Extension of the ζ -method for calculating deflections of two-way slabs based on linear elastic finite element analysis

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1 **ABSTRACT**

2 Deflections are among the most complex aspects of reinforced concrete behaviour. Furthermore, deflection

3 control is increasingly a governing criterion in design, especially in the case of two-way members such as flat

4 slabs or edge-supported slabs. However, simple and theoretically sound and adequately verified methods for

5 calculating deflections of two-way reinforced concrete members have been lacking. Therefore, in this study, a

6 novel method is proposed for deflection control of two-way slabs. The method is an extension of the ζ -method

7 (as defined by Eurocode 2 and the *fib* Model Code 2010) of interpolating deflections between the uncracked

8 and fully cracked state. As a basis, the method uses elastic deflections that are obtained through linear elastic

9 analyses in finite element software. The elastic deflection is then adequately transformed based on the

10 properties of a representative section, to account for cracking, shrinkage and creep according to a modified

11 version of the ζ -method. Finally, the method is validated against available experimental results from literature,

12 obtaining satisfactory precision and accuracy. The method proposed in this study aims at providing an easy-to-

13 use tool for designers carrying out deflection control of two-way reinforced concrete slabs.

14

15 **Keywords:**

16 Deflection, serviceability, two-way slab, design, flat slab, edge-supported slab, FEM, Model Code

1. Introduction

Traditionally, theoretical advances in structural analysis and design have focused on resistance models for ultimate limit states (ULS) and the majority of research in the field of new concrete types has concentrated on increasing the strength of the materials. This has led to the fact that increasingly high-strength concretes are used in structural applications, accompanied by increasingly efficient ULS design methods. However, an increase in concrete strength is not accompanied by a proportional increase in its modulus of elasticity or tensile strength (FIB, 2013; FIB Bulletin 92, 2019). This means that the stiffness of concrete structures and their resistance to cracking have not increased as much as their strength. Therefore, serviceability limit states (SLS) such as deflections and cracking have become critical in design. In fact, a global trend over the past several decades of increasing strengths of both concrete and reinforcement, reduction of partial safety factors for ULS and a demand for faster construction and longer spans has meant that concrete structures today are “four or more times more flexible than those designed 50 years ago” (FIB Bulletin 52, 2010).

Among SLS, deflection control is especially important but also difficult for modelling due to the large number of influencing factors as well as uncertainties associated with these factors (Beeby and Narayanan, 2005; Pecić, 2012). Methods for calculating (or better, “estimating”) deflections of one-way members (beams and one-way slabs) have been improved over time in the long tradition of CEB and the *fib* (CEB-FIP, 1991, 1978; CEB, 1985; FIB, 2013) taking on the form of more or less complex methods of interpolating curvatures or deflections calculated for sections (or entire members) in uncracked and fully cracked states. Today, very useful, simple, precise and accurate models of varying levels of complexity are available for calculating deflections of one-way members (FIB Bulletin 92, 2019; Mari et al., 2019, 2010; Mari, 2000).

What is still lacking is a theoretically justified, but simple-to-use, method for calculating deflections of two-way members. In fact, slabs of residential and commercial buildings constitute the largest horizontal load-bearing members of buildings and precisely for these types of elements deflection control is critical in design (their in-service load can be very close to their cracking load). However, the fact that they have a bi-directional structural behaviour associated with significant redistribution in the two load-carrying directions has prevented researchers so far from postulating simple methods for calculating their deflections. Furthermore, in order to validate any model, experimental results are necessary. At the same time, full-scale long-term experimental testing of two-way slabs is extremely difficult, associated with large uncertainties, a

large number of influencing factors and large costs. Therefore, literature on the topic is scarce. Most of the literature deals with in situ measurements of two-way slab deflections in residential or commercial buildings, sometimes from the time of construction, sometimes only after complaints by occupants (Jokinen and Scanlon, 1987; Taylor and Heiman, 1977; Vollum et al., 2002). In other instances, only short-term deflections are reported (Gardner and Shao, 1996) or specific loading is applied (Gardner and Fu, 1987). Herein, only four laboratory experimental programmes on two-way slabs were identified by the authors as suitable for model validation (Gilbert and Guo, 2005; Jaccoud and Favre, 1982; Radnić and Matešan, 2008; Tellenbach and Favre, 1985) and these will be discussed later in more detail.

The general lack of experimental data has led to researchers mostly proposing models based on theoretical considerations, empirical approximations and extension of the approaches available for one-way elements. Hence, the so-called “crossing beam” or “strip” method is one of the most widely recognized methods for deflections of two-way slabs, formulated by numerous researchers and standards (ACI 435R-95, 2003; Ghali et al., 2002; Gilbert and Ranzi, 2011; Scanlon and Suprenant, 2011). Generally, it consists of isolating strips or “beams” from a two-way slab, approximating its boundary conditions and then using one of existing methods for one-way members, possibly with special provisions for flat slabs when the results of several strips need to be superimposed. Nonetheless, these methods work for a limited range of standard statical systems with crude approximations and a time-consuming calculation procedure.

Besides these methods, the advances in finite element method (FEM) software (FIB Task Group 4.4, 2008) have enabled fully nonlinear deflection calculations. Various commercial and open-source software exist with wide capabilities for modelling time-dependent concrete behaviour (Kabir, 1976; Lee, 2007; Mari, 2000; Scordelis, 1984; Tošić and Kurama, 2020). For example, Lee (2007) developed a user-defined model in ABAQUS (Simulia, 2018) and successfully modelled instantaneous and long-term deflections of two-way slabs. However, this type of analysis is still not widespread in use among practitioners, especially for the cases of residential and commercial buildings, even with commercially-developed software because of the complexities and difficulties associated with nonlinear analysis of the time-dependent behaviour of concrete. However, even in these cases, FEM software is increasingly used but only for the purpose of carrying out linear elastic analyses. Therefore, the time has come to take advantage of the benefits offered by the widespread use of linear elastic analyses in FEM software to propose a deflection control method for two-way

slabs. Such an approach was envisioned even in 1985 in the CEB Design Manual on Cracking and Deformations (CEB, 1985) where a use of an elastic deflection was the basis for a series of transformations in order to obtain an estimate of the time-dependent deflection (considering cracking, creep and shrinkage). However, back at that time, the difficulty was precisely in obtaining this elastic deflection and the fact that a designer had to rely on a series of graphs instead of having a closed-form solution that can easily be implemented in spreadsheets or programming scripts.

Therefore, in this study, a method for deflection control is proposed based on using linear elastic deflections determined from a FEM analysis and a series of transformations to obtain deflections due to load (including the time-dependent effect of creep) and shrinkage in uncracked and fully cracked states that can be interpolated using the existing ζ -method (FIB, 2013). Furthermore, the method is validated against available experimental results under laboratory conditions, showing satisfactory agreement with results and stability of the obtained solutions. The proposed method aims at providing practicing engineers with a simple and reliable tool for deflection control of two-way reinforced concrete slabs.

2. Extending the ζ -method to deflection control of two-way slabs

2.1. General approach

The use of FEM software in every-day work of design engineers has become prevalent in recent decades. Performing structural analysis for different types of structures has become ubiquitous, especially in the case of designing commercial and residential buildings (precisely the structures in which two-way slabs are dominant structural elements and critical in terms of deflection control); in such cases, a wide variety of commercial software is available. Typically, designers will perform a linear elastic FEM analysis (LFEA) in the software that they use and the results of this analysis will form a basis for ULS design. However, SLS design, and especially deflection control, has still not incorporated the many advantages that LFEA can bring. Therefore, the time has come to take full advantage of the benefits brought by this technology.

Clearly, LFEA has numerous advantages over any of the “manual” deflection control methods for two-way slabs described earlier:

- ability to model arbitrary slab geometries (in plan, variations in thickness, openings) and variations in mechanical properties;
- ability to model arbitrary support conditions (beams, walls, columns);

- ability to model arbitrary distribution of loads on the structure, load combinations and loading sequences;
- possibility of using the same FEM model for both ULS and SLS.

