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Mechanical Model for the Shear Strength of Steel Fibers Reinforced Concrete Beams (SFRC) without Stirrups

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ABSTRACT

Numerous studies have shown that adding steel fibers to the concrete mixture improves both the shear strength and ductility of reinforced concrete beams. Most current shear design formulations for steel fiber reinforced concrete beams are empirically based and their predictions provide acceptable results when compared with tests results, but only in limited ranges of the parameters involved. On the other hand, many shear theoretical models suggested in the literature are derived by extending previous formulations for conventional reinforced concrete beams, just introducing the stresses transferred across the critical shear crack. Since the effects of steel fibers on the others shear resisting mechanisms are not accounted for, “adjusting” empirical factors must be used to fit the experimental results. There is, therefore, the need for developing mechanical models capable to rationally account for the effects of steel fibers on the global shear strength and on each shear resisting mechanisms.

In this paper, the previously derived and validated Multi Action Shear Model for RC elements is extended to steel fibers reinforced concrete beams without stirrups. The effects of steel fibers on each resisting mechanism have been identified and incorporated in the formulation of each shear component and in the Multi-Action Shear Model equilibrium equations. The residual tensile stresses of fibrous

concrete, obtained through a simple formulation, has been used to: capture the enhancement of the compression chord contribution; the shear transferred by bridging effect of the fibres along the critical shear crack and by dowel action.

The proposed model is shown to properly estimate the shear resistance of a large set of available test data, being able to account for most influencing parameters, like fibers types and amounts, concrete strength, longitudinal reinforcement ratios and beams geometry.

Keywords: Steel Fiber Reinforced concrete (SFRC), Shear Strength, Multi Action Shear Model, Shear resisting mechanisms, Mechanical model

1. INTRODUCTION

The brittle behavior of plain concrete in tension leads to achieve a shear failure in reinforced concrete (RC) beams without transversal reinforcement in rapid manner and without any warning. The lack of post-cracking ductility in tension is a characteristic of concrete, thus several researchers pointed out the tensile strength of material as a key parameter to predict the shear failure of RC beams without stirrups. Many codes connect the tensile strength with the shear strength though the equations are usually derived in a semi-empirically way.

Over the past decades, there has been a wide interest in using Steel Fiber Reinforced Concrete (SFRC) to improve the performance of RC beams, especially in tension and shear. The presence of fibers in the mixture of concrete helps to delay crack initiation and to control the post-cracking behavior in tension of the new material. Then, these mechanical properties of the new composite material, enhanced by steel fibers, lead to largely improve the shear strength of RC beams.

Many experimental results reported in the literature confirm these beneficial effects in terms of shear strength due to steel fibers [1], as also shown in the database recently constructed by Lantsoght [2]. In most of the available studies, the effect of the parameters that influence the shear resistance of SFRC beams, as well as the fiber amount and characteristics, have been largely investigated in small and medium scale specimens.

In the last ten years, the attention has moved to study the size-effect in SFRC beams, and several investigations with large-scale specimens have been carried out by different researchers [3–6]. They provided a series of interesting information in investigating the shear behavior of SFRC beams. Zarrinpour and Chao [6] conducted an experimental campaign to identify the shear-enhancement and failure mechanisms behind the ultimate shear strength of SFRC full-scale slender beams by using the full field-deformation-measuring capability of Digital Image Correlation (DIC) technology. By the analysis of their experimental results, they observed that the greater shear strength in SFRC beams was due to the fiber bridging effect, which delays the propagation of the cracks into the compression zone, whose shear strength is enhanced by the compressive stresses induced by the higher load. Furthermore, according to their interpretation, the slow progression of the cracks keeps the compression zone depth

large, thereby enabling it to contribute to a higher shear resistance [6]. This is in partial contrast with most of the existing models for predicting the shear resistance of SFRC beams, which are usually based on existing models for RC beams by introducing only the shear contribution of SFRC residual tensile stress across the shear critical crack. Moreover, only the simple sum of each single shear contributions (concrete matrix in tension and steel fibers across the crack) is usually proposed in the updated equations for SFRC specimens [3, 7, 8], especially in code formulations [9, 10].

More recently, some deep theoretical models, able to take into account the interaction between the various resistant mechanisms (concrete matrix in tension, in compression, and bridging stress along the critical crack), originally proposed to predict the shear strength of conventional RC beams, have been updated to the case of SFRC specimens through the addition of the fiber component to the shear resistance of RC beams. These kind of formulations are based on sophisticated theory, as the Modified Compression Field Theory (MCFT) [11–13], the plasticity equations for concrete [14–17], and the critical shear band concept [18].

In this context, Marí et al. [19] developed a conceptual model for the prediction of the shear-flexural strength of slender RC beams with and without transverse reinforcement, namely Multi-Action Shear Model (MASM). The model incorporates the contributions to shear strength provided by the un-cracked compressed concrete chord (V_c), by the diagonally cracked web (V_{cw}), including the residual stresses transferred along the crack's length by fracture energy of concrete, by the stirrups crossing the critical shear crack (V_s), if they are present and, in that case, by the longitudinal reinforcement (V_l). After the development of the first branch of the critical shear crack, failure is considered to occur when the stresses at any point of the concrete compression chord reach the assumed biaxial stress failure envelope, as proposed by Kupfer [20].

The MASM, and a subsequent simplification of it, called the Compression Chord Capacity Model (CCCM) [21], have proved to adequately predict the shear strength of reinforced and Prestressed Concrete (PC) beams, with and without stirrups, having rectangular, T and I cross section shapes [22] [23], being slender or non-slender beams [24], and even reinforced with Fibre Reinforced Polymers (FRP) [25]. In this paper, the MASM has been extended to allow capturing the shear strength of SFRC beams, which is briefly summarized in the following sections. Note that a preliminary adaptation of the

CCCM for its use for SFRC beams had been already presented [21], but in this case the model was just adapted by introducing a shear contribution of SFRC residual tensile stress across the shear critical crack. The adaptation proposed in this paper of the MASM clearly improves the previous works thanks to its strong mechanical background.

The ability of SFRC to maintain a residual tensile stress after cracking has been the key to model 1) the reduction of damage in the compression chord, by suitably estimating the neutral axis depth, 2) the bridging effect of the fibres across the critical shear crack and, 3) the enhancement of the dowel action in the longitudinal reinforcement.

In order to adequately modelling the effects of fibres on the residual tensile strength of the hardened composite, the residual tensile stress is evaluated in a simple manner by using the drop-down constant tensile stress model as proposed by Lim et al. [22] and adapted by Spinella et al. [15]. This simplified approach allows to avoid some cumbersome calculations, thus resulting in a more suitable method for design purposes.

