A multiple-reference complex-based controller for power converters

Federico M. Serra, Arnau Dòria-Cerezo, and Marc Bodson, *Fellow, IEEE*,

Abstract—A multiple-reference complex-based controller is proposed for three-phase power converters feeding nonlinear and unbalanced loads. The control scheme incorporates a stable multiple-complex coefficient filter with bandwidths that are set arbitrarily and independently for every harmonic under consideration. The multiple-reference complex-based control scheme is applied to an uninterruptible power supply system with a voltage source converter. Each harmonic is controlled using a standard complex proportional-resonant controller that is designed for stability and robustness using the Nyquist criterion. Similar stability and robustness properties follow for the overall system due to the frequency-domain properties of the filter and of the controllers. The proposed methodology is validated in simulation and experimental tests.

Index Terms—Complex-valued transfer functions, multiple-complex coefficient filter, proportional-resonant complex controller, complex Nyquist plots, three-phase voltage source converter.

I. INTRODUCTION

Three-phase power converters often operate in environments with nonlinear elements and under unbalanced conditions. Nonlinearities induce higher harmonic components so that periodic signals are not longer sinusoidal, while unbalanced systems involve positive and negative sequences. Control algorithms for power converters have to deal with the control of multiple (positive and negative) frequencies [1].

The representation of physical systems using complex-valued dynamic models was considered for induction machines in [2] and, in general, for three-phase electrical systems in [3], considering its benefits when used in real applications. The main advantage of this representation is that the analysis is considerably simplified, especially when two-input two-output systems are converted to single-input single-output systems. Extensions of classic control techniques have been proposed in the last years to deal with complex-valued systems. Further, control theory tools for complex-valued systems have been generalized such as the Routh-Hurwitz test for complex polynomials [4][5][6], the root locus technique [7], frequency-domain analysis methods [8], stability conditions for time-varying complex-valued systems [9], or the sliding mode control technique [10][11]. Increasingly, the complex domain representation is used not only to simplify the equations of

a system, but also to design and analyze control systems in industrial applications. Examples include electrical machines [12][13][14], power converters [15][16][17], and others [8]. Complex controllers also have advantages in applications where harmonics with different sequences need to be controlled, because each sequence component can be treated independently [18][19].

In applications, the use of an intermediate filter stage to separate the individual components from noisy signals is often advantageous. Complex filters that are commonly used for signal processing [20] have been recently proposed for electrical power systems. One of the most successful scheme is a set of first-order complex-coefficient filters (CCF) known as the multiple-complex coefficient filter (MCCF) [21]. The main advantage of using a MCCF compared to classical band-pass filters using real coefficients is the polarity-selective property, which makes it possible to extract the positive or negative sequence components in a straightforward manner [22]. This algorithm is used for grid synchronization, but is also valuable for obtaining magnitude and phase information on the respective harmonic components. Applications of the MCCF algorithm include shunt active filters [23][24] and DSTATCOMs [25].

In this work, the Multiple-Reference Complex-Based Control (MRCC) scheme is proposed for the control of three-phase power converters. The control algorithm uses the ability of the MCCF of extracting the harmonic components, including positive and negative sequences. For each harmonic component, a complex proportional-resonant (PR) controller is proposed. The overall scheme looks similar to the set of reduced-order generalized integrators (ROGIs) presented in [26], but enables the tuning of the control gains for each harmonic independently. As an example, the MRCC scheme is applied to a uninterruptible power supply (UPS) consisting of a three-phase voltage source converter (VSC) feeding a nonlinear unbalanced load. This example makes it possible to demonstrate the applicability of the proposed controller. The stability of the closed-loop system, including the filter dynamics that are disregarded in many papers, is analysed via the Nyquist criterion applied to transfer functions with complex coefficients [8].

The main contributions are:

- A new proof of stability of the MCCF. The MCCF is shown to be stable for *all* values of the cutoff frequencies of the CCFs. This theoretical result is obtained through a relatively simple proof made possible by the complex-domain representation. The result removes the constraint that a single cutoff frequency be used for filtering all harmonics.

F. M. Serra is with the Department of Engineering and the Automatic Control Laboratory, Universidad Nacional de San Luis and CONICET, 5730, Villa Mercedes, San Luis, Argentina. fmserra@unsl.edu.ar.

A. Dòria-Cerezo is with the Department of Electrical Engineering and the Institute of Industrial and Control Engineering, Universitat Politècnica de Catalunya, 08028 Barcelona, Spain arnau.doria@upc.edu.

M. Bodson is with the Department of Electrical and Computer Engineering, University of Utah, Salt Lake City, UT 84112-9206 USA. marc.bodson@utah.edu.

- The main difference with respect to other papers in the literature is the inclusion of the MCCF dynamics as a part of the control loop for design purposes. Then, the design and tuning procedure is made by considering the overall dynamics, including the filter. This is not common and, usually, the filter dynamics are disregarded for stability analysis, see recent examples using the MCCF in [27][28].
- A multiple-reference complex-based control strategy for three-phase power converters. Thanks to the use of the MCCF that extracts the information for each harmonic, the proposed methodology makes it possible to individually analyse the dynamics of each harmonic and, compared with traditional approaches, facilitates the design by using the complex Nyquist criterion.
- A practical example is illustrated by a three-phase VSC. The example includes a detailed design description, as well as simulation and experimental results.

This paper is organized as follows. In Section II, the complex notation is presented. The MCCF and its stability properties are introduced in Section III. Section IV presents the overall MRCC scheme, and is applied to a VSC in Section V. Then, in Section VI, the proposed control algorithm is validated by simulations and experimental tests. Finally, conclusions are summarized in Section VII.

II. COMPLEX REPRESENTATION OF THREE-PHASE SIGNALS

Consider a set of three-phase variables (voltages) $\mathbf{v}_{abc}(t) = (v_a(t), v_b(t), v_c(t))^T$. Then, $\mathbf{v}_{abc}(t)$ can be transformed into a complex variable, $v(t) \in \mathbb{C}$, as

$$v(t) = T\mathbf{v}_{abc}(t), \quad (1)$$

where $T \in \mathbb{C}^3$ is defined as

$$T = c \begin{pmatrix} 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} \end{pmatrix} e^{-j\theta(t)}, \quad (2)$$

for a constant c and an angle $\theta(t)$ to be defined. Different values of c are made in the literature, with $c = \sqrt{\frac{2}{3}}$ preserving the definition of power and $c = \frac{2}{3}$ maintaining the signal amplitude. The transformation (1) is not bijective and reconstruction of $\mathbf{v}_{abc}(t)$ from $v(t)$ generally requires additional information. In electrical three-phase systems, this information comes from the homopolar component that is the sum of the components of $\mathbf{v}_{abc}(t)$,

$$v_h(t) = v_a(t) + v_b(t) + v_c(t). \quad (3)$$

Often, this component is assumed to be zero (and this is forced to be true for currents in three-wire devices).

