FRACTAL BEHAVIOUR OF DISTANCES BETWEEN CONSECUTIVE AFTERSHOCKS: THE EXAMPLES OF LANDERS, NORTHRIDGE AND HECTOR MINE SOUTHERN CALIFORNIA MAINSHOCKS

--Manuscript Draft--

Manuscript Number:	GEOJ-D-18-00375R1
Full Title:	FRACTAL BEHAVIOUR OF DISTANCES BETWEEN CONSECUTIVE AFTERSHOCKS: THE EXAMPLES OF LANDERS, NORTHRIDGE AND HECTOR MINE SOUTHERN CALIFORNIA MAINSHOCKS
Short Title:	Fractal structure of distances between consecutive aftershocks
Article Type:	Article
Manuscript Classifications:	460: Seismology
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Funding Information:	
Abstract:	Distances between consecutive aftershocks are analysed by means of mono- and multifractal theory with the aim of quantifying the complexity of the physical mechanism governing them, as well as their predictability and predictive instability. Hausdorff, H_a , and Hurst, H , exponents are determined by semivariograms and rescaled analysis, respectively. The exponent β of the power law describing power spectral contents is also quantified. These three parameters permit a generation of fractional Gaussian noise, fGn, simulating distances. The complexity and predictive instability of physical mechanism generating the series of distances is quantified by means of the correlation dimension, μ^* , the Kolmogorov entropy, κ , and the Lyapunov exponents, λ_i , which are based on the reconstruction theorem formulation. Additionally, the multifractal detrended fluctuation analysis, MF-DFA, contributes with a different point of view to quantify the complexity of the series, in terms of fractal spectral width, W , spectral asymmetry, B , and the critical Hölder exponent, α_0 . By one hand, the MF-DFA is applied to the complete set of distances characterising the whole aftershock process. By the other hand, the MF-DFA is applied to segments of the series of distances with the aim of determining the evolution of the complexity since the mainshock up to the end of the stress relaxation process. Finally, an ARIMA multilinear regression process is applied to obtain some improvements, in comparison with fGn simulations, on the prediction of distances. The database for this analysis is obtained from the Southern California Seismic Network (SCSN) catalogue. Three series of aftershocks equalling to or exceeding magnitudes of 2.0, assuring seismic catalogue completeness, and associated with Landers (06/28/1992), Northridge (01/17/1994) and Hector Mine (10/16/1999) mainshocks are obtained. It is worth mentioning that common mono-multifractal behaviour for the three aftershocks series is not detected, whatever aftershock periods or segments of them are considered.
Keywords:	Interevent distances, mutifractality, ARIMA process, Southern California
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Question	Response
Does this manuscript belong to a special issue?	No
Response to Reviewers:	<p>Reviewer 1</p> <p>1)A better description of future contributions on earthquake forecasting, based on results derived from the present manuscript, are developed in pages 29-30 (lines 715-725), including the reference of the abstract published in the European Geophysical Union, EGU, congress of 2018.</p> <p>2)With respect to the notation of the Hurst exponent, it has been finally described all along the manuscript by H instead of H_u, with the aim of avoiding misunderstandings with respect to the Hausssdorf exponent, H_a, (see also first point of reviewer 2 report).</p> <p>3)Dots (periods) in the equations have been deleted when they are not followed by new paragraphs.</p> <p>4)The meaning of symbol A_m is briefly explained in line 264 and its effects on the computation of k are briefly remembered in lines 299-300.</p> <p>5)The paragraph corresponding to the beginning of section 3.4.2. is revised for a better understanding of multifractal concepts.</p> <p>6)The way how the slope $\mu(m)$ is quantified with a good degree of accuracy is briefly explained in Pages 11-12, lines 276-279.</p> <p>7)A more detailed meaning of the Kolmogorov entropy is developed in lines 530-540 (page 21), being cited that MAD index values obtained on the multiregression process are coherent with Kolmogorov coefficient values (lines 662-666).</p> <p>8)Labels with the name of every seismic episode are included within figures depicting results for the three seismic sequences.</p> <p>9)After enlarging horizontally Figure 1, real and FFG-noise series could be more easily compared.</p> <p>10)The symbol $m > 2.0$ has been finally chosen to define in the corresponding figures the minimum seismic magnitude assuring data completeness.</p> <p>Reviewer 2</p> <p>1)The right use of characters H, H_a, β, μ and , respectively designing Hurst and Hausssdorf coefficients, the exponent of the power spectrum, the correlation dimension and the Kolmogorov exponent, has been accurately revised along the manuscript.</p> <p>2)Line 54 (old version): effectively, "magnitude events" is obvious and it is removed.</p> <p>3)Lines 65-67 (old version): it has been transformed to single sentence.</p> <p>4)Stress release, as consequence of changes on tectonic stresses, is used sometimes. Nevertheless, "strain energy release" is a much more appropriate description in this case and it is finally used along the text.</p> <p>5)The small details corresponding to lines 118, 140 and 526 have been amended. Additionally, "Hector mine" and "Hectormine" words have been finally substituted in the text and in the figure labels by Hector Mine.</p> <p>6)Lines 690-696: the fragmentation of the paragraph is avoided, in agreement with the reviewer's comment.</p>

1 **Fractal behaviour of distances between consecutive aftershocks.**
2 **The examples of Landers, Northridge and Hector Mine**
3 **Southern California mainshocks**

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20 **Running Title:**

21 Fractal structure of distances between consecutive aftershocks

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ABSTRACT: Distances between consecutive aftershocks are analysed by means of mono- and multifractal theory with the aim of quantifying the complexity of the physical mechanism governing them, as well as their predictability and predictive instability. Hausdorff, Ha , and Hurst, H , exponents are determined by semivariograms and rescaled analysis, respectively. The exponent β of the power law describing power spectral contents is also quantified. These three parameters permit a generation of fractional Gaussian noise, fGn, simulating distances. The complexity and predictive instability of physical mechanism generating the series of distances is quantified by means of the correlation dimension, μ^* , the Kolmogorov entropy, κ , and the Lyapunov exponents, λ_i , which are based on the reconstruction theorem formulation. Additionally, the multifractal detrended fluctuation analysis, MF-DFA, contributes with a different point of view to quantify the complexity of the series, in terms of fractal spectral width, W , spectral asymmetry, B , and the critical Hölder exponent, α_0 . By one hand, the MF-DFA is applied to the complete set of distances characterising the whole aftershock process. By the other hand, the MF-DFA is applied to segments of the series of distances with the aim of determining the evolution of the complexity since the mainshock up to the end of the stress relaxation process. Finally, an ARIMA multilinear regression process is applied to obtain some improvements, in comparison with fGn simulations, on the prediction of distances. The database for this analysis is obtained from the Southern California Seismic Network (SCSN) catalogue. Three series of aftershocks equalling to or exceeding magnitudes of 2.0, assuring seismic catalogue completeness, and associated with Landers (06/28/1992), Northridge (01/17/1994) and Hector Mine (10/16/1999) mainshocks are obtained. It is worth mentioning that common mono-multifractal behaviour for the three aftershocks series is not detected, whatever aftershock periods or segments of them are considered.