2. CHARACTERIZATION OF THE RESIDUAL TENSILE STRESS OF SFRC

2.1. Post-cracking tensile behavior of SFRC

The addition of steel fibers in the range up to 1.5% by volume fraction to concrete (V_f), as normally used in SFRC structural beams, provides residual tensile stresses at large displacements without significantly improving the tensile strength. This constitutive behavior depends on the mechanical properties of cement matrix, the amount (V_f) and the characteristics of fibers added, like length (L_f), diameter (d_f) and steel yield strength (σ_{sy}).

The parameters characterizing the SFRC in tension are usually obtained by means of flexural tests, to avoid large scatter typical of pure axial tensile test. However, the obtained stress values cannot be directly considered the tensile stress transferred across the critical shear crack, and they need to be transformed by theoretical or empirical equations [23–25].

For example, the Model Code 2010 (MC2010) [25] suggests two simplified stress-crack opening constitutive laws (plastic rigid or linear post-cracking behavior) depending by f_{Fts} and f_{Ftu} , which are the tensile stress levels representative of Service (SLS) and Ultimate Limit State (ULS), respectively.

According to EN 14651, the post-cracking tensile stress can be determined as a function of the residual flexural tensile strength corresponding to Crack Mouth Opening Displacement (CMOD) = 0.5 and 2.5 mm (f_{R1} and f_{R3} , respectively) measured on a three-points bending test [25].

Similarly, the ASTM 1609 [26] proposes three-point bending tests, without notch mouth, to estimate the flexural tensile strength of SFRC. Dinh et al. [3] suggested a procedure to determine the average residual tensile stress from ASTM 1609 tests, which is limited to ASTM 1609 beams exhibiting a single flexural crack (i.e., deflection softening behavior), by modelling the specimen as two rigid blocks rotating an angle with respect to each other.

Some analytical models to reproduce the tensile stress constitutive behavior are also available in the literature.

Voo and Foster [27] proposed the Variable Engagement Model (VEM), which assumes that slip between fibers and concrete matrix occurs before the full bond stress is developed, and the fibers fracture themselves before being pulled out across a crack. Then, some slip between the matrix and the fibers must occur before the anchorage is engaged. The VEM has been adopted in previous works to estimate the shear strength of slender SFRC beams by plasticity approach [15], and also to reproduce the overall response-behavior of SFRC beams by Non-Linear Finite Element Analysis (NLFEA) [13].

Lee et al. [28, 29] proposed the Diverse Embedment Model (DEM), which is derived with consideration given to all possible fiber orientations and embedment lengths and is influenced by the member's finite dimensions. The model considers the frictional bond behavior, mechanical anchorage effects of fibers and the influence of finite member dimensions.

These analytical models are able to provide detailed information about the constitutive behavior of SFRC under uniaxial tensile stress, but require itemized information about matrix, fibers and, mostly, their interaction behavior. Moreover, to use one of these analytical constitutive laws, the knowledge of the crack width growth is needed.

In fact, in SFRC beams before the shear failure, the tensile force transferred across the critical diagonal crack through fiber tension depends on the shear crack width. In order to estimate the magnitude of this force, the use of a constant tension stress would be a handy tool, rather than calculating its overall distribution.

Therefore, in this work, a simple analytical equation for the estimation of the average residual tensile stress (σ_{tu}), constant along the shear crack, is used [15, 22].

2.2. Estimation of the average residual tensile stress of SFRC

As known, the post-cracking tensile stress of SFRC is, in general, proportional to the term $V_f (L_f/d_f)$, being L_f and d_f the fiber length and diameter, respectively, and V_f the volumetric fraction of fiber. Accounting for bond effects by means of a bond factor β_τ which depends on the type of fiber (i.e., straight, hooked, crimped, etc.), the fiber factor F_τ is obtained as:

$$F_\tau = \beta_\tau V_f \frac{L_f}{d_f} \quad (1)$$

Usually, the β_τ value has an empirical origin. In this work, though, the fiber bond factor has a physical meaning: $\beta_\tau = \tau/f_{ct}$ is the ratio between the mean fiber-matrix shear stress and the strength under direct tension of the concrete matrix [15]. The shear stress τ is calculated as function of concrete matrix tensile strength (f_{ct}), shape of fiber (hooked, plain or other), and type of matrix (concrete or mortar) as suggested by Voo and Foster [27]. In the typical case of concrete matrix, τ adopts the value $2.5 f_{ct}$ for hooked fibers and $1.2 f_{ct}$ for straight fibers [27].

The formulation of the average residual tensile stress for fibrous concrete in tension is derived by the constitutive law proposed by Lim et al. [22]. In that model, the post-cracking behavior follows a horizontal branch, and the ultimate tensile strength (σ_{tu}) can be estimated as:

$$\sigma_{tu} = 2\eta_0\eta_l V_f \frac{L_f}{d_f} \quad (2)$$

Introducing the fiber factor of Eq. (1) in Eq. (2), σ_{tu} becomes:

$$\frac{\sigma_{tu}}{f_{ct}} = 2\eta_0\eta_l F_\tau \quad (3)$$

where $\eta_0 = 0.405$ is the fiber orientation factor, η_l is the length efficiency factor for fiber, which depends on the critical length $l_c = (\sigma_{sy} d_f) / (2 \tau_f)$. If the fiber length, L_f , is less or equal to l_c then $\eta_l = 0.5$, else $\eta_l = 1 - l_c / (2 L_f)$.

It should be observed that the maximum allowable value of Eq. (3) is 1, then the fiber factor cannot be over than $(2\eta_0\eta_l)^{-1}$ [16]. Furthermore, the residual tensile strength of fibrous concrete depends on the

tensile strength of plain concrete (f_{ct}), which, according to Spinella [16] should be evaluated as $0.45 f_{cm}^{0.4}$ (in MPa), and being f_{cm} the average value of the plain concrete compressive strength.