The transformation (1) includes the well-known $\alpha\beta$ transformation as a special case with $\theta(t) = 0$ and where $v_\alpha(t)$, $v_\beta(t)$ are the real and imaginary parts of $v(t)$. The dq -transformation is obtained with $\theta(t)$ corresponding to the angle of the d axis and $v_d(t)$, $v_q(t)$ being the real and imaginary parts of $v(t)$ [17]. In particular, with $\frac{d\theta}{dt} = \omega$, $v(t)$ is constant if and only if $v_a(t)$, $v_b(t)$, $v_c(t)$ are positive-sequence balanced sinusoidal signals of frequency ω . However, $v(t)$ is defined regardless of steady-state or balanced conditions,

and generalizes the concept of phasor for $v_a(t)$ (as well as $v_b(t)$, $v_c(t)$ with $\frac{2\pi}{3}$ phase shifts).

A periodic three-phase voltage $\mathbf{v}_{abc}(t)$ can be expressed as a sum of multiple frequencies,

$$\mathbf{v}_{abc}(t) = \sum_{l \in \mathcal{H}_r} \mathbf{v}_l(t) = \sum_{l \in \mathcal{H}_r} \begin{pmatrix} V_{al} \cos(l\omega_o t + \phi_{al}) \\ V_{bl} \cos(l\omega_o t - \frac{2\pi}{3} + \phi_{bl}) \\ V_{cl} \cos(l\omega_o t + \frac{2\pi}{3} + \phi_{cl}) \end{pmatrix}, \quad (4)$$

where ω_o is the fundamental angular frequency, $\mathcal{H}_r = \{h_{1r}, h_{2r}, \dots, h_{pr}\} \subset \mathbb{N}^p$ is the set of harmonics of $\mathbf{v}_{abc}(t)$, and V_{xl} , ϕ_{xl} , with $x = a, b, c$, are the amplitude and phase of the l -th harmonic, respectively.

Using (1), with $\theta = 0$ and $c = \frac{2}{3}$, the (scalar) complex variable corresponding to the (vector) three-phase symmetric periodic signal in (4) can be written as the,

$$v(t) = \sum_{l \in \mathcal{H}_r} (a_l e^{j l \omega_o t} + b_l e^{-j l \omega_o t}), \quad (5)$$

where

$$a_l = \frac{V_{al} e^{j\phi_{al}} + V_{bl} e^{j\phi_{bl}} + V_{cl} e^{j\phi_{cl}}}{3}, \quad (6)$$

$$b_l = \frac{V_{al} e^{-j\phi_{al}} + V_{bl} e^{-j(\phi_{bl} + \frac{2\pi}{3})} + V_{cl} e^{-j(\phi_{cl} - \frac{2\pi}{3})}}{3}, \quad (7)$$

are the coefficients of the well known positive and negative sequences. Then, the complex signal in (5) can be written as the sum of v_k 's complex signals, with positive and negative frequencies, as

$$v(t) = \sum_{k \in \mathcal{H}_c} v_k(t), \quad (8)$$

with a new set of (positive and negative) harmonics defined as $\mathcal{H}_c \subset \mathbb{Z}^{2p}$, and

$$v_k(t) = V_k e^{j\phi_k} e^{j k \omega_o t} \in \mathbb{C}, \quad (9)$$

where the complex values $V_k e^{j\phi_k}$ correspond to (6) or (7), depending if $k = l$ or $k = -l$.

Remark 1: In a symmetric and balanced case, $V_{al} = V_{bl} = V_{cl} = V_l$ and $\phi_{al} = \phi_{bl} = \phi_{cl} = \phi_l$. Then, coefficients in (6)-(7) are

$$a_l = V_l e^{j\phi_l}, \quad b_l = 0, \quad (10)$$

and no negative frequencies appear.

III. THE MULTIPLE-COMPLEX COEFFICIENT FILTER

The MCCF is composed by a set of complex-coefficient filters (CCF) that process $\alpha\beta$ signals to extract the components for each harmonic. In the complex notation used in Section II, this approach corresponds to estimating some terms of $v(t)$ defined in (8). In [21], all the CCF modules have the same feedback gains, resulting in identical cutoff frequencies. In this paper, the MCCF scheme is generalized considering different cutoff frequencies for the CCF modules. Figure 1 shows the MCCF scheme using the complex notation, where $\mathcal{H} = \{h_1, h_2, \dots, h_n\} \subset \mathbb{Z}^n$ is the set of considered harmonics. $\mathcal{H}$ is usually defined as a subset of $\mathcal{H}_c$, including the most representative harmonics in the system, i.e., $n \leq 2p$.

Fig. 1: Scheme of the Multiple-Complex Coefficient Filter.

A. State-space description of the MCCF

Consider an MCCF for n harmonics defined by the set $\mathcal{H} \subset \mathbb{Z}^n$. Then, the CCF for the m -th harmonic is,

$$\frac{\hat{v}_m(s)}{\bar{v}_m(s)} = \frac{\gamma_m}{s - jm\omega_o + \gamma_m}, \quad m \in \mathcal{H} \quad (11)$$

where $\bar{v}_m$ and $\hat{v}_m$ denote the complex input and output signals of the filter for the m -th harmonic, ω_o is the fundamental frequency and γ_m is the cutoff frequency, which impacts the bandwidth associated with the m -th harmonic.

The differential form of (11) is,

$$\frac{d\hat{v}_m}{dt} = (jm\omega_o - \gamma_m)\hat{v}_m + \gamma_m\bar{v}_m, \quad (12)$$

and, from Figure 1, the input for each CCF module is,

$$\bar{v}_m = \hat{v}_m - \sum_{k \in \mathcal{H}} \hat{v}_k + v, \quad (13)$$

where v is the complex input signal. From (12), the overall model of the MCCF in compact form is,

$$\dot{\hat{\mathbf{v}}} = \mathbf{A}\hat{\mathbf{v}} + \mathbf{B}v, \quad (14)$$

where $\hat{\mathbf{v}} = (\hat{v}_{h_1}, \dots, \hat{v}_{h_n})^T$, and the matrices $\mathbf{A} \in \mathbb{C}^{n \times n}$ and $\mathbf{B} \in \mathbb{C}^{n \times 1}$ are given by

$$\mathbf{A} = \begin{pmatrix} -\gamma_{h_1} + jh_1\omega_o & -\gamma_{h_1} & \dots & -\gamma_{h_1} \\ -\gamma_{h_2} & -\gamma_{h_2} + jh_2\omega_o & \dots & -\gamma_{h_2} \\ \vdots & \vdots & \ddots & \vdots \\ -\gamma_{h_n} & -\gamma_{h_n} & \dots & -\gamma_{h_n} + jh_n\omega_o \end{pmatrix}, \quad (15)$$

and,

$$\mathbf{B}^T = (\gamma_{h_1} \quad \gamma_{h_2} \quad \dots \quad \gamma_{h_n}), \quad (16)$$

respectively.

Equation (14) is a complex description and a generalization of the state-space form of the MCCF presented in [21].

B. Transfer function of the MCCF

The transfer function of a single CCF has been studied in [21], and references therein. However, the MCCF structure has a more complicated behavior than the CCF. In this section, the transfer function relating the m -th output $\hat{v}_m$ to the filter input v is obtained, and the stability properties of the MCCF are analyzed.

Define the m -th output equation for the MCCF as

$$\hat{v}_m = \mathbf{C}_m \hat{\mathbf{v}}, \quad (17)$$

where

$$\mathbf{C}_m = (c_{m,1} \quad c_{m,2} \quad \dots \quad c_{m,n}) \in \mathbb{R}^{1 \times n}, \quad (18)$$

and

$$c_{m,k} = \begin{cases} 1 & \text{if } k = m \\ 0 & \text{otherwise} \end{cases}. \quad (19)$$

Let $H_m(s)$ be the transfer function relating the filter input v to the m -th output $\hat{v}_m$, so that

$$\hat{v}_m(s) = H_m(s)v(s). \quad (20)$$

The transfer function is obtained from (14), with (15)-(16) and (18), resulting in

$$H_m(s) = \frac{N_{H_m}(s)}{D_H(s)}, \quad (21)$$

where

$$N_{H_m}(s) = \gamma_m \prod_{k \in \mathcal{H}, k \neq m} (s - jh_k\omega_o), \quad (22)$$

$$D_H(s) = \prod_{k \in \mathcal{H}} (s - jh_k\omega_o) + \sum_{k \in \mathcal{H}} \gamma_k \prod_{l \in \mathcal{H}, l \neq k} (s - jh_l\omega_o). \quad (23)$$

Therefore,

$$H_m(jh_l\omega_o) = 0 \quad \text{if } l \neq m \quad (24)$$

$$= 1 \quad \text{if } l = m \quad (25)$$

In other words, the m -th harmonic is estimated exactly by the m -th component while other harmonics are completely rejected. This property can be observed in Figure 2, that shows the gain plots of transfer functions (21) with $\mathcal{H} = \{-11, -5, -1, 1, 7, 13\}$ ¹. Note that, since the transfer functions $H_m(s)$ have complex coefficients, plots are not symmetric for positive and negative frequencies. The total number of harmonics, as well as the harmonic components, shape the gain plots, but the properties of (24) and (25) are always satisfied.

The stability of the MCCF is given by the eigenvalues of (15). Figure 3 shows the result of calculating the n eigenvalues of $\mathbf{A}$ for 5000 different cutoff values. The parameters used for the computation were $\omega_o = 100\pi$ rad/s, six harmonics $\mathcal{H} = \{-11, -5, -1, 1, 7, 13\}$ and $\gamma_m > 0$ for all $m \in \mathcal{H}$. One finds that the MCCF remains stable for all values of the parameters γ_m . This observation suggests that it may be possible to prove

¹The selected set of harmonics is representative of the harmonic content present in the voltage signals due to the nonlinear load used in this work. See the harmonic spectrum in Figure 16 of the experimental validation.

Fig. 2: Gain plots for a MCCF for $\omega_o = 100\pi$ rad/s, harmonic components $\mathcal{H} = \{-11, -5, -1, 1, 7, 13\}$, and $\gamma_m = 0.88$ for all harmonics.

Fig. 3: Eigenvalues of $\mathbf{A}$ in (15), for $\omega_o = 100\pi$ rad/s and $\mathcal{H} = \{-11, -5, -1, 1, 7, 13\}$. Batch of 5000 randomly selected $\gamma_m > 0$ values.

the stability of the algorithm for arbitrary cutoff frequencies. Indeed, the following result specifies that, if the MCCF (14) does not contain any repeated harmonic, the filter is stable for all choices of cutoff frequencies $\gamma_{h_k} > 0$, $k \in \mathcal{H}$, and the result is true even if the gains are different for distinct harmonics.

Proposition 1: Assume that $h_k \neq h_l$ for all $k \neq l$ and $\gamma_{h_k} > 0$ for all k . Then, the polynomial $D_H(s)$ has all roots in the open left half-plane.

Proof. Using the Matrix Determinant Lemma (see Theorem 18.1.1. in [29]), the Characteristic Polynomial of (15) given by (23) can be written as

$$D_H(s) = P(s)(s - jh_1\omega_o) \dots (s - jh_n\omega_o), \quad (26)$$

where

$$P(s) = 1 + \frac{\gamma_{h_1}}{s - jh_1\omega_o} + \dots + \frac{\gamma_{h_n}}{s - jh_n\omega_o}. \quad (27)$$

The remainder of the proof has two parts:

Part 1: the roots of $D_H(s)$ are distinct from $s = jh_k\omega_o$ for all k . Indeed, for $s = jh_1\omega_o$, $D_H(jh_1\omega_o) = \gamma_{h_1}(j\omega_o)^{n-1}(h_1 - h_2) \dots (h_1 - h_n)$, which is different from zero under the assumptions. Repeating for other values of k , the result is obtained.

Part 2: the roots of $D_H(s)$ are in the open left half-plane. Given Part 1, the roots of $D_H(s)$ are the same as the roots of $P(s)$ and are in the open left half-plane if and only if $P(s) \neq 0$ for all values of s with $\text{Re}(s) \geq 0$. Values of s

Fig. 4: Scheme of the Multiple-Reference Complex-based Controller.

such that $s = jh_k\omega_o$ do not need to be considered. In other words, one needs $P(a + jb) \neq 0$ for all $a \geq 0$, b arbitrary, and $a + jb \neq jh_k\omega_o$ for all k . One has,

$$P(a + jb) = 1 + \frac{\gamma_{h_1}(a - j(b - h_1\omega_o))}{a^2 + (b - h_1\omega_o)^2} + \dots + \frac{\gamma_{h_n}(a - j(b - h_n\omega_o))}{a^2 + (b - h_n\omega_o)^2}. \quad (28)$$

Under the assumptions, $a^2 + (b - h_i\omega_o)^2 \neq 0$ for all i and,

$$\text{Re}(P(a + jb)) = 1 + \frac{\gamma_{h_1}a}{a^2 + (b - h_1\omega_o)^2} + \dots + \frac{\gamma_{h_n}a}{a^2 + (b - h_n\omega_o)^2} \geq 1, \quad (29)$$

which implies that $P(a + jb) \neq 0$. $\triangleleft$

IV. THE MULTIPLE-REFERENCE COMPLEX-BASED CONTROL

A. Controller structure

The Multiple-Reference Complex-based Control (MRCC) is based on Figure 4. It consists in extracting the harmonic components of a signal, $\hat{v}(t)$, by means of the MCCF,

$$v(t) \approx \hat{v}(t) = \sum_{k \in \mathcal{H}} \hat{v}_k(t). \quad (30)$$

Then, the control action is composed of the sum of individual control actions corresponding to each harmonic,

$$u(s) = \sum_{k \in \mathcal{H}} C_k(s)(v_k^d(s) - \hat{v}_k(s)). \quad (31)$$

The reference signal for the m -th harmonic is

$$v_m^d(s) = \frac{V_m}{s - jm\omega_o}, \quad (32)$$

where V_m the desired amplitude of the m -th harmonic. The main difference with respect to the set of ROGIs presented in [26] is the inclusion of the CCF modules in Figure 4 to extract the harmonic components and facilitate the control design.

The controller $C_m(s)$ for harmonic m consists of a standard complex proportional-resonant (PR) controller with the form [30],

$$C_m(s) = K_{pm} + \frac{K_{im}}{s - jm\omega_o}, \quad (33)$$

where $K_{pm}, K_{im} \in \mathbb{C}$ are the proportional and resonant gains, respectively. The second term, also known as a ROGI [26],

is used to ensure zero error when tracking sinusoidal signals [31] and rejecting disturbances, thanks to the Internal Model Principle. Defining $K_m = K_{pm}$ and $\alpha_m = \frac{K_{im}}{K_{pm}}$, the PR controller in (33), can be written as

$$C_m(s) = K_m \frac{s + \alpha_m - jm\omega_o}{s - jm\omega_o}, \quad (34)$$

where $K_m, \alpha_m \in \mathbb{C}$ are the complex gains that will be selected in the design of the controller.

B. Closed-loop dynamics

Using (20) in (31), the control input can be written as

$$u(s) = \sum_{k \in \mathcal{H}} C_k(s) v_k^d(s) - \sum_{k \in \mathcal{H}} C_k(s) H_k(s) v(s), \quad (35)$$

so that the output signal is

$$v(s) = \frac{\sum_{k \in \mathcal{H}} C_k(s) G(s) v_k^d(s)}{1 + \sum_{k \in \mathcal{H}} C_k(s) H_k(s) G(s)}, \quad (36)$$

where $G(s)$ is the transfer function of the power converter (that may have complex coefficients). Closed-loop stability of (36) is determined by the roots of

$$1 + L(s) = 0, \quad (37)$$

where $L(s)$ is the loop transfer function

$$L(s) = \sum_{k \in \mathcal{H}} L_k(s), \quad (38)$$

where, for a given harmonic m ,

$$L_m(s) = C_m(s) H_m(s) G(s). \quad (39)$$

$L_m(s)$ is the loop transfer function that would be obtained if a single harmonic was cancelled. The overall loop transfer function is the sum of the loop transfer functions associated with the different harmonics.

Remark 2: Notice the difference of the closed loop dynamics in (36) with respect to the standard approach consisting of a set of PR (without the MCCF) that would result in

$$v(s) = \frac{\sum_{k \in \mathcal{H}} C_k(s) G(s)}{1 + \sum_{k \in \mathcal{H}} C_k(s) G(s)} v^d(s). \quad (40)$$

- In the denominator: the filter transfer function, $H_k(s)$, appears in the closed loop transfer function (36) affecting only the poles and, thus, the stability compared to (40).
- In the numerator: in (40), the overall reference $v^d(s)$ multiplies the closed-loop transfer function. In contrast, in (36) each reference component $v_k^d(s)$ only multiplies its corresponding PR controller, $C_k(s)$. This feature offers more possibilities for adjusting the response to each harmonic component.

C. Stability analysis

The Nyquist Criterion is a useful tool to determine the stability of feedback systems. Through the complex transformation, the system has become a single-input single-output system, where the Nyquist diagram is most powerful for stability and robustness analysis. The criterion, based on the argument principle which is valid for any meromorphic function, applies to transfer functions with complex coefficients mostly as in the real case, but with the difference that the complex Nyquist curve is no longer symmetric for negative and positive frequencies. See further details in [32].

Consider first the case where a single harmonic is controlled, and $L(s) = L_m(s)$. Assume that the converter transfer function $G(s)$ is stable. Given that $H_m(s)$ is stable for all $\gamma_m > 0$, the loop transfer function $L_m(s)$ is stable, except for a pole at $s = jm\omega_o$. A modified Nyquist contour must be used, where the portion of the imaginary axis close to $\omega = m\omega_o$ is replaced by a semicircle of a small radius. Specifically, the segment $\omega = m\omega_o + \delta\omega$, where $\delta\omega$ varies from $-\varepsilon$ to ε and ε is small is replaced by a semicircle where $\delta\omega = \varepsilon e^{j\alpha}$ and α varies from $-\pi/2$ to $\pi/2$. The number of encirclements is counted using the image of the modified contour, noting that the loop transfer function does not have any pole within the modified contour.

