Pre-filtered synchronization structure for grid-connected power converters to reduce the stability impact of PLL dynamics

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Abstract—Grid-connected converters under weak grid conditions could present instability associated with the synchronization elements and not directly with the current control loop. The impedance-based stability criterion is an effective tool to address the problem. To solve the negative effect of the synchronous reference frame phase-locked loop (SRF-PLL) on grid current control under weak grid conditions, a synchronization structure with pre-filter, based on a filtered quadrature signal generator (FQSG), is proposed in this paper to improve the dynamics of the controller's performance. The small-signal impedance model, implemented as a multi-input multi-output system in the dq -frame, characterizes the effect of the FQSG-PLL dynamics on the current control loop. From it, the impedance-based analysis shows the reduction of the stability issues of the converter. The analysis results were verified by time-domain simulations and by experimental tests in a full-scale 2 MW converter test-bench.

Keywords—PLL dynamics, impedance-based analysis, small-signal stability, pre-filtering stage, multiple-input multiple-output (MIMO)

I. INTRODUCTION

With the increasing development of renewable energy sources and their grid integration, international standards require that generation plants reach the same levels of reliability and performance as conventional generation. Power generation technologies should provide new control solutions to contribute to grid regulation, improving the voltage and frequency dynamics in problematic, highly disturbed or weak grids. One of the main stability issues for grid-connected voltage source converters (VSCs) is small-signal stability analysis, with two widely used approaches, the state-space model in the time-domain and the impedance-based model in the frequency domain [1].

State-space models require a detailed definition of the dynamic equations of all elements of the system but provide a lot of information about the oscillation modes and the dependence of the system parameters. In the case of VSCs connected to the grid, this is a great challenge [2], [3]. To solve this problem, one of the most interesting trends is the modelling of all elements through equivalent impedances in the frequency domain.

To obtain the impedance-based model, the system must be linearized over a specific operating point and provides information about the frequency response and stability margins at that point [4]. Then, the control loop stability impact of grid current, synchronization structures or dc-link voltage is analysed based on the impedance stability criterion [5].

This paper addresses the dynamic effect of the synchronization loop of a grid-connected converter system under weak grid conditions. The classical synchronization solutions applied to the field of three-phase power generation were based on simple synchronous reference frame phase-locked loops (SRF-PLL) [6], [7]. They operate satisfactorily under steady-state and balanced conditions; however, their response is unacceptably deficient under unbalanced or distorted conditions. To solve the main drawbacks of the SRF-PLL, different synchronization structures were published and patented [8]–[10]. Some of these new structures proposed the integration of blocks with filtering capabilities that improve the harmonic immunity of the PLL. Almost all these improved and advanced grid synchronization methods were also frequency adaptive, thus permitting good synchronization with the fundamental frequency, even under frequency variations [11].

Most of the previously mentioned synchronization schemes have the asymmetry phase tracking PLL in the synchronous reference frame in common because only the q-axis component is regulated. PLL introduces small-signal disturbance terms in the current loop and the analysis of its influence on the output impedance is developed in the dq and $\alpha\beta$ domains [12]. To capture the asymmetrical dynamics, the impedance model must be based on impedance matrices and the generalized Nyquist criterion (GNC) is used for the stability assessment of these MIMO dynamic systems. Analytical studies, simulations and experimental tests in low power converters show the problem for fast synchronization systems [13], [14].

Recently, some methods of reducing the impedance matrices to single-input and single-output (SISO) equivalents have been proposed to facilitate stability analysis. Some authors simplify the impedance matrices since there are no couplings between axes, assuming inaccurate stability implications [13]–[16]. Others establish the SISO equivalent based on certain assumptions that do not always occur in grids with a large presence of converters [17], [18]. There are proposals based on the alteration of the PLL control structure, as in [19] where the concept of symmetric PLL allows

establishing the small-signal impedance model with SISO complex transfer functions.

Various solutions have been proposed to mitigate the negative effect of PLL on current control. A common solution in early studies about this issue was reducing the PLL bandwidth [4], [13]. Later, different converter stability reports incorporate advanced synchronization systems to analyse their dynamics influence. Some solutions for single-phase converters were presented combined with the phase tracking PLL with different quadrature signal generators (QSG) and [20] specifically incorporated delayed signal cancellation (DSC), second-order generalized integrator (SOGI) and Park's inverse transformations. In the synchronous reference system, a decoupled double SRF (DDSRF-PLL) structure was included in impedance-based stability studies [21] and, in the stationary reference system, a detailed impedance model based on a frequency-locked loop (FLL) synchronization system was analysed [22].