A typical LFEA in commercial FEM software will yield, as results, internal forces in the elements, as well as linear elastic displacements. The obtained linear elastic displacements are particularly useful, not only because of the reasons described earlier, but also because this type of FEM analysis is able to take into account both the bending and torsional stiffness of the slab.

Therefore, a method that takes advantage of LFEA must be based on utilizing the calculated linear elastic displacement and transform it into an estimated deflection. In essence, simplified methods for deflections of RC members are based on a transformation of a linear elastic deflection. Consider for example the simplified method proposed in Eurocode 2 and the *fib* Model Code 2010, based on interpolating deflections (EN 1992-1-1, 2004; FIB, 2013):

$$a = \zeta \cdot a_2 + (1 - \zeta) \cdot a_1 \tag{1}$$

where a is the estimated deflection, ζ is the distribution coefficient, a_1 and a_2 are the deflections in states 1 and 2, respectively, i.e. in the uncracked and fully-cracked states. Equation (1) should be used for calculating deflection of cracked members; deflections of uncracked members can be determined by adopting $\zeta = 0$. Typically, the distribution coefficient ζ has the following form:

$$\zeta = 1 - \beta \cdot \left(\frac{M_{cr}}{M} \right)^2 \tag{2}$$

where M_{cr} is the cracking moment, M is typically the maximum moment along the span of a member (or at its end in the case of cantilevers), and β is an empirical factor equal to 1 for first-time loading and 0.5 for repeated or sustained loading. Furthermore, the value of 0.5 for sustained loading also covers the effects of shrinkage and temperature, as well as previous loading and unloading of live loads (CEB Bulletin 235, 1997). The formulation was first introduced in the CEB-FIP Model Code 1978 (CEB-FIP, 1978) based on an empirical model of the behaviour of cracked concrete elements under axial tension (Jaccoud and Favre, 1982). In the original formulation, there were two β coefficients, β_1 and β_2 , where β_1 was used to take into account bond (=0.5 for smooth bars and 1.0 for ribbed/high bond bars). As only ribbed bars are used today, coefficient β_1 was eliminated from the formulation and only $\beta = \beta_2$ remained, considering the nature/history of loading.

Both deflections a_1 and a_2 —the ones due to load—can be written in the following way:

$$a_i = K \cdot \frac{M \cdot L^2}{E_{c,ef} \cdot I_i^*} \quad (3)$$

where K is a coefficient dependent on the statical system and loading (e.g., =0.104 for uniformly distributed load on a simply supported beam), M is the maximum moment over the element span for the considered load combination, L is the span, $E_{c,ef}$ is the (effective) concrete modulus of elasticity (i.e. effective modulus for long-term deflections), and I_i^* is the transformed second moment of area of the representative (or “dominating”) cross-section (where M is acting) for state $i = 1, 2$.

A linear elastic deflection of the same beam could be written as

$$a_{el} = K \cdot \frac{M \cdot L^2}{E_c \cdot I_c} \quad (4)$$

where E_c is the modulus of elasticity and I_c is the second moment of area. This deflection, a_{el} , can easily be calculated using LFEA. Then, a transformation can be performed:

$$a_i = a_{el} \cdot \frac{E_c \cdot I_c}{E_{c,ef} \cdot I_i^*} \quad (5)$$

Therefore, simply by normalising linear elastic deflections using different stiffness, estimated deflections in states 1 and 2 can be obtained and interpolation using Equation (1) can be performed. This is valid for instantaneous and time-dependent deflections caused by load.

Deflections are also significantly influenced by shrinkage. For example, the deflection in states 1 and 2 can be calculated from the following expression:

$$a_{cs,i} = \delta_{cs} \cdot \varepsilon_{cs}(t, t_s) \cdot \frac{S_i^*}{I_i^*} \cdot \frac{L^2}{8} \quad (6)$$

where δ_{cs} is a coefficient dependent on the statical system (e.g., =1 for a simply supported beam), $\varepsilon_{cs}(t, t_s)$ is the shrinkage strain at time t for a concrete cured until time t_s , and S_i^* is the transformed first moment of area of the reinforcement in state i (where i is 1 or 2) relative to the transformed section’s centroid. At first, it may seem that there is no direct link to a_{el} . However, simplified methods estimate deflections based on one representative cross-section. In that cross-section, the curvatures from shrinkage and from load have a certain ratio:

$$\left(\frac{1}{r}\right)_{cs,i} / \left(\frac{1}{r}\right)_{load,i} = \left(\varepsilon_{cs}(t, t_s) \cdot \frac{S_i^*}{I_i^*} \right) / \left(\frac{M}{E_{c,ef} \cdot I_i^*} \right) \quad (7)$$

Because the deflection is estimated based on the curvature of a single cross-section, a reasonable assumption is that the ratio between curvatures is translatable to a ratio between deflections:

$$a_{cs,i} = a_{load,i} \cdot \left(\frac{1}{r}\right)_{cs,i} / \left(\frac{1}{r}\right)_{load,i} \quad (8)$$

Substituting Equation (5) for $a_{load,i}$, we finally obtain

$$a_{cs,i} = a_{el} \cdot \frac{E_c \cdot S_i^*}{M} \cdot \varepsilon_{cs}(t, t_s) \cdot \frac{I_c}{I_i^*} \quad (9)$$

Thus, a consistent formulation of estimated deflections, both from load and shrinkage, can be obtained based on a transformation of the linear elastic deflection (obtained either using Equation (4) or LFEA).

The derivations shown above demonstrate the path which must be taken towards a theoretically justified method for calculating deflections of two-way slabs using LFEA. The following subsection describes in detail a first-ever proposal of such a method and the further challenges that it solves.

2.2. Full description of the proposed method

The proposed method is based on a *choice of a unidirectional strip of unit width* for which deflections (Figure 1) and internal forces (Figure 2) can be obtained. This means that “the estimated deflection of any point should be calculated based on a transformation of the elastic deflection of that point, as well as of the support points of the considered strips.” The basic idea developed is to calculate slab deflections using a single strip with end and mid-span moments calculated using an elastic FEM program.

Bending moments and deflections are determined using LFEA (linear, elastic, uncracked FEM).

The uncracked deflection due to load $a_{l,1}$ and fully cracked deflection $a_{l,2}$ due to load are determined by scaling using relevant transformed stiffnesses EI^* . Shrinkage deflections (Equation (6)) are determined by beam analogy from the curvature at a single cross-section.

Finally, deflections in the partially cracked state are obtained by interpolating between uncracked and fully cracked deflections using a modified version of the coefficients ζ .

2.2.1. Choice of elastic deflection

The method for calculating deflection of two-way slabs proposed in this paper is fundamentally an adaptation and extension of the simplified ζ -method (based on directly interpolating deflections instead of curvatures), as defined in the *fib* Model Code 2010 (FIB, 2013). Herein, the proposed method will be referred to as the “extended ζ -method for two-way members”. Furthermore, the method is fundamentally based on a general approach proposed in the CEB Design Manual on Cracking and Deformations (CEB, 1985) but making full use of modern FEM capabilities for linear elastic analyses as well as offering closed-form solutions instead of the use of different graphs.

The starting point for the method is the LFEA that results in linear elastic displacements. As an example, a flat slab, supported on continuous supports (walls) on its perimeter and interior point supports (columns), is shown in Figure 1 with its linear elastic displacements under quasi-permanent load. The corresponding bending moments are shown in Figure 2. If the corner panel is analysed, the *total displacement* is 3.33 mm. Considering a midspan strip in the x -direction, Figure 1, the displacement is zero at the slab perimeter but 2.85 mm at the end of the panel span. Hence, the displacement of 3.33 mm actually consists of two components: (1) a kinematic component due to the displacement of the strip ends and (2) a deflection component (produced by the curvature along the strip). In this case, the kinematic component is $(0 + 2.85)/2 = 1.43$ mm. Hence the deflection is $3.33 - 1.43 = 1.90$ mm.