Let,

$$L_m(s) = \frac{L_{mo}(s)}{s - jm\omega_o}, \quad (41)$$

where $L_{mo}(s)$ is stable with no poles on the $j\omega$ -axis. For infinitesimal ε , the image of the semicircle is

$$L_m(jm\omega_o + \varepsilon e^{j\alpha}) \simeq \frac{L_{mo}(jm\omega_o)}{\varepsilon} e^{-j\alpha}, \quad (42)$$

which is also a semicircle, but with large radius and connecting the frequency response from $\omega = m\omega_o - \varepsilon$ to $\omega = m\omega_o + \varepsilon$ in the clockwise direction.

A necessary and sufficient condition for closed-loop stability is that the Nyquist curve does not encircle the $-1 + j0$ coordinate. A sufficient condition is that the Nyquist curve does not cross the real axis for $\text{Re}(s) < -1$. In particular, the large semicircle crosses the real axis in the right-hand side of the plane if the angle of the asymptote for $\omega \rightarrow m\omega_o^-$ belongs to $(0, \pi)$ (or, equivalently, the asymptote for $\omega \rightarrow m\omega_o^+$ belongs to $(0, -\pi)$). As will be shown, controller gains can always be found to satisfy this condition.

When the controllers for the separate harmonics are used together, stability is determined by the frequency response of the loop transfer function $L(s)$, which is the sum of the individual loop transfer functions $L_m(s)$. The modified Nyquist contour includes multiple semicircles. However, due to the bandpass property of $H_m(s)$ and to the resonant nature of $C_m(s)$, $L(j\omega)$ is close to one of the $L_m(j\omega)$ whenever its magnitude is large. In fact, the angle of the asymptote of $L(j\omega)$ for ω converging $m\omega_o^-$ (or $m\omega_o^+$) is the same as the angle of the asymptote of $L_m(j\omega)$, because $H_l(jm\omega_o) = 0$ for $l \neq m$.

As a result, the design of the control system can be performed one harmonic at a time, making sure that the Nyquist curve does not cross the real axis for $\text{Re}(s) < -1$. To check

the stability of the overall feedback system, one only needs to verify that the finite part of the Nyquist curve does not intersect the real axis for $\text{Re}(s) < -1$. No adjustment is typically needed, however, because the frequency response is dominated by one of the components, unless its magnitude is small. Interestingly, the MCCF not only serves to extract the harmonic components, but also to separate the frequency bands of the individual controllers. The application of the concepts will be further explained in the practical design considered in the next section.

V. DESIGN OF THE MRCC

In this section, the design of the MRCC is demonstrated for a UPS, which consists in a three-phase VSC feeding a nonlinear unbalanced load. First, the complex-based model of the VSC is presented. Then, parameters are selected for each component of the controller. Afterwards, the individual controllers are combined and the stability and robustness properties of the overall controller are evaluated on the basis of the Nyquist curve.

A. Complex representation of the VSC

Figure 5 shows the electrical scheme of the three-phase VSC, where L , C and R represent the inductance, capacitance, and inductor losses of one phase of the LC filter, respectively. When the load is unbalanced and nonlinear, currents are expected to have a high harmonic content. The dynamics

Fig. 5: Three-phase VSC with an LC filter.

associated with the VSC are described in abc coordinates by,

$$L \frac{d\mathbf{i}_{abc}}{dt} = -R\mathbf{i}_{abc} - \mathbf{M}\mathbf{v}_{abc} + \mathbf{M}\mathbf{u}_{abc}, \quad (43)$$

$$C \frac{d\mathbf{v}_{abc}}{dt} = \mathbf{i}_{abc} - \mathbf{i}_{Labc}, \quad (44)$$

where, $\mathbf{v}_{abc}^T(t) = (v_a(t), v_b(t), v_c(t))$ is the vector of output voltages (i.e., the voltages on the filter capacitors), $\mathbf{i}_{abc}^T(t) = (i_a(t), i_b(t), i_c(t))$ is the vector of currents in the inductors, $\mathbf{u}_{abc}^T(t) = (u_a(t), u_b(t), u_c(t))$ is the vector of voltages computed by the switching policy (that will be used as control inputs), $\mathbf{i}_{Labc}^T(t) = (i_{La}(t), i_{Lb}(t), i_{Lc}(t))$ is the vector of load currents, and the $\mathbf{M}$ matrix is,

$$\mathbf{M} = \frac{1}{3} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}. \quad (45)$$

Remark 3: The model (43)-(44) assumes that the load currents are independent from the voltages, so that the load

currents can be treated as disturbances. Practically, load currents depend on the voltages and constitute an additional feedback path. The effect of this feedback will be considered in Section V-D.

The control objective is to ensure balanced three-phase voltages on the capacitors, that is,

$$\mathbf{v}_{abc}^d(t) = V^d \begin{pmatrix} \cos(\omega_o t) \\ \cos(\omega_o t - \frac{2\pi}{3}) \\ \cos(\omega_o t + \frac{2\pi}{3}) \end{pmatrix}, \quad (46)$$

where V^d and ω_o are the desired amplitude and frequency, respectively.

The three-phase to complex transformation (1) is then used not only to define equivalent complex signals but also a complex equivalent VSC system. In particular, assuming a three-phase connection without neutral (i.e., the homopolar component is zero), the VSC dynamics in (43)-(44) can be transformed into the complex equivalent system (letting $\theta = 0$)

$$L \frac{di}{dt} = -Ri - v + u, \quad (47)$$

$$C \frac{dv}{dt} = i - i_L, \quad (48)$$

where $v, i \in \mathbb{C}$ are the complex voltage and current, $u \in \mathbb{C}$ is the complex control input, and $i_L \in \mathbb{C}$ is the complex load current (all are scalar variables). Similarly to (8), the load current is written as

$$i_L(t) = \sum_{k \in \mathcal{H}} i_{Lk}(t) = \sum_{k \in \mathcal{H}} I_{Lk} e^{jk\omega_o t}, \quad (49)$$

where the modulus and phase of $I_{Lk} \in \mathbb{C}$ are the amplitude and phase of $i_{Lk}(t)$.