An advanced grid synchronization structure, detailed and evaluated in [23], incorporates a pre-filter FQSG that estimates the instantaneous symmetrical components of the grid voltage in the $\alpha\beta$ -domain, adding filtering capacity with no need for any synchronous reference frame. This paper addresses the FQSG impact in the stability, not directly associated with the current loop, of a VSC connected to low short circuit ratio (SCR) grids. It is approached with stability criteria based on an impedance model in dq -frame, incorporating small-signal transfer functions of the different synchronization elements.

The contributions of this paper are as follows:

- 1) It develops a dq -frame small-signal model of the pre-filter FQSG stage.
- 2) It proposes a MIMO model of a grid-connected VSC using the FQSG-PLL synchronization structure, which enables analysing the converter stability impact using the well-known GNC.
- 3) It verifies the reduction in stability issues of the converter under weak grid condition with the pre-filtered synchronization structure.

The rest of the paper is structured as follows. Section II describes the topology of the system under study and derives its impedance-based model with PLL effect in the dq -domain. Section III explains the structure of the proposed synchronization system, FQSG-PLL. Section IV presents the derivation of the asymmetric admittance model to which GNC is applied to analyse the small-signal stability. Simulation tests are carried out in section V to validate the analysis. Section VI covers the performance of the proposed synchronization system in a 2 MW full converter provided by Ingeteam Power Technology, S.A.. Conclusions are drawn in Section VII.

II. SYNCHRONOUS FRAME IMPEDANCE MODEL

A. DQ -frame current control impedance model

This section develops the impedance model of a three-phase grid-connected VSC current control in the synchronous reference system, without the influence of PLL dynamics and with a basic representation of the filter and grid [12].

Fig. 1 shows the base representation of the inverter with a simplified one-line diagram. The most significant elements, in addition to the inverter, are the output filter, the grid with its impedance and the current control implemented in a rotating synchronous reference frame, dq -frame, with proportional-integral (PI) controllers $H_{PI,dq}(s)$. SRF-PLL was used to synchronize the voltage at the point of common coupling (PCC) to the grid. The stability impact of the dc-link voltage control, detailed in [24], is outside the scope of this paper, so the dc-link voltage V_{dc} was assumed to be constant.

Italic letters are used to represent real vectors and bold letters denote complex vectors. The same criteria are used for transfer functions. A subscript ' dq ' is added for vectors and transfer functions referred to the dq -frame and later, ' $\alpha\beta$ ' for the $\alpha\beta$ -frame.

Fig. 1. Base representation of the grid-connected VSC

The impedance model analysis was based on the converter with only an inverter-side inductor L_1 and connected to an ideal voltage source without impedance. Initially, it was assumed that the PLL was correctly synchronized with the positive sequence grid voltage and its dynamics did not affect the current control. Thus, to obtain the angle signal to apply the Park transformation, we considered a grid of constant frequency and equal to the nominal frequency, ω_1 .

Fig. 2 shows the Laplace block diagram of the L-filtered three-phase VSC in the synchronous reference system. $H_{PI,dq}(s)$ represents the PI controller transfer function and $K_{inv}(s)$ is the time delay of the digital control systems [25].

$Y_{L1,dq}(s)$ is the admittance of the inductor L_1 . The frequency translation of $s \rightarrow s + j\omega_1$ between $\alpha\beta$ and dq frame allows obtaining the transfer function $Y_{L1,dq}(s) = Y_{L1,\alpha\beta}(s + j\omega_1)$ [26].

$Y_{OL,dq}(s)$ is a transfer function, initially associated with the inverter-side filter inductor, $Y_{OL,dq}(s) = Y_{L1,dq}(s)$, which must be extended if we consider feedforward voltage or some variant of active damping.

Moreover, i_{dq}^* is the reference current given to the converter, i_{dq} is the real current output through L_1 and v_{dq} is the voltage of the PCC point.

Fig. 2. Block diagram of the current controller (dq -frame)

The main transfer functions, deduced from Fig. 2 by applying control-block reduction techniques, are the open-loop and closed-loop current controller, $G_{OL,dq}(s)$ and $G_{CL,dq}(s)$, and the output admittance of the converter seen from the PCC point, $Y_{CL,dq}(s)$. With these functions, an equivalent model in dq -frame can be established, as shown in Fig. 3.