One of the goals of this research work is to provide a mechanically-based model for shear strength of SFRC without stirrups that can be used in daily engineering design as an alternative to empirically-based formulations provided by current codes. Then, for the sake of consistency with European codes, the mean value of the tensile strength of plain concrete given by the MC2010 [30] is used in this work ($f_{ct} = 0.3 f_{ck}^{2/3}$ for $f_{ck} \leq 50$ MPa and $f_{ct} = 2.12 \ln[1 + 0.1(f_{ck} + 8 \text{ MPa})]$ for $f_{ck} > 50$ MPa). Therefore, the value of the average residual tensile stress given by Eq. (3) has to be adapted to consider a different formulation of f_{ct} , with respect to the one used in [16] and originally proposed by Bentz [30]. This is done by multiplying factor $\beta_\tau = \tau_f/f_{ct}$ by the ratio between the tensile strength proposed by Bentz and by MC2010, which is, approximately, 0.58 (for $f_{ck} > 10$ MPa). Then, to use the plain concrete strength formulation of MC2010 in the proposed shear model, the fiber bond factor value is herein modified and calculated as $\beta_\tau = \tau_f/f_{ct} \cong 0.58 \times 2.5 = 1.45$, and $= 0.58 \times 1.2 \cong 0.70$ for hooked and straight fibers, respectively.

In Figure 1, the numerical non-dimensional tensile stress-crack opening curves, calculated by the Variable Engagement Model (VEM), the MC2010, and the Eq. (3) are plotted for specimen NSC2-FRC1 [1]. A volumetric percentage of 0.38% of steel fibers was used in the concrete mixture, and they were hooked with an aspect ratio (L_f/d_f) equal to 1. As reported by the authors of the test, the tensile stress levels at SLS and ULS, experimentally obtained by bending test, were $f_{Fts} = 1.17$ MPa and $f_{Ftu} = 0.84$ MPa, respectively.

Figure 1. Numerical non-dimensional tensile stress-crack opening curves obtained for specimen NSC2-FRC1 [1].

3. GENERAL OVERVIEW OF THE MULTI-ACTION SHEAR MODEL (MASM)

It is generally agreed that as the load increases in a RC member failing in shear, damage concentrates around a critical shear crack, originally a flexural crack. A first branch of this crack develops inclined across the web and until the neighborhood of the flexural neutral axis (Fig. 2). Under incremental loading, a second branch of the crack develops inside the un-cracked concrete chord, which will connect the first branch of the crack and the load application point, producing failure.

Figure 2. Critical shear crack evolution [19].

The main assumption of the MASM, backed by the empirical observation of many authors [31–33] is that when the second branch of the critical crack develops, the load does not significantly increase, as softening of the concrete in the compression zone initiates. During the crack propagation inside the flexural compression zone, redistribution of internal forces may occur, affecting the relative importance of the different shear resisting actions.

To associate the initiation of the shear failure to the propagation of the second branch of the critical crack results in a significant simplification of the problem, since it allows formulating a failure criterion expressed in terms of concrete stresses in the compression chord (Kupfer's biaxial failure envelope [20]), (Fig. 3). This failure criterion depends on the compression and tensile strength of concrete, which have less scatter than other parameters associated to kinematical failure criteria.

Figure 3. Kupfer's failure envelope [19].

The MASM [19] proposes explicit equations for the different shear transfer actions considering that the tip of the shear critical crack has propagated until the flexural neutral axis. The shear strength, V_u in Eq. (4), is the sum of the shear resisted by the transverse reinforcement, if it exists, V_s , and by the shear resisted in the un-cracked compression chord, V_c , the shear transferred across web cracks, V_w , and the dowel action in the longitudinal reinforcement, V_l (Fig. 4).

Figure 4. Shear strength components in a RC beam according to MASM.

Table 1 shows the dimensionless equations (shear force divided by $f_{ct} b d$) governing each contributing component. Detailed information related to these equations, factors, and parameters can be found in Cladera et al. [34].

$$V_u = (V_c + V_{cw} + V_l) + V_s = f_{ct} b d [(v_c + v_{cw} + v_l) + v_s] \quad (4)$$

In order to derive V_c , shear, normal and principal stresses distributions inside the un-cracked concrete chord are adopted in the model. Since the shear failure takes place for moderate stresses in the critical section (placed where the critical crack reaches the flexural neutral axis), a linear distribution of compression stresses is assumed (Fig. 5). Furthermore, the shear stresses along the critical shear crack concentrate near the crack tip, where crack opening is lower, and decrease as the load increases. For this reason, it is assumed that shear stresses are fully concentrated in the un-cracked concrete chord, having a parabolic distribution with zero values at its top and bottom fibers.

Figure 5. Stresses distributions in the compression chord, according to MASM.

Furthermore, based on experimental observations made by the authors and summarized by Marí et al. [19] and Cladera et al. [35], the horizontal projection of the first branch of the flexural-shear critical crack is considered to be equal to $0.85d$, where d is the effective depth of the beam.

Based on the above assumptions, V_c is obtained by relating forces and stresses in the compression chord when failure takes place, according to Kupfer's criterion, and setting the equilibrium between forces, moments and the shear resisting components in the portion of the beam placed above the first branch of the critical crack (see Figure 4). The solution of the resulting equation for V_c can be very well approached by Eq. (5) in Table 1.

The shear resisted in the web, V_{cw} , is obtained by integrating the residual tensile stresses transferred across the critical crack, derived by fracture mechanics in function of the strain in the longitudinal reinforcement, among other factors. Given its reduced magnitude compared to V_c , a value of V_{cw} associated to an average strain at the reinforcement at shear failure (around $\varepsilon_{sb} = 0.0009$) is adopted, see Eq. (6) in Table 1.

The shear transferred by the longitudinal reinforcement, V_l , is neglected in beams without stirrups, due to the low capacity of the plain concrete cover to resist the push-off force due to shear. In case of the presence of stirrups, the longitudinal bars are considered fixed on them and subjected to a differential

imposed vertical movement due to shear deformations. Then, V_l depends on the number and diameter of the longitudinal bars, stirrups spacing and other factors, but, again, its value is very reduced in comparison with V_c . Therefore, a simple formula [Eq. (7b)] which depends on the longitudinal reinforcement ratio, is adopted.

The shear resisted due to the shear reinforcement, V_s , is given by Eq. (8), derived considering the vertical force of the stirrups intersected by the first branch of the critical crack.