In general, the *linear elastic deflection* a_{el} , can be written in the following form:

$$a_{el} = Z_{\max} - 0.5 \cdot (Z_{s,1} + Z_{s,2}) \quad (10)$$

where $Z_{\max}$ is the maximum displacement over the considered span and $Z_{s,1}$ and $Z_{s,2}$ are the displacements of the “support lines” of the strip. Note that for the vertical strip in the y -direction, the elastic displacement is equal to the deflection $a_{el} = Z_{\max} = 2.85$ mm as the displacement of the strip’s support points (column and outer wall) are zero.

In the case of the midspan point of the corner panel in Figure 1, the estimation of the deflection at midspan would require the transformation of $a_{el} = 1.90$ mm, and its *superposition* to the estimated deflections at its support points (done on $a_{el,1}$ and $a_{el,2}$ equal to 0 and 2.85 mm, respectively). Importantly, the choice of this strip can be arbitrary (but optimally, the simplest/shortest strip can be chosen). In other words, for calculating the centre-span deflection, strips can be chosen either as shown in Figure 1 (x -direction midspan

strip and y-direction support strips), or perpendicularly (y-direction midspan strip and x-direction support strips).

2.2.2. Geometric and mechanical properties

Once the strip direction is selected (e.g. as in Figure 1), its geometric properties should be calculated (i.e. second moment of area and first moment of area of reinforcement, for states 1 and 2) considering a cross-section *unit width* (i.e. $b = 1$ m). Although the method is based on a representative cross-section, the distribution of reinforcement along the strip has significant effect on deflections. Therefore, when calculating geometric properties, the “average reinforcement ratio” for both tensile and compression reinforcement (ρ_m and ρ'_m , respectively) should be used, see CEB (1985) and Marí et al. (2010) and Figure 3.

$$\rho_m = \frac{\rho_A l_A + \rho_C l_C + \rho_B l_B}{l} \quad (11)$$

$$\rho'_m = \frac{\rho'_A l_A + \rho'_C l_C + \rho'_B l_B}{l} \quad (12)$$

where ρ_A and ρ'_A are the reinforcement ratios of tensile and compressive reinforcement at support A; ρ_B and ρ'_B are the reinforcement ratios of tensile and compressive reinforcement at support B; ρ_C and ρ'_C are the reinforcement ratios of tensile and compressive reinforcement at midspan; l_A and l_B are lengths of those parts subjected to negative moments; and l_C is the length of the part subjected to positive moments.

As for modelling time-dependent behaviour of concrete, the effective modulus of elasticity can be used:

$$E_{c,ef} = \frac{E_c}{1 + \varphi(t, t_0)} \quad (13)$$

where E_c is the short-term modulus of elasticity of concrete, φ is the creep coefficient (e.g. calculated according to the *fib* Model Code 2010 model), t is the time at which deflections are evaluated and t_0 is the concrete age at load application. Equation (13) assumes a single load applied at t_0 ; for changes in load over time (loading/unloading) an average creep coefficient can be used or methods such as the age-adjusted effective modulus or Volterra integration are possible (FIB, 2013). Finally, to calculate the transformed geometric properties, the effective modular ratio is used:

$$\alpha_e = \frac{E_s}{E_{c,ef}} \quad (14)$$

where E_s is the modulus of elasticity of reinforcement.

2.2.3. Calculation of deflections

As was explained in subsection 2.1, deflections due to load are based on a transformation of the linear elastic deflection a_{el} . The linear elastic deflection obtained using LFEA will be denoted a_{LFEA} , and the estimated deflections due to load in states 1 and 2 $a_{l,1}$ and $a_{l,2}$, respectively. Then, the following applies:

$$a_{l,1} = a_{LFEA} \cdot \frac{E_c}{E_{c,ef}} \cdot \frac{I_c}{I_1^*} \quad (15)$$

$$a_{l,2} = a_{LFEA} \cdot \frac{E_c}{E_{c,ef}} \cdot \frac{I_c}{I_2^*} \quad (16)$$

If the aim is to estimate an initial deflection, then $E_{c,ef}$ can be replaced by E_c .

Deflections due to shrinkage— $a_{cs,1}$ and $a_{cs,2}$ for states 1 and 2, respectively—are formulated based on the general principles explained in subsection 2.1, and are proposed in the following form:

$$a_{cs,1} = a_{LFEA} \cdot \frac{E_c \cdot S_1^*}{M} \cdot \varepsilon_{cs}(t, t_s) \cdot \frac{I_c}{I_1^*} \quad (17)$$

$$a_{cs,2} = a_{LFEA} \cdot \frac{E_c \cdot S_2^*}{M} \cdot \varepsilon_{cs}(t, t_s) \cdot \frac{I_c}{I_2^*} \quad (18)$$

where M is the maximum moment over the strip length under the considered load combination (typically quasi-permanent). For the slab shown in Figure 1, the bending moment diagram is shown in Figure 2 (in this case $M = 21.77$ kNm/m).

Therefore, deflections in states 1 and 2, a_1 and a_2 are

$$a_1 = a_{l,1} + a_{cs,1} \quad (19)$$

$$a_2 = a_{l,2} + a_{cs,2} \quad (20)$$

The next step is controlling whether the slab is cracked. The cracking moment M_{cr} is calculated as

$$M_{cr} = W_{c,1} \cdot f_{ct} \quad (21)$$

where $W_{c,1}$ is the gross section modulus and f_{ct} is the concrete tensile strength. Typically, the parameter most naturally associated with the cracking moment is the concrete flexural strength $f_{ct,fl}$. Nonetheless, the choice of tensile strength for Equation (21) should be based on considerations of the potential effect of shrinkage on the reduction of tensile strength. As a general recommendation and unless more precise data is available, the axial tensile strength f_{ctm} can be used (EN 1992-1-1, 2004). For example, Pecić (2012) determined that this is a sufficiently accurate way of accounting for the reduction of flexural strength due to shrinkage.

In current methods, Equation (2) is used to calculate ζ . However, this formulation is not adequate for two-way slabs for two reasons. First, two-way slabs possess a greater possibility for redistribution compared with one-way members; hence, the effect of uncracked parts of the slab can be greater. This can be taken into account by substituting M in Equation (2) by a geometric mean of M_{cr} and M (i.e. $\sqrt{M_{cr} \cdot M}$), as was already proposed in the “bilinear” method (CEB, 1985). Reorganizing Equation (2) leads to the following formulation:

$$\zeta = 1 - \beta \cdot \frac{M_{cr}}{M} \quad (22)$$

Secondly, each representative section of a two-way slab that is considered has moments in two orthogonal directions, e.g. M_x and M_y . Since the choice of strip is arbitrary, this means that there are also two distribution coefficients, ζ_x and ζ_y :

$$\zeta_x = 1 - \beta \cdot \frac{M_{cr}}{M_x} \quad (23)$$

$$\zeta_y = 1 - \beta \cdot \frac{M_{cr}}{M_y} \quad (24)$$

As explained previously, existing “strip” or “crossing beam” methods suggested calculating deflections twice, once in the x direction, once in the y direction and adopting an average value of deflections as the estimate (Gilbert and Ranzi, 2011). In this, method, the deflection is calculated only in one direction (arbitrarily chosen) and deflections in states 1 and 2 are interpolated using an average of the distribution coefficients ζ_x and ζ_y :

$$\zeta_{mod} = \frac{\zeta_x + \zeta_y}{2} \geq 0.75 \cdot \max\{\zeta_x, \zeta_y\} \quad (25)$$

However, a lower bound should be imposed on the coefficient ζ_{mod} . This is done in order to ensure that the proposed method has a relatively smooth transition towards one-way slabs. Namely, as the ratio of spans increases, so does the ratio of moments in x and y directions; consequently, ζ_{mod} will become increasingly lower than the larger of the values of ζ_x and ζ_y until one of them becomes zero and ζ_{mod} becomes $0.5 \cdot \max\{\zeta_x, \zeta_y\}$. Therefore, for span ratios very close to one-way slabs ($L_x/L_y \approx 2$) the proposed method would give unconservative results.