Fig. 3. Grid-connected VSC dq -frame impedance model

This model can then be extrapolated to any impedance or external filters settings, without having to modify the analysis of the converter. According to Fig. 3, the output converter in dq -frame can be derived.

$$i_{dq} = G_{CL,dq}(s) i_{dq}^* - Y_{CL,dq}(s) v_{dq} \quad (1)$$

B. SRF-PLL impact on the impedance model

This section defines the admittance functions that must be incorporated into the converter impedance model to include the PLL dynamics in future stability analysis with impedance-based methods.

In this first approach to PLL dynamics, a conventional structure with phase tracking in a synchronous reference system was considered, in which the small-signal model takes us to a second-order dynamic system [9], [12]:

$$\Delta\theta = \frac{H_{PLL,dq}(s)}{s + H_{PLL,dq}(s)V_{1d}} \Delta v_q = F_{PLL}(s) \Delta v_q \quad (2)$$

where $V_{1,dq} = V_{1d} + jV_{1q}$ is the voltage at the measurement point, Δv_{dq} is the voltage disturbance in dq -frame at steady state, aligned with axis dq , $H_{PLL,dq}(s) = k_{p,PLL} + k_{i,PLL}/s$ represents the PI controller transfer function for the PLL and $\Delta\theta$ is the phase variation detected.

The PLL dynamics, in the synchronous reference system, affect the current reference of the controller ($\Delta i_{P,dq}^*$) and the voltage reference of the modulator ($\Delta u_{P,dq}$). These dynamic

effects can be included in a similar way to the linearization used for the PLL model. Considering $I_{1,dq} = I_{1d} + jI_{1q}$, the current of the converter and $U_{1,dq} = U_{1d} + jU_{1q}$, the modulation voltage, both in steady-state, $\Delta i_{P,dq}^*$ and $\Delta u_{P,dq}$ can be deduced taking into account the response of the detected phase corresponding to the voltage perturbation $\theta = \omega_1 t + \Delta\theta$. Both functions depend on the voltage disturbance in q . In this way, to consider the PLL dynamics in the model, it is necessary to include asymmetric transfer matrices with cross-couplings between the d and q axis components. With these functions $G_{iPm}(s)$ and $G_{uPm}(s)$, denoted with an additional subscript 'm', the impedance model must be represented by admittance matrices to capture the asymmetrical dynamics getting a MIMO dynamic system.

$$\begin{bmatrix} \Delta i_{P,d}^* \\ \Delta i_{P,q}^* \end{bmatrix} = \begin{bmatrix} 0 & -F_{PLL}(s)I_{1q} \\ 0 & F_{PLL}(s)I_{1d} \end{bmatrix} \begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix} = G_{iPm}(s) \begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} \Delta u_{P,d} \\ \Delta u_{P,q} \end{bmatrix} = \begin{bmatrix} 0 & -F_{PLL}(s)U_{1q} \\ 0 & F_{PLL}(s)U_{1d} \end{bmatrix} \begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix} = G_{uPm}(s) \begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix} \quad (4)$$

C. Impedance model with PLL effect

This section deals with the integration of the PLL effect in the impedance model and its influence on the frequency response of the converter admittance. To obtain the closed-loop response of the current controller in the dq -frame, the block diagram of transfer matrices in Fig. 4 was considered.

Fig. 4. Block diagram of the transfer matrices (dq -frame) with PLL effect

The development of the impedance model incorporates symmetric transfer matrices for current control in dq -frame, $H_{PI,m}(s)$, time delay $K_{inv,m}(s)$, with third-order Padé approximation, and inverter-side inductor L_i , $Y_{LL,m}(s)$. The influence of the PLL is completely incorporated in the study of the converter admittance with the matrix functions $G_{iPm}(s)$ and $G_{uPm}(s)$. With the control-block reduction techniques in Fig. 4, the converter output admittance function can be obtained. $Y_{CL,m,dq}(s)$ is the admittance matrix modified by the effect of the PLL dynamics and based on real space vectors.

The modified output current, including the influence of the PLL dynamics, is given by:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = G_{CL,m}(s) \begin{bmatrix} i_d^* \\ i_q^* \end{bmatrix} - Y_{CL,m}(s) \begin{bmatrix} v_d \\ v_q \end{bmatrix} \quad (5)$$

Table I shows the converter parameters used in this work.