The input/output response is determined by the complex transfer function of the system from u to v , or

$$G(s) = \frac{1}{LCs^2 + RCs + 1}. \quad (50)$$

From (46) with (1), the control objective is to track the complex signal

$$v^d(t) = V_1 e^{j\omega_o t}, \quad (51)$$

where $V_1 \in \mathbb{C}$ accounts for both the amplitude and the phase of the reference

$$V_1 = V^d e^{j\phi}. \quad (52)$$

By applying the Laplace transform, (51) results in

$$v^d(s) = \frac{V_1}{s - j\omega_o}. \quad (53)$$

The reference signals as defined in (32) are given by

$$V_m = \begin{cases} V_1 & \text{if } m = 1 \\ 0 & \text{otherwise} \end{cases}. \quad (54)$$

B. Control design for the m -th harmonic

We begin with a study of the influence of the control parameters on the complex Nyquist plot. From (34), one has

$$C_m(j\omega) = K_m \frac{\alpha_m + j(\omega - m\omega_o)}{j(\omega - m\omega_o)}, \quad (55)$$

Fig. 6: Complex Nyquist plots of $L_1(j\omega)$. Influence of control parameters. **Left top plot:** with $\alpha_1 = 2000 + j0$ and $\phi_{K_m} = 0$, variation of $|K_1| = (0.1, 0.2, 0.3)$ in blue, red and yellow, respectively. **Right top plot:** with $|K_1| = 0.1$ and $\alpha_1 = 2000 + j0$, variation of $\phi_{K_1} = (0, -\frac{\pi}{6}, \frac{\pi}{6})$ in blue, red and yellow, respectively. **Left bottom plot:** with $K_1 = 0.1 + j0$ and $\text{Im}(\alpha_1) = 0$, variation of $\text{Re}(\alpha_1) = (1000, 2000, 3000)$ in blue, red and yellow, respectively. **Right bottom plot:** with $K_1 = 0.1 + j0$ and $\text{Re}(\alpha_1) = 2000$, variation of $\text{Im}(\alpha_1) = (0, -\omega_o, \omega_o)$ in blue, red and yellow, respectively.

with the magnitude and phase equations

$$|C_m(j\omega)| = |K_m| \frac{\sqrt{\text{Re}(\alpha_m)^2 + (\text{Im}(\alpha_m) + \omega - m\omega_o)^2}}{|\omega - m\omega_o|}, \quad (56)$$

and

$$\angle C_m(j\omega) = \phi_{K_m} - \frac{\pi}{2} \text{sign}(\omega - m\omega_o) + \arctan\left(\frac{\text{Im}(\alpha_m) + \omega - m\omega_o}{\text{Re}(\alpha_m)}\right), \quad (57)$$

where ϕ_{K_m} denotes the phase of the complex number K_m . Note that the magnitude and the phase of K_m have important and independent impacts in equations (56)-(57), respectively. On the other hand, equation (57) suggests that α_m can be used for a fine-tuning at frequencies away from $m\omega_o$.

The discussion is illustrated for $m = 1$ on Figure 6, which shows complex Nyquist plots for the loop transfer function $L_1(s)$ and various choices of control parameters. The VSC and MCCF parameters are the ones used in Section VI-A. The magnitude and phase of K_1 directly affect the magnitude and the phase of the loop transfer function, as shown in the top plots of Figure 6. The effect of α_1 is shown in the bottom plots. Since changing the imaginary part of α_1 is similar to modifying the phase of K_1 , the real part of α_1 reduces the arc of the unit circumference defined by the crossing points with the curves for $\omega < m\omega_o$ and $\omega > m\omega_o$. On the other hand, $\text{Re}(\alpha_1)$ can be used to adjust the gain margin.

m	-11	-5	-1	1	7	13
$\text{Re}(\alpha_m)$	2000	5000	2000	2000	5000	7000
$\text{Im}(\alpha_m)$	0	0	0	0	0	0
$ K_m $	0.02	0.02	0.04	0.04	0.01	0.01
ϕ_{K_m}	π	0	0	0	0	π

TABLE I: Control parameters

To ensure the stability of the feedback system, one needs to calculate $\lim_{\omega \rightarrow m\omega_o^-} \angle L_m(j\omega)$, where

$$\angle L_m(j\omega) = \angle C_m(j\omega) + \angle G(j\omega) + \angle H_m(j\omega). \quad (58)$$

By definition of the MCCF, $\lim_{\omega \rightarrow m\omega_o^-} \angle H_m(j\omega) = 0$. Then, it follows from (50) and (57), that

$$\lim_{\omega \rightarrow m\omega_o^-} \angle L_m(j\omega) = \phi_{K_m} - \frac{\pi}{2} + \arctan\left(\frac{m\omega_o RC}{1 - m^2\omega_o^2 LC}\right), \quad (59)$$

and two different cases appear:

$$\begin{aligned} m^2\omega_o^2 < \frac{1}{LC} &\implies \arctan\left(\frac{m\omega_o RC}{1 - m^2\omega_o^2 LC}\right) \in (0, \pi), \quad (60) \\ m^2\omega_o^2 > \frac{1}{LC} &\implies \arctan\left(\frac{m\omega_o RC}{1 - m^2\omega_o^2 LC}\right) \in (0, -\pi). \quad (61) \end{aligned}$$

Conditions (60)-(61) are directly related to the resonance frequency of the LC filter, given by $\omega_r = \frac{1}{\sqrt{LC}}$. The result shows that ϕ_{K_m} should be used to compensate the negative phase of the third term in (59), depending on whether the harmonic frequency is smaller or larger than the LC resonance frequency.

Table I shows the control gains for the numerical example used in Section VI-A. The set of harmonics considered, $\mathcal{H} = \{-11, -5, -1, 1, 7, 13\}$, is representative for the load used in the experimental validation in Section VI, see Figure 16a. Notice that, since $\omega_o = 100\pi$ rad/s and the VSC has a resonance frequency at $\omega_r = 3.1427 \cdot 10^3$ rad/s, harmonics with $|m| > 10$ require $\phi_{K_m} = \pi$ rad, which is equivalent to a reversal of feedback polarity. The complex Nyquist plots for each harmonic are shown in Figure 7.

C. Stability and robustness of the overall system

In Figure 8, the complex Nyquist plot of $L(j\omega)$ is shown. Given that there are no encirclements of $-1 + j0$, the closed-loop system is stable. Besides, closed-loop poles on the imaginary axis are avoided because the Nyquist plot does not pass through the $(-1, 0)$ point.