Table 1. Summary of dimensionless shear contributing components for slender beams and factors used (MASM)

Shear resisting action	Dimensionless equations
Compression chord	$v_c = \xi \left[(0.88 + 0.7v_s) \frac{x}{d} + 0.02 \right]$ (5)
Cracked concrete web	$v_w = 167 \frac{f_{ctm}}{E_c} \left(1 + \frac{2G_f E_c}{f_{ctm}^2 d} \right)$ (6)
Longitudinal reinforcement	$if\ v_s > 0 \rightarrow v_l = 0.23 \frac{n \cdot \rho_l}{1-x/d}$ (7a)
	$if\ v_s = 0 \rightarrow v_l = 0$ (7b)
Shear reinforcement	$v_s = (d - x) \cot \theta \frac{A_{sw} f_{yw}}{s f_{ctm} b \cdot d}$ (8)
Struts crushing	$V_{max} = \alpha_{cw} b z v_1 f_{cm} \frac{\cot \theta}{1 + \cot^2 \theta} \approx 0.225 f_{cm} b \cdot d$ (9)
Factors	Expressions
Size and slenderness effects	$\xi = \frac{2}{\sqrt{1 + \frac{d_0}{200}}} \left(\frac{d}{a} \right)^{0.2} \leq 0.45$ d and d_0 in mm (10)
Neutral axis depth	$\frac{x}{d} = \alpha \rho \left(-1 + \sqrt{1 + \frac{2}{n\rho}} \right) \approx 0.75(\alpha \rho)^{1/3}$ (11)
Crack inclination	$\cot \theta = \frac{0.85d}{d-x} = \frac{0.85}{1-\frac{x}{d}} \leq 2.5$ (12)

Struts crushing [Eq. (9)] may occur in cases when a large contribution of V_s exists, so this check is necessary. For this cases, the formulation of current EC-2 [36], derived from plasticity models was adopted, but assuming that the angle of the compression strut is equal to the angle of the critical crack. A combined size and slenderness factor is defined by Eq. (10) and includes the size effect factor proposed by the ACI Committee 44, as proposed by Bazant et al. [37].

The flexural relative neutral axis depth x/d to be used in the MASM equations is obtained assuming zero tensile strength of concrete and linear behavior of concrete in compression, and is given by Eq. (11). It

depends mainly on the homogenized longitudinal reinforcement ratio $n\rho$ and, therefore, the angle of inclination of the crack, given by Eq. (12), depends on x/d . As the longitudinal ratio increases, x/d increases and also $\cot\theta$, thus increasing the contribution of the shear reinforcement V_s and also the contribution of the un-cracked concrete chord, V_c .

For the determination of f_{ct} using the EC-2 equations, the characteristic concrete compressive strength f_{ck} shall not be taken greater than 60 MPa. This limitation is provided due to the larger observed variability in shear strength of High Strength Concrete (HSC) members.

In the case of deep beams, in which the shear span – to effective depth ratio (a/d) is less than 2.5, the shear strength is enhanced due to arch effect. This is taken into account in the MASM by incorporating the differential aspects of the observed structural behavior of non-slender beams with respect to slender beams, on its formulation [38]. As a result, the value of the neutral axis depth x/d , and the shear component V_c have been modified, becoming functions of a/d , as follows:

$$\frac{x_1}{d} = \frac{x}{d} + \left(1 - \frac{x}{d}\right) \left(1 - 0.4 \frac{a}{d}\right)^2 \leq 1 \quad (12)$$

$$V_{c1} = V_c \left[1 + \left(2.5 - \frac{a}{d}\right)^2\right] \quad (13)$$

4. PROPOSED MODEL FOR THE SHEAR STRENGTH OF SFRC BEAMS WITHOUT STIRRUPS

4.1. Influence of steel fibers on the considered shear resisting mechanisms

The capacity of SFRC to resist a residual tensile stresses affects the three shear resisting mechanisms adopted by the MASM for beams without stirrups.

4.1.1. Shear resisted along the critical shear crack.

The bridging effect of the steel fibers allows transferring tensile stresses across the critical shear crack, providing a resultant force T_f , whose component in the direction normal to the beam axis is the shear force V_{cf} . Force T_f is considered normal to the critical shear crack since, due to the increment of shear strength provided by the steel fibers, the crack width at ULS is large enough to considerably reduce the aggregate interlock, as experimentally observed by Zarrinpour and Chao [6]. In this work, according to section 2.1, a constant tensile stress σ_{tu} extended along the whole first branch of the critical crack is

used, which is considered representative of the residual tensile strength at failure (Fig. 6). Such value is determined by means of Eq. (3), without the need of tensile characterization tests, what is a great practical advantage. Then, the non-dimensional shear strength v_{cf} can be obtained by Eq (14):

$$v_{cf} = \frac{V_{cf}}{f_{ct} b d} = \frac{0.85d \sigma_{tu} b}{f_{ct} b d} = 0.85 \frac{\sigma_{tu}}{f_{ct}} \quad (14)$$

Figure 6. Contribution of steel fibers to the shear resisted in the web and by dowel action.

4.1.2. Dowel action in the longitudinal reinforcement

In beams without stirrups, the vertical displacement of the longitudinal reinforcement outwards the beam is constrained only by the concrete in contact to the longitudinal reinforcement. In conventional RC beams, the resistance offered by the plain concrete to prevent the outwards displacement of the longitudinal reinforcement is small. Furthermore, once the tensile strength is reached at any point, a quick propagation of a horizontal crack takes place driving to a brittle failure.

However, in the case of SFRC, the ductile post-cracking behavior allows developing residual tensile stresses normal to the reinforcing bar, in a certain length, providing a non-negligible shear resistance. In this work, the constant tensile residual strength σ_{tu} is considered to develop in a length l_{cr} equal to the distance from the critical shear crack to the support (see Fig 7).

Figure 7. Dowel action in the longitudinal reinforcement of SFRC beam

According to the MASM, the critical shear crack initiates where, for the load producing the ultimate shear force V_u , the bending moment law reaches the cracking moment, M_{cr} . Therefore, the contribution of SFRC to the non-dimensional shear resisted by dowel action in the longitudinal reinforcement can be determined by Eq. 15:

$$v_l = \frac{V_l}{f_{ct} b d} = \frac{l_{cr} \sigma_{tu} b}{f_{ct} b d} = \frac{\sigma_{tu} l_{cr}}{f_{ct} d} = \frac{\sigma_f M_{cr}}{f_{ct} V_u d} \cong \frac{0.2 \sigma_{tu}}{v_u f_{ct}} \quad (15)$$

Where a value $d = 0.9h$ has been assumed, so the cracking moment results $M_{cr} \cong 0.2 f_{ct} b \cdot d^2$, and v_u is the non-dimensional shear strength of the beam. Observe that v_l depends on the total shear resisted v_u , which is not known “a priori”, so an iterative procedure should be necessary to calculate it.

4.1.3. Shear resisted by the un-cracked compression concrete chord.

In addition to the direct contribution of fibers to the shear resisted in the web V_{cf} , and by dowel action V_l , the horizontal component of force $T_f (= V_{cf} \tan \theta)$ increases the compression force in the un-cracked chord, due to equilibrium of forces and moments, thus increasing the neutral axis depth, and, therefore, the shear capacity of the un-cracked concrete chord.