TABLE I CONVERTER PARAMETERS FOR SIMULATION

Symbol	Description	Value
S_n	Rated Power	2 MVA
U_n	Grid Voltage	690 V
f_i	Rated frequency	50 Hz
f_{sw}	Switching frequency	2.5 kHz
f_s	Control sampling frequency	5 kHz
L_f	Filter inductor	100 μ H
I_{d0}	Steady-state output current (d-axis)	2485 A
I_{q0}	Steady-state output current (q-axis)	0 A

Initially, it was considered that the dynamics of the PLL is slow enough to neglect its effect on the stability of the converter. Then, the increase of the PLL bandwidth and its effect on the total admittance of the converter was analysed. Fig. 5 shows the Bode plots of the admittance matrix components for three synchronization systems with different bandwidth (20, 150 and 300 Hz) over a range of 2 kHz positive frequencies.

Fig. 5. Inverter admittance Y_{CLM} with PLL effect, $f_{PLLBW} = 20, 150$ and 300 Hz

In the case that does not consider the PLL dynamics effect, the admittance matrix is symmetric, $Y_{dd}(s) = Y_{qq}(s)$ and $Y_{qd}(s) = -Y_{dq}(s)$ [26].

In the other scenarios considered, with the PLL dynamics effect, the admittance transfer matrix is asymmetric due to the influence, mainly, on the elements Y_{dq} and Y_{qq} .

The Y_{qq} admittance shows how the magnitude is limited and the phase exceeds 90° within the frequency range of the PLL bandwidth, which implies a behaviour such as negative resistance of the converter output, which can lead to instabilities, depending on the external grid to which it is connected. According to passivity-based stability assessment, a negative resistance can increase the possibility of resonances or increase the magnitude of these [27]. The Bode plots show

that a higher PLL bandwidth will produce a wider frequency range for the negative resistor.

There is also an obvious influence in the phase of the term Y_{dq} , although in magnitude is much lower, which leads many studies to simplify the matrix of admittance, ignoring these magnitudes of cross-coupling components [15], [21]. This approach may have implications for converter stability.

III. FILTERED QUADRATURE SIGNAL GENERATOR

One advanced grid synchronization structure, detailed and evaluated in [23], incorporates a pre-filter FQSG which takes the real measured input signal and, with an estimated frequency, delivers the instantaneous symmetrical components of the grid voltage in the $\alpha\beta$ -domain, adding filtering capacity, without needing a synchronous reference frame.

The structure of the proposed FQSG is shown in Fig. 6.

Fig. 6. FQSG structure

The FQSG structure needs to estimate the positive and negative sequences for proper system operation, especially in an unbalanced grid. The transfer function from the input voltage vector to the positive-negative sequence components detected by the FQSG is given by:

$$\begin{bmatrix} v_{\alpha}^{+} \\ v_{\beta}^{+} \\ v_{\alpha}^{-} \\ v_{\beta}^{-} \end{bmatrix} = \frac{k\omega'}{s^2 + 2k\omega's + \omega'^2} \begin{bmatrix} s & -\omega' \\ \omega' & s \end{bmatrix} \begin{bmatrix} v_{\alpha} \\ v_{\beta} \end{bmatrix} \quad (6)$$

where ω' and k set the resonance frequency and damping factor, respectively.

Regarding filtering capability, both happen to be second-order systems with harmonic attenuation capacity. By changing the value of the gain k , it is possible to increase the filtering capacity, limiting the dynamic response. The recommended value for k is 0.35. This value, although a faster response could be achieved, offers a good trade-off between the bandwidth of the FQSG and its sensitivity in front of harmonics and noisy signals affecting the synchronization system.

The pre-filter reduces the equivalent bandwidth of the PLL but the dynamic performances of the FQSG-PLL are not equivalent to the performances of a standard SRF-PLL with a lower bandwidth, quite deficient under unbalanced or distorted

conditions. The small signal model of SRF-PLL is a second order filter with adjustable bandwidth. On the other hand, the FQSG_PLL, in addition to adjusting the bandwidth, rejects the negative sequence. In distorted grids, and especially during asymmetric sags, the proportion of negative sequence in the grid voltage can be very high and cause disturbances in the frequency estimation of the SRF-PLL.

The FQSG is integrated into the structure of the synchronization system and the positive sequence estimator, FQSG⁺, was used as the input for the PLL.

To perform advanced current regulation in grid-connected systems that also require negative sequence control, it is convenient to implement a detection system capable of performing an appropriate measure of the positive and negative sequence. To know the exact value of the phase of the negative sequence voltage, the positive and negative sequence estimators and two PLL blocks are needed, one for each sequence.

IV. THE STABILITY IMPACT OF THE FQSG-PLL

A. FQSG transfer function in dq-frame

To analyse the influence of the pre-filter on converter stability, it is necessary to develop its small-signal model. The filtering stage is defined in the stationary reference system with voltage input in $\alpha\beta$; the first step is to establish its transfer function in the synchronous reference system (dq) according to the synchronization structure shown in Fig. 7.