In Figure 9, the complex Nyquist plots of $L_m(j\omega)$ and $L(j\omega)$ are compared. One finds that the loop frequency response appears as the combination of the loop frequency responses. In particular, $L(j\omega)$ approaches one of the $L_m(j\omega)$ along the asymptotes and close to the unit circle. This result is obtained each frequency response $C_m(j\omega)H_m(j\omega)$ becomes large in a different frequency band, not only because of the resonant mode of $C_m(s)$, but also because of the selectivity of the filter $H_m(s)$ (see Figure 2). The closed loop poles of $L(j\omega)$ and $L_m(j\omega)$ are shown in Figure 10, which suggests that the collective behavior of the individual loops is representative of the overall system in terms of pole locations as well.

Fig. 7: Complex Nyquist plots of $L_m(j\omega)$ with the control parameters in Table I, and the delay margin, T_{dm} , associated to each plot.

Fig. 8: Complex Nyquist plot of $L(j\omega)$ with the control parameters in Table I.

Fig. 9: Complex Nyquist plots of $L(j\omega)$ shown in Figure 8 (in blue) compared with all the complex Nyquist plots of $L_m(j\omega)$ shown in Figure 7 (in red).

Fig. 10: Comparison of the closed loop poles of $L(j\omega)$ (in blue) and $L_m(j\omega)$ (in red).

The Nyquist curve can also be used to evaluate the robustness margins of the system. Gain and phase margins can be measured directly, but one should remember that they apply to the complex system, or to the real system if the same gain and phase changes are applied to the different components (a , b , and c). The delay margin is defined as the maximum time delay that may be applied while preserving stability, and can be calculated as

$$L(j\omega_{gc})e^{-j\omega_{gc}T_{dm}} = -1, \quad (62)$$

where ω_{gc} is the gain crossover frequency (where the magnitude of the loop frequency response is equal to 1). Note that both positive and negative frequencies can determine the delay margin. If multiple frequencies are present, the smallest value must be used. In Figure 7, the result of the computation of the delay margin is given for each individual harmonic. The minimum is $T_{dm} = 0.397$ ms for $m = 13$. The value is close to the value obtained with the overall Nyquist plot of $L(j\omega)$ in Figure 8, which gives $T_{dm} = 0.347$ ms. The delay margin is more than sufficient for the sampling rate of 0.05 ms used in the experimental stage in Section VI-B.

Fig. 11: Influence on R_L on the asymptotes of $\angle L_m(j\omega)$, with $\phi_{K_m} = 0$.

D. Robustness to load feedback

As pointed out in Section V-A, the model (48) considers the load current i_L as a disturbance. In general, this current depends on the voltage v and constitutes a feedback path. It is not possible to consider all possible load dynamics, but the effect of a static (resistive) load is studied more carefully in this section. From the model (47)-(48) with $i_L = \frac{1}{R_L}v$, the complex transfer function from u to v is

$$G(s) = \frac{R_L}{R_L LC s^2 + (R_L RC + L)s + R + R_L}. \quad (63)$$

Note that, when $R_L \rightarrow \infty$, the transfer function (50) is obtained.

Following the analysis of the asymptotes in the previous section, we have now

$$\lim_{\omega \rightarrow m\omega_o^-} \angle L_m(j\omega) = \phi_{K_m} - \frac{\pi}{2} + \arctan \left(\frac{m\omega_o(R_L RC + L)}{R + R_L(1 - m^2\omega_o^2 LC)} \right), \quad (64)$$

that shows how different values of the resistive load affect the phase for frequencies close to the resonant frequency. In particular, note that when $R_L \rightarrow \infty$, the condition (59) is recovered. Conversely, for $R_L = 0$

$$\lim_{\omega \rightarrow m\omega_o^-} \angle L_m(j\omega) = \phi_{K_m} - \frac{\pi}{2} - \arctan \left(\frac{m\omega_o L}{R} \right). \quad (65)$$

Figure 11 shows how the asymptotes in (64) evolve with R_L . For higher loads (when $R_L \rightarrow 0$), the last term in (64) converges to the angle of the impedance $Z_m = R + jm\omega_o L$, which is close to $\frac{\pi}{2}$. This effect can also be observed in the complex Nyquist plots, see Figure 12. In contrast, for high values of R_L , the asymptotes of the complex Nyquist plots tend to move in the counter-clockwise direction, for harmonics in the range defined by the resonance frequency, ω_r , (clockwise for other harmonics), reducing the phase margin. In conclusion, the worst-case scenario is when $R_L \rightarrow \infty$, which was the one considered for design in Section V-B

Fig. 12: Complex Nyquist plots of $L_m(j\omega)$. Influence of the load resistance, R_L . The control parameters are the ones in Table I and the load resistance varies between $R_L = 2000, 15$ and 10Ω , in blue, red and green, respectively.

VI. EXPERIMENTAL VALIDATION

The MRCC proposed in the previous section was validated through simulations and experiments. The system parameters were $v_{dc} = 200$ V, $L = 2.75$ mH, $R = 0.3 \Omega$, $C = 45 \mu\text{F}$, with a 20 kHz switching frequency. The desired three-phase voltages are balanced with phase amplitude 100 V and 50 Hz frequency. Therefore, from (51), (52) and (54), $V^d = 100$ V, $\phi = 0^\circ$ and $\omega_o = 2\pi 50$ rad/s.

The VSC feeds a nonlinear unbalanced load, consisting of three single-phase unbalanced rectifiers connected to each phase and a three-phase rectifier feeding a resistive load. The main harmonic content is $\mathcal{H} = \{-11, -5, -1, 1, 7, 13\}$, see Figure 16a.

A. Simulation results

Simulations have been performed using *SimPowerSystems* of *Matlab/Simulink* with a realistic model of the power converter including losses, switching effects, and noise in the measured signals. This model does not emulate the microcontroller (and its problems related to execution times or quantification), nor thermal or electromagnetic interferences.

Fig. 13: Simulation results: In plots (a) to (f), behavior of the errors for each harmonic component, real and imaginary parts of $e_m(t)$ in blue and red, respectively. In plot (g), the three-phase output voltages $v_{abc}(t)$.