In order to calculate the neutral axis depth in SFRC, $x_f > x$, a flexural analysis of a section is performed, assuming plane section deformation and a constant value of the residual tensile stress of SFRC in the cracked zone (Fig. 6). By setting the equilibrium of forces and moments a four degree polynomial equation on x_f/d is derived and solved numerically. Due to the presence of fibers, equilibrium equations

are coupled, so x_f/d depends on the moment M at the section placed at abscissa ξ_{crit} (Fig. 6). Figure 8 shows the solution for x_f/d , for the case of $M = 2 M_{cr}$. The theoretical solution can be very well approached (with an error less than 3%) by the following equation:

$$\frac{x_f}{d} = \frac{x}{d} + 0.42 \frac{M_{cr}}{M} (1 - 1.66 n \rho) \frac{\sigma_{tu}}{f_{ct}} \quad (16)$$

Figure 8. Non-dimensional neutral axis depth for SFRC beam, in function of σ_{tu}/f_{ct} and ρ , for

$$M = 2 M_{cr}$$

Once the neutral axis depth is known, the shear resisted by the compression chord is obtained by integration of the shear stresses. For this purpose, the maximum shear resisted at the critical point and the distribution of the shear stresses inside the compression chord must be determined.

According to the Mohr's circle for stresses, the principal tensile (σ_1) and compressive (σ_2) stresses produced by the state of normal (σ_x) and shear (τ) stresses at any point of the concrete compression chord are:

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} \quad (17)$$

Based on experimental observations [39], it is assumed that shear strength in RC beams without stirrups is associated to the tensile shear failure at a critical shear section close to the support. Then, solving

Eq. (17), the shear stress, τ , at any point of the compression chord, can be estimated in function of the normal and principal tensile stresses as:

$$\tau = \sigma_1 \sqrt{1 - \frac{\sigma_x}{\sigma_1}} \quad (18)$$

The critical point where failure of the compression chord is considered to occur, is assumed to be placed at the critical section, defined as the section where the critical shear crack reaches the neutral axis. The critical point belongs to such section and is assumed to be placed at a distance $y = 0.425x$ from the neutral axis. Such position, which depends on the normal and shear stresses distributions, was determined by Mari et al [19] as the point where Kupfer's failure envelop is first reached.

Previous research works have studied the effects of adding steel fibers on the multiaxial stress strength of SFRC [40]. Even though the steel fibers increment the strength of SFRC in a biaxial compression stress state, not a relevant influence of the strength branch on the compression-tension branch has been found in the literature. Since failure in the compression chord is mainly produced in this branch, Kupfer's biaxial failure envelope will be used also for SFRC members when predicting the shear failure according to the MASM.

The compression-tension branch of Kupfer's failure envelope is considered a straight line when the principal compressive stress σ_2 is less than 80% of the uniaxial compression concrete strength (see Fig. 6), given by Eq. (19):

$$\frac{\sigma_1}{f_{ct}} + 0.8 \frac{\sigma_2}{f_{cc}} = 1 \quad (19)$$

Then, the principal tensile stress of the critical point in the compression chord can be obtained as:

$$\sigma_1 = \left(1 - 0.8 \frac{\sigma_2}{f_{cc}}\right) f_{ct} = R_t f_{ct} \quad (20)$$

Where R_t is the tensile strength reduction factor due to the presence of the compressive principal stress, σ_2 . By substituting Eq. (20) into Eq. (18), the maximum shear stress τ_u that can be resisted is:

$$\tau_u = R_t f_{ct} \sqrt{1 - \frac{\sigma_x}{R_t f_{ct}}} \quad (21)$$

Once the value of τ_u and its position along the compression chord are known, it is possible to define the equation of the parabolic distribution of the shear stresses inside the un-cracked chord. Then, the shear

resisted by the un-cracked concrete chord is obtained by integrating the shear stresses inside it, resulting the following expression in non-dimensional terms:

$$v_c = \frac{V_c}{f_{ct} b d} = \frac{\int_0^x \tau(y) b dy}{f_{ct} b d} \cong 0.682 \frac{x}{d} R_t \sqrt{1 - \frac{\sigma_x}{R_t f_{ct}}} \quad (22)$$

where the integration factor 0.682 corresponds to the above defined position of the critical point (if the point was placed at $y = 0.5x$, the integration factor had been 2/3).

4.2. Modification of the MASM equations for SFRC beams

The shear resisted by the compression chord, V_c [Eq (22)] is expressed in terms of stresses, while for design purposes it should be expressed in terms of internal forces. For this purpose, the equilibrium of forces and moments of the portion of beam placed above the first branch of the critical shear crack are set (Fig. 6), thus allowing to express the internal forces in function of the stress resultants, as follows:

$$\sum F_h = 0 \rightarrow C = T_l + V_{cf} \tan \theta \quad (23)$$

$$\sum F_v = 0 \rightarrow V = V_c + V_{cf} + V_l \quad (24)$$

$$\sum M_A = 0 \rightarrow C(d - \frac{x_f}{3}) = M_{cr} + V_c 0.85d + V_{cf} 0.425d(1 + \tan^2 \theta) \quad (25)$$

In addition, the normal stress at the critical point depth, $\sigma_x = \sigma_{c,crit}$, is expressed in function of the compression force C in the un-cracked chord:

$$\frac{1}{2} \sigma_{c,max} b x_f = C \rightarrow \sigma_{c,crit} = 0.425 \sigma_{c,max} = \frac{0.85 C}{b x_f} \quad (26)$$

Being $\sigma_{c,max}$ the compression stress of concrete top fiber.

Then, and substituting Eq. (26) into Eq. (22), the following expression for the non-dimensional shear resisted by the un-cracked concrete chord is obtained:

$$v_c = 0.682 \frac{x_f}{d} R_t \sqrt{1 - \frac{0.85 [0.2 + v_c + 0.5 v_f (1 + \tan^2 \theta)]}{R_t f_{ct} \frac{x_f}{d} (1 - \frac{x_f}{3d})}} \quad (27)$$

Where a non-dimensional cracking moment $\mu_{cr} = 0.2$ and a flexural lever arm $z = d - x_f/3$ have been adopted. Eq. (27) is a second order polynomial equation in v_c , however it needs to be solved iteratively, since R_t is a function of the principal stress σ_2 [Eq. (19)] that are not known a priori. The influence of

steel fibers on v_c is introduced through the increment of the neutral axis depth [Eq. (16)], and through the equilibrium of moments [Eqs. (25) and (27)].