Fig. 7. Synchronization structure with pre-filter FQSG

The angle signal estimated to apply the Park transformation, θ^+ , is independent of the frequency tuning required by the FQSG block since it is synchronized with a constant frequency grid and equal to the nominal frequency, ω_1 .

From (6), the transfer function, in $\alpha\beta$ -frame, from the input voltage vector, $v_{\alpha\beta}$, to the positive sequence components detected by the FQSG block, $v_{\alpha\beta}^+$, is obtained.

$$\frac{v_{\alpha\beta}^+}{v_{\alpha\beta}} = \frac{k\omega_1}{s^2 + 2k\omega_1 s + \omega_1^2} \begin{bmatrix} s & -\omega_1 \\ \omega_1 & s \end{bmatrix} = F_{\alpha\beta}(s) \quad (7)$$

$F_{\alpha\beta}(s)$ can be expressed as a complex transfer function [3]:

$$F_{\alpha\beta}(s) = \frac{k\omega_1(s + j\omega_1)}{s^2 + 2k\omega_1 s + \omega_1^2} \quad (8)$$

To translate the transfer function of the FQSG from the $\alpha\beta$ to dq frame, it is necessary to apply the Park transformation to both voltage terms of (7). Thus, the following relationship between the measured dq voltage, v_{dq}^+ , and the estimated one from the angle of the PLL, v_{dq}^{*+} , is established.

$$v_{dq}^{*+} = e^{-j\theta^+} F_{\alpha\beta}(s) e^{j\theta^+} v_{dq}^+ = e^{-j\Delta\theta} e^{j\omega_1 t} F_{\alpha\beta}(s) e^{j\omega_1 t} v_{dq}^+ \quad (9)$$

$$v_{dq}^{*+} = e^{-j\Delta\theta} F_{\alpha\beta}(s + j\omega_1) v_{dq}^+ = e^{-j\Delta\theta} F_{dq}(s) v_{dq}^+ \quad (10)$$

Next, starting from (8), the transfer matrix $F_{\alpha\beta}(s + j\omega_1)$ is developed,

$$\begin{aligned} F_{\alpha\beta}(s + j\omega_1) &= F_{dq}(s) = \\ &= \frac{k\omega_1(s + j2\omega_1)[(s^2 + 2sk\omega_1) - j(2s\omega_1 + 2k\omega_1^2)]}{s^4 + 4k\omega_1 s^3 + (4k^2\omega_1^2 + 4\omega_1^2)s^2 + 8ks\omega_1^3 + 4k^2\omega_1^4} \end{aligned} \quad (11)$$

Finally, the pre-filter FQSG matrix transfer function, in dq -frame and based on real space vectors, is obtained.

$$F_{\alpha\beta}(s + j\omega_1) = F_{dq}(s) = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} = \frac{v_{dq}^+}{v_{dq}^+} \Big|_{\theta^+ = \omega_1 t} \quad (12)$$

$$F_{11}(s) = F_F(s)(s^3 + 2k\omega_1 s^2 + 4s\omega_1^2 + 4k\omega_1^3) = F_{22}(s) \quad (13)$$

$$F_{12}(s) = -F_F(s)2ks\omega_1^2 = -F_{21}(s) \quad (14)$$

$$F_F(s) = \frac{k\omega_1}{s^4 + 4k\omega_1 s^3 + (4k^2\omega_1^2 + 4\omega_1^2)s^2 + 8ks\omega_1^3 + 4k^2\omega_1^4} \quad (15)$$

The transfer functions obtained describe the performance of the FQSG on the synchronous reference frame.

B. FQSG-PLL impact on the impedance model

From the pre-filter FQSG transfer function in dq -frame, voltage disturbances, Δv_d and Δv_q , are considered to obtain the small-signal modelling of the synchronization structure FQSG-PLL. Applying (9) with a small angle approximation of trigonometric functions, yields:

$$\begin{bmatrix} V_d' + \Delta v_d' \\ V_q' + \Delta v_q' \end{bmatrix} = \begin{bmatrix} 1 & \Delta\theta \\ -\Delta\theta & 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} V_{1d} + \Delta v_d \\ V_{1q} + \Delta v_q \end{bmatrix} \quad (16)$$

A conventional PLL small-signal model, in the synchronous reference system, establishes the following relationship between the voltage error $\Delta v_q'$ caused by the perturbation Δv_q , and the phase variation detected by the PLL, $\Delta\theta$.