Hence, a lower limit of a 25% reduction of ζ relative to the larger value of ζ_x and ζ_y is imposed in Equation (25); this limit is also valid if the considered strip is uncracked in one of the directions (i.e. if ζ_x or ζ_y are equal to 0). The effect of this lower limit is the limitation on the increase of “effective stiffness,” $(EI)_{eff}$, of

the system through the reduction of ζ to ζ_{mod} . This limit depends on the larger value of ζ_x and ζ_y and the stiffness of the system in states 1 and 2. For slabs where the ratio of stiffness in state 2 to state 1 is typically 0.15–0.30 and for ζ value 0.7–0.9, the limit on effective stiffness increase imposed by Equation (25) is approximately 25%. Considering the overall precision of deflection control methods, such a difference for cases close to one-way slabs is acceptable.

Finally, the deflection is estimated by interpolating between a_1 and a_2 as

$$a = (1 - \zeta_{\text{mod}}) \cdot a_1 + \zeta_{\text{mod}} \cdot a_2 \quad (26)$$

3. Verification of the proposed ζ -method

3.1. Experimental studies on long-term deflections of two-way slabs

As with any other method, a verification against experimental results is essential to assess the model's performance, its accuracy (closeness of predictions and experimental values) and its precision (scatter of the ratio between predicted and experimentally obtained values). However, experiments on long-term deflections of two-way slabs are extremely rare. The authors were able to find only four studies in literature with sufficiently detailed reports that would enable calculating deflections (Gilbert and Guo, 2005; Jaccoud and Favre, 1982; Radnić and Matešan, 2008; Tellenbach and Favre, 1985).

The study by Gilbert and Guo (2006) is the more comprehensive and representative study of the four. The authors tested a total of seven flat slabs (labelled S1–S7) with four 3×3 m spans, supported on nine 1.25-m high columns with 200×200 mm cross-sections. The slabs had depths of 100 and 90 mm with two different reinforcement layouts (designated I and II in Figure 4 and used for slabs S1–S5 and S6–S7, respectively). Considering the slab dimensions, the specimens could be considered to represent realistic slabs in a 1:2 scale. The slabs were cast from five concrete batches: single batches were used for slabs S1–S3 and then slabs S4–S5 and S6–S7 were cast from the same batches. Importantly, compressive strength, modulus of elasticity, flexural strength, creep and shrinkage were also measured. The concretes were relatively low-strength with 28-day strengths of different batches varying between 18 and 38 MPa. Creep and shrinkage were moderately high with 450-day values of the creep coefficient and shrinkage strain between 2.34 and 2.89 and 0.56‰ and 0.84‰, respectively, for different batches.

Different loading histories were used for the specimens, Figure 5. Slab S2 was excluded since it was described that the slab underwent a hail storm during the experiment which had a significant effect on the

evolution of deflections. Within their study, Gilbert and Guo (2006) performed an extensive measurement of deflections at different points on the slabs, shown as points #1–16 in [Figure 6](#). However, the authors did not report results for the 16 points separately but performed an averaging according to the slab's symmetry: deflections are reported as Δ_1 (average of points #4, #6, #11, and #13), Δ_2 (average of points #8 and #9), Δ_3 (average of points #1, #2, #15, and #16), Δ_4 (average of points #5 and #12), and Δ_5 (average of points #3, #7, #10, #14). The deflection time curves of slabs S4–S7 are shown in [Figure 7](#).

As for Radnić and Matešan (2008), they tested only one corner-supported 1×1 m slab with a thickness of only 30 mm, [Figure 8](#). In this regard, the slab can be considered to be in approximately 1:6 scale. The slab was produced from a concrete with relatively high 28-day compressive strength of 55 MPa. The authors reported measured compressive strength, modulus of elasticity and flexural strength. Shrinkage and creep was also measured but reported in another study (Radnić and Matešan, 2010) from which the data was obtained. The slab was loaded with a total load of 10 kN distributed over a central area of 0.9×0.9 m. Importantly, loading was done after 92 days when a significant part of the shrinkage had already occurred (Radnić and Matešan, 2010).

Deflections were measured in three points (MD1, MD2, and MD3) and the results are shown in [Figure 9](#). It should be noted that points MD2 and MD3 are on slab edges whose behaviour is actually more representative of one-way slab behaviour; e.g. at point MD3 there is no bending moment in the x -direction.

Within their extensive experimental programme, Jaccoud and Favre (1982) tested two series of two-way slabs: series B (consisting of slabs B1, B2, and B3) and series D (consisting of slabs D1, D2, and D3); however, slabs from series D are reported in more detail by Tellenbach and Favre (1985).

Slabs from series B were rectangular slabs with a thickness of 120 mm and spans of 4 m, continuously simply supported along the edges. Slabs B1 and B2 had reinforcement consisting of $\varnothing 10$ mm bars spaced at 150 mm in both directions, whereas slab B3 had reinforcement $\varnothing 12/150$ mm and $\varnothing 8/150$ mm in orthogonal directions. Slab B1 was loaded with a load of 10.85 kN/m^2 , and slabs B2 and B3 with a load of 18.76 kN/m^2 . Slabs were loaded at the age of 14 days and deflections were measured over a one-year period and reported for the centre point of the slabs. Compressive strength, tensile strength, modulus of elasticity, creep coefficient, and shrinkage strain values are reported (Jaccoud and Favre, 1982).

Slabs from series D were also rectangular slabs with a thickness of 110 mm and spans of 2.7 m. Slabs D1 and D2 were continuously simply supported along the edges, whereas slab D3 was point-supported in the corners. Slabs D1 and D3 had a reinforcement consisting of $\varnothing 10/150$ mm in both directions. Slab D2 had the same reinforcement, however, it was placed diagonal to the edges. Therefore, slab D2 was excluded from the analysis and only slabs D1 and D3 were considered. Slabs D1 and D3 were loaded at an age of 21 days with 26.6 and 8.9 kN/m², respectively, and deflections were measured during two years and reported for the centre point of the slabs. Compressive strength, tensile strength, modulus of elasticity, creep coefficient, and shrinkage strain values are reported (Tellenbach and Favre, 1985).

3.2. LFEA and calculation of deflections

All of the analysed slabs were modelled using the Tower 8 software (Radimpex, Serbia). The slabs were modelled using the default thin-plate elements available in the software. Mesh size was selected as $\sim 2 \cdot h$, where h is the slab depth. Supports were modelled as point supports and load was applied as uniformly distributed. The only mechanical property necessary for modelling was the modulus of elasticity, E_c , which was entered as the experimentally measured value at age of loading as reported in the corresponding studies.

Following an LFEA, bending moments and elastic displacements were obtained for the slabs. An example of LFEA results for slab S4 is shown in Figure 10 and 11. The results necessary for calculation of deflections are summarized in Tables 1–4. Deflections at the centres of panels were considered from both directions (x and y) in order to demonstrate the insensitiveness of the methods to the choice of strip direction. This means that centre-panel deflections were calculated twice. For example, for slabs S1, S3, S4, S5, S6, and S7 this meant calculating the deflection at point Δ_1 using an x -direction strip and superimposing that deflection on to deflections calculated for points Δ_2 and Δ_3 (this was labelled Δ_1 - x), and then again calculating the deflection at point Δ_1 using a y -direction strip and superimposing that deflection on to deflections calculated for points Δ_4 and Δ_5 (this was labelled Δ_1 - y).

For shrinkage and creep, experimental measurements were used as a starting point. Using the reported data, the *fib* Model Code 2010 (FIB, 2013) creep and shrinkage models were fitted to the experimental values and extrapolated to obtain values at different times, where necessary. For example, in the case of (Gilbert and Guo, 2005), values were reported only up to 450 days and had to be extrapolated to later times. Furthermore, slabs S1 and S3 were loaded in phases at different times and creep coefficients were reported only for loading

at 14 days. In these cases, the fitted *fib* Model Code 2010 models were used for later loading ages as well, and the effective creep coefficient principle was used to normalize the different loading phases. In the case of (Radnić and Matešan, 2008), most of the shrinkage had already occurred by the time of loading (92 days); however, this shrinkage was taken into account when calculating the initial deflections. As for the cracking moment, the reported tensile strengths were used. Specifically, in the case of (Gilbert and Guo, 2005), full flexural strength reported at the time of loading, i.e. $f_{ct,fl}$, was used since the slabs were cured until loading and the effect of shrinkage could therefore be neglected.