Once v_c is known, the total shear resisted by a SFRC beam is obtained by adding the contributions of the different shear strength mechanisms, $v_u = v_c + v_{cf} + v_l$ [Eq. (24)].

In Figure 10 the total non-dimensional shear resisted is plotted in function of the neutral axis depth x/d obtained by means of Eq. (10) - without considering the effect of fibers - for different values of σ_{tu}/f_{ct} .

It can be observed the linear dependency between the neutral axis depth, so the theoretical solution is very closely approached by the following simplified equation:

$$v_u = v_c + v_{cf} + v_l = \left(0.84 - 0.10 \frac{\sigma_{tu}}{f_{ct}}\right) \frac{x}{d} + 0.08 + 1.10 \frac{\sigma_{tu}}{f_{ct}} \quad (25)$$

Figure 9. Total non-dimensional shear resisted by a SFRC beam in function of x/d and σ_{tu}/f_{ct}

When predicting shear tests results, the size effect in the compression chord ξ , given by Eq. (9), must be included and the mean tensile strength of concrete f_{ctm} should be used. Then, the absolute value of the shear resisted is given by:

$$V_u = \left[\xi \left(0.84 - 0.10 \frac{\sigma_{tu}}{f_{ctm}} \right) \frac{x}{d} + 0.08 + 1.10 \frac{\sigma_{tu}}{f_{ctm}} \right] f_{ctm} b d \quad (26)$$

Note that, for the sake of simplicity, Eq. (26) depends on x/d , the relative neutral axis depth of an identical beams without steel fibers, and not on x_f/d . Once V_u and V_{cf} are known through Eqs. (26) and

(14), respectively, V_l can be determined from Eq. (15) and then, $V_c = V_u - (V_{cf} + V_l)$. Thus, the influence of different parameters on the contribution of each shear resisting mechanism can be studied. As a matter of examples, Figures 10 and 11 show the influence of ρ and σ_{tu}/f_{ctm} on the relative contribution of each component, respectively.

Figure 10. Relative shear strength contribution of each component in function of ρ for $\sigma_{tu}/f_{ctm} = 0.60$

It can be seen in Figure 10 that, for the same amount of steel fibers, as the longitudinal reinforcement increases, the relative contribution of V_c increases, while the relative contribution of the web and the dowel action slightly decrease.

Figure 11 shows that, for the same amount of longitudinal reinforcement, the higher tensile residual strength relative to concrete, σ_{tu}/f_{ct} , the higher relative contribution of the web that, for a value $\sigma_{tu}/f_{ct} = 0.40$, approximately, becomes higher than V_c . The dowel action reaches a maximum for $\sigma_{tu}/f_{ct} \cong 0.25$ and after this value slightly decreases. This is due to the fact that as σ_{tu}/f_{ct} increases, so does V_u , and length l_{cr} decreases (Fig. 9). Therefore, the favorable effect of an increment of SFRC residual strength is balanced by the decrement of l_{cr} .

Figure 11. Relative shear strength contribution of each component in function of σ_{tu}/f_{ctm} for

$$\rho = 0.125\%$$

4.3. Design Equations

Eq. (26) is derived based on mean values and can be used for predicting the results of shear tests. However, for design purposes, the design values of the concrete and steel strengths (characteristic values divided by materials partial factor coefficients), should be used to account for the scatter in the materials production, dimensions of the members, construction procedures, etc., and to fulfil the safety level required by the design codes. Thus the following design equation is proposed:

$$V_u = \left[\zeta \left(0.84 - 0.10 \frac{\sigma_{tu}}{f_{ctm}} \right) \frac{x}{d} + 0.08 + 1.10 \frac{\sigma_{tu}}{f_{ctm}} \right] 0.3 f_{cd}^{2/3} b d \quad (27)$$

Where the tensile strength is expressed in terms of the compression design strength of the concrete.

In order to verify the level of safety of the formulation or to determine the suitable partial coefficients which allow fulfilling the reliability index required, a reliability analysis should be performed, that is not the objective of this paper.

5. EXPERIMENTAL VERIFICATION

In literature, several experimental studies about the shear behavior of SFRC beams without stirrups are available. However, many experiments do not provide a detailed description of all input parameters, and, as expected, they are not cataloged according to a common scheme.

Recently, Lantsoght [2] has compiled a database of 488 experiments reported in the literature, and the full spreadsheet is available as supplementary file in .xlsx format available in the public domain [41]. The structural parameters covered by the database vary over a wide range, however the mechanical characterization of SFRC was not reported. In fact, when experimental results about post-cracking tensile stress of SFRC are available, they are generally obtained by using different experimental protocols, then it makes difficult to homogenize the data [2].

Contestually, Cuenca et al. [23] presented a material-performance-based shear database for FRC elements. The database is composed by 93 FRC specimens with a broad range of selected parameters, and the post-cracking resistance ($f_{R,1}$ and $f_{R,3}$) is also given according to EN 14651 standard.

An analysis of the predictions provided by the proposal and also by different formulations available in literature for the shear strength of SFRC beams has been carried out. Both the database above described have been used.

In addition to the proposal of this work, the formulation suggested by Dinh et al. [3] and two different analytical models recently included in MC2010 [25] have been used in the predictions. The choice of these formulations between the several ones available in literature is due to the fact that the model of Dinh et al. [3] is similar to the proposed one, and, as for the MC2010 models, the fiber contribution is provided by the experimental result of a bending test. Therefore, for the database of Cuenca et al. [23], the formulations are used both using the reported value of f_{Ftu} and also using the calculated value of σ_{fu} in this work [Eq. (3)].

4.2. *Dinh et al.* [3]

This model is conceptually similar to the proposed one, indeed a SFRC beam without stirrups is assumed to fail along an idealized inclined crack line and over the compressed concrete. The inclined crack is assumed to extend from the reinforcement level up to its tip, at the neutral axis level. According to this model, the shear strength of SFRC beams can be calculated as sum of shear force across the compression region and the vertical component of the diagonal tension resistance provided by the fibers: $V_u = V_c + V_{cf}$. The dowel action contribute is neglected. The contribution of the compression region is evaluated as a function of the yielding force of longitudinal reinforcement ($= 0.13 A_s f_y$), while the

contribution of fiber tension to the shear strength can be expressed as $V_{cf} = b (d - x) \sigma_{tu}$, extrapolating from ASTM 1609 four-point bending test the average tensile stress σ_{tu} and assuming an inclination of the shear crack equal to 45° (that is $\cot\theta = 1$) [3].

4.3. Model Code 2010 (MC1)

The MC2010 recommended that the shear resistance of SFRC slender beams be calculated by the sum of the contributions of stirrups, if present, and concrete matrix-fibers mixture. The fiber component is treated as additional longitudinal reinforcement smeared over the cross-section of the beam and is adapted to the context of the EC-2 [36] model for shear in RC beams. For non prestressed elements and without transverse reinforcement, the shear strength is:

$$V_u = 0.18k bd \left[100\rho f_{cm} \left(1 + 7.5 \frac{f_{Ftu}}{f_{ct}} \right) \right]^{1/3} \quad (28)$$

where k is the size effect parameter $[= 1 + (200/d)^{0.5} \leq 2]$. In this paper this model is referred to as the MC1 model.

4.4. Model Code 2010 (MC2)

An alternative formulation is suggested in the MC2010. It has been proposed by Foster [11] and it is based on the simplified MCFT [39]. The contribution of concrete matrix-fibers mixture is calculated as follows:

$$V_u = (k_v \sqrt{f_{cm}} + 0.8 f_{Ftu} \cot\theta) b 0.9d \quad (29)$$

being $k_v = 0.4/(1 + 1500\varepsilon_x) \times 1300/(1000 + k_{dg} 0.9d)$ the strain-aggregate size parameter, derived from the MCFT and determines the capacity of the cross-section to resist aggregate interlock, with $k_{dg} = 32/(16 + d_g) \leq 0.75$; and the crack angle is $\theta = 29^\circ + 7000\varepsilon_x$.

Both k_v and θ are estimated as function of the critical crack width ($w_u = 0.2 + 1000\varepsilon_x \geq 0.125$ mm) and of the longitudinal strain at centroid level (ε_x), then they must be solved simultaneously by an iterative procedure. In this study the model is notated as MC2. For further information on the derivation of this model, the reader is referred to [11, 25].

4.5. Comparison

Figure 12 presents the correlation between experimental (V_{Exp}) and the predicted (V_{Pre}) shear strength using the four considered models and the specimens in the two different databases. A tolerance of 15% has been represented in the graphs. It can be seen that the proposed rational method presents a very good correlation with tests. Table 2 summarizes the statistical results for all models considered.

For Lantsoht [2] database, the fiber contribution has been calculated by using the σ_{tu} in Eq. (3) for all formulations (f_{Ftu} is not available in this database). The most accurate model is the proposal presented in this paper [Mean = $V_{Exp}/V_{Pre} = 1.15$, CoV = 0.22] followed by the MC2. The less accurate method is Dinh et al. [3]. The trend is confirmed both in terms of mean and Coefficient of Variation (CoV). The CoV is a measure of precision of the model, and the proposal provides 0.22 for the full database considered. Lantsoht [2] provided also a comparison between some code predictions. The f_{Ftu} value in MC1 was estimated as suggested by German guideline; the limitation of $\rho < 0.02$ was neglected; and the reduction factor $\beta = a/2d$ for $0.5d \leq a \leq 2d$ was used. With these assumptions, the MC1 gives a mean of 1.24 and a CoV of 0.29 [2].

For Cuenca et al. [23] database, it should be noticed that the prestressed beams in the database are not considered in the following comparison. The fiber contribution has been calculated by using both the σ_{tu} in Eq. (3) and the f_{Ftu} value reported in this database. By introducing the average tensile strength value for SFRC calculated by Eq. (3) in all formulations, the proposal is the model which provides the best performance also for this database [$V_{Exp}/V_{Pre} = 1.17$ (0.24)]. The MC1 also estimates the shear strength with enough accuracy [$V_{Exp}/V_{Pre} = 1.19$ (0.24)]. The use of f_{Ftu} to calculate the fiber contribution to the shear strength has led to different results. The model with the mean value of the experimental-numerical ratio closer to one is the MC2 [$V_{Exp}/V_{Pre} = 0.99$ (0.24)], but the proposal and the MC1 formulations provide the best conservative prediction: 1.04 (0.22) and 1.04 (0.23), respectively. Dinh et al. [3] model gives the worst statistical results, and it could be due to the different bending test suggested to estimate the average tensile strength of SFRC and to the conservative assumption of $\cot\theta = 1$.

Figure 12. Correlation between the test value and the prediction value for the studied models and the two considered databases [2, 23].

Table 2. Comparison between experimental (V_{Exp}) and prediction (V_{Pre}) shear strength value for SFRC beams.

Database	#	V_{Exp}/V_{Pre}							
		Proposal		Dinh et al. [3]		MC1		MC2	
		f_{Ftu}	σ_{tu} Eq. (3)	f_{Ftu}	σ_{tu} Eq. (3)	f_{Ftu}	σ_{tu} Eq. (3)	f_{Ftu}	σ_{tu} Eq. (3)
Cuenca [23]	82	1.04 (0.22)	1.17 (0.24)	1.18 (0.29)	1.50 (0.33)	1.04 (0.23)	1.19 (0.24)	0.99 (0.24)	1.28 (0.24)
Lantsoght [2]	488		1.15 (0.22)		1.70 (0.50)		1.58 (0.49)		1.43 (0.39)

In Figure 13, the values of V_{Exp}/V_{Pre} for the studied models are presented as a function of σ_{tu}/f_{ct} , for the specimens of both the databases considered. It highlights that the proposal of this paper presents good

results for the two databases, and that the dispersion is very limited. By contrast, the other formulations used to predict the tests results provide too safe results and higher dispersion of the numerical results. Finally, from Figure 13 it looks that the ratio V_{Exp}/V_{Pre} is independent on the ratio σ_{tu}/f_{ct} .

Figure 13. Correlation between the experimental and the numerical value for the studied models and the considered database [2, 23].

