ANALYSIS OF BRICK MASONRY WALLS STRENGTHENED WITH FIBRE REINFORCED POLYMERS AND SUBJECTED TO ECCENTRIC COMPRESSIVE LOADS

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ABSTRACT

Fibre Reinforced Polymers laminates are currently used to strengthen brick masonry walls. However, no specific evidences of the structural response of this strengthening system when brick walls are subjected to eccentric compressive loads, which might lead to second order bending effects, have been found. An experimental campaign on real-size walls has been carried out and the results have been compared with a numerical model, which predicted the observed shear/compressive masonry failure, and a new analytical approach, which has been used to accurately calculate the load-bearing capacity of the walls. Useful considerations for the strengthening design are also presented.

KEYWORDS

FRP, masonry, FEA, analytic, second order

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1. Introduction

Mechanism formation or shear failure are common collapse modes of masonry structures which are determined by the low tensile strength of this material. In this line, the researches by Yokel [1,2] are key pieces of the study of the bending/buckling failure of masonry. To prevent this phenomena, several strengthening technologies have been developed to improve the tensile response of the masonry. Among others, the most extended methodology might be the application of Fibre Reinforced Polymers (FRP), as presented by Ascione et al. in [3] and using Textile Reinforced Mortar (TRM), as studied by Babaeidarabad et al. [4], Harajli et al. [5] or Ortlepp et al. [6]. In particular, the use of TRM is significantly influenced by the composite response of the strengthening system, which has been studied, for example, by Larrinaga et al. [7]. Other researchers pointed out the advantages of placing plastic bands to improve the masonry tensile strength [8], studied alternative strengthening techniques (Engineered Cementitious Composites) against blast and impact loads [9] or considered the performance of Fibre Reinforced Concrete (FRC) and Mineral Based Composites (MBC) [10].

Strengthening masonry structures with FRP is a common actuation among the conservation and maintenance tasks carried out on the masonry buildings which require structural strengthening (see [11,12]). Although the drawbacks of this system (water vapour impermeability and insufficient mechanical compatibility with the masonry), which have been pointed out by several authors (see, for example the work by Baratta et al. [13], or the articles by Papanicolaou et al. [14,15]), FRP is still used for strengthening masonry structures because of its outstanding performance (justified in works like [11,12]) and the existence of consolidated guides and standards like [16,17]. In relation with the performance of the FRP, this strengthening technology is specially indicated for those interventions which require limiting the lateral deflection of the walls and look for a stiffer out-of-plane response of the structure. Moreover, it is also important to assess the second order out-of-plane response of those walls which have required an intervention using FRP in order to strengthen them against seismic events. Thus, the use of FRP is justified in some specific cases although it might not be the preferable option for general masonry interventions.

One of the most studied applications of the FRP on masonry structures is to strengthen masonry walls against seismic loads [18–21]. In addition, there are comprehensive researches about the out-of-plane shear behaviour of FRP-strengthened walls [22], the in-plane shear response [23] or the out-of-plane flexural response [24]. However, it is not typically analysed how these strengthening systems might affect the response and the load-bearing capacity of the walls, in particular when vertical compressive loads are applied. Moreover, it has to be mentioned that these vertical loads are usually applied with some eccentricity, which contributes to develop sudden mechanism failure collapses [25], which are related with the second order bending effects associated with the slenderness of the walls and the eccentricity of the load. Thus, the analysis of the influence of FRP strengthening systems on the in-plane compressive load-bearing capacity is a topic that needs to be addressed to better understand these structures and assess their mechanical resistance.

There are a few experimental studies about the influence of strengthening systems on the mixed bending/buckling response which leads to a mechanism formation failure mode. Among

them, the work by Dr. Garcia [26] has to be mentioned because of the novelty of strengthening multi-wythes ancient-like masonry walls subjected to eccentric load. Another research in this line [27] proved that superficial strengthening techniques might be effective at increasing the load-bearing capacity of brick masonry walls subjected to eccentric axial loads. However, the bibliographic research has not provided comprehensive experimental evidences about the performance of FRP strengthened walls subjected to this loading configuration.

On the other hand, numerical models are commonly developed on the basis of experimental results (used for calibrating and validating them) and applied to extrapolate those experimental evidences to a wider range of cases. With this purpose, the work by Cecchi et al. [28] modelled the in-plane response of FRP-strengthened masonry, the research by Milani [29] modelled the out-of-plane response and afterwards modelled the response of masonry elements strengthened with FRP grids [30] for in-plane and out-of-plane configurations. However, there have not been found specific models for studying the influence of FRP strengthening laminates on the response of eccentrically loaded brick masonry walls. Nevertheless, researches focused on other strengthening systems, like TRM (see [27]), might be considered as a starting point for the development of a numerical simulation to model this structural case.

The experimental and numerical approaches are useful to understand the structural response of materials and particular areas of a structure but easier approaches are usually preferred to deal with the design or assessment of full structures. In this line, the analytical approach presented in the codes [16,17] might be taken as a valuable reference. For more specific models, the works by Hamed et al. [24,31] are significant for addressing the out-of-plane response of FRP-strengthened masonry walls. These papers consider experimental and analytical approaches and take into account the influence of geometric parameters like the slenderness of the wall. Similarly, the recent research by Caggiano et al. [32] states an analytical approach for the bonding analysis of the FRP. However, the problem of the eccentrically loaded brick-masonry walls strengthened with FRP has not been particularly addressed from an analytical approach yet.