Key-words: Interevent distances, mutifractality, ARIMA process, Southern California

1. INTRODUCTION

Aftershock series have been analysed from very different points of view since the beginning of the 20th century up to nowadays by considering different relationships in seismology ([Kanamori and Anderson, 1975](#)). Bath's law, establishing the expected maximum magnitude of the aftershocks, and the verification of the Gutenberg-Richter law for aftershock series, are worthy of mention. Concerning these questions, it can be cited ([Shcherbakov et al., 2005](#)), among others, who accurately analysed these questions for the three aftershock series considered at the present paper. Additionally, pioneering studies of Omori ([Omori, 1894](#)) lead to the well-known Omori's modified law ([Utsu, 1961](#); [Ogata, 1983](#); [Utsu et al., 1995](#); [Lolli and Gasperini, 2006](#)), among many others, which has permitted to propose several mechanisms concerning strain energy release along the aftershock process and some models explaining the aftershock sequence mechanisms. A couple of them could be cited. By one hand, the ETAS model ([Ogata, 1988 and 1998](#); [Ogata and Zhuang, 2006](#)), among others, with every aftershock generating its own aftershock process. By the other hand, the segregation of the aftershocks into leading aftershocks, following the Omori's modified law, and short aftershock episodes, known as cascades. Whereas the ETAS model is nowadays of universal acceptance, the segregation into leading aftershocks and cascade episodes, as far as we know, has been only considered by [Correig et al. \(1997\)](#) to analyse a seismic crisis of relative low magnitude at Eastern Pyrenees (Europe) and by [Monterrubio et al. \(2016\)](#), who analysed three seismic crisis of high magnitude in Southern California. In spite of a certain controversy with respect to the reliability of this segregation mechanism, its main patterns were confirmed by [Correig et al., \(1997\)](#), [Moreno et al., \(2001\)](#) and [Monterrubio et al. \(2015\)](#), who obtained very similar results by simulating aftershock sequences taking as reference cellular automata algorithms ([Bak et al., 1988 and 1990](#); [Barriere and Turcotte, 1994](#)).

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77 All these strategies to achieve a better knowledge of the aftershock mechanisms,
78 basically the strain energy release and stress propagation processes, are based on the
79 elapsed time between consecutive aftershock events. An alternative to this point of view
80 could be the analysis of distances between consecutive events. These series of distances
81 should be governed by the rearrangement of the state of stresses (Main, 1996), bearing in
82 mind (Kilb et al., 2002) the distinction between the dynamic and static compounds of the
83 stress change after a mainshock. The dynamic compound would depend on the spatial and
84 temporal evolution of the rupture or, in other words, on radiated seismic waves and on fault
85 displacement. Conversely, the static compound would only depend on fault displacement.
86 It is also worth of mention that, according to Kilb et al. (2002), who analysed stress
87 changes after Landers (1992) mainshock at Southern California, and King et al. (1994),
88 dynamic and static stress changes are similarly relevant in triggering aftershocks and new
89 mainshocks after the passage of seismic waves. In short, it is assumed in this paper the
90 hypothesis that the complexity of the strain energy release process after a mainshock could
91 not be only described by the Omori's law and alternative physical models just mentioned.
92 Even though the strain energy release with time is universally accepted, the starting point
93 previous to the mainshock is a heterogeneous and complex spatial distribution of stresses,
94 interacting with lithospheric fractures. Additionally, it is also accepted that these fractures
95 (faults) are not homogeneously distributed in the lithosphere, but with a spatial fractal
96 structure. According to this assumption, the epicentres of the seismic activity and
97 distances between consecutive events should be also fractal distributed. For instance, in
98 agreement with Turcotte (1997), the fractal dimension of regional or worldwide seismicity
99 would be very approximately equal to twice the b-parameter of the Gutenberg-Richter law.

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In agreement with [Davidsen and Paczusky \(2005\)](#), among others, it can be assumed that the statistical distribution of distances between consecutive aftershocks is quite well described by a power law. This fact suggests by one hand a mono- or multifractal structure of these interevent distance series. By the other hand, cellular automata models, based on yet mentioned self-organised criticality theory and power laws, could lead to algorithms to simulate interevent distances. It is also worth mentioning that the fractal structure of the seismicity has been also analysed and interpreted by [Robertson et al. \(1995\)](#) in terms of percolation theory ([Stinchcombe and Watson, 1976](#)) to explain fractal dimensions obtained for spatial distribution of epicentres and hypocenters of seismic events.

This manuscript explores the mono- and multifractal structure of three distance series corresponding to the aftershocks generated by three mainshocks of notable Richter magnitude (Landers, 06/28/1992; Northridge, 01/17/1994; Hector Mine, 10/16/1999) in Southern California. Rescaled range analysis and computation of semivariograms and power spectra permit to obtain the Hurst, H , and Hausdorff, Ha , exponents and the exponent β of the power law describing the mentioned spectra. In agreement with [Turcotte \(1997\)](#) and [Malamud and Turcotte \(1999\)](#), these three parameters suggest that fractional Gaussian noise, fGn, series could be the first approach to the real distance series. The complexity of a nonlinear physical mechanism governing distances between consecutive events is analysed by means of the reconstruction theorem ([Diks, 1999](#)) and more concretely, with the concepts of correlation dimension, μ , and Kolmogorov entropy, κ . The predictive instability of distances is evaluated, again in terms of the reconstruction theorem, by mean of the Lyapunov exponents ([Wiggins, 2003](#)) and the Kaplan-Yorke dimension ([Kaplan and Yorke, 1979](#)).

Another relevant point of view of the complexity of the physical mechanism governing the series of distances is the multifractal spectrum analysis, which is based on

the multifractal detrended fluctuation analysis, MF-DFA (Kantelhardt et al., 2002), specifically designed for multifractal analyses of data series. The spectral width, W , the spectral asymmetry, B , and the critical Hölder exponent, α_0 , for which the multifractal spectrum reaches a maximum permit to define a complexity index, which has been used before in different scientific fields such as climatology, more specifically thermometric regimes, (Burgueño et al., 2014) or health sciences (Shimizu et al., 2002). It is worth mentioning that every seismic crisis depicts its own complexity pattern, indistinctly if the whole aftershock period or segments of this seismic activity are analysed. In consequence, it should be assumed that the physical mechanism (basically a stress relaxation process and a state of stresses propagation) governing the inter event distances has to be notably complex and distinct for every seismic crisis.