6. CONCLUSIONS

In this paper, a mechanical model for predicting the shear strength of SFRC beams without stirrups, has been presented. The model is an extension of the Multi-Action Shear Model, in which the SFRC effects on each shear resistant mechanism have been identified and incorporated into the governing equations of Multi-Action Shear Model. The following main conclusions of the work done can be drawn:

- 1) In the proposed model, the total shear resisted by a SFRC beam without stirrups is composed by the sum of the contributions of the un-cracked concrete chord, V_c , the shear transferred across the critical shear crack V_{cf} , and the shear resisted by dowel action in the longitudinal reinforcement, V_l . A compact and simple formulation has been derived, including the sum of these components, each of which can be also determined separately.
- 2) The increment of shear strength of SFRC beams with respect to conventional RC beams is essentially due to the bridging capacity of SFRC, that allows resisting a tensile residual stress σ_{tu} across a crack. Such residual stresses contribute directly to resist shear in the web and in the longitudinal reinforcement by dowel action, and indirectly to increase the shear resisted in the compression concrete chord through the increment of the neutral axis depth.
- 3) The residual tensile stress across a crack depends on the crack width and usually needs to be determined by means of flexural beam tests. However, in this work, a uniform ultimate tensile strength σ_{tu} , not depending on the crack width, has been used, based on previous works of some of the authors. The value of σ_{tu} is a function of the aspect ratio, shape, volumetric percentage, and the plain concrete tensile strength, which are values known by the designer.
- 4) The additional contribution of fibers to V_c is mainly due to the increment of compression force in the un-cracked chord, necessary to equilibrate the moment produced by the resultant of the tensile stresses along the critical crack. Such increment of compression force produces an increment of the neutral axis depth and, therefore, an increment of the shear resisted by the un-cracked chord, V_c .
- 5) A simple, but accurate equation to obtain the increment of the flexural neutral axis depth x_f in function of the applied moment, the longitudinal reinforcement ratio and the ratio between the applied and the beam's cracking moment has been obtained.
- 6) The shear transferred across the crack, V_{cf} , is the vertical component of the force transferred by residual stresses across the whole length of the critical shear crack below the neutral axis.
- 7) It is considered that the shear transferred by dowel action in the longitudinal reinforcement, V_l , is due to the vertical residual stresses along the horizontal crack in the interface between the

longitudinal reinforcement and the SFRC. The contributing length to calculate V_l , is considered the distance l_{cr} , between the critical crack opening and the supports, which according to the Multi-Action Shear Model is where at shear failure the bending moment is equals the cracking moment ($l_{cr} = M_{cr}/V_u$), but not higher than the shear crack spacing, assumed as $0.85d$ in a simplified way.

8) The model has been verified by comparing its predictions with the results of shear tests included in two different already published databases: a) a large data based recently published, containing 488 tests; and b) a selected database containing only those tests where SFRC post-cracking was characterized by means of flexural beam tests. The results obtained are summarized as follows:

a. For the large database, the proposal presented in this paper provides good results both in terms of mean (1.15) and CoV (0.22). It can be concluded that, in spite the simplification made to determine the residual tensile strength (what has evident advantages for design), the results of the predictions are better than any of the existing models, including the MC2010 code.

b. For the selected database, the results of the experimental-numerical ratio have been calculated by using both analytical (σ_{tu}) and experimental (f_{Ftu}) residual tensile strength. In the first case, the proposed model and the MC1 formulation of MC2010 give accurate predictions. In the second case, the MC2 equation of MC2010 gives the most accurate prediction (Mean = 0.99 and CoV = 0.24), but the proposed model and the MC1 provide safer results (Mean = 1.04 for both; CoV = 0.22 and 0.23, respectively).

9) The relative contribution of each component depends on the longitudinal reinforcement ratio, ρ and on the SFRC residual stress to the concrete tensile strength ratio σ_{tu}/f_{ctm} . The higher ρ , the higher relative value of V_c , while the higher σ_{tu}/f_{ctm} , the higher relative contribution of V_{cf} .

10) Further investigations should be carry out by introducing a residual tensile stress of SFRC in function of the critical shear crack width, and improving the Kupfer's failure envelope in the tension/compression branch for the case of SFRC.

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Compliance with Ethical Standards

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709 **NOTATIONS**

710	A_s, A_{sw}	Longitudinal rebar and stirrups area
711	C	Resultant of compression force
712	E_c	Young modulus of plain concrete
713	F_τ	Fiber factor
714	G_f	Energy fracture of plain concrete
715	L_f	Fiber length
716	M, M_{cr}	Bending moment and cracking bending moment
717	R_τ	Reduction factor
718	T_f	Horizontal component of fiber resultant force
719	T_l	Resultant force of longitudinal rebar
720	V_{Exp}, V_{Pre}	Experimental and predicted shear strength
721	V_f	Volumetric percentage of fibers
722	$V_c, V_{cw}, V_s, V_l, V_{cf}$	Shear strength contribution: concrete, concrete web, stirrups, dowel action, and fibers
723	fibers	
724	V_u	Ultimate shear strength
725	a	Shear span length
726	b, d, h	Width, effective depth and total depth of beam cross-section
727	d_f	Fiber diameter
728	d_g	Maximum aggregate size
729	d_0	Effective depth of the cross-section (d), but not less than 100 mm
730	f_{ck}, f_{cm}	Characteristic and average value of the plain concrete compressive strength
731	f_{ct}	Tensile strength under direct tension of plain concrete
732	f_{Fis}, f_{Ftu}	Tensile stress level representative of SLS and ULS
733	f_{R1}, f_{R3}	Residual flexural tensile strength corresponding to CMOD = 0.5 and 2.5 mm
734	f_y, f_{yw}	Steel rebar and steel stirrups yield strength
735	k	Size effect parameter
736	k_{dg}	Average crack specimen factor

737	k_v	Strain-aggregate size parameter
738	l_c	Critical fiber length
739	l_{cr}	Distance from shear critical crack to the support
740	n	Homogenization factor
741	$v_c, v_{cw}, v_s, v_l, v_{cf}$	Non-dimensional shear strength contribution: concrete, concrete web, stirrups, dowel
742		action, and fibers
743	v_u	Non-dimensional ultimate shear strength
744	x, x_f	Neutral axis depth of plain concrete and SFRC
745	y	Vertical abscissa from neutral axis
746	w_u	Critical crack width
747	z	Inner lever arm
748	β	Reduction factor
749	β_τ	Fiber bond factor
750	$\varepsilon_s, \varepsilon_{sb}$	Average normal strain of concrete and of longitudinal reinforcement
751	η_0, η_l	Fiber orientation factor and length efficiency factor for fiber
752	μ_{cr}	Non-dimensional cracking moment
753	θ	Shear crack inclination angle
754	ρ	Geometrical percentage of longitudinal reinforcement
755	σ_1, σ_2	Principal tensile stresses
756	$\sigma_{c,crit}$	Normal stress at the critical point
757	$\sigma_{c,max}$	Normal stress at the top fiber of cross-section
758	σ_{sy}	Steel fiber yield strength
759	σ_{tu}	Average residual tensile stress of SFRC
760	σ_x, σ_y	Tensile stresses along x and y direction
761	τ, τ_u	Shear stress and ultimate shear stress
762	τ_f	Mean fiber-matrix shear stress
763	ξ	Size and slenderness factor

764 ζ Abscissa along the longitudinal axis beam