$$\Delta\theta = H_{PLL,dq}(s) \Delta v_q' \frac{1}{s} \quad (17)$$

Substituting (17) into (16), cancelling the steady-state values and neglecting the second-order terms of the errors, the following equation shows the angle error as a function of the disturbance in the grid voltage in the dq -frame, Δv_{dq} :

$$\Delta\theta = \frac{H_{PLL,dq}(s) F_{21} \Delta v_d + H_{PLL,dq}(s) F_{22} \Delta v_q}{s + H_{PLL,dq}(s) (F_{11} V_{1d} + F_{12} V_{1q})} \quad (18)$$

$$\Delta\theta = F_{PLL1}\Delta v_d + F_{PLL2}\Delta v_q \quad (19)$$

To consider the PLL dynamics in the model, it is necessary to include two new transfer matrices associated with the influence on the current control input reference $\Delta i_{F,dq}^*$ and on the voltage modulation input reference $\Delta u_{F,dq}$.

$$\begin{bmatrix} \Delta i_{F,d}^* \\ \Delta i_{F,q}^* \end{bmatrix} = \underbrace{\begin{bmatrix} -F_{PLL1}(s)I_{1q} & -F_{PLL2}(s)I_{1q} \\ F_{PLL1}(s)I_{1d} & F_{PLL2}(s)I_{1d} \end{bmatrix}}_{G_{iFm}(s)} \begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix} \quad (20)$$

$$\begin{bmatrix} \Delta u_{F,d} \\ \Delta u_{F,q} \end{bmatrix} = \underbrace{\begin{bmatrix} F_{PLL1}(s)V_{c1q} & F_{PLL2}(s)V_{c1q} \\ -F_{PLL1}(s)V_{c1d} & -F_{PLL2}(s)V_{c1d} \end{bmatrix}}_{G_{uFm}(s)} \begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix} \quad (21)$$

Incorporating $G_{iFm}(s)$ and $G_{uFm}(s)$ matrices in the current control scheme, presented in Fig. 4, a modified admittance $Y_{Fm}(s)$ based on real space vectors can be derived.

$$Y_{OLFm}(s) = Y_{OLm}(s) - G_{uFm}(s)K_{Inv}(s)Y_{LI,dq}(s) \quad (22)$$

$$Y_{Fm} = [I_m + G_{OLm}(s)]^{-1} Y_{OLFm}(s) - G_{CLm}(s)G_{iFm}(s) \quad (23)$$

This admittance function captures the asymmetrical dynamics caused by the FQSG-PLL synchronization system and modifies the converter output current i_{dq} :

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = G_{CLm}(s) \begin{bmatrix} i_d^* \\ i_q^* \end{bmatrix} - Y_{Fm}(s) \begin{bmatrix} v_d \\ v_q \end{bmatrix} \quad (24)$$

Considering a PLL with a bandwidth of 80 Hz, Fig. 8 shows the Bode plots of the admittance components $Y_{dd}(s)$ and $Y_{qq}(s)$ before and after including the pre-filter.

Fig. 8. Inverter admittances with pre-filter effect (Y_{dd} y Y_{qq}), $f_{PLLbw} = 80$ Hz

With SRF-PLL effect included, a wide frequency range with phase greater than 90° , associated with the PLL bandwidth, shows the negative resistor behaviour of $Y_{qq}(s)$. However, the frequency response with FQSG-PLL clearly demonstrates a considerable decrease of phase within this frequency range. In the range of frequencies of interest, the

phase of the admittance has become closer to an element with positive resistance, because while it is between $+90^\circ$ and -90° , the real part of the admittance will be positive and the system will tend to attenuate any external resonance [27]. Therefore, the FQSG eliminates the possible instabilities that the negative resistance behaviour could produce in frequencies within the bandwidth margin of the PLL.

C. Impedance-based Stability Analysis

In addition to the input admittance matrix of the converter in the previous sections, it is necessary to obtain the impedance of the grid side in dq -frame. According to Fig. 1, the calculation of the total grid impedance must include the grid side filter inductance, L_2 , the transformer impedance, Z_{2t} , the grid impedance, Z_g , and the admittance of the filter capacitor, C_f . In this way, the total grid impedance, reducing inductive terms to L_g , is obtained.

$$Z_{g,dq}(s) = \frac{(sL_g + R_g) + j\omega_1 L_g}{((sL_g + R_g) + j\omega_1 L_g)(sC_f + j\omega_1 C_f) + 1} \quad (25)$$

The impedance matrix transfer function is obtained in the dq -frame, $Z_{g,mdq}(s)$, transforming the complex formulation to its equivalent matrix formulation with real functions [26].