To sum up, it is clear that FRP is a commonly used strengthening system for masonry structures and, although it has been deeply studied for some loading configurations, there is a knowledge gap around the structural response of brick masonry walls strengthened with FRP (called FRPW in the paper) and subjected to eccentric compressive loads, which usually lead to a mechanism formation failure mode developed due to second order bending effects. In fact, experimental, numerical and analytical analyses might contribute to better understand this structural case and it is the aim of this paper to present the research carried out in this direction. Thus, experimental, numerical and analytical approaches have been used to analyse the influence of FRP on brick masonry walls subjected to eccentric vertical loads.

2. Methodology

The research is divided in three lines: the experimental campaign, the numerical simulation of the tested cases and the implementation of an analytical approach to calculate the load-bearing capacity of the FRPW (FRP strengthened masonry walls). The applicability of the

solutions presented herein are enhanced by taking into account these three approaches simultaneously.

Each one of these activities is described with detail in this section. Then, the corresponding results are presented, compared and discussed to bring the final conclusions in the next sections.

2.1. Description of the walls. FRPW cases

Seventeen walls have been built and tested. Fifteen of these walls have been strengthened with carbon FRP laminates using five different configurations (see Figure 1). Thus, there are three repetitions for each strengthening pattern and there are two control unreinforced brick masonry walls (called W#13 and W#15). The geometric properties of the tested walls are presented in Table 1. The first column of this table corresponds to the walls' series, which describes the pattern of the strengthening system (or the absence of strengthening for the control specimens). These patterns are detailed in columns 7th to 10th by presenting the number of vertical laminates uniformly distributed on the width of the strengthened face (7th), the number of horizontal laminates uniformly distributed on the height of the strengthened face (8th) and the number of inclined laminates placed at the same section – constant height – (9th), considering two directions at an angle of $\pm\alpha$ (°) (10th column). The third column presents the real eccentricity of the applied load at mid-height, whereas columns 4th to 6th show the height, thickness and width of the walls respectively. It has to be highlighted that the thickness of the control walls is slightly larger than the thickness of the strengthened ones. This difference is due to a change of the geometry imposed by the producer of the bricks, who certified the same mechanical properties and used the same raw materials to produce these pieces. In fact, the shape of the bricks is not so significant for the value of the mechanical properties because several experimental characterisation tests, like the ones to obtain the compressive strength and the water absorption coefficient, were carried out on samples which were cut from full bricks, as mentioned in [33]. The different effective height of the control walls is related with the manual execution of the walls. Finally, the experimentally obtained load-bearing capacity of each wall is presented in the last column.

Solid clay fired bricks produced by “Calvente Hermanos SL” and common Portland cement mortar produced by “Durland cementos SL” have been used to produce the masonry. The compressive strength and flexural strength of the mortar are 3.7MPa and 1.25MPa respectively (see [25]). The used FRP corresponds to the BASF's MBrace[®] system, composed of an epoxy primer called MBrace Primer, an epoxy adhesive called MBrace Laminate Adhesive HT and $b_{FRP}=80\text{mm}$ wide and $t_{FRP}=1.2\text{mm}$ thick carbon fibre laminates.

The density, ρ , of the masonry was calculated from the weight and the dimensions of the masonry samples tested to characterise its mechanical properties. The flexural strength of the masonry, f_x , have been experimentally obtained performing bond wrench tests that followed the procedure described in [33]. The compressive strength, f_m , and Young's modulus, E_{m0} , of the masonry for the control cases have been experimentally obtained by testing piles of 5 bricks as described in [33], with a similar approach of that presented in [34] by Brencich et al. In contrast, the design compressive strength associated with the eccentric loading configuration characteristic of the considered FRP-strengthened cases has been calculated on

the basis of the work by Brencich et al. [35], who suggested that the eccentric compressive strength (21.3 MPa) might be calculated from the concentric compressive strength (18.1 MPa) using the analytical expression presented in equation 15 of [35]. The concentric compressive strength was experimentally determined by testing piles of 5 bricks and 4 joints in a uniform compression test like the ones presented in [33]. The increase of the compressive strength when the load is eccentrically applied is a controversial issue which might be explained in terms of simplified masonry definitions, like the intentionally used one in the developed numerical model, which do not consider the plastic response of this material. In this line, a representative relationship between the Young's modulus and the compressive strength, which is defined in the fourth table of the work by Brencich et al. [36], is used too. A ratio of $E_m/f_m = 120$ is assumed taking into account the data presented in this reference and a Young's modulus of 2560 MPa is obtained applying this ratio. Although the phenomenon of the compressive strength increase was not experimentally investigated in the current experimental campaign, this assumption and the corresponding values of compressive strength and Young's modulus improve the results of the numerical model. Thus, the bibliographic evidences and the obtained results seem to point out that this is a reasonable and simplified approach for modelling the compressive response of masonry elements subjected to bending-compression forces.

Regarding other mechanical parameters, the tensile strength of the bricks used in all walls, $f_{tb} = 2.8 \text{ MPa}$, has been indirectly obtained from flexural experimental tests by using a relationship initially developed to correlate the tensile and flexural strengths of concrete [37] but successfully used by other researches [38] with the purpose of obtaining the tensile strength of solid clay bricks from their flexural strength. In contrast, the shear strength of the masonry associated with the indirect tensile failure of bricks and joints (inclined crack observable in Figure 2), f_s , has been calculated using the corresponding formulation in Eurocode-6 [39] for the strengthened walls. In contrast, it was experimentally obtained for the control walls using the experimental results presented in [33].

Regarding the tensile, bonding and sliding strength of the FRP and its Young's modulus (f_{tFRP} , f_{bFRP} , f_{sFRP} and E_{FRP} respectively), these parameters have been provided by the manufacturer of the strengthening system. The bonding and sliding strengths are referred to the interface between masonry and FRP.

The energy of fracture of the masonry joints due to debonding (G_f^I), the energy of fracture of the bricks in tension ($G_{fb}^I = 100 \text{ N}\cdot\text{m}$), the energy of fracture of the masonry due to sliding along an inclined compressive-shear crack (G_f^{II}), and the first and second modes energy of fracture of the masonry-FRP interface (G_{fint}^I and G_{fint}^{II} associated to the bonding and sliding strength respectively) have been calculated taking into account the corresponding strength (flexural strength of masonry, tensile strength of bricks, shear strength of masonry, bonding strength of FRP-masonry interface and sliding strength of FRP-masonry interface respectively) and using the equation (Eq. 1). This expression has been adjusted using the experimental results presented in [40] and has been previously used in other researches [25,33]. The mechanical properties of the masonry and FRP are summarised in Table 2 and Table 3 respectively.

$$(N/m) \quad G_f^I = 36.65 \cdot f_{xt} \quad (MPa) \quad (\text{Eq. 1})$$

Finishing with the samples definition, it has to be remarked that the layers of epoxy adhesive were thicker than recommended by the producers because of the irregular surface of the masonry, even in laboratory conditions. Thus, it is expected that onsite applications will result in thicker layers of adhesive, which might affect the structural response as discussed later on.

It has to be remarked that laminates were only installed on one side of the walls, corresponding with the tensile-bending face. The side where tensile forces would appear was controlled by the eccentricity of the load at the ends of the walls in the laboratory. For real applications the sign of the eccentricity would not be controllable so both sides of the walls should be strengthened.

2.2. Description of the experimental tests

The test setup used for testing the walls of this research has been successfully applied in previous experimental campaigns. See [25,27] for details about the test configuration showed in (Figure 3) and the corresponding procedure. The main characteristic of these tests was the pinned-pinned configuration with equal fixed eccentricity of the load at both ends of the walls. This structural configuration was set by means of two hinges. The load was applied using a hydraulic jack (0-1000kN range) and measured with a pressure transmitter (0-1000 bar range, 0.2% linearity). The test was force controlled at an average load rate of 3kN/s (it was manually controlled). The movements of the wall were measured with potentiometric displacement sensors (in-plane descending of the top of the wall) and laser displacement sensors (out-of-plane, at three different heights). In addition, the rotation of the hinges was also indirectly measured by using displacement potentiometers. All data was simultaneously recorded at a sample rate of 50Hz. The initial geometry was also measured before testing the walls. Finally, a high speed camera was used to capture the failure mode of the walls.

In comparison with previous experimental campaigns, the use of strain gages installed on the FRP laminates to monitor the response of this strengthening system is the main difference for the experiments presented herein. All strain gages were installed in the direction of the fibres of the corresponding laminate. These measurements were necessary to perform an analysis about the effectiveness of the FRP laminates depending on their position and orientation. The position of the strain gages is presented in Figure 1 for all strengthening patterns. The strain gages used for this purpose were 350Ω resistance, 6mm of measuring length and a gage factor of 2.04±1.0%.

2.3. Description of the Numerical Simulation

Although there are reference publications about the numerical simulation of FRP-strengthened masonry (e.g. [29]), another approach aimed to be implemented in commercial Finite Element Analysis packages is preferred for extending the availability to practitioners. ANSYS v.14 ® has been used in this research.

The implemented numerical model is a bidimensional plane strain simplification of the real problem. The plane strain approach has been considered for two reasons: this is more

representative of real walls, which are commonly characterised by larger width than the other dimensions and the presence of movement restrains at the lateral edges, and the negligible strain measurements on the horizontal laminates (see the experimental results in section 3.1), which support the plane strain simplification. However, using a plane stress simplification might have provided better results for the tested walls or in the case of analysing FRP-strengthened piers but, as long as the aim of the implemented numerical tool is to cover the most representative situation of walls, the plane strain approach seemed the more suitable one. In addition, taking into account the experimental evidences, this simplification is not in contradiction with the strain measurements in the direction perpendicular to the analysis plane.

Moreover, a simplified micromodelling approach have been used, so the structure is represented by different parts which represent the masonry, the FRP-and-adhesive or the hinges independently. Then, zero-thickness interface contact elements are defined between these parts. Hence, the mortar of the masonry or the adhesive of the FRP-masonry contact are not modelled as independent parts but included in the masonry and FRP definition respectively. These interface elements are used to control the tensile or shear response of these composite materials (including the mortar joints or epoxy adhesive) whereas the compressive behaviour is described by the homogeneous material properties of the masonry or FRP. It has to be highlighted that no homogenisation process has been required because the properties of the masonry as an equivalent material have been experimentally determined or deduced from experiments and the properties of the installed FRP (this is taking into account the adhesive influence) were provided by the producer. The different features of the model are described in detail below, including the geometry, the material, the meshing, the boundary conditions, the loading pattern and the calculation procedure.

It has to be highlighted that the numerical model used for simulating the control cases is the one previously presented in [25].

2.3.1. Geometry

Regarding the geometry, it has to be remarked that the overall geometry of each modelled wall is defined to accurately represent the geometry of the corresponding real wall. With this aim, the vertical alignment of each wall was measured every two masonry rows with a laser sensor mounted on a vertical guiding rail. These experimental measurements were used to reproduce a realistic geometry in which each masonry row is represented by a rectangular part. Moreover, each hinge is represented by a triangle placed at the corresponding end of the wall. The FRP laminates are represented as a uniform strengthening layer along the width of the wall. Thus, the FRP is modelled as a group of rectangles (1.2mm thickness) in the bidimensional section for all the possible strengthening patterns. These rectangles correspond to the division of the strengthening layer in as many parts as masonry rows are defined. This division allows implementing a different masonry-FRP contact definition for each row and it is used to model the restraining effect of the crossed strengthening patterns (2V2H, 3V5H and 3I3I cases). Depending on the particular properties of each contact between the rectangles representing the FRP and the rectangles representing the masonry, it is possible to model different FRP patterns. Finally, and accordingly with the failure modes observed in

strengthened walls of previous campaigns [27] and the experimental results of this own research, it is decided to define fictitious inclined contacts in the masonry at the ends of the walls to allow the compression/shear failure mode. This approach makes it possible to keep a simple material definition. An example sketch of the geometric model is presented in Figure 3.

2.3.2. Material

Masonry is defined as isotropic linear elastic perfect-plastic material in compression. This is a simplification of the real response, which is more similar to a parabolic strain-stress curve (see [41]). However, for the studied cases, in which the failure mode is due to second order bending effects and the compressive response is not so influential, this approach achieves accurate results (according with the data presented later on) and keeps the material definition simpler than a more realistic description. FRP is also defined as isotropic linear elastic in compression, although this assumption is not significant since the FRP is only considered to be placed in the side of the wall where tensile efforts are developed.

Regarding the tensile and shear response of these materials, both masonry and FRP are geometrically defined as groups of parts with contacts between them and it is the definition of these contacts, which is based on the work by Alfano and Crisfield [42], what controls the tensile/shear response using a Cohesive Zone Model (CZM). The CZM defines an elastic response of the contact up to the fragile failure of the joint by separation or sliding. The elastic branch depends on the elastic modulus of the less stiff material involved in the contact and the fragile failure is defined by the corresponding strength (in normal or tangential direction) and the associated critical fracture energy (mode I, G_f^I , for the normal direction and mode II, G_f^{II} , for the tangential one). If the normal and tangential failure of the contact are simultaneously possible, the failure criteria of the contact is defined by a power law energy criterion presented in the following equation (Eq. 2), where G^I and G^{II} are the normal and tangential energy of fracture respectively and G_f^I and G_f^{II} are the critical energy of fracture for each mode independently considered.

$$\left(\frac{G^I}{G_f^I}\right) + \left(\frac{G^{II}}{G_f^{II}}\right) = 1 \quad (\text{Eq. 2})$$

In addition, it has to be mentioned that the tangential slip possibility under compressive normal contact stress is considered only for the mixed-mode debonding cases (tensile and sliding) by activating this option in the definition of the contact, which sets a predefined frictional response to control the sliding process under compressive stress.

Thus, for each contact definition using the CZM, the tensile and/or shear strength and the corresponding energies of fracture are required. If considering the whole structure, the contacts to be defined are: real contacts between the masonry and the hinges, real contacts between the masonry rows, real contacts between the masonry and the FRP, fictitious contacts between FRP parts and fictitious contacts between masonry parts along the fictitious inclined contacts which have been geometrically defined before. The particularities for each particular contact, whose position might be observed in Figure 4, are:

- Real, masonry-hinge: Bonded, because no separation has been observed in any experimental test.
- Real, masonry-masonry rows: Not sliding allowed in the tangential direction. In the normal direction: $f_x = 0.36$ MPa and $G_f^I = 13$ N·m. This definition is based on the experimental evidence that the sliding along the mortar joint was not observed.
- Real, masonry-FRP (Not applicable for control cases): Sliding (f_{sFRP} , $G_{f_{int}}^{II}$) and debonding (f_{bFRP} , $G_{f_{int}}^I$) are allowed. The value of the parameters, as described in the next paragraph, is presented in 2nd-5th columns of Table 3.
- Fictitious, FRP-FRP: Bonded, because no tensile failure of the FRP was observed.
- Fictitious, masonry-masonry inclined: The sliding failure mode is characterised by f_s and G_f^{II} in Table 2 and the normal opening of the contact is characterised by the tensile strength of the bricks $f_{tb} = 2.8$ MPa and the corresponding fracture energy $G_{fb}^I = 100$ N·m calculated with (Eq. 1).

Moreover, considering the bidimensional simplification of the FRP strengthening system requires a non-conventional material definition. In this line, the density (ρ_{FRP}), Young's modulus (E_{FRP}), FRP-masonry bonding strength (f_{bFRP}), FRP-masonry tangential strength (f_{sbFRP}) and tensile strength of the FRP (f_{tFRP}) are modified to represent the real amount of FRP laminate distributed along the width of the wall. Only the vertical laminates (or part of the vertical component of the inclined ones) are considered in this simplification accordingly with the results presented in section 3.1. The modified variables are calculated from the original ones using equation (Eq. 3) or (Eq. 4) depending if the laminates were vertical or inclined respectively. In this last case, only $\frac{3}{4}$ parts of the capacity of the installed laminates is considered in agreement with the results presented later on in section 3.1, which show that the inclined laminates that were not in contact with the masonry along all their length mobilise half the strain than the laminates in perfect contact with the masonry, averaging 75% overall. In (Eq. 3) and (Eq. 4), K represents the generic original property, and K^* the modified one. These equations are valid for all the previously listed properties except the density: E_{FRP} , f_{bFRP} , f_{sbFRP} and f_{tFRP} . The modification of the density for the inclined cases does not need the $\frac{3}{4}$ factor. The variable b_{FRP} is the width of the laminate (80mm) and $\bar{b}$ is the average width of the walls (833mm). It has to be noticed that assuming a linear distribution on the width of the wall of the FRP-masonry tensile and shear strengths is a first approach arisen from the bidimensional simplification that should be taking into account when analysing the results.

$$K^* = K \cdot \frac{\#_{v,FRP} \cdot b_{FRP}}{\bar{b}} \quad (\text{Eq. 3}) \quad K^* = K \cdot \frac{3}{4} \cdot \frac{\#_{i,FRP} \cdot b_{FRP}}{\bar{b} \cdot \sin \alpha} \quad (\text{Eq. 4})$$

In the case of inclined laminates, (Eq. 4) is based on the fact that the original strength properties are defined along the inclined direction. In addition, the component of the vertical force which follows the direction of the laminates might be calculated as the vertical force (F), divided by the number of inclined laminates ($\#_i$) and multiplied by the factor $\sin \alpha$. Finally, it is assumed that the vertical force supported by the distributed modified FRP should be equal to the summation of the vertical component of the force bore by each inclined laminate. Thus, F is the same for equations (Eq. 5) and (Eq. 6), which represent the calculation of the real stress on the laminate, σ , and the corresponding value for the modified parameter assuming a uniform distribution along the walls' width, σ^* , respectively. In these equations, t_{FRP} represents

the thickness of the FPR laminate. Combining equations (Eq. 5) and (Eq. 6) the formulation presented in (Eq. 4) is justified except for the $\frac{3}{4}$ factor which is related with the experimental evidences mentioned before and justified in section 3.1.

$$\sigma = \frac{\frac{F}{\#_i} \cdot \sin \alpha}{b_{FRP} \cdot t_{FRP}} \quad (\text{Eq. 5})$$

$$\sigma^* = \frac{F}{\bar{b} \cdot t_{FRP}} \quad (\text{Eq. 6})$$

The calculated values of the mechanical equivalent properties are summarised in Table 3. In agreement with the experimental results, the horizontal laminates are not considered to directly contribute at the resistance of the walls against the bending/buckling phenomena. It is also necessary to remark that the fracture energies corresponding to the tensile, shear or bonding strengths of the FRP are calculated using (Eq. 1) and these results are also presented in Table 3. About this procedure, it has to be highlighted that using the same equation to calculate all fracture energies is not accurate and might be associated with possible errors of the numerical model. However, no bibliographic references which explain how to calculate the fracture energies required by the implemented numerical model have been found. Thus, (Eq. 1) has been used as a first approach and with the aim of avoiding guessing arbitrary values for these fracture energy parameters. This limitation has to be taken into account when analysing the results.

Finally, it is assumed that the contact between masonry and FRP is perfectly bonded (not sliding nor opening of the contact is allowed) at the positions (height) where laminates cross ones over others. This hypothesis is oriented to model the restraining effect that laminates might develop ones over others when they are applied following a crossed pattern (2V2H, 3V5H and 3I3I cases).

2.3.3. Mesh

Both masonry and FRP are automatically meshed (see Figure 4 left) with 8-nodes uniform structured quadrilateral elements with quadratic integration of the displacements. The hinges are meshed using an unstructured mesh with 6-nodes triangular and 8-nodes quadrilateral uniform elements with quadratic integration of displacements. The average size of the elements representing the masonry and the hinges is 5mm whereas it is 2mm for the elements of the FRP. The sizes of the elements have been set after a sensitivity analysis, which assured that considering half the size caused a slight (around 3%) change of the calculated load-bearing capacity. In addition, performing the calculations in a reasonable time was a requirement. Thus, the calculation time for each wall was limited to 60 minutes in an Intel(R) Core(TM)2 Duo CPU E8400 @3.00GHz with 3.00GB of RAM memory working with Windows7 32 bits.

Two types of elements are used to mesh the contacts: one is a bidimensional solid element assigned to the contact part and overlaying it, whereas the other one is a set of segment elements assigned to the target part by following its edge. The assigning of contact or target element is automatic for each pair of parts. The contact between the two parts occurs when the contact element penetrates through the target element.

2.3.4. Boundary conditions and loading pattern

The experimental pinned-pinned configuration is represented by the boundary conditions in the numerical model. All displacements of the bottom vertex of the lower hinge are restrained. Similarly, the horizontal displacement of the top vertex of the triangular part representing the upper hinge is also restrained. The rotation around these vertexes is free, representing the real structural boundary conditions.

The vertical force is indirectly applied through the imposition of a vertical descending displacement of the top vertex of the upper hinge. In addition, the position of the vertexes allows representing the real eccentricity of the load in an accurate way. Finally, the vertical reaction at the point where the vertical displacement is imposed represents the compressive applied force to be compared with the experimental results.

The calculation procedure consists of applying the imposed displacement step by step, so each imposed deformation is applied on the deformed shape of the wall due to the previous loading steps. This process activates the second order bending effects. In addition, the displacement-controlled simulation allows calculating some loading steps after reaching the maximum applied force and this fact enables a more accurate interpretation of the obtained results in relation with the qualitative analysis of the failure mode.

2.4. Description of the Analytical approach

Although the numerical model presented herein can be implemented in general-purpose commercial finite element packages, it is known that practitioners are more familiar with simpler tools aimed to perform quick designs or checking predefined solutions. Thus, it is necessary to provide accurate solutions at a lower computational cost. Because of this, an analytical approach is presented in this section.

The methodology, which is described in detail later on, is based on the following assumptions: a) the bending moment which causes the failure of a FRP-strengthened masonry section due to reaching the material strength can be calculated using the properties of the section and the value of a predefined (fixed) compressive load. Simultaneously, b) the deflection of the wall at mid-height (where it is maximum for the pinned-pinned configuration) can be calculated using the geometric and material properties of the structure and a fixed value of the compressive load. In this case, the associated bending moment can be obtained by multiplying the applied load by the total eccentricity (initial eccentricity + deflection). Finally, c) if the bending moment calculated using these two methodologies converge for the same compressive load, this is the load-bearing capacity of the FRP-strengthened studied wall.

2.4.1. Input variables

The input variables required by the proposed analytical approach might be divided in material properties, geometric variables and variables to describe the strengthening pattern. Regarding the material properties, the compressive strength of the masonry, f_m , the ultimate compressive strain of the masonry, $\epsilon_{m,ult}$, the Young's modulus of the FRP, E_{FRP} , and the corresponding tensile ultimate strain, $\epsilon_{FRP,ult}$ are required.

In relation with the geometry, the thickness (t), width (b) and effective height (H_{ef}) of the wall are required. H_{ef} is a fundamental parameter which controls the second order bending

response and the possible buckling failure of the walls. In the current research this variable coincides with the distance between the hinges placed at both extremes of the wall (top and bottom). In addition, the eccentricity of the load at mid-height, e_0 , is also a defining variable. The width (b_{FRP}) and thickness (t_{FRP}) of the FRP laminates, the number of vertical ($\#_{v,FRP}$) and inclined ($\#_{i,FRP}$) laminates which are installed, and the orientation of the inclined ones (α) are also input parameters.

2.4.2. Calculation basis

The calculation is based on assuming a failure mode of the wall (masonry crushing in compression or debonding failure of the FRP) and seeking for the compressive load that simultaneously satisfies this failure criteria (in terms of strain) and the force and bending moment equilibrium equations together with the strain compatibility expressions.

The analytical method presented herein has been developed on the following hypothesis:

- Linear strain distribution across the thickness of the section.
- Rectangular stress diagram for the compressed masonry. The constant value of the stress in the masonry corresponds to its compressive strength, f_m . Taking into account the usual performance of FRP, it is likely that the masonry reaches its plasticity state, so the hypothesis is consistent with this experience. This compressive strength is applied in 0.8 times the length of the neutral axis depth.
- Masonry bears no tensile stresses. The tensile force is born by the FRP.
- The stresses in the laminate are uniformly distributed and the corresponding resultant punctual force is taken into account for the calculation instead of the stress distribution.
- The thickness of the adhesive is neglected because the spacing between masonry and FRP is not significant for the current analytical model.
- The compressive failure of the masonry is reached when the maximum strain meets the ultimate compressive strain of the masonry, $\epsilon_{m,ult}$. In this case, it has to be checked that the strain of the FRP, ϵ_{FRP} , is below the critical one, which is set to 45% of the ultimate strain according with ACI 440.7R-10 [16], $\epsilon_{FRP,ef} = 0.45 \cdot \epsilon_{FRP,ult}$.
- The debonding failure of the FRP is reached when the laminate meets its critical strain, $\epsilon_{FRP,ef}$. In this case, it has to be checked that the strain in the masonry is below the ultimate one, $\epsilon_{m,ult}$.

2.4.3. Calculation procedure and formulation

A schematic diagram of the calculation procedure is presented in Figure 5. This includes:

- a) Calculating the constant parameters. This step includes calculating the inertia modulus of the composed section, I , using (Eq. 7) or (Eq. 8) depending on the direction of the laminates, vertical or inclined respectively. The variable x is the position of the gravity centre of the homogenised section calculated as presented in (Eq. 9) or (Eq. 10), if the laminates are vertical or inclined respectively. In addition, the ratio n of the modified Young's modulus of the FRP (E_{FRP}^*) over theoretical secant Young's modulus of the

masonry for the first calculation step ($E_{m0}^{Theoretical}$) is defined in (Eq. 11).
 $E_{m0}^{Theoretical}$ is calculated as presented in (Eq. 12).

$$I = \frac{1}{12}bx^3 + \frac{1}{12}b(t-x)^3 + \frac{1}{12}nb_{FRP}\#_{v,FRP}t_{FRP}^3 + b\frac{x^3}{4} + b\frac{(t-x)^3}{4} + nb_{FRP}\#_{v,FRP}t_{FRP}\left(\frac{t_{FRP}}{2} + t - x\right)^2 \quad (\text{Eq. 7})$$

$$I = \frac{1}{12}bx^3 + \frac{1}{12}b(t-x)^3 + \frac{1}{12}nb_{FRP}\frac{\#_{i,FRP}}{\sin\alpha}t_{FRP}^3 + b\frac{x^3}{4} + b\frac{(t-x)^3}{4} + nb_{FRP}\frac{\#_{i,FRP}}{\sin\alpha}t_{FRP}\left(\frac{t_{FRP}}{2} + t - x\right)^2 \quad (\text{Eq. 8})$$

$$x = \frac{bt + nb_{FRP}\#_{v,FRP}t_{FRP}}{2b} \quad (\text{Eq. 9})$$

$$x = \frac{t}{2} + \frac{nb_{FRP}\#_{i,FRP}t_{FRP}}{2b\sin\alpha} \quad (\text{Eq. 10})$$

$$n = \frac{E_{FRP}^*}{E_{m0}^{Theoretical}} \quad (\text{Eq. 11})$$

$$E_{m0}^{Theoretical} = \frac{f_m}{\varepsilon_{m,ult}} \quad (\text{Eq. 12})$$

- b) Choosing a value for the compressive eccentric load, N . This variable is iteratively actualised in the next steps up to the convergence criteria defined later on.
- c) Calculating, using (Eq. 13), the total eccentricity, e , which takes into account the initial eccentricity, e_0 , and the second order deflection of the wall at mid-height. This formulation, which is derived from the classical buckling theory of elastic beams, has been used in previous researches and documents [33,43,44]. The Young's modulus of the masonry used in this equation is updated at each step according with (Eq. 14), which considers that only the compressed part of the masonry contributes to the lateral stiffness of the wall, reducing the value of the equivalent Young's modulus linearly with the compressed area. This is performing an elastic calculation of the properties but considering only the compressed area of the cross-section. For the first calculating step, $E_m^{Theoretical} = E_{m0}^{Theoretical}$.

$$e = e_0 \left(\frac{1}{\cos\left(\frac{H_{ef}}{2} \sqrt{\frac{N}{E_m I}}\right)} \right) \quad (\text{Eq. 13})$$

$$E_m^{Theoretical} = E_{m0}^{Theoretical} \frac{t-c}{t} \quad (\text{Eq. 14})$$

- d) Choosing a failure mode (masonry crushing or debonding failure of FRP) and calculating the position of the neutral axis, c , using the formulation which corresponds to the selected failure mode.
 In the case of choosing the masonry crushing mode, (Eq. 15) and (Eq. 16) are used. The first one represents the force equilibrium in the section and the second one is the strain compatibility equation.

$$N = f_m b 0.8c - \varepsilon_{FRP} A_{FRP} E_{FRP} \quad (\text{Eq. 15})$$

$$\frac{\varepsilon_{m,ult}}{c} = \frac{\varepsilon_{FRP}}{t + \frac{t_{FRP}}{2} - c} \quad (\text{Eq. 16})$$

where A_{FRP} , which is the area of the section of the laminates, is calculated using (Eq. 17) or (Eq. 18) if the laminates are vertical or inclined respectively.

$$A_{FRP} = b_{FRP} \#_{v,FRP} t_{FRP} \quad (\text{Eq. 17})$$

$$A_{FRP} = b_{FRP} \frac{\#_{i,FRP}}{\sin \alpha} t_{FRP} \quad (\text{Eq. 18})$$

In this case of assuming the masonry compressive failure, it has to be checked that $\varepsilon_{FRP} \leq \varepsilon_{FRP,ef}$ ($=0.013$ in the studied case). If this requirement is not fulfilled, the FRP debonding failure has to be assumed.

In the case of choosing the FRP debonding failure, (Eq. 19) is used to calculate the position of the neutral axis, c . It has to be remarked that, in this case, $\varepsilon_{FRP} = \varepsilon_{FRP,ef}$ because the debonding failure of FRP is assumed and the corresponding maximum critical strain is considered. Afterwards, (Eq. 20) is used to obtain the maximum strain in the masonry, ε_m , which has to be compared with the ultimate strain of the masonry in compression. If $\varepsilon_m \leq \varepsilon_{m,ult}$, the procedure continues to the next step. Otherwise, the masonry compressive failure has to be chosen as described before.

$$N = f_m b 0.8c - \varepsilon_{FRP,ef} A_{FRP} E_{FRP} \quad (\text{Eq. 19})$$

$$\frac{\varepsilon_m}{c} = \frac{\varepsilon_{FRP,ef}}{t + \frac{t_{FRP}}{2} - c} \quad (\text{Eq. 20})$$

- e) Calculating the bending moment, M , associated with the stress distribution that arises from the previous hypothesis. Thus, (Eq. 21) is used to obtain the value of M . It has to be remarked that, in the case of assuming the debonding failure of the FRP, $\varepsilon_{FRP} = \varepsilon_{FRP,ef}$ in (Eq. 21).

$$M = f_m b 0.8c (x - 0.4c) + \varepsilon_{FRP} A_{FRP} E_{FRP} \left(t + \frac{t_{FRP}}{2} - x \right) \quad (\text{Eq. 21})$$

- f) Calculating the eccentricity associated with the bending moment, e_M , using (Eq. 22). This formulation assumes that the bending moment is only originated by the eccentricity of the axial load.

$$e_M = \frac{M}{N} \quad (\text{Eq. 22})$$

- g) Comparing the total eccentricity due to second order bending effects, e , with the eccentricity associated with the bending moment, e_M . If these two variables are equal, the value of the compressive load, N , assumed in the step b) corresponds to the load-bearing capacity of the studied wall. Otherwise, the value of N is changed and the procedure restarts in step b) up to the convergence between e and e_M .

3. Results

The results of this research have been divided in three parts: the qualitative and quantitative experimental results, the results of the implemented numerical model and the results of the proposed analytical approach. Dimensionless variables have been defined to ease the comparison between cases. These are: the dimensionless lateral displacement (h/t), which has been calculated as the ratio of the lateral displacement, h , out of the thickness of the wall, t ; the dimensionless vertical descending displacement of the top of the wall (v/H_{ef}), which has been calculated as the ratio of the vertical displacement, v , out of the effective height of the wall, H_{ef} ; and the dimensionless applied force, ϕ , which has been calculated as the applied force, N , out of the maximum force that the cross section of the wall could bear in uniform compression configuration, $N_{ult} = f_m \cdot t \cdot b$.

3.1. Experimental results

The qualitative observation of the tests has brought significant evidences. First of all, the FRP has not debonded from the masonry in any case. This might be due to an excellent adhesion between the rough surface of the masonry and the primer. This behaviour might be also related with using strengthening patterns which provide enough amount of FRP to assure that the masonry compressive/shear failure will happen before any debonding process. Secondly, it has been noted that the failure mode is a compression/shear mixed collapse of the masonry near the ends of the walls (see Figure 2) for all cases. Because of this, it might be recommendable to check the compressive and lateral shear capacity of the masonry when installing an FRP strengthening system.

Before presenting the comparison between the unreinforced and FRP-strengthened walls, it is important to remark that the larger thickness of the control walls and their different height makes it difficult to numerically compare the unreinforced and strengthened cases. However, a significant qualitative comparison can be carried out if taking into account that the smaller thickness of the strengthened walls might cause larger lateral displacements and might be associated with reaching less load-bearing capacity. In addition, the average effective height of the FRP-strengthened walls is greater than for the unreinforced ones, which should be associated with larger lateral displacements and lower load-bearing capacity of the strengthened walls. Thus, the observed load-bearing capacity increase associated with the installation of FRP laminates is a lower bound of the performance possibility of this technique.

Taking into account the experimental results, it is observed that the structural response of the FRP-strengthened walls shows less scattering (6.7%) than comparable cases of unreinforced masonry walls subjected to eccentric compressive loads. For example, the scattering of comparable tests on unreinforced masonry walls included in [25] (Walls W#10 to W#16) was 22.1%. In addition, this fact can be observed in Figure 6, where the lateral displacement at mid-height is represented versus the applied load for all tested walls. Each graph in Figure 6 corresponds to a strengthening pattern (or the control case). It is noticed that the three repetitions of each strengthening case are practically overlapped except for wall 3VOH_1 which was loaded up to half its capacity, unloaded and loaded again up to the failure due to a technical problem with the hydraulic jack during the test. This fact might explain the linear response of the wall 3VOH_1 up to larger loads than the other cases. It is also important to highlight that all strengthened walls show a similar lateral stiffness if comparing the different

graphs. Thus, the FRP-strengthening makes the out-of-plane structural response more uniform. The value of the scattering of the experimental results for each series is summarised in the seventh column of Table 4, which also shows other experimental results. The vertical displacement measured when the maximum load was applied is presented in the third column, whereas the fourth and fifth columns summarise the lateral displacement for the maximum load and the dimensionless measure of this force respectively. The load-bearing capacity for each case is presented in the sixth column. The corresponding load-bearing increase, in comparison with the average of the control cases, is summarised in the last column of this table. These values have to be analysed carefully because of the geometric differences between unreinforced and FRP-strengthened walls pointed out before. However, these differences would cause an underestimation of the load-bearing increase, so the values in the last column of Table 4 are a lower bound of the effectiveness of the FRP system. Thