It is observed that the increase in the fibre content leads to a reduction in the CV_I estimated. In general, this reduction is more noticeable between the contents of 30 kg/m³ and 45 kg/m³ than between the contents of 45 kg/m³ and 60 kg/m³.

Both outcomes are also related with the likelihood of obtaining big variations. Likewise described in the case of the height and the width, the increase on the fibre content produces a consequent upraise in the number of fibres crossing the cracked section. This makes the final scatter less dependent of the position of a single fibre, whose variations are diluted by the others also present.

Notice that on the average the CV_I of the number of fibres in the case of IC (Figure 8.a) is 20% higher than that of AC (Figure 8.b). Nevertheless, the differences in terms of the angle with the cracked plane are approximately 0%. This suggests that the degree of anisotropy of concrete has a major influence on the CV_I of the number of fibres and much less influence on the CV_I of the angle.

Since in AC the fibres tend to align perpendicular to the casting direction, the average projected length of the fibre perpendicular to the cracking plane tend to increase. Therefore, the number of fibres crossing the cracked plane increases, reducing the scatter of this result. On the contrary, the higher probability of fibre alignment reduces the average angle formed between the fibre and the perpendicular to the cracked plane. Consequently, the reduction in the variation due to the bigger number of fibres is compensated by the decrease on the average angle, leading practically to the same CV_I for IC and AC.

It is important to remark that all variables considered in the parametric study affect the fibre distribution profile. However, the influence of the parameters that determine the area of the cracked surface (height and width) is considerably higher than that of the fibre content or of the type of concrete used.

4.2 Scatter on residual strength

Even though the complete load-CMOD curve is obtained for each model processed, the analysis of the scatter on the residual strength is conducted for the $f_{R,1}$ and $f_{R,4}$, which are the basis for the estimation of the constitutive equation of FRC in many codes and standards. Figure 9 shows the intrinsic CV_I obtained for both residual strengths, considering as an example elements with hooked fibres and AC. The curves present a shape similar to the described for the scatter on fibre distribution, showing a considerable reduction of the CV_I as either the width or the height of the cracked section increases. However, the CV_I for the residual strength reach values approximately twice as big as the estimated in section 4.1.

This outcome was expected since the mechanical behaviour is simultaneously affected by the number of fibres, their embedded length and their angle. The combined interaction of these aspects explains the higher CV_I obtained for the residual strength that reaches values around 12% for the smallest dimensions and fibre content. It is noteworthy the reduction experienced by the intrinsic CV_I when the dimensions of the cracked section are increased. In some cases, the small scale elements present a scatter approximately 3 times bigger than the obtained for the largest ones.

Figure 10 presents the influence of the type of fibre on the CV_I for the contents of 30 kg/m³, 45 kg/m³ and 60 kg/m³. Even though the results of only some models are included, the same trend was observed in all cases from the parametric study. For the same content, the CV_I of hooked fibres is on the average 28% and 8% higher than the CV_I of the straight fibres for $f_{R,1}$ and $f_{R,4}$, respectively.

The fact that the difference for $f_{R,1}$ is 3.5 times higher than for $f_{R,4}$ provides an important clue for explaining this behaviour. In the case of the hooked fibres, some of the fibres that cross the cracked section present a partially embedded hook. It implies an additional softness since the hook must be

bended and straightened before these fibres become active, which will happen only after a certain crack opening has taken place. In other words, the number of active fibres is smaller for the case of hooked fibres, thus leading to a bigger CV_I . As the CMOD increases, part of the hooked fibres start to contribute to the residual strength so that the difference in terms of the total number of active fibres reduces. Consequently, the difference between CV_I estimated for models with hooked and straight fibres decreases for $f_{R,4}$.

Figure 10 also indicates that the CV_I obtained for $f_{R,1}$ is slightly higher than the estimated for $f_{R,4}$ in the case of hooked fibres, whereas the opposite happens in the case of straight fibres. The most intuitive result would be to observe an increase in the scatter as the CMOD increases since part of the fibres are pulled out of the concrete matrix, thus reducing the number of fibres that contribute to the residual strength. However, in the case of hooked fibres, this effect is compensated by the activation of part of the fibres as the CMOD increases.

Figure 11 depicts the CV_I depending on the type of concrete for elements with hooked fibre. The scatter obtained for AC is smaller than the estimated for the IC, regardless of the fibre content and the CMOD considered. This result is attributed to the higher likelihood of fibres aligning perpendicular to the cracking plane in the case of AC, which consequently increases the number of fibres that contribute to the residual strength and consequently reduces the scatter for both $f_{R,1}$ and $f_{R,4}$.

It is also observed that the CV_I decreases as the fibre content increases. For instance, in elements with 150 mm of width, 125 mm of height and hooked fibres, the CV_I for mixes with 30 kg/m³ is approximately 19% higher than the obtained for mixes with 45 kg/m³, which in turn is 17% higher than the assessed for 60 kg/m³. A similar trend was also observed in the data from Figure 1 and in the extensive experimental results from Barr et al. [31, 32].

Figure 12 shows the influence of the concrete compressive strength on the scatter estimated in the model with 150 mm of width, 125 mm of height, IC and hooked fibres. For the same fibre dosage, the difference between the CV_I in elements with the compressive strength 50 MPa in comparison with those with 30 MPa is on the average equal to 0.29% and -0.37% for $f_{R,1}$ and $f_{R,4}$, respectively. Such a small difference indicates that the compressive strength practically does not affect the intrinsic scatter of the material. This agrees with the results reported by Barr et al. for normal and high strength concrete [31, 32].

Figure 13 illustrates how the CV_I of the $f_{R,4}$ is related with the CV_I of the number of fibres crossing the cracked plane, the CV_I of the fibre shortest embedded length and the CV_I of the angle between the fibre and a line perpendicular to the crack. Only the results of the elements with AC and 60kg/m³ are included. A high R^2 of the linear regression is obtained with all results, thus indicating that the mechanical response is directly influenced by the parameters that characterize the fibre distribution.

4.3 Simplified equation to estimate the CV

The data obtained in the parametric study may be used to fit a simplified equation to estimate the CV_I depending on the main parameters that affect the intrinsic scatter. For that, a multivariable linear regression was performed. In order to simplify the proposal of this equation, the average value of CV_I obtained for AC and IC was used since in practice the consistency of the FRC tends to be between the ideal isotropy of IC and the anisotropy.

After several trials conducted to identify a general formulation that yields a good fit, (4) was proposed. It takes into account that the CV_I is closely related with the cracked area, obtained by the multiplication of the width (W) and the height (H). However, the parametric study also showed that the width and the height of the cracked section do not have the same repercussion in the CV_I . To consider that, the powers b and c are included in the equation as well as the parameter a .

$$CV_I = a \cdot H^b \cdot W^c \quad (4)$$

The back analysis conducted shows that a , b and c are related with the content of fibre (represented by the equivalent number N_{eq}), as shown in Table 1. Notice that c was considered approximately constant since the influence of the fibre content is minimal.

The equivalent number N_{eq} is introduced in order to make the equations proposed generalizable for fibres with length different from the considered in this study (60 mm). It is calculated according with (5) as a function of the number of fibres (N) in a cubic meter of concrete and of the length of the fibre (l) in mm. In turn, the variable N may be estimated through a simple equivalence of volume depending of the weighted content of fibre per cubic meter (Cf), the specific weight (γ) of the steel used to produce the fibre, the diameter (d) and the length (l) of the fibre.

$$N_{eq} = \frac{l}{60} \cdot N = \frac{l}{60} \cdot \frac{4 \cdot Cf}{\gamma \cdot \pi \cdot d^2 \cdot l} = \frac{Cf}{15 \cdot \pi \cdot \gamma \cdot d^2} \quad (5)$$

Figure 14 presents the fit between the CV_I calculate in the parametric study and the estimated with Eq. 1 for $F_{R,1}$ and $F_{R,4}$. In both cases a R^2 obtained is approximately equal to 0.99. The good fit supports the use of the simplified equation proposed to estimate the intrinsic value CV_I .

5 NEW FORMULATION FOR THE CHARACTERISTIC VALUE

The extensive parametric study conducted clearly confirms that the intrinsic CV_I on the residual strength assessed in the simulation of the bending test may be many times higher than the estimated for elements with a bigger cracked area. In other words, the characteristic residual strength obtained

from the bending test should be smaller than that of real scale elements executed and tested in a similar way.

To guarantee more representativeness and to avoid penalizing the material in the structural design, a new formulation for the estimation of the characteristic residual strength is required considering the particularities of FRC. This formulation should be capable of providing a reasonable prediction of the characteristic value for the real structure based on the results of the bending test.

The generally accepted equation used to estimate the characteristic residual strength ($f_{R,k}$) considers that the results follow a normal distribution given by an average value ($\overline{f_R}$), the standard deviation of the population (σ) and the constant Z that accounts for the confidence interval used (approximately 1.64 for 95% probability), as shown in (6). This equation may be written in terms of the coefficient of variation of the population (CV).

$$f_{R,k} = \overline{f_R} - Z \cdot \sigma = \overline{f_R}(1 - Z \cdot CV) \quad (6)$$

In practice, the scatter is commonly estimated for a limited sample and may not coincide with the obtained for the population that requires the assessment of a significant number of elements. Therefore, a prediction of σ is performed according with (7) as a function of the standard deviation of the limited sample (σ') and the parameter k that accounts for the uncertainty involved in the estimation. Notice that (7) may also be written in terms of the CV of the limited sample (CV').

$$\sigma = k \cdot \sigma' = k \cdot (\overline{f_R} \cdot CV') \quad (7)$$

In general, k is calculated for a 75% confidence interval depending on the number of specimens tested, which yields the values summarized in Table 2. As the size of the sample and the certainty related with the estimation of σ increases, k approaches 1. This distinction between the scatter of the sample and of the population is of great relevance and should be taken into account. From this point on in the paper, variables that appear with an apostrophe are obtained as the result of the test of a limited sample, whereas variables without the apostrophe are the scatter calculated for a significantly big number of elements that characterize a population.

According with the classification proposed, the CV may be expressed as the sum of the three components illustrated in Figure 2. Each one of them should approach a normal distribution if the resulting distribution also does so. Following classic deductions for the sum of normal distributions, the CV may be calculated through (8). In this equation, CV_I represents the part of the scatter intrinsic to the material, whereas CV_O the other sources of scatter corresponding, for example, to the test execution of the test and the production of the sample. Notice that the subscript U is added to all

variables. By convention, it is substituted by “BT” when the scatter refers to a elements with the same cross section of the bending test and by “RS” when it refers to the real scale element.

$$CV_U^2 = CV_{I,U}^2 + CV_{O,U}^2 \quad (8)$$

It is important to remark that (8) is the basis to estimate the CV for the real scale element (CV_{RS}) from the CV of the bending test (CV_{BT}). For that, it is also necessary to know the CV from the different sources of scatter (CV_O). The estimation of CV_O is a complex subject that entails the use of an extensive database of experimental results. Given the limited number of experimental programs from the literature - especially with large scale elements - it is not possible to perform a direct quantification.

However, some assumptions may be done in order to derive CV_{RS} , which could be used in (6) for a reasonable prediction of the characteristic residual strength. Two formulations are proposed to estimate the CV_{RS} based on whether the different sources of scatter are independent or dependent.

5.1 Independent scatter

The first formulation is deduced under the assumption that the different sources of scatter are independent from each other so that the phenomenon that affects one of them does not influence the others. It is assumed that the scatter experienced due to testing and production should be similar in small scale and large scale elements if the tests and the production are performed the same way for both of them. Therefore, following a more generalist approach, it may be considered that $CV_{O,RS}=CV_{O,BT}$. Combining this consideration and (8) for each element, it is possible to deduct (9) for the estimation of the CV_{RS} of the real scale element. This may be performed either with the scatter obtained for the population (CV_{BT}) or with the scatter obtained for the limited (CV'_{BT}) sample and the parameter k given in Table 2.

$$CV_{RS} = \sqrt{CV_{BT}^2 - CV_{BT,I}^2 + CV_{RS,I}^2} = \sqrt{(k \cdot CV'_{BT})^2 - CV_{BT,I}^2 + CV_{RS,I}^2} \quad (9)$$

The $CV_{BT,I}$ and $CV_{RS,I}$ may be calculated through (4). Substituting the latter and the typical dimensions of the bending test in (9) gives the final equation to estimate the global coefficient of variation of the real scale element (10). It is important to remark that in the formulation proposed, the CV only depends of the results of the bending test of a limited sample ($\overline{f_R}$ and CV'_{BT}) the content of fibre and the dimensions of the real scale element.

$$CV_{RS} = \sqrt{(k \cdot CV'_{BT})^2 - a^2 \cdot (125^{2 \cdot b} \cdot 150^{2 \cdot c} - H^{2 \cdot b} \cdot W^{2 \cdot c})} \quad (10)$$

5.2 Dependent scatter

The second formulation considers that the intrinsic scatter and the scatters due to other sources such as the production process are, in nature, related with the distribution of the fibres in the cracked section. Therefore, even though the specific cause and the intensity of the disturbance may vary, these scatters are governed by the geometry of the element, the fibre content and distribution. In this context, the $CV_{U,O}$ may be written according with (11) as a function of $CV_{U,I}$ and a proportionality constant (φ) that accounts for the difference in terms of intensity.

$$CV_{U,O} = \varphi \cdot CV_{U,I} \quad (11)$$

If the proportionality is maintained regardless of the scale, (8) and (11) may be combined to obtain (12). Substituting (4) for $CV_{BT,I}$ and $CV_{RS,I}$ gives (13) that allows the direct estimation of the scatter from the real scale element based on the scatter of the bending test of a limited sample (CV'_{BT}).

$$CV_{RS} = CV_{BT} \cdot \frac{CV_{RS,I}}{CV_{BT,I}} = k \cdot CV'_{BT} \cdot \frac{CV_{RS,I}}{CV_{BT,I}} \quad (12)$$

$$CV_{RS} = k \cdot CV'_{BT} \cdot \frac{H^b \cdot W^c}{125^b \cdot 150^c} \quad (13)$$

5.3 Limit of application

The CV'_{BT} measured from the bending test of a limited number of specimens may vary considerably, albeit it is usually around 18%. In some cases, due to the probabilistic aspects involved, some of the samples may even present CV'_{BT} close to 0%. However, it was demonstrated that an intrinsic CV_I will always be present if a sufficiently big number of specimens are considered. In these cases, the use of the small CV'_{BT} obtained from the test might lead to an unsafe prediction of the characteristic residual strength.

Based on the results obtained in previous sections, it is possible to reduce the impact of this problem by detecting unreasonable CV'_{BT} and by giving fair predictions of the value to be used in the

estimation of the characteristic residual strength. For that, some limitations to the application of either (10) and (13) are suggested. These limitations take into account that the CV_{RS} could never be smaller than $CV_{RS,I}$. Since other sources of scatter might increase the CV even more, the minimum value is obtained by a superposition of two times $CV_{RS,I}$. Substituting this in (8) gives (14), which allows the estimation of the minimum CV_{RS} that should be used regardless of the result estimated with the new formulations proposed.

$$CV_{RS} \geq \sqrt{2} \cdot CV_{RS,I} \geq \sqrt{2} \cdot a \cdot H^{2 \cdot b} \cdot W^{2 \cdot c} \quad (14)$$

6 SENSIBILITY ANALYSIS

A sensibility analysis is conducted to evaluate the repercussions and the applicability of the alternative formulations to estimate the characteristic residual strength in comparison with the current calculation procedure. For the analysis, it is assumed that a sample with 4 specimens was tested providing an average residual strength and a CV'_{BT} equal to 5 MPa and 15%, respectively. In order to obtain the curves, first the CV_{RS} for the whole population is estimated according with the currently used formulation ($k \cdot CV'_{BT}$). Moreover, estimations are also made with the dependent (10) and the independent (11) formulations. The values obtained are then applied in (6) to estimate the $f_{R,k}$. Figure 15 presents the influence of the width of the element on the final CV_{RS} and the $f_{R,k}$ estimated for an element with 125 mm of height and 30 kg/m³ of fibres of the same type considered in the numerical simulation.

As shown in Figure 15.a, a constant CV_{RS} of 24.05% is estimated for the current formulation since it does not account for the influence of the width of the element. The independent formulation leads to a maximum reduction of approximately 13% in this value for a width of 2000 mm. For the same width, the dependent formulation yields an even bigger reduction (71%). In terms of the characteristic residual strength, the difference between the formulations is less noticeable than for the CV_{RS} (see Figure 15.b). In this case, the current formulation provides a constant $f_{R,k}$ of 3 MPa. The lower scatter calculated in the formulations proposed lead to an increase in the characteristic residual strength of 9.3% and 47.3% for the independent and the dependent approach, respectively.

It is evident that the different principles considered for the deduction are the responsible for the results obtained for the independent and the dependent. The assumption that the scatter due to other sources is proportional to that intrinsic to the material favour a much smaller CV_{RS} in the case of dependent formulation in comparison with that estimated with independent one that takes into account that the other sources of scatter are constant. Based on this outcome, the question regarding which of them is the most representative arises.

The answer to this question, although not definitive, comes from the analysis of the reports from other authors that tested bigger specimens or real scale elements. As mentioned before, several authors indicate that the CV measured in the tensile residual response of real scale or larger elements is between 2% and 10% [5, 6, 11, 12]. This range is considerably below the considered for the bending test that commonly shows CV between 10% and 30% [2, 10].

Therefore, a great difference is expected between the scatter of the bending test and of the large scale element. Due to its mathematical basis, the independent formulation tends to present a relatively small difference with the CV assumed for the bending test. This scatter is usually above 12% hence providing results that are far over the reported in the literature for the real large scale elements. On the contrary, the CV obtained in the dependent formulation usually falls within the typical range of CV reported, thus indicating that it is more representative of the reality found in the test of bigger FRC elements conducted by other authors.

Figure 16.a shows how the CV_{RS} is affected by the number of specimens tested in each sample. The figure corresponds to an element with 300 mm of height, 500 mm of width and 30 kg/m³ of fibres. The CV'_{BT} and the average residual strengths are the identical to those of the previous example. The increase in the number of the specimens tested produces a decrease in the CV_{RS} , attributed to the bigger certainty regarding the average values and the scatter estimated for the population. This is especially evident when the number of specimens is increase from 3 to 6, thus leading to a CV_{RS} 38% smaller on the average. The reduction observed for bigger samples is considerably smaller.

Again, the current formulation provides the highest values of CV_{RS} that reaches 15.8% when 30 specimens are tested. For the same condition, the independent formulation shows a CV_{RS} of approximately 13%. The smallest results are obtained for the dependent formulation, reaching a minimum of 7.4%. The $f_{R,k}$ calculated is shown in Figure 16.b. It indicates that the dependent formulation proposed in this paper leads to an increase in the $f_{R,k}$ between 18.7% and 54.7%. Interestingly, the difference is more evident when the number of specimens decreases. This suggests that the independent formulation is less sensible to the uncertainties related with the number of specimens tested in a sample.

Figure 17 shows how the CV_{RS} and the $f_{R,k}$ are influenced by the coefficient of variation measured in the bending test of a limited number of specimens. The curves were obtained for an element with 200 mm of height, 500 mm of width and 45 kg/m³ of fibres. It is considered that a sample with 4 specimens was tested, providing an average residual strength equal to 5 MPa.

The CV_{RS} calculated with the current formulation is linearly proportional to the CV'_{BT} measured in the bending test of a limited sample. The slope of the straight line equals the constant k (Table 2) that for a set of 4 specimens is 1.6. On the other hand, an approximately bilinear trend is

obtained for the alternative formulations proposed. The first branch of the curve presents a constant CV_{RS} given by the restriction defined in section 5.3. In the second branch, the CV_{RS} increases with the scatter obtained in the bending test. The slope of the second branch is higher in the case of the independent formulation, which approaches the current formulation as the CV'_{BT} of the bending test increases.

Notice that on the average, for the normal range of scatter of the bending test, the independent formulation gives a CV_{RS} around 25%. This confirms that the approach adopted for the deduction in this case provides over conservative results in comparison with the CV_{RS} measured in real scale structures, despite being smaller than that of the current formulation. A more reasonable result is obtained with the dependent formulation that presents a slope 40% smaller for the second branch.

For values of scatter CV'_{BT} in the bending test below 5%, the current formulation provides a CV_{RS} that is bigger than the obtained with the alternative proposals. This indicates a zone of the graph in which the direct assessment of the scatter from the bending test is likely to lead to an unsafe prediction of the CV_{RS} . The limitation defined according with the parametric study help reduce this problem.

The trends regarding $f_{R,k}$ are the reflex of those already highlighted for the CV_{RS} . Again, a remarkable difference is observed between the current and the dependent formulations. For typical values of scatter in the bending test (18%), the latter provides a characteristic residual strength 49% higher than the former.

7 CONCLUSIONS

The approach proposed in this study to simulate the probabilistic distribution and the mechanical response of each fibre is a robust alternative to the traditional simulation of a homogeneous material or complex finite element models. It allowed the study of a fundamental aspect of the FRC: its intrinsic variability. Based on the results obtained, the following conclusions may be derived.

- A significant scatter was observed in the distribution of the fibres. This scatter clearly increases as the fibre content or the cracked area decreases.
- The scatter in the mechanical response follow the same trend observed for the fibre distribution, reaching higher values due to the summed effect of the variation in the fibre angle, embedded length and in the number of fibres crossing the cracked section.

- The new view proposed about the sources of scatter allowed the quantification of the part of the variability intrinsic to the material. A back analysis provided a simple expression to estimate the intrinsic CV_I depending on the fibre content and the cracked area.
- The equations and the philosophy devised in this study allowed the proposal of an alternative formulation to estimate the CV and the $f_{R,k}$ expected in the real scale element. The sensibility analysis performed in this paper shows that, in the majority of cases, the bending test of a small scale element provides an overestimation of the CV and an underestimation of the characteristic residual strength expected in the real scale structure. This confirms the result obtained by other authors.
- The direct consideration of the bending test of a limited number of specimens might give a CV bellow the one intrinsic to the material, which was thoroughly estimated in this study. In this case, the use of the small scatter obtained from the test might lead to an unsafe prediction of the characteristic residual strength. The use of the limitation proposed in (14) must be taken into account to reduce this problem.
- In almost all cases, the CV estimated with the alternative formulation proposed for real scale elements is smaller than the obtained directly from small scale test. The use of this new CV more representative of the real structure denotes a significant contribution towards an optimized design of FRC. Furthermore, it might contribute to reduce the incidents observed in the quality control of the material.

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Dear Editor,

We are pleased to submit our original research paper entitled “Intrinsic Scatter of FRC: An Alternative Philosophy to Estimate Characteristic Values” to Materials and Structures. The paper deals with an up-to-date issue related with the design and the quality control of FRC.

It is known that the high scatter found in small scale tests tend to reduce if larger specimens are tested. In this study, an alternative approach is proposed to quantify the influence of the size of the cracked section on the scatter expected. The extensive parametric study conducted shows that the formulation currently used to calculate the characteristic residual strength penalizes the material. Based on that, alternative formulations are proposed to estimate this values considering the scatter expected in the real structure.

We believe that the results derived from this study represent a contribution towards a more efficient design of structures and the reduction of the non-conformities in the quality control of the FRC. We put ourselves entirely at your service to perform any changes or improvements you consider necessary.

Sincerely yours,

Sergio H. P. Cavalaro
Antonio Aguado

Figure 1.- Coefficient of variance measured in the bending test EN 14651

Figure 2.- Types of scatter

Figure 3
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Figure 3.- Methodology to estimate the scatter

Figure 4.- Typical fibre distribution in a beam

Figure 5.- Pull-out key points of the aligned straight and hooked fibres

Figure 6.- Comparison between FEM results and the experimental results of Blanco [29]

Figure 7
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Figure 7.- CVI for number of fibre (a), embedded length (b) and angle (c) depending on the width of elements with IC

Figure 8
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Figure 8.- CVI for IC (a) and for AC (b)

Figure 9
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Figure 9.- CV_I for $f_{R,1}$ (a) and for $f_{R,4}$ (b) depending on the width of elements with hooked fibres and AC

Figure 10
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Figure 10.- CV_f depending on the type of fibre

Figure 11
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Figure 11.- CV_1 depending on the type of concrete

Figure 12.- CV₁ depending on the concrete compressive strength

Figure 13.- Relation between the CV_I of $f_{R,4}$ and the CV of number of fibre (a), embedded length (b) and angle (c) for elements with AC and 60 kg/m³

Figure 14.- Fit of the prediction of CV_I for $f_{R,1}$ (a) and $f_{R,4}$ (b)

Figure 15.- Influence of width of the element on the final CV_{RS} (a) and the $f_{R,k}$ (b)

Figure 16.- Influence of the number of specimens tested on the final CV_{RS} (a) and the $F_{R,k}$ (b)

Figure 17.- Influence of the result of the bending test on the final CV_{RS} (a) and the $f_{R,k}$ (b)

Table 1.- Parameters a , b and c used to estimate the CV_I for $f_{R,1}$ and $f_{R,2}$

Parameter	CV_I for $f_{R,1}$	CV_I for $f_{R,4}$
a	$4184.1 - 645.9 \bullet \ln N_{eq}$	$4125.5 - 618.9 \bullet \ln N_{eq}$
b	$(N_{eq} - 1215) / 2700$	$(N_{eq} - 2115) / 4500$
c	-0.48	-0.47

Table 2.- Parameters k depending on the number of specimens tested

Number of specimens	3	4	5	6	8	10	20	30	∞
k	2.05	1.60	1.42	1.33	1.22	1.17	1.07	1.05	1.00
Z	1.64								
$k \bullet Z$	3.37	2.63	2.33	2.18	2.00	1.92	1.76	1.73	1.64