Given the converter admittance and the grid impedance, the system, with asymmetric impedance matrices, is formulated as a MIMO system, which can be analysed with impedance-based methods. There are several common stability analysis methods based on impedance for these systems, among them the Bode diagrams and the GNC.

Both methods require calculating the return-ratio matrix:

$$L_{dq}(s) = Z_{g,mdq}(s)Y_{Fmdq}(s) \quad (26)$$

The impedance specification on Bode diagrams has some limitations when applied to MIMO systems. It can only work when both the converter admittance matrix and the grid impedance matrix are diagonal dominant. If the impedance matrices are asymmetric, the stability should be analysed on the Bode diagram of the eigenvalues of the return ratio matrix $L_{dq}(s)$.

To obtain the frequency response of the eigenvalues of that matrix, a grid that presents the parameters shown in Table II was considered.

TABLE II GRID PARAMETERS FOR SIMULATION

Symbol	Description	Value
SCR	Short Circuit Ratio	4
X/R	Grid X/R Ratio	10
L_g	Grid Inductor	121.6 μ H
R_g	Grid Resistance	0.0038 Ω
C_f	Filter capacitor	800 μ F
L_{2t}	Transformer Inductor	33.2 μ H
R_{2t}	Transformer Resistance	0.0012 Ω

Three different cases were analysed for a 150Hz PLL bandwidth. In Fig. 9(a) the influence of the PLL dynamics is not included. In this case, the phase of both eigenvalues is never cross -180° which implies that the VSC system is stable. In Fig. 9(b) one of the eigenvalues in red remains stable while the other shows instability. The magnitude of the eigenvalue in blue is positive with 0.76 dB when the phase cross -180° at 121 Hz. This indicates that the system is unstable but quite near the stable boundary, which implies that the system diverges slowly with resonances nearby 121 Hz. With the use of the pre-filter FQSG, it can be seen in Fig. 9(c) that the converter becomes stable again since the negative magnitude is maintained when the phase of the eigenvalue in blue crosses -180° .

Fig. 9. Frequency response of the eigenvalues in the dq -frame. (a) No effect of the PLL dynamics. (b) PLL dynamics effect. (c) FQSG-PLL dynamics effect

GNSC is the most generalized method to analyse the stability of the MIMO system. For GNSC, the system is stable if and only if the eigenvalue loci of $L_{dq}(s)$ do not encircle the critical point $(-1,0)$ based on the condition that both subsystems are independently stable.

Taking these same cases into account, Fig. 10 shows the Nyquist diagrams for the eigenvalues of the return ratio matrix, showing that only in case with the influence of PLL dynamics without a pre-filter stage, Fig. 10(b), the system turns unstable with encircling $(-1,0)$.

Fig. 10. Nyquist diagrams for the eigenvalues of the impedance ratio. (a) No effect of PLL dynamics. (b) PLL dynamics effect. (c) FQSG-PLL dynamics effect

V. SIMULATION RESULTS

Before experimental testing, the digitization of all the FQSG-PLL processes and algorithms and their translation into programming code to be directly applicable to the control system was carried out. The simulation of the discrete system model confirms that there were no errors in the discretization process.

During the simulation stage, operating the synchronization structure in the presence of PLL with increasing bandwidth was verified. The converter and grid parameters included in

Tables I and II configured the simulation scenario. The bandwidth of the PLL was modified from 30 Hz to 200 Hz.

Fig. 11 shows the simulation results of PCC voltage and converter current with SRF-PLL. It can be seen in Fig. 11(a) that the system is stable for a 30 Hz PLL bandwidth. When increasing the PLL bandwidth to 150Hz, oscillations can be seen in Fig. 11(b), which is consistent with the frequency domain analysis in Fig. 9(b).

Fig. 11. Simulated PCC voltage and VSC current for dq -frame current control with SRF-PLL. (a) $f_{PLL BW} = 20$ Hz. (b) $f_{PLL BW} = 150$ Hz. (c) $f_{PLL BW} = 200$ Hz

Fig. 12. Harmonic spectrum of the simulated waveforms ($f_{PLL BW} = 150$ Hz)

To further confirm the stability prediction, Fig. 12 shows the harmonic spectrum of the Fig. 11(b), where two main resonant components at 72 Hz and 172 Hz can be observed. If the system contains an undamped oscillation at 172 Hz in the

$\alpha\beta$ -frame, this oscillation frequency will be shifted to 122 Hz in the dq -frame. Due to the frequency coupling effect of the PLL [12], the one single-frequency perturbation imposed on the grid-connected converter will generate responses at two frequencies that are separated by twice fundamental frequency (100 Hz). Therefore, another oscillation will be generated at 72 Hz. The two frequencies shift compared with the one identified in Fig. 9(b), 121 Hz in the dq -frame, verify the frequency-coupling effect. Fig. 11(c) shows the step change of the PLL bandwidth from 30 to 200 Hz at the time instant of 2.2 s where the VSC system turns to be unstable.

The characteristic scenarios affected in stability by the dynamics of the PLL have been selected for evaluating the synchronization capability of the FQSG-PLL presented in this paper. To test the response of the FQSG-PLL, the simulation scenarios shown in Fig. 11 was repeated with the pre-filter stage set at the recommended value for $k = 0.35$. Fig. 13 illustrates the stable behaviour of the FQSG-PLL, with 150 Hz PLL bandwidth close to the stability boundary of the SRF-PLL, and verifies its harmonics rejection capability.

Fig. 13. Simulated PCC voltage and VSC current for the dq -frame current control with FQSG-PLL ($k_{FQSG} = 0.35$)

VI. EXPERIMENTAL RESULTS

To test the synchronization structure and observe its behaviour under different real grid conditions, an experimental system was used in the laboratory.

A. Experimental Setup

The experimental tests were carried out at a test-bench that was built by Ingeteam Power Technology S.A., a market leader specialising in the development of converters for wind power generation.

Fig. 14. Full converter topology test-bench Ingeteam Power Technology

The test-bench allows testing on full converter (FC) topology in low voltage (LV, 400 Vrms). Fig. 14 shows the

general scheme of the platform, where a 2 MW Ingecon Wind converter is connected to a VSC virtual grid. The switching frequency of the power converter was set to 2.5 kHz with a current reference of 2000 A. The system replicates the grid parameters included in Tables I and II.

B. Pre-filter effect

Fig. 15 shows the measured waveforms of the PCC voltages and VSC currents for the dq -frame current control, initially with the proposed synchronization system with pre-filter. The test was performed with 150 Hz PLL bandwidth, close to the stability boundary of the SRF-PLL. FQSG-PLL offers a good response but when the pre-filter is removed, the voltage and current registers show the harmonic distortion predicted with the simulation results in Fig. 11(b).

Fig. 15. Experimental waveforms with pre-filter effect ($f_{PLL BW} = 150$ Hz). (a) VSC currents: (800A/div, 20ms/div); (b) PCC voltages: (200V/div, 20ms/div)

C. Voltage sags

The capability of the FQSG-PLL synchronization system to perform a fast and accurate synchronization was tested in the test-bench under different grid fault scenarios with a current reference of 1000 A.

Fig. 16 shows the measured waveforms of the PCC voltage (phase to phase, v_{rs} and v_{ro}) and injected currents to the grid in case of unbalanced voltage sag (100%). The FQSG filter has a fast performance and allows the detection of the positive and negative sequence of the voltage in 20–25 ms.

Fig. 16. Experimental waveforms in case of unbalanced voltage sag. (a) VSC currents: (700A/div, 50ms/div); (b) PCC voltages: (160V/div, 50ms/div)

Fig. 17 shows the measured waveforms of the PCC voltage (phase to phase, v_{rs} and v_{st}) and injected currents to the grid in the case of three-phase symmetrical voltage sag (100%). The response of the FQSG-PLL in this case also shows good behaviour with fast detection of the positive sequence

amplitude, at about 20ms, despite the severe reduction of the three-phase voltages.

Fig. 17. Experimental waveforms in the case of three-phase voltage sag. (a) VSC currents: (700A/div, 50ms/div); (b) PCC voltages: (160V/div, 50ms/div)

VII. CONCLUSION

This paper addresses the stability issues associated with the synchronization elements of grid-connected converters under weak grid conditions.

The analysis of the SRF-PLL dynamics effect, in dq -frame, reveals instability in weak grids. The small-signal model of pre-filter FQSG and its interaction with the current loop control and grid impedance is detailed and an asymmetric impedance matrices model, formulated as a MIMO system, is analysed based on the impedance stability criterion. The dynamic response of the FQSG-PLL shows good behaviour and confirms it as a suitable solution to enhance stability for grid-connected VSC systems.

Conclusions drawn from the theoretical study of stability were ratified using computer simulations and the experimental results show that a 2 MW grid-connected inverter can be operated stably with the proposed synchronization system under different grid conditions.

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