The proposed test consists in three scenarios. First, only the fundamental component is controlled (controllers $C_m(s)$ for $m \neq 1$ are switched off). Then, at $t = 0.2$ s, the complete MRCC is activated (with zero reference for the harmonic components). Finally, at $t = 0.4$ s, a reference change is applied to the fundamental component from 100 V to 80 V amplitude. Figures 13 to 16 show the results. For simplicity, the initial transient is omitted and only reference changes are discussed.

During the first part of the test, the fundamental component tracks its reference. See in Figure 13a that $e_1(t)$ remains close to zero, but other harmonic components are still present because they are not controlled. See also errors in Figure 13 and voltage/current waveforms in Figure 14. At $t = 0.2$ s, all the harmonic components are controlled and their errors converge to zero, see Figure 13. In Figure 14, the voltage waveforms are improved. Finally, when the reference changes at $t = 0.4$ s, the fundamental component achieves the desired value in approximately 30 ms (one cycle and a half of the output voltage), and the errors of other harmonics also come down to zero after a short transient, see Figure 13.

Frequency spectrums for the three-phase output voltages are shown for the first and second scenarios in Figure 16. The Total Harmonic Distortion (THD) when the full MRCC is used

Fig. 14: Simulation results: Detail for control change. (a) Three-phase voltage, (b) voltage (blue) and load current (red) of phase a.

Fig. 15: Simulation results: Detail for reference change. (a) Three-phase voltage, (b) voltage (blue) and load current (red) of phase a.

drops from 14.12% to 3.01%, which is a low value compared with to common standards for UPS.

B. Experimental results

The experimental platform consists in the LC three-phase VSC described earlier. The converter was assembled from a SEMiX-101GD12E4s IGBT module of Semikron and the controller algorithm was implemented in a TMS320F28335 floating point DSP of Texas Instrument. The experimental testbed is shown in Figure 17.

The testing procedure includes the same scenarios described with the numerical simulations. Figure 18a shows the voltage signals during the first scenario (when only the control of the fundamental component is running with $V^d = 100$ V). The amplitude and frequency of the three-phase voltages (CH1 to CH3) are controlled, but the waveforms are distorted because

Fig. 16: Simulation results: Voltage harmonic spectrum, positive (blue) and negative (red) harmonic components. (a) Control of fundamental harmonic, (b) MRCC.

Fig. 17: Experimental testbed.

there is no harmonic compensation of the components of the load current (see phase b in CH4). In contrast, the voltage is improved when all the significant harmonic components are controlled, see Figure 18b.

Finally, Figures 18c and 18d show the transient behavior when the full MRCC is engaged, and under an amplitude reference change, respectively. In Figure 18c the waveform of the voltage is improved in approximately one cycle and a half, and the reference of the amplitude and frequency are maintained during the change. In Figure 18d, the amplitude of the fundamental component changes from 100 V to 80 V, and the new reference value is reached in less than one period and a half.

VII. CONCLUSIONS

In this paper, a new scheme is presented for controlling three-phase power converters. The methodology is based on complex-parameter models, which transform the systems to single-input single-output systems and enable powerful analysis tools. The proposed controller is based on a MCCF combined with multiple reference controllers. An intermediate contribution is a new proof of stability of the MCCF for arbitrary values of the gains. Filtering of separate harmonic components can be tuned independently. As opposed to traditional schemes, the design of the controllers is simplified by considering one harmonic at a time, a procedure that is made possible by the frequency selectivity of the combined MCCF and reference controllers. Original contributions are the inclusion of the dynamics of the MCCF during the control design, and a guarantee of stability of the overall feedback system. The analysis using the Nyquist diagram also makes it possible to evaluate the robustness margins of the system. Due to the frequency separation, the margins are close to the ones predicted from the designs carried out for separate harmonics.

Fig. 18: Experimental results. (a) Three-phase output voltages (CH1 to CH3) and current of phase b (CH4) for control of the fundamental component. (b) Three-phase output voltages (CH1 to CH3) and current of phase b (CH4) for the proposed MRCC. (c) Three-phase voltages for control change test. (d) Three-phase voltages for reference change test.

Simulation and experimental results validated the proposed control methodology in a practical converter design.

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Científicas y Técnicas (CONICET). His research interests include modeling and advanced control of power converters in applications of microgrids, electric vehicles and renewable energy conversion systems.

Arnau Dòria-Cerezo was born in Barcelona, Spain, in 1974. He received the M.S. degree in electromechanical engineering from the Universitat Politècnica de Catalunya (UPC), Barcelona, Spain, in 2001, the DEA degree in industrial automation from the Institut National des Sciences Appliquées de Lyon, Villeurbanne, France, in 2001, and the Ph.D. degree in advanced automation and robotics from UPC, in 2006.

He is currently an Associate Professor with the Department of Electrical Engineering, UPC. He carries out his research with the research group on Advanced Control of Energy Systems, Institute of Industrial and Control Engineering, UPC. From 2003 to 2004, he was a Control Training Site-Research Fellow with the Laboratoire des Signaux et Systèmes, Supélec, France. In 2010, he was a Visitor with the Technische Universiteit Delft, Delft, The Netherlands. His research interests include modeling and control of electrical systems and automotive applications.

Dr. Dòria-Cerezo has been an Associate Editor for the Control Engineering Practice since 2017.

Marc Bodson (F'06) received the Ingénieur Civil Mécanicien et Electricien degree from the Université Libre de Bruxelles, Brussels, Belgium, in 1980, two M.S. degrees, one in electrical engineering and computer science and one in aeronautics and astronautics, from Massachusetts Institute of Technology, Cambridge, MA, USA, in 1982, and the Ph.D. degree in electrical engineering and computer science from the University of California at Berkeley, Berkeley, CA, USA, in 1986.

Currently, he is a Professor of Electrical and Computer Engineering with the University of Utah, Salt Lake City, UT, USA. He was Chair of the Department Electrical and Computer Engineering from 2003 to 2009. His research interests include adaptive control with applications to electromechanical systems and aerospace.

Prof. Bodson was elected Associate Fellow of the AIAA in 2013. He was a Belgian American Educational Foundation Fellow in 1980 and a Lady Davis Fellow with the Technion, Haifa, Israel, in 1990. He was the Editor-in-Chief of the IEEE Transactions on Control Systems Technology from 2000 to 2003.