The last step of the analysis could be considered as a return to predictive algorithms. Besides simulations based on fGn series and quantification of complexity and predictive instability through results derived from the reconstruction theorem, a multilinear autoregressive, ARIMA, process (Box and Jenkins, 1976) is applied to the series of distances. Then, the prediction of distances between consecutive aftershocks can be compared with results derived from the fGn simulations.

The contents of the paper are distributed as follows: the most relevant characteristics of the three mainshocks and their aftershock series are briefly described in the section *Database*. A brief mathematical and conceptual description of the mono and multifractal algorithms is developed in the section *Methodology*. The main results of these algorithms applied to the series of distances between consecutive aftershocks and their implications are introduced in the section *Results*. The section *Prediction of Distances* includes a brief explanation of the ARIMA process and results of the fGn simulations compared with predictions obtained from the ARIMA process. Advances on the knowledge

concerning the structure of distances between consecutive events, as well as of not yet solved questions, are summarised in the section *Conclusions*.

2. DATABASE

The analysis of three seismic crises for large and shallow Southern California earthquakes (Landers, 1992; Northridge, 1994; Hector Mine, 1999), thus generating a high number of aftershocks, is expected permitting to investigate with detail properties and mechanisms governing series of distances between consecutive aftershocks. The aftershock series are provided by the Southern California Seismic Network (SCSN) catalogue, which is available at the address <http://www.dat.scec.org/ftp/catalogs/SCSN>. The most relevant features of the mainshocks and their aftershock sequences are summarized in Table 1, according to Shcherbakov et al. (2006), among others, who defined the aftershock zones according to Kagan (2002). (*Table 1 is about here*) In agreement with fits to the Gutenberg-Richter law (GR-law) obtained by Shcherbakov et al. (2005), and Lana et al. (2017), a minimum magnitude of completeness equal to 2.0 is chosen. This threshold magnitude would be in agreement with results derived by Wyss and Wiemer (2000) and Woessner and Wiemer (2005). The recording aftershocks period has been limited to one year after the mainshock, including in the three cases the highest generation rates of aftershocks (Lana et al, 2017).

Many studies concerning these three seismic crises have been published, especially for Landers mainshock and its aftershock series. A summary of many characteristics of these three seismic episodes can be found in Shcherbakov et al. (2005). With respect to Northridge, it has to be cited Hauksson et al. (1995), who revised seismological and tectonic aspects related to this seismic crisis, and Thio and Kanamori (1996), who evaluated the complexity of this earthquake and analyzed possible relationships with

aftershock mechanisms. The complex system of strike-slip faulting associated with the Hector Mine crisis was investigated by [Hauksson et al. \(2002\)](#) and implications for aftershock hazard were proposed by [Wiemer et al. \(2002\)](#). Additionally, the postseismic fault healing was analyzed by [Li et al., \(2003\)](#). Landers main event and its aftershock series have been more extensively studied than those of the other two seismic crisis. [Gomberg \(1996\)](#) analysed stress/strain changes and seismicity triggering seismicity after the mainshock and [Gross and Kisslinger \(1997\)](#) estimate tectonic stress rate by means of Landers aftershocks. It is also worth mentioning the complete revision of Landers earthquake sequence by [Hauksson et al. \(1993\)](#) and the evaluation of earthquake probabilities by taking into account foreshocks and aftershock series ([Jones, 1994](#)). Additionally, a valuable summary of the Landers seismic crisis was compiled in a special issue of the Bulletin of the Seismological Society of America in 1994. Among others, can be cited [Wald and Heaton \(1994\)](#), [Cohee and Beroza \(1994\)](#), [Wyatt et al. \(1994\)](#), [Roquemore et al. \(1994\)](#), [Hauksson \(1994\)](#), [King et al. \(1994\)](#), who analyzed slip distribution, crustal deformation, foreshocks and aftershocks and state of stresses before and after the mainshock.

3. METHODOLOGY

3.1. Rescaled-range Analysis

The first elementary step of a fractal analysis could be the process known as the rescaled-range analysis ([Korvin, 1992](#); [Goltz, 1997](#); [Diks, 1999](#); [Dimri, 2005](#)), which is quantified by the power law

$$R(\tau)/S(\tau) \propto \tau^H \quad (1)$$

being $R(\tau)$ and $S(\tau)$ the range of variation and standard deviation respectively of time series segments of length τ . If equation (1) is well accomplished, signs of self-similar or self-affine behavior of the analyzed series have to be accepted. The relevant parameter of this analysis is the Hurst exponent, H , which determine the random character of the series ($H \approx 0.5$), their persistence (H clearly exceeding 0.5) or anti-persistence (H clearly lowering 0.5). Besides the verification of the self-similar/affine character, the numeric values of H permit to decide if the randomness of the series discourages forecasting strategies or the persistent/anti-persistent character facilitates successful application of forecasting algorithms. Whereas time trends would be a first basic predictive tool in case of strong persistence, an average of previous elements would be another predicting approach for evident anti-persistence. Besides these first approaches to the predictive problem, algorithms based on fractional Gaussian noise simulations and autoregressive ARIMA process are also tested in Sections 3.5 and 4.1.

3.2. Self-affine Character

Basically, two procedures can be considered to confirm the self-affine behaviour of a series. One of them, according to [Turcotte \(1997\)](#), consists on the box-counting method, many times considered in fractal computations. Nevertheless, a question concerning anisotropy, and inherent to the self-affine character, has to be carefully taken into account. The fractal anisotropy of the analysed series implies that some of its properties, $g(x,y)$, in a two dimensional space do not scale as $g(rx,ry)$, with r the factor scale, but as $g(rx,r^{H_a}y)$, being H_a the Hausdorff exponent. Then, the Hausdorff exponent would be easily quantified by $H_a = 2 - D$, being D the fractal dimension derived from the mentioned box-counting process. An alternative, used in this paper, to the box-counting process is given by the

computation of the semivariogram $\gamma(\tau)$ of the series. Given that the semivariogram depends on the segment length τ (Malamud and Turcotte, 1999) as

$$\gamma(\tau) = \gamma_0 \tau^{2H_a} \quad (2)$$

an evaluation of H_a is straightforward by means of graphs of $\gamma(\tau)$ in a log-log scale, the slope of the expected straight line being two times the Hausdorff exponent.

3.3. Reconstruction Theorem

The reconstruction theorem (Diks, 1999) is a valuable fractal theory which permits to quantify the complexity and predictive instability of a physical process. With respect to the complexity, and possible chaotic behaviour, the minimum number of nonlinear equations, μ , which describe a physical mechanism, the embedding dimension, d_E , necessary to obtain an asymptotic value, μ^* , of the correlation dimension μ and the Kolmogorov entropy, κ , a measure of the loss of memory of the physical mechanism along the process, are fundamental parameters.

The mathematical aspect of the reconstruction theorem process is based on the generation of m-dimensional space vectors:

$$z(i) = \{x(i), x(i+1), \dots, x(i+m-1)\}, \quad i = 1, \dots, N \quad (3)$$

where $\{x(i)\}$ are empiric data. In terms of $z(i)$ vectors, the correlation integral, following the Grassberger-Procaccia formulation (Grassberger and Procaccia, 1983a, b), can be written as

$$C(m, r) = \lim_{N \rightarrow \infty} \frac{1}{N^2} \sum_{i,j=1}^N H\{r - \|z(i) - z(j)\|\} \quad (4)$$

with r an Euclidean distance in the m -dimensional space and $H\{.\}$ the Heaviside function. Equation (4) is the starting point to obtain numeric estimations of parameters μ and κ . If it is accepted that

$$C(m, r) = A_m e^{-m\kappa} r^{\mu(m)} \quad (5,a)$$

with A_m being the correlation amplitude for the reconstruction dimension m . After plotting this correlation integral (Equation 5,a) in terms of r on log-log scales,

$$\log\{C(m, r)\} = \log(A_m) - m\kappa + \mu(m) \log(r) \quad (5,b)$$

the slope $\mu(m)$ is straightforwardly obtained for every reconstruction dimension m . As mentioned in others papers related to this concept of correlation dimension, see for instance Burgueño et al. (2014), two factors which could lead to wrong estimations of $\mu(m)$ have to be carefully revised. One of them is the phenomenon of lacunarity (Turcotte, 1997), detected for small values of r . The other is the saturation of $C(m, r)$ for high values of r , whatever the value of m in both cases. A wrong diminished value of $\mu(m)$ would be then estimated if segments of r corresponding to lacunarity and saturation phenomena are included. This shortcoming concerning $\mu(m)$ can be easily solved by determining the r

interval for which $\log\{C(r)\}$ is strongly linear dependent on $\log(r)$ and, at the same time, the highest square regression coefficient is detected. In this way, the slope of the log-log linear dependence will be a good estimation of $\mu(m)$. After the right quantification of the correlation dimension for every reconstruction dimension m , it is relevant to observe that $\mu(m)$ tends asymptotically to a value, μ^* , which is interpreted as the minimum number of non-linear equations describing the mechanism that governs empiric data $\{x\}$, becoming μ^* a first evaluation of the complexity of the analysed time series. It has to be remembered that a high maximum dimension m or, in other words, embedding dimension d_E , to achieve the asymptotic value of $\mu(m)$, is usually related to a random behaviour of the analysed time series.

Together with the degree of complexity and the possible random component of the physical mechanism, the loss of memory of the physical system is another valuable parameter. This loss of memory is quantified by the Kolmogorov entropy, κ , which can be estimated as follows. According to Equation (5,b) and (Lana et al., 2010), naming $\log\{C(m,r)\} - \mu(m)\log(r)$, as $\alpha(m)$, we obtain

$$\alpha(m) = \log(A_m) - m\kappa \quad (5,c)$$

Equation (5,c) permits a fast and accurate estimation of κ by a least square regression of empiric $\alpha(m)$ in terms of m . Nevertheless, $\log(A_m)$ should be a constant. Empiric data confirms this behaviour only for m tending to ∞ (in practice, for high reconstruction dimensions m), being then assumed that A_{m+1}/A_m tends to 1.0. In other words, correlation amplitudes tend to be very similar for high reconstruction dimensions.

Another very important application of the reconstruction theorem is the quantification of the predictive instability. Taking advantage of m -dimensional vectors generated according Equation (3), in agreement with [Wiggins \(2003\)](#), the Lyapunov exponents, λ_j ($j=1, 2, \dots, m$), can be computed according to the algorithms proposed by [Eckmann \(1986\)](#) and [Stopp and Meier \(1988\)](#). Assuming that the addition of all the m Lyapunov exponents is negative, hypothesis which is not contradictory with respect to the existence of some positive ones generating predictive instability, the trajectory in the m -dimensional space is described by aperiodic orbits around a strange attractor defined by the Kaplan-Yorke dimension, D_{KY} ([Kaplan and Yorke, 1979](#)). Quantitatively,

$$D_{KY} = \ell_0 + \frac{1}{|\lambda_{\ell_0+1}|} \sum_{j=1}^{\ell_0} \lambda_j \quad (6)$$

being ℓ_0 the maximum number of positive and negative Lyapunov exponents, in decreasing order, accomplishing $\lambda_1 + \lambda_2 + \dots + \lambda_{\ell_0} \geq 0$.

3.4. Multifractal Spectrum

The concept of multifractal spectrum is necessary for a more complete understanding of the complexity of a non-linear system. Whereas the reconstruction theorem formulation permits a first approach to the complexity of a physical system and very valuable information about its loss of memory (Kolmogorov entropy) and predictive instability (Lyapunov exponents), parameters describing a multifractal spectrum lead to a new quantification of the system complexity and its expected degree of resolution.

3.4.1. Multifractal detrended fluctuation

The multifractal detrended fluctuation analysis (MF-DFA) represents a reliable way of characterization of multifractal non-stationary and stationary time series (Kantelhardt et al., 2002), being the main objective a quantification of the rescaled structure of the q -order moments of the analysed series, which is a process inherent to a multifractal structure analysis. It is worth of mention that the MF-DFA offers higher quality and simplicity in comparison with other algorithms with similar objectives (Feder, 1988; Muzy et al., 1994) and it has been applied to very different scientific fields (Shimizu et al., 2002; Feng et al., 2009; Dutta, 2010; Ghosh et al., 2012; Burgueño et al., 2014; Mali, 2014) such as human health, biology or Earth sciences (seismology and climatology), among others.

The MF-DFA method can be summarised by 5 steps as follows:

1. Determination of the profile of the time series. In other words, residuals from the average of the whole series are computed.
2. Segmentation of the profile. N_s non-overlapping segments of the profile, with equal length s , are generated. With the aim of improving computational accuracy and taking advantage of the whole data series, this second step is usually repeated starting from both extremes of the profile. In this way, $2N_s$ segments are available.
3. Computation of the local variance for each one of the $2N_s$ segments. A least-square polynomial fit is computed for every segment and the residual variance of this fitting process is quantified for every segment. In this way, the removal of non-stationarity from original series is assured. In agreement with Koscielny-Bunde et al. (2006), polynomial degrees varying from 2 to 5 could be convenient, being chosen in the present paper a fourth-order polynomial.
4. Computation of the q th-order fluctuation function, $F_q(s)$. This function is defined as

$$F_q(s) \equiv \left\{ \frac{1}{2 \cdot N_s} \sum_{v=1}^{2 \cdot N_s} [F^2(s, v)]^{q/2} \right\}^{1/q}, \quad q \neq 0, \quad q \in \mathfrak{R} \quad (7,a)$$

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355 $F^2(s, \gamma)$ represents the local variance for every one of the $2N_s$ segments and parameter q is
 356 chosen varying within the $+15, -15$ range. As a particular case, for zero-order ($q = 0$)

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$$F_0(s) \equiv \exp \left\{ \frac{1}{4 \cdot N_s} \sum_{v=1}^{2 \cdot N_s} \ln [F^2(s, v)] \right\} \quad (7,b)$$

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360 This last equation has to be applied to avoid some computational problems leading to
 361 singularities. At this step of the procedure it should be remembered that steps 2 to 4 have to
 362 be repeated for several segment lengths s . In agreement with [Kantelhardt et al. \(2002\)](#)
 363 convenient values of s would be in the $m+2 \leq s < N/4$ interval, where m is the selected
 364 order for polynomial fits of step 3.

365 5. Determination of the scaling behavior of $F_q(s)$. Under the hypothesis that the analyzed
 366 series are long-range power-law correlated, log-log plots of $F_q(s)$ versus s for each value of
 367 q would accomplish the power-law

368

$$F_q(s) \approx s^{h(q)} \quad (8)$$

370

371 with the exponent $h(q)$ depending on q .

372

373 If the analyzed series is non-stationary or noisy, (fractal Brownian signals for example), the
 374 exponent $h(q=2)$ will be larger than unity and it will satisfy $h(2) = H + 1$, where H is the
 375 well-known Hurst exponent ([Movahed and Hermanis, 2008](#); [Ge and Leung, 2012](#)),
 376 previously introduced in section 3.1. Conversely, for stationary time series, as daily

extreme temperatures (Burgueño et al., 2014) or monthly atmospheric circulation indices (Lana et al., 2016), the exponent $h(2)$ is exactly the Hurst exponent. By this reason, the exponent $h(q)$ is many times also cited as the generalized Hurst exponent.

At this point, it is important to differentiate monofractal and multifractal behaviours. Whereas monofractal behaviours do not contribute very relevantly to obtain more information about the structure of the mechanism governing the analysed series, details offered by multifractal behaviours are much more complete and interesting. In the first case, variances $F^2(s, \nu)$ are identical or very similar for all segments and the generalized Hurst exponent is reduced to the Hurst exponent of Section 3.1 quantifying randomness, persistence or anti-persistence for the whole series length. For multifractal behavior, segments with large variance dominate the q -order fluctuation function for positive q . Thus, the generalized Hurst exponent, $h(q)$, describes the scaling behavior of the segments with large fluctuations (Movahed and Hermanis, 2008). Conversely, for negative values of q , small fluctuations govern the q -order fluctuation function and then $h(q)$ describes the scaling behavior of the segments with small fluctuations (Movahed and Hermanis, 2008).

3.4.2. Singularity spectrum

The singularity spectrum $f(\alpha)$ is closely related to the q -order fluctuation function, $F_q(s)$, in terms of the generalized Hurst exponent, $h(q)$, and the Legendre transform

$$\alpha = h(q) + q \frac{d[h(q)]}{dq} \stackrel{\text{Legendre}}{\leftrightarrow} f(\alpha) = q[\alpha - h(q)] + 1 \quad (9)$$

where α is identified as the singularity strength or Hölder exponent and $f(\alpha)$ represents the fractal dimension of the different subset of the series. The multifractal scaling exponent, $\tau(q)$, is defined as

$$\tau(q) = qh(q) - 1 \quad (10)$$

and the Hölder exponent α is quantified as

$$\alpha = \frac{d\tau(q)}{dq}. \quad (11)$$

The singularity spectrum provides new viewpoints of the multifractal structure of a series, given that $f(\alpha)$ quantifies the fractal dimensions of subsets of the series associated with the same singularity strength α . Several relevant fractal parameters related to the singularity spectrum have to be mentioned. The first one is the critical Hölder exponent α_o , which corresponds to the maximum of $f(\alpha)$. A small value of α_o implies that the “fine-structure” of the physical mechanism cannot be recovered from empiric data. Conversely, a large value of α_o confirms the possibility of recovering the “fine-structure”. In agreement with these behaviors, the Hurst exponent, $h(q=2)$, and α_o shows a clear linear relationship, confirmed for instance by [Burgueño et al. \(2014\)](#).

The other two relevant parameters, spectral asymmetry and spectral width, are related to the standard mathematical structure of $f(\alpha)$. The shape of this function is well fitted to a quadratic function around the position α_o

$$f(\alpha) = A(\alpha - \alpha_o)^2 + B(\alpha - \alpha_o) + C \quad (12)$$

C is an additive constant theoretically equal to 1, given that $f(\alpha)$ is defined as a normalized function with a maximum equal to 1.0, and B quantifies the asymmetry of the spectrum, ($B = 0$ for a symmetric spectrum). A right-skewed spectrum, $B > 0$, indicates “fine-structure”, in correspondence with high values of the critical Hölder exponent, while a left-skewed spectrum, $B < 0$, suggests “smooth-structure”, in agreement with low values of α_o .

The spectrum width, W , defined as the range of α is computed as

$$W = \alpha_1 - \alpha_2 \quad (13)$$

with $f(\alpha_1) = f(\alpha_2) = 0$, being α_1 larger than α_2 . Given that zeros of the singularity spectrum are theoretically obtained for q -orders tending to $\pm\infty$, α_1 and α_2 are estimated by extrapolating the fitted curve of equation (12) to zero. Then, the wider the range of the Hölder exponent, the stronger is the multifractality. Similarly, the wider the range of α , the “richer” is the structure of the physical process. As a summary, a series with a high value of α_o , a wide range of fractal exponents and a right-skewed shape is more complex than one with the opposite characteristics (Shimizu et al. 2002). In terms of physical mechanism, a fine-structure could be analyzed provide that parameters α_o , B and W confirm high complexity. On the contrary, only a smooth-structure of the physical

mechanism would be obtained if the values of these three parameters suggest low complexity.

Finally, it is straightforward to conclude that monofractality will be characterized by a singularity spectrum $f(\alpha) = 1.0$ of null width. This fact implies that the dependence of $h(q)$ on q disappear, $h(q)$ is reduced to the Hurst exponent H , which at the same time is coincident with the single Hölder exponent α . It is also evident that a monofractal structure has to be assumed less complex than a multifractal.

3.5. Autoregressive Processes

At the present paper a simplified version of the autoregressive integrated moving average ARIMA($p, d, 0$) model (Box and Jenkins, 1976) has been chosen. Instead of first differences series, the ARIMA($p, 1, 0$) algorithm, also known as autoregressive AR(p) process, is directly applied to the elements of the series, being established the system of equations

$$x(i) = \theta + \sum_{k=1}^p \delta_k x(i-k) + a_i, \quad i = p+1, \dots, N \quad (14)$$

where it is assumed that, using an autoregressive order p , element $p+1$ of the series should be predicted with some level of uncertainty by the previous p samples. $\{x\}$ is the set of N empirical data, $\{\theta, \delta_1, \dots, \delta_p\}$ are the parameters of the autoregressive process of order p and $\{a\}$ is a noise series. The procedure leading to the estimation of optimal values of the parameters, based on matrix formulation and Crout's algorithm (Press et al., 1992), as well as the detection of the optimum autoregressive order, OAO, is detailed for instance in (Lana et al., 2016).

A convincing solution of the system of equations (14) needs for some criterion to decide when the optimum autoregression order, OAO, has been achieved. As in other applications of autoregressive concepts, the decision can be taken by searching for the order p minimizing the mean absolute deviation, MAD, (Stephenson et al. 2000), which is defined by

$$MAD = \frac{1.483}{N} \sum_{j=1}^N |x(j) - x^*(j)| \quad (15)$$

with $x^*(j)$ representing elements of the empiric series and $x(j)$ elements predicted according to the AR(p) process. It is worth of mention that the same MAD concept can be used to quantify the degree of predictability based on fGn simulations and to compare the degree of predictive accuracy of AR(p) and fGn methods.

4. RESULTS

4.1. Simulation by fGn Series.

The first objective of the analysis is to explore the possibility that fractional Gaussian noise, fGn, series (Mandelbrot et al. 1968; Mandelbrot and Wallis, 1969) could simulate empiric series. fGn series can be generated according to rules proposed for instance by Turcotte (1997), which include some constraints about values of the Hurst and Hausdorff exponents and the β exponent of the power law $S(\omega) \approx \omega^{-\beta}$ describing the power spectrum $S(\omega)$ of empiric series in terms of frequency ω . The self-affine character of the three series of distances is characterized, in agreement with Table 2, by a low Hausdorff

exponent, especially for Landers and Northridge series. According to the obtained results, the fractal anisotropy of the three series is very low as the three Hausdorff exponents are close to zero. With respect to the rescaled analysis (Table 2), almost equal Hurst exponents are derived for the three series, which are characterized by very moderate signs of persistence. The good fit of empiric rescaled data to a power law is qualitatively verified and quantitatively confirmed by a square regression coefficient close to 0.99. Similar good fits are observed for Northridge and Hector Mine series. Exponents β are also very similar, especially when comparing values for Landers and Hector Mine series. In short, in agreement with Turcotte (1997), the three series could be modeled by fGn series and, at the same time, in agreement with values of the Hurst exponent and the parameter β , characterized by moderate persistence and signs of stationary behavior.

(Figure 1 is about here)

Figure 1 shows a qualitative comparison between empiric and fGn series for a fragment of the whole recorded data. It has to be recognized several relevant shortcomings with respect to these simulations. First, it is quite evident that peaks of distances between consecutive events are not well reproduced by fGn series. Second, there is not a single time shift to obtain better coincidence of empiric and simulate peaks. Third, discrepancies between both series, which are quantified by the MAD index, (Stephenson et al., 2000) are always a quantity surpassing the standard deviation of the empiric series. Consequently, these first analyses only permit qualify the persistence and stationary character of the empiric series and reproduce some of their properties (degrees of persistence, stationary character and fractal anisotropy), by means of fGn series. *(Table 2 is about here)*

4.2. Consequences of the Reconstruction Theorem

As mentioned before, the reconstruction theorem permits an evaluation of the complexity of the physical mechanism (number of nonlinear equations and embedding dimension) and its loss of memory (Kolmogorov entropy), together with the predictive instability (Lyapunov exponents) and the complexity of the strange attractor (Kaplan-Yorke dimension) around which the reconstructed vectors describe aperiodic orbits in the multidimensional phase space. The results concerning all these parameters are summarized in [Table 2](#) and examples of correlation integral curves, correlation dimension, μ^* , embedding dimension, d_E , Kolmogorov entropy, κ , and Lyapunov exponents λ_i for the series of Landers are shown in [Figures 2a-2e](#). *(Figure 2 is about here)* As a general overview, it could be concluded that the complexity of the three series is very similar, given the almost equal number of non-linear differential equations needed to analyze the three distance series, determined by μ^* , which ranges from 13.1 to 13.5. The predictive instability is also quite similar for the three cases, with the first Lyapunov exponent, the most relevant component causing instability, ranging from 0.126 to 0.134. A quite different behavior is suggested for the loss of memory of the physical mechanism governing the distance series. According to the Kolmogorov entropy values, an increasing loss of memory is observed from Northridge ($\kappa = 1.292$) to Hector Mine ($\kappa = 1.425$) and to Landers ($\kappa = 1.823$). It is worth of mention the notable loss of memory for the seismic activity of Hector Mine and, especially, of Landers, characterized by their intense aftershock activities, and the agreement of the highest Kolmogorov entropy values with the highest MAD indexes of the multilinear regression (section 4.4).

4.3. Multifractality

Taking into account that the reconstruction theorem is applied to obtain only a global overview of all aftershock sequence process, a more detailed analysis could be convenient. With the aim of obtaining more information about patterns of the aftershock mechanisms, the multifractality of the three series is analyzed by means of the MF-DFA algorithm at two different time scales. First, it is applied to the whole series length. Figure 3 shows an example of the MF-DFA process for the Hector Mine series and Figure 4 depicts the multifractal spectra for the three series. *(Figures 3 and 4 are about here)* Second, the MF-DFA is applied to consecutive segments of the series, defined by moving windows with a fixed number of aftershocks for every step. Due to the quite different aftershock activity of Landers and Hector Mine areas when comparing with Northridge area, different window lengths and shifts have been chosen (lengths of 3,000 consecutive distances for Landers and Hector Mine series and a shift of 100 distances and lengths of 2,000 distances and a shift of 50 distances for Northridge). In short, a description of the multifractal characteristics of the distance between consecutive events along the stress relaxation process is obtained.

In agreement with Table 2, where multifractal parameters for every one of the whole series are summarised, a dependence of the multifractality (spectral width, W) on the aftershock intensity should be discarded, given that Landers depicts the lower value of W but Hector Mine the highest. On the contrary, the spectral asymmetry, B , diminishes with the aftershock activity, ranging from almost zero for Landers to small negative values for Hector Mine and clearly negative for Northridge. Consequently, only for Landers would be possible to detect fine-structures. Conversely, Hector Mine series would offer a smooth-structure description of the aftershock process, behaviour that would be even more evident for Northridge series. With respect to the Hölder critical exponent, α_0 , unclear conclusions can be obtained about the complexity of the distances, excepting that the less complex

series would be that corresponding to Hector Mine. This conclusion would not be in agreement with that derived from the reconstruction theorem, which qualify the three series of very similar complexity. Another relative shortcoming is some discrepancy for Hector Mine series between the Hurst exponent derived from the rescaled analysis and that estimated by $h(q=2)$. In spite of the quite clear similarity between α_0 and $h(q=2)$, also observed in other Earth sciences fields (see for instance [Burgueño et al., 2014](#)), whereas the Hurst exponent derived from rescaled analysis depicts some signs of persistence, that derived from the MF-DFA algorithm is linked to a clear random process.

A deeper insight in the aftershock process generating distance series can be obtained, as mentioned before, by analysing segments of a series of distances with the same window length and window shift. Instead of length measured as elapsed times since the main event, a fixed number of events have been chosen, following the strategy proposed by ([Shcherbakov et al., 2006](#)) for analysis based on the concepts of correlation integral and correlation length. [Figure 5a](#) depicts the evolution of the Hurst exponent, derived from $h(q=2) \approx H$, and the critical Hölder exponent for the different segments of the analysed series corresponding to Landers, Northridge and Hector Mine areas. A common pattern for the three series is the almost equal values of both parameters, step by step, along the series length. It should be remembered that this pattern has been yet observed by analysing the complete series and summarised in [Table 2](#). Besides this common pattern, it is noticeable the different behaviours along the aftershock processes. Whereas the moderate persistence is almost stationary along the strain energy release process for Northridge, the Hector Mine series is characterised by a decreasing trend of the persistence, reaching a level close to randomness, with small signs of anti-persistence, at the end of the aftershock process. The Landers series depicts a different behaviour, with fluctuations from a moderate persistence to predominant randomness and a notable

increase of the persistence at the end of the strain energy release. The degree of multifractality is quantified by the spectral width W (Figure 5b), taking as reference that W equal to zero would imply pure monofractality. Similarly to the critic Hölder and Hurst exponents, Landers aftershock series is characterised by notable fluctuations from almost zero (monofractality) to spectral widths close to 0.40. Northridge series shows an increase of the multifractality since the beginning of the aftershock process. Nevertheless, after achieving a maximum width close to 0.45 (the highest of the three series), a clear decreasing trend is observed at the end of the strain energy release process. The third example, corresponding to Hector Mine is characterised by an evident increasing trend, from $W = 0.20$ (beginning of the strain energy release process after the main shock), to $W = 0.35$ (at the end of the strain energy release process). If the two extreme multifractal exponents, α_1 and α_2 , are analysed (Figure 5c) it can be detected which are the most relevant Hölder exponents contributing to the multifractality of the analysed series. (Figures 5a,b,c are about here) Landers series are characterised by a like-step evolution of α_1 , being the fluctuations of W attributable to similar fluctuations on α_2 . The evolution of W for Northridge series would be a direct consequence of the evolution of α_2 . In other words, the increase of the multifractality and the final tendency towards monofractality would be an almost stationary evolution of α_1 with more relevant changes on α_2 . Finally, Hector mine depicts a quite evident stationary evolution of α_1 and a clear decreasing trend for α_2 , which would be responsible of the diminishing multifractal character, yet manifested in Figure 5b by the decreasing trend on W .

A synthesis of these multifractal parameters is given by the complexity index, CI (Shimizu et al., 2002; Burgueño et al., 2014) which is generated by adding normalized contributions of the spectral width, the critical Hölder exponent and the spectral asymmetry. In agreement with Shimizu et al. (2002), positive values of CI indicate that the

finest structure of the mechanism (strain energy release and stress propagation) governing the distance series would be determined. A detailed description of the stress field governing the aftershock process then could be possible. Conversely, only a smooth structure could be determined if the corresponding CI were negative. *(Figure 6 is about here)* Figure 6 depicts the evolution of this index for the three series. For Landers, a smooth structure could be obtained for a great length of the aftershock period. Only at the end of the strain energy release process, due to a notable increase of CI toward positive values, the fine structure could be determined. The Northridge series depicts a quite different evolution, with probable determination of smooth structures at the beginning and at the end of the aftershock process and fine-structure detection out of these two time intervals. Finally, Hector Mine series is characterised by a notable evolution from smooth- (beginning of the strain energy release and aftershock activity) to fine-structure (end of the aftershock activity).

4.4. Multilinear Regression

With the aim of obtaining some predictive procedure, as well as an alternative to simulations made by fGn series, the ARIMA process has been applied to reduce uncertainties on predicted distances between consecutive events. Figure 7 shows the evolution of the MAD index with the multi regression order p . For the three examples, especially for Landers and Hector Mine, a change can be observed on the slope of the MAD index in terms of p . A detailed search of local MAD minima for every case leads to optimal autoregression order within the 150 -200 range. *(Figure 7 is about here)* In spite of trends suggesting higher values of p to obtain deeper minima of MAD, shortcomings of the algorithm constrain the computations to values not exceeding $p = 200$. The required high number p of previous elements to achieve a prediction for the real element $p+1$ of the

series could be in some way expected by several reasons. By one hand the results of the reconstruction theorem suggest complex nonlinear systems of equations and loss of memory of the system. By the other hand, both the rescaled as the multifractal analyses lead to estimation of Hurst exponents with signs of time trends but without a clear persistence. In consequence, the applied multilinear regression could be assumed as an alternative to a more complex system of nonlinear equations describing the spatial and temporal stress distribution since the mainshock up to the end of the aftershock period, taking as fundamental data the distance between two consecutive aftershocks.

(Table 3 is about here)

In spite of the just mentioned shortcomings of the multilinear regression, it represents an improvement on the prediction of the inter event distances, as Table 3 shows. Values of MAD index for the ARIMA process are systematically lesser than those derived from the fGn series simulation and very similar to standard deviations of empiric data. It is also worth of mention that whereas the highest MAD indexes close to $p = 200$ are obtained for Landers and Hector Mine (close to 26.0 and 18.0 and with the highest Kolmogorov entropies), the lowest MAD index (close to 8.5) is obtained for Northridge, with the lowest Kolmogorov entropy. Additional details of the multilinear regression are shown in Figure 8. *(Figure 8 is about here)* The percentage of under/ overestimated cases exceeding $\pm\sigma$ is quite similar (15.1 to 17.2% and 16.3 to 20.4%) for the three multilinear regression and the percentage of cases with predictive errors exceeding 40 km are not very numerous and only detected on the series of Landers and Hector Mine. Nevertheless, the prediction residuals do not show a strong symmetry and do not follow a Gaussian distribution, as Figure 8 depicts, especially for Landers. In spite of this, the expected values of the residuals are small and predicted distances could not be submitted to relevant biases.

5. CONCLUSIONS

The complexity of the physical mechanism governing the series of distances between consecutive aftershocks, or in other words the strain energy release process after a mainshock, becomes evident by taking into consideration several aspects of the fractal analyses:

- The required high embedding dimensions, d_E , and the corresponding high correlation dimensions, μ^* , for the three series.

- The loss of memory of the physical process, which is a pattern confirmed after revising the Kolmogorov entropies. In spite of the values of μ^* are similar for the three seismic crisis, the highest values of κ are associated with aftershock processes with the highest generation rates (Landers and Hector Mine).

- Opposite to the behavior of the Kolmogorov entropy, a notably similar predictive instability, governed by the first Lyapunov exponent, is detected for the three seismic crises. Confirming this behavior, every strange attractor is characterized by a very similar D_{KY} fractal dimension.

This relatively simple dependence of the complexity on the generation rates of the aftershock activity becomes not so easy to explain when the multifractal analysis is applied to the three series given that:

- A single pattern of multifractal behavior, based on parameters α_0 , B and W , is not observed and a clear dependence of these parameters on aftershock generation rates cannot be assessed.

- The beginning, continuation and finishing of any of the three strain energy release processes depict different patterns of complexity, being quite difficult to establish a simple relationship between multifractal parameters and aftershock intensity.

In consequence, every aftershock process would be governed by its specific strain energy release and stress propagation characteristics. This fact would be true even when comparing Landers and Hector Mine examples, characterized by their spatial vicinity and with several studies assuming that Hector Mine could be triggered by Landers mainshock and its aftershock sequence.

The ARIMA process cannot be considered as an ultimate solution to the prediction of distances between consecutive events but only an improvement with respect to simulations based on fGn series. Additionally, the necessity of a high number p of empiric distances to make a prediction of distance $p+1$ would be in agreement with the main features of the mono- multifractal analyses and the complexity due to the superposition of two strain energy release distribution. By one hand, the well-known strain energy release along the aftershock processes (Omori's law), and by the other hand, the complex and heterogeneous spatial distribution and propagation of stresses along the aftershock process, subsequently activating faulting within the lithosphere.

Finally, it is worth of mention that the results obtained for the aftershock activity, especially those related to the multifractal spectrum parameters, would suggest analysing the behaviour of these parameters for the complete seismic series, including background seismicity, low magnitude shocks, mainshocks and the corresponding aftershocks. This would permit to search for possible characteristic patterns which could constitute some kind of predictive sign (or warning) of forthcoming mainshocks. Specifically, preliminary analyses developed by [Martínez and Lana \(2018\)](#) for the seismic activity of Southern

California, using the moving window strategy, detected some changes in the evolution of multifractal parameters along the different stages of the seismic activity, even during the aftershock process. Further analyses would be necessary to establish whether these changes could provide predictive signs of forthcoming earthquakes.

ACKNOWLEDGEMENTS

This research was supported by the Spanish Government through the project CGL2015-65913-P.

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LIST OF TABLES AND FIGURES

Table 1. Main shock magnitudes, M_W , coordinates (θ, λ) and dates of the seismic crisis of Landers (1), Northridge (2) and Hector Mine (3). L designs the aftershocks episode length (in days), m_0 the threshold magnitude to define the series of distances between consecutive events and N the number of aftershocks constrained by L and m_0 values

Table 2. Hausdorff and Hurst exponents (H_a and H), correlation and embedding dimensions (μ^* , d_E), Kolmogorov entropy, κ , the two first Lyapunov exponents, λ_1 and λ_2 , the Kaplan-Yorke dimension, D_{KY} , the exponent β of the power spectrum and the Hölder exponent, α_0 , spectral width, W , and asymmetry, B , of the multifractal spectrum for the seismic crisis of 1) Landers, 2) Northridge and 3) Hector Mine. An additional estimation of the Hurst exponent is obtained by imposing a q -order equal to 2

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Fig. 1. Normalised distances compared with the corresponding fGn series for the seismic crisis of Landers, Northridge and Hector Mine.

Fig. 2. Reconstruction theorem applied to Landers series. a) Correlation integral. b) Correlation dimension, μ^* . c) Kolmogorov entropy, κ . d) The three first Lyapunov exponents for the embedding dimension and one thousand iterations. e) Evolution of the three first Lyapunov exponents with the reconstruction dimension m .

Fig. 3. q -order fluctuation function, $f_q(s)$, generalised Hurst exponent, $h(q)$, multifractal scaling exponent, $\tau(q)$, and Hölder exponent, $\alpha(q)$, for the series of distances between consecutive events pertaining to the Hector Mine aftershock process.

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Fig. 6. Evolution of the complexity index, CI , for the different moving windows applied to Landers, Northridge and Hector Mine series.

Fig. 7. Evolution of MAD index depending on the autoregressive parameter, p , for the three series corresponding to the seismic crisis of Landers, Northridge and Hector Mine.

Fig. 8. Histogram of the three series of residuals.

Table 1 Landers (1), Northridge (2) and Hector Mine (3). L designs the aftershocks episode length (in days), m the threshold magnitude to define the series of distances between consecutive events and N the number of aftershocks constrained by L and m values

	M_w	θ (°N)	λ (°W)	Date	L	m	N
1	7.3	34:12.00	116:26.22	06/28/1992	365	2.0	12,264
2	6.7	34:12.78	118:32.22	01/17/1994	365	2.0	2,875
3	7.1	34:35.64	116:16.26	10/16/1999	365	2.0	5,447

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954 **Table 2.** Hausdorff and Hurst exponents (Ha and H), correlation and embedding
 955 dimensions (μ^* , d_E), Kolmogorov entropy, κ , the two first Lyapunov exponents, λ_1 and λ_2 ,
 956 the Kaplan-Yorke dimension, D_{KY} , the exponent β of the power spectrum and the Hölder
 957 exponent, α_0 , spectral width, W , and asymmetry, B , of the multifractal spectrum for the
 958 seismic crisis of 1) Landers, 2) Northridge and 3) Hector Mine. An additional estimation of
 959 the Hurst exponent, $h(q=2)$, is obtained by imposing a q-order equal to 2

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	Ha	H	$\mu^*(d_E)$	K	λ_1	λ_2	D_{KY}	β	α_0	W	B	$h(Q=2)$
1	0.002	0.65	13.1 (27)	1.823	0.126	0.098	12.24	0.159	0.65	0.18	-0.05	0.64
2	0.004	0.64	13.5 (31)	1.292	0.131	0.095	12.18	0.099	0.60	0.34	-1.57	0.65
3	0.031	0.65	13.2 (32)	1.425	0.134	0.105	12.60	0.173	0.53	0.41	-1.03	0.51

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Table 3.Standard deviation of empiric data, σ_E , and MAD values obtained after the fGn simulations, $MAD(fGn)$ and the multilinear regression, $MAD(mlr)$ for 1) Landers, 2) Northridge and 3) Hector Mine series

	σ_E	$MAD(fGn)$	$MAD(mlr)$
1	28.8	33.1	21.2
2	7.6	11.9	7.1
3	16.0	23.9	15.3

Figure 1.

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Figure 2

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1058 **Figure 3**

Figure 4

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1142 **Figure 6**

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Figure 7

Figure 8