Considering that the experimental programmes tested slabs for relatively short durations, the deflection measurements were biased towards shorter measurement times. This is not representative of general practice where deflections would be calculated for a service life of up to 50 years. In particular, the coefficient β in Equation (2) theoretically changes from 1.0 to 0.5 immediately after loading (representing a simplified and easy approach), causing a discontinuity when the evolution of deflections over time is modelled. Previously, researchers have proposed that β should be a function of time decreasing from 1.0 to 0.5 (Tošić, 2018). Therefore, when modelling the experimental results from (Gilbert and Guo, 2005; Jaccoud and Favre, 1982; Radnić and Matešan, 2008; Tellenbach and Favre, 1985), a linear decrease of β from 1.0 to 0.5 over 100 days was adopted, remaining at 0.5 thereafter.

As for modelling tension stiffening in those tests that had non-constant load using Equation (22), the coefficient ζ was taken as the maximum obtained using M_{cr} and M at the time of application of the loads and at every time t of calculation (where a reduced M_{cr} was calculated by the assumed simplification of using the factor β). For example, for a maximum load applied at time t_0 and afterwards reduced to a lower sustained value, the maximum ζ was found between two options: using full M_{cr} ($\beta = 1$) and maximum M at time t_0 , and using a reduced $\beta \cdot M_{cr}$ ($\beta < 1$) with sustained M at time t .

In order to facilitate understanding of the method, a detailed calculation of the long-term deflection for point Δ_2 of slab S4 is provided in the Appendix.

3.3. Calculated deflections and comparison with experiments

After calculating all of the deflections, the results are analysed in terms of the calculated-to-experimental deflection ratio, a_{calc}/a_{exp} . The a_{calc}/a_{exp} ratio for all the slabs and all measurement points is plotted against time and shown in Figures 12 and 13.

Overall, the method is able to satisfactorily capture the evolution of deflections over time. This is especially important considering that the trends are very similar for all considered experiments. Since these cover a wide range of slab geometries, reinforcements, loading procedures and durations, this finding is very important and confirms the versatility and robustness of the method. The mean value of the $a_{\text{calc}}/a_{\text{exp}}$ is 1.33 with a coefficient of variation of 32.9%. Considering the range of parameters covered by the considered slabs and the associated uncertainties with experimental measurements and deflection methods in general, this can be considered a satisfactory result.

As seen by comparing Figures 12 and 13, the method performs somewhat better on simply supported slabs (Figures 13) relative to continuous systems (Figures 12). This is expected, as basing deflection calculations of continuous systems on the linear elastic moment distribution can lead to overestimation (Pecić, 2012). A further improvement of the method in this direction, based on detailed nonlinear numerical analyses is a possibility left for future work. Furthermore, slightly inferior performance of the method is seen for earlier times (<100 days), after which the method's performance "stabilizes." This can be partially explained by the fact that this is the period of major crack development – something that is difficult to accurately capture with a simplified method as this one. This is also evident from the predictions for slab S5 – the extent of cracking due to an early and short application of a very large load (relative to the sustained load) cannot be fully captured by the proposed method. At the same time, the case of slab S5 is not representative of live-to-dead load ratios of buildings. Nonetheless, for the general case of long-term deflections—which are in any case the major interest in reinforced concrete slabs—the method performs satisfactorily well.

4. Conclusions

In this study, a novel method for calculating deflections of two-way slabs—the "extended ζ -method for two-way members"—was proposed and validated. The method is based on using elastic deflections of slabs that are easily obtainable using linear elastic analysis in any FEM software. Then, through a series of transformations, the time-dependent deflection, taking into account cracking, shrinkage, and creep is obtained. Based on the method formulation and validation presented in this paper, the following conclusions are drawn:

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- 1)
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- It was shown that elastic deflections obtained through linear elastic analyses in FEM software can be a
- basis for successfully estimating instantaneous and time-dependent deflections of two-way slabs (either
- flat slabs or edge-supported slabs, of arbitrary geometry, support and load conditions).
- The method is successfully derived when considering strips to obtain bending moment values in
- orthogonal directions and when using the average reinforcement ratio along the selected strip to calculate
- geometric properties. Furthermore, the method is robust with regards to the choice of strip direction.
- Time-dependent deflections due to load and shrinkage can easily be obtained by a transforming the
- stiffness properties of a representative section of a slab used in the FEM software to account for cracking,
- creep and shrinkage.
- The verification of the proposed model against the limited number of available experimental results shows
- satisfactory agreement with measured values. Although the method, in general, predicts higher
- instantaneous deflections, the predictions of time-dependent deflections are more consistent with
- experimental results. More experimental studies and nonlinear numerical analyses are necessary in the
- future to further study the deflection behaviour of slabs in the initial period after loading, the evolution of
- cracking and the effect of uncracked sections of slabs on the overall deflection.

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Even though the verification showed a satisfactory performance of the method, the results are

dependent on the range of parameters in the considered experiments. Nonetheless, the proposed extended ζ -

method for two-way members can become a relevant contribution for new generations of codes and a practical

tool for designers.

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α_e

β

δ_{cs}

ε_{cs}

ζ

ζ_{mod}

ρ

modulus ratio

empirical factor for cracking moment reduction

coefficient for deflections due to shrinkage dependent on the statical system

concrete shrinkage strain

distribution coefficient

modified distribution coefficient

tension reinforcement coefficient

1	1	ρ_m	average tension reinforcement coefficient
2			
3	2	ρ'	compression reinforcement coefficient
4			
5	3	ρ_m'	average compression reinforcement coefficient
6			
7	4	φ	concrete creep coefficient
8			
9	5	$(1/r)_{cs}$	curvature of a concrete section due to shrinkage
10			
11	6	$(1/r)_{load}$	curvature of a concrete section due to load
12			
13	7	a	deflection
14			
15	8	a_1	deflection of a concrete member in the uncracked state
16			
17	9	a_2	deflection a concrete member in the fully cracked state
18			
19	10	a_{calc}	calculated deflection
20			
21	11	a_{cs}	deflection of a concrete member due to shrinkage
22			
23	12	$a_{cs,1}$	deflection of a concrete member due to shrinkage in the uncracked state
24			
25	13	$a_{cs,2}$	deflection of a concrete member due to shrinkage in the fully cracked state
26			
27	14	a_{el}	elastic deflection
28			
29	15	a_{exp}	experimentally measured deflection
30			
31	16	$a_{LFEA,qp}$	deflection of a two-way slab obtained using LFEA
32			
33	17	a_{load}	deflection of a concrete member due to load
34			
35	18	$a_{l,1}$	deflection of a concrete member due to load in the uncracked state deflection of
36			
37	19	$a_{l,2}$	a concrete member due to load in the fully cracked state cross-section width
38			
39	20	b	flexural strength of concrete
40			
41	21	f_{ct}	time
42			
43	22	t	age of concrete at load application
44			
45	23	t_0	age of concrete at the onset of drying
46			
47	24	t_s	modulus of elasticity of concrete
48			
49	25	E_c	effective modulus of elasticity of concrete
50			
51	26	$E_{c,ef}$	modulus of elasticity of steel
52			
53	27	E_s	transformed second moment of area in the uncracked state
54			
55	28	I_1^*	transformed second moment of area in the fully cracked state
56			
57	29	I_2^*	
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2	1	I_c	gross concrete section second moment of area
3			
4	2	K	coefficient for deflections due to load dependent on the statically system
5			
6	3	L	span
7			
8	4	M	bending moment
9			
10	5	M_{cr}	cracking moment
11			
12	6	M	maximum moment acting on a member along its length
13			
14	7	S_1^*	moment of area of reinforcement in the uncracked state
15			
16	8	S_2^*	moment of area of reinforcement in the fully cracked state
17			
18	9	$W_{c,1}$	gross section modulus of concrete cross-section
19			
20	10	Z	elastic vertical displacement
21			
22	11	Z_{max}	maximum elastic vertical displacement of a strip of a two-way concrete slab
23			
24	12	Z_s	elastic vertical displacement of the support strip of strip of a two-way concrete slab
25			
26	13	CoV	coefficient of variation
27			
28	14	FEM	finite element method
29			
30	15	LFEA	linear finite element analysis
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33	16	SLS	Serviceability Limit State
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35	17	ULS	Ultimate Limit State
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References

- ACI 435R-95, 2003. Control of deflection in concrete Structures, Deflection in Concrete Structures. American Concrete Institute, Farmington Hills, MI.
- Beeby, A.W., Narayanan, S., 2005. Designer's guide to Eurocode 2: Design of concrete structures. Thomas Telford, London.
- CEB-FIP, 1991. Model Code 1990. Comité Euro-International Du Béton, Paris. <https://doi.org/10.1680/ceb-fipmc1990.35430>
- CEB-FIP, 1978. Model Code 1978. Comité Euro-International Du Béton, Paris.
- CEB, 1985. Design Manual on Cracking and Deformations. Comité Euro-International Du Béton, Lausanne.
- CEB Bulletin 235, 1997. Serviceability Models - Behaviour and Modelling in Serviceability Limit States including repeated and sustained loads - Progress Report.
- EN 1992-1-1, 2004. Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings. CEN, Brussels.
- FIB, 2013. fib Model Code for Concrete Structures 2010. International Federation for Structural Concrete (fib), Lausanne. <https://doi.org/10.1002/9783433604090>
- FIB Bulletin 52, 2010. Structural Concrete - Textbook on Behaviour, Design and Performance, Volume 2 - Basis of design. International Federation for Structural Concrete (fib), Lausanne. <https://doi.org/10.1017/CBO9781107415324.004>
- FIB Bulletin 92, 2019. Serviceability Limit States of Concrete Structures. International Federation for Structural Concrete (fib), Lausanne.
- FIB Task Group 4.4, 2008. Practitioner's guide to finite element modelling of reinforced concrete structures. FIB, Lausanne.
- Gardner, N.J., Fu, H.C., 1987. Effects of high construction loads on the long-term deflections of flat slabs. ACI Struct. J. 84, 349–360. <https://doi.org/10.14359/1641>
- Gardner, N.J., Shao, X.Y., 1996. Punching shear of continuous flat reinforced concrete slabs. ACI Struct. J. 93, 219–228. <https://doi.org/10.14359/1494>
- Ghali, A., Favre, R., Eldbadry, M., 2002. Concrete Structures. Stresses and Deformation. Taylor & Francis, New York.
- Gilbert, R.I., Guo, X.H., 2005. Time-dependent deflection and deformation of reinforced concrete flat slabs -

- 1 An experimental study. ACI Struct. J. 103, 304.
- 2
- 3 2 Gilbert, R.I., Ranzi, G., 2011. Time-dependent behaviour of concrete structures. Spon Press, New York.
- 4
- 5 3 Jaccoud, J.-P., Favre, R., 1982. Fleche des structures en béton armé: Verification experimentale d'une
- 6
- 7 4 methode de calcul. Lausanne.
- 8
- 9 5 Jokinen, E.P., Scanlon, A., 1987. Field measured two-way slab deflections. Can. J. Civ. Eng. 14, 807–819.
- 10
- 11 6 <https://doi.org/10.1139/l87-119>
- 12
- 13 7 Kabir, A.F., 1976. Nonlinear analysis of reinforced concrete panels, slabs and shell for time dependent effects.
- 14
- 15 8 Lee, J. Il, 2007. Role of early-age concrete properties and construction loading on slab serviceability.
- 16
- 17 9 Pennsylvania State University.
- 18
- 19 10 Marí, A., Torres, L., Oller, E., Barris, C., 2019. Performance-based slenderness limits for deformations and
- 20
- 21 11 crack control of reinforced concrete flexural members. Eng. Struct. 187, 267–279.
- 22
- 23 12 <https://doi.org/10.1016/j.engstruct.2019.02.045>
- 24
- 25 13 Marí, A.R., 2000. Numerical simulation of the segmental construction of three dimensional concrete frames.
- 26
- 27 14 Eng. Struct. 22, 585–596. [https://doi.org/10.1016/S0141-0296\(99\)00009-7](https://doi.org/10.1016/S0141-0296(99)00009-7)
- 28
- 29 15 Marí, A.R., Bairán, J.M., Duarte, N., 2010. Long-term deflections in cracked reinforced concrete flexural
- 30
- 31 16 members. Eng. Struct. 32, 829–842. <https://doi.org/10.1016/j.engstruct.2009.12.009>
- 32
- 33 17 Pecić, N., 2012. Improved method for deflection control of reinforced concrete structures. Faculty of Civil
- 34
- 35 18 Engineering, Belgrade University.
- 36
- 37 19 Radnić, J., Matešan, D., 2010. Testing of prestressed concrete shell under long-term load and unload. Exp.
- 38
- 39 20 Mech. 50, 575–588. <https://doi.org/10.1007/s11340-009-9242-9>
- 40
- 41 21 Radnić, J., Matešan, D., 2008. Experimental testing of reinforced concrete slab behaviour under long-term
- 42
- 43 22 load. Materwiss. Werksttech. 39, 157–161. <https://doi.org/10.1002/mawe.200700261>
- 44
- 45 23 Scanlon, A., Suprenant, B.A., 2011. Estimating Two-Way Slab Deflections. Concr. Int. 33, 29–34.
- 46
- 47 24 Scordelis, A.C., 1984. Computer models for nonlinear analysis of reinforced and prestressed concrete
- 48
- 49 25 structures. PCI J. 29, 116–135. <https://doi.org/10.15554/pcij.11011984.116.135>
- 50
- 51 26 Simulia, 2018. Abaqus 6.14, Abaqus 6.14 Analysis User's Guide. Dassault Systèmes.
- 52
- 53 27 Taylor, P.J., Heiman, J.L., 1977. Long-term deflection of reinforced concrete flat slabs and plates. ACI J.
- 54
- 55 28 Proc. 74, 556–561. <https://doi.org/10.14359/11046>
- 56
- 57 29 Tellenbach, A., Favre, R., 1985. Flèches à long terme des dalles en béton à l'état fissuré. Ingénieurs Archit.
- 58
- 59
- 60

- 1
2 1 suisses 111, 219–225.
3
4 2 Tošić, N., 2018. Behaviour of reinforced concrete beams made with recycled and waste materials under long-
5
6 3 term loading. University of Belgrade.
7
8 4 Tošić, N., Kurama, Y., 2020. Parametric numerical study on service-load deflections of reinforced recycled
9
10 5 aggregate concrete slabs and beams based on fib Model Code 2010. Struct. Concr.
11
12 6 <https://doi.org/10.1002/suco.202000015>
13
14 7 Vollum, R.L., Moss, R.M., Hossain, T.R., 2002. Slab deflections in the Cardington in-situ concrete frame
15
16 8 building. Mag. Concr. Res. 54, 23–34. <https://doi.org/10.1680/macr.2002.54.1.23>
17
18 9
19
20
21
22
23
24
25
26
27
28
29
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1 List of tables:

2 Table 1. LFEA bending moment results for slabs from (Gilbert and Guo, 2005)

Slab	Point	M_x (kNm/m)	M_y (kNm/m)
S1*	Δ_1	2.82 (4.40)	3.09 (4.82)
	Δ_2	-1.49 (-2.32)	4.41 (6.88)
	Δ_3	0.83 (1.30)	3.84 (5.99)
	Δ_4	4.05 (6.32)	-1.89 (-2.95)
	Δ_5	3.89 (6.07)	0.74 (1.15)
S3*	Δ_1	1.10 (2.68)	1.20 (2.93)
	Δ_2	-0.58 (-1.41)	1.71 (4.18)
	Δ_3	0.32 (0.78)	1.50 (3.64)
	Δ_4	1.58 (3.84)	-0.73 (-1.79)
	Δ_5	1.51 (3.68)	0.29 (0.70)
S4	Δ_1	4.29	4.65
	Δ_2	-1.96	6.66
	Δ_3	1.30	5.79
	Δ_4	6.03	-2.62
	Δ_5	5.94	0.28
S5 ²	Δ_1	1.11	1.19
	Δ_2	-0.50	1.72
	Δ_3	0.34	1.49
	Δ_4	1.55	-0.67
	Δ_5	1.53	0.19
S6	Δ_1	2.67	2.89
	Δ_2	-1.23	4.18
	Δ_3	0.82	3.63
	Δ_4	3.78	-1.64
	Δ_5	3.72	0.46
S7	Δ_1	2.83	3.06
	Δ_2	-1.30	4.42
	Δ_3	0.87	3.84
	Δ_4	4.00	-1.65
	Δ_5	3.94	0.49

¹ values in parentheses are for the second loading stage

² moments due to maximum applied load at t_0 are equal to those of S4

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Table 2. LFEA bending moment results for slabs from (Jaccoud and Favre, 1982; Radnić and Matešan, 2008; Tellenbach and Favre, 1985)

Slab	Point	M_x (kNm/m)	M_y (kNm/m)
B1	centre-panel	7.57	7.57
B2 & B3	centre-panel	13.09	13.09
D1	centre-panel	8.55	8.55
D3	centre-panel	7.10	7.10
	edge-x	9.88	0
	edge-y	0	9.88
MD	MD1	1.30	1.30
	MD2	1.74	0
	MD3	0	1.74

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1 Table 3. LFEA elastic displacements and deflections for slabs from (Gilbert and Guo, 2005)

Slab	Point	$Z_{\max}$ (mm)	$Z_{s,1}$ (mm)	$Z_{s,2}$ (mm)	$a_{\text{LFEA},qp}$ (mm)
S1 ¹	Δ_1-x	1.79 (2.79)	1.41 (2.20)	1.05 (1.64)	0.56 (0.87)
	Δ_1-y	1.79 (2.79)	1.28 (2.00)	1.07 (1.67)	0.62 (0.97)
	Δ_2	1.41 (2.20)	0	0	1.41 (2.20)
	Δ_3	1.05 (1.64)	0	0	1.05 (1.64)
	Δ_4	1.28 (2.00)	0	0	1.28 (2.00)
	Δ_5	1.07 (1.67)	0	0	1.07 (1.67)
S3 ¹	Δ_1-x	0.93 (2.25)	0.73 (1.78)	0.54 (1.32)	0.29 (0.70)
	Δ_1-y	0.93 (2.25)	0.66 (1.61)	0.55 (1.34)	0.32 (0.78)
	Δ_2	0.73 (1.78)	0	0	0.73 (1.78)
	Δ_3	0.54 (1.32)	0	0	0.54 (1.32)
	Δ_4	0.66 (1.61)	0	0	0.66 (1.61)
	Δ_5	0.55 (1.34)	0	0	0.55 (1.34)
S4	Δ_1-x	5.20	4.10	3.04	1.63
	Δ_1-y	5.20	3.71	3.14	1.78
	Δ_2	4.10	0	0	4.10
	Δ_3	3.04	0	0	3.04
	Δ_4	3.71	0	0	3.71
	Δ_5	3.14	0	0	3.14
S5	Δ_1-x	1.34	1.06	0.78	0.42
	Δ_1-y	1.34	0.96	0.81	0.46
	Δ_2	1.06	0	0	1.06
	Δ_3	0.78	0	0	0.78
	Δ_4	0.96	0	0	0.96
	Δ_5	0.81	0	0	0.81
S6	Δ_1-x	3.78	2.97	2.20	1.19
	Δ_1-y	3.78	2.69	2.28	1.29
	Δ_2	2.97	0	0	2.97
	Δ_3	2.20	0	0	2.20
	Δ_4	2.69	0	0	2.69
	Δ_5	2.28	0	0	2.28
S7	Δ_1-x	3.99	3.15	2.33	1.25
	Δ_1-y	3.99	2.85	2.41	1.36
	Δ_2	3.15	0	0	3.15
	Δ_3	2.33	0	0	2.33
	Δ_4	2.85	0	0	2.85
	Δ_5	2.41	0	0	2.41

¹ values in parentheses are for the second loading stage

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Table 4. LFEA elastic displacements and deflections for slabs from (Jaccoud and Favre, 1982; Radnić and Matešan, 2008; Tellenbach and Favre, 1985)

Slab	Point	$Z_{\max}$ (mm)	$Z_{s,1}$ (mm)	$Z_{s,2}$ (mm)	$a_{\text{LFEA,qp}}$ (mm)
B1	centre-x	2.61	0	0	2.61
	centre-y	2.61	0	0	2.61
B2	centre-x	4.52	0	0	4.52
	centre-y	4.52	0	0	4.52
B3	centre-x	4.52	0	0	4.52
	centre-y	4.52	0	0	4.52
D1	centre-x	1.61	0	0	1.61
	centre-y	1.61	0	0	1.61
D3	centre-x	3.43	2.28	2.28	1.15
	centre-y	3.43	2.28	2.28	1.15
MD	MD1-x	3.73	2.43	2.43	1.30
	MD1-y	3.73	2.43	2.43	1.30
	MD2	2.43	0	0	2.43
	MD3	2.43	0	0	2.43

APPENDIX: Worked example of deflection calculation

Example description

This Appendix presents a detailed calculation example for the application of the proposed ζ -method extension to two way slab deflection calculation. The example is a detailed calculation of one of the results presented in the associated article. Herein, a step-by-step procedure is shown for the slab S4 (Gilbert and Guo, 2005) and measurement point Δ_2 .

First, the finite element model is described and input data are presented. Then, deflections after 776 days are calculated for measurement point Δ_2 .

Finite element model

The slab is described in detail in (Gilbert and Guo, 2005) and in the article with which this Appendix is associated. The slab has a thickness of 90 mm, reinforced with reinforcement layout I per (Gilbert and Guo, 2005) (Figure 4) and is modelled with a modulus of elasticity of 22010 MPa.

The data necessary for using the model are the modulus of elasticity and gross concrete second moment of area of a 1-m strip:

$$E_c = 22010 \text{ MPa}; I_c = \frac{b \cdot h^3}{12} = \frac{1000 \cdot 90^3}{12} = 60750000 \text{ mm}^4$$

The slab is loaded with a uniformly distributed load of 5.26 kN/m² at 14 days and the load is maintained until the age of 776 days (total duration of load is 762).

The results of a linear elastic finite element analysis in terms of bending moments and displacements is shown in Figures 10 and 11, respectively.

Calculation of average reinforcement ratios

The strip considered for measurement point Δ_2 is identified in Figure 10 from which corresponding lengths of positive and negative bending moments necessary for average reinforcement calculation using Equations (11) and (12) were calculated. Reinforcement of slab S4 is shown in Figure 4.

Average positive and negative reinforcement calculation is shown in Table A1, respectively. Effective depth was taken as 77 mm per Figure 3 in (Gilbert and Guo, 2005).

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1 Table A1. Average reinforcement ratios for calculating deflection Δ₂ of slab S4

	<i>l_A</i> (m)	<i>l_B</i> (m)	<i>l_C</i> (m)	<i>ρ_A</i> (%)	<i>ρ_B</i> (%)	<i>ρ_C</i> (%)	<i>ρ_m</i> (%)	<i>ρ_A</i> ' (%)	<i>ρ_B</i> ' (%)	<i>ρ_C</i> ' (%)	<i>ρ_m</i> ' (%)
Δ ₂	0.15	0.70	2.15	0.41	0.73	0.46	0.52	0.46	0.46	0	0.13

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2 **Calculation of deflection Δ₂ after 776 days**

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Input data for calculating deflection Δ₂ is shown below. For creep and shrinkage, the experimentally measured values in (Gilbert and Guo, 2005) up to 450 days were fitted to the *fib* Model Code 2010 creep and shrinkage models (FIB, 2013) and extrapolated to 776 days. The following data is obtained from LFEA:

M_y = 6.66 kNm; *M_x* = 1.96 kNm; *a_{LFEA}* = 4.10 mm

and the following is input data from Gilbert and Guo (2005):

φ = 3.53; *ε_{cs}* = 0.94‰; *f_{ct}* = 2.76 MPa; *E_s* = 200000 MPa

A_s = 402.2 mm²; *A_s*' = 101.1 mm²; *d₁* = *d₂* = 13 mm

Then, the following can be calculated:

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$$E_{c,ef} = \frac{E_c}{(1 + \varphi)} = \frac{22010}{1 + 3.53} = 4912 \text{ MPa}; \alpha_e = \frac{E_s}{E_{c,ef}} = \frac{220000}{4912} = 40.71$$

12
$$M_{cr} = W_c \cdot f_{ct} = \frac{1000 \cdot 90^2}{6} \cdot 2.76 = 3.73 \text{ kNm}$$

13 Geometric properties for state 1 are calculated using the following set of equations:

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$$A_1^* = A_c + (\alpha_e \cdot A_s) + (\alpha_e \cdot A_s') = 110489 \text{ mm}^2$$

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$$z_{1,1}^* = \frac{A_c \cdot z_{c,1} + (\alpha_e \cdot A_s) \cdot d_1 + (\alpha_e \cdot A_s') \cdot (h - d_2)}{A_1^*} = 41.45 \text{ mm}$$

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$$I_1^* = I_c + A_c \cdot (z_{c,1} - z_{1,1}^*)^2 + (\alpha_e \cdot A_s) \cdot (d_1 - z_{1,1}^*)^2 + (\alpha_e \cdot A_s') \cdot (h - d_2 - z_{1,1}^*)^2 = 80338561 \text{ mm}^4$$

17
$$S_{1,s}^* = (\alpha_e \cdot A_s) \cdot (z_{1,1}^* - d_1) - (\alpha_e \cdot A_s') \cdot (h - d_2 - z_{1,1}^*) = 319512 \text{ mm}^3$$

18 For state 2 it is first necessary to determine the position of the neutral axis:

$$\xi = \alpha_e \cdot (\rho_m + \rho'_m) \cdot \left(-1 + \sqrt{1 + \frac{2 \cdot \left(\rho_m + \rho'_m \cdot \left(\frac{d_2}{d} \right) \right)}{\alpha_e \cdot (\rho_m + \rho'_m)^2}} \right) = 0.451$$

Then, geometric properties are determined as

$$A_2^* = A_{2c} + (\alpha_e \cdot A_s) + (\alpha_e \cdot A'_s) = 55208 \text{ mm}^2$$

$$z_{2,1}^* = \frac{A_{2c} \cdot z_{2c,1} + (\alpha_e \cdot A_s) \cdot d_1 + (\alpha_e \cdot A'_s) \cdot (h - d_2)}{A_2^*} = 55.28 \text{ mm}$$

$$I_2^* = I_{2c} + A_{2c} \cdot (z_{2c,1} - z_{2,1}^*)^2 + (\alpha_e \cdot A_s) \cdot (d_1 - z_{2,1}^*)^2 + (\alpha_e \cdot A'_s) \cdot (h - d_2 - z_{2,1}^*)^2 = 45161899 \text{ mm}^4$$

$$S_{2,s}^* = (\alpha_e \cdot A_s) \cdot (z_{2,1}^* - d_1) - (\alpha_e \cdot A'_s) \cdot (h - d_2 - z_{2,1}^*) = 602879 \text{ mm}^3$$

Now, deflections due to load in states 1 and 2 can be calculated using [Equations \(15\) and \(16\)](#), respectively:

$$a_{l,1} = a_{\text{LFEA}} \cdot \frac{E_c}{E_{\text{c,ef}}} \cdot \frac{I_c}{I_1^*} = 4.1 \cdot \frac{22010}{4912} \cdot \frac{60750000}{80338561} = 13.89 \text{ mm}$$

$$a_{l,2} = a_{\text{LFEA}} \cdot \frac{E_c}{E_{\text{c,ef}}} \cdot \frac{I_c}{I_2^*} = 4.1 \cdot \frac{22010}{4912} \cdot \frac{60750000}{45161899} = 24.71 \text{ mm}$$

Deflections due to shrinkage in states 1 and 2 are calculated using [Equations \(18\) and \(19\)](#), respectively:

$$a_{\text{cs},1} = a_{\text{LFEA}} \cdot \frac{E_c \cdot S_1^*}{M_y} \cdot \varepsilon_{\text{cs}}(t, t_s) \cdot \frac{I_c}{I_1^*} = 4.1 \cdot \frac{22010 \cdot 319512}{6.66 \cdot 10^6} \cdot 0.94 \cdot 10^{-3} \cdot \frac{60750000}{80338561} = 3.08 \text{ mm}$$

$$a_{\text{cs},2} = a_{\text{LFEA}} \cdot \frac{E_c \cdot S_2^*}{M_y} \cdot \varepsilon_{\text{cs}}(t, t_s) \cdot \frac{I_c}{I_2^*} = 4.1 \cdot \frac{22010 \cdot 602879}{6.66 \cdot 10^6} \cdot 0.94 \cdot 10^{-3} \cdot \frac{60750000}{45161899} = 10.34 \text{ mm}$$

Deflections in states 1 and 2 are

$$a_1 = a_{l,1} + a_{\text{cs},1} = 13.89 + 3.08 = 16.97 \text{ mm}; a_2 = a_{l,2} + a_{\text{cs},2} = 24.71 + 10.34 = 35.05 \text{ mm}$$

The distribution coefficients ζ for directions x and y are

$$\zeta_x = 1 - \beta \cdot \frac{M_{\text{cr}}}{M_x} = 1 - 0.5 \cdot \frac{3.73}{1.96} < 0 \Rightarrow \zeta_x = 0$$

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1 1 $\zeta_y = 1 - \beta \cdot \frac{M_{cr}}{M_y} = 1 - 0.5 \cdot \frac{3.73}{6.66} = 0.72$

2 2 $\zeta_{mod} = \frac{\zeta_x + \zeta_y}{2} \geq 0.75 \cdot \max\{\zeta_x, \zeta_y\} \Rightarrow \zeta_{mod} = 0.75 \cdot \zeta_y = 0.54$

3 Finally, the total deflection of point Δ_2 after 776 days is

4 4 $a_{\Delta 2} = \zeta_{mod} \cdot a_{\Delta 2,2} + (1 - \zeta_{mod}) \cdot a_{\Delta 2,1} = 0.54 \cdot 35.05 + (1 - 0.54) \cdot 16.97 = 26.73 \text{ mm}$

Figure 1. LFEA displacements for a flat slab (left) and its corner span (right)

160x117mm (300 x 300 DPI)

Figure 2. LFEA bending moments in the slab corner span for the x-direction midspan strip
160x103mm (300 x 300 DPI)

Figure 3. Bending moment diagram defining l_A , l_B and l_C (Marí et al., 2010)

79x62mm (330 x 330 DPI)

Figure 4. Reinforcement plan for slabs S1–S7 (authorized reprint from Gilbert and Guo (2006))

84x70mm (600 x 600 DPI)

Figure 5. Loading histories for slabs S1-S7 (authorized reprint from Gilbert and Guo (2006))

84x81mm (600 x 600 DPI)

Figure 6. Deflection measurement points on slabs S1–S7 (authorized reprint from Gilbert and Guo (2006))

84x71mm (600 x 600 DPI)

Figure 7. Deflection time histories for slabs S4, S6, and S7 (authorized reprint from (Gilbert and Guo, 2006))

177x162mm (600 x 600 DPI)

Figure 8. Reinforcement plan of slab MD (authorized reprint from Radnić and Matešan (2008))

150x93mm (96 x 96 DPI)

Figure 9. Deflection time curves of slab MD (authorized reprint from Radnić and Matešan (2008))

187x73mm (96 x 96 DPI)

Figure 11. LFEA elastic displacements Z, slab S4

160x82mm (300 x 300 DPI)

Figure 12. Evolution of the $a_{\text{calc}}/a_{\text{exp}}$ ratio over time for slabs S1, S3, S4, S5, S6, and S7 (Gilbert and Guo, 2005)

Figure 13. Evolution of the $a_{\text{calc}}/a_{\text{exp}}$ ratio over time for slab MD (Radnić and Matešan, 2008), slabs B1, B2, and B3 (Jaccoud and Favre, 1982), and slabs D1 and D3 (Tellenbach and Favre, 1985)

80x147mm (300 x 300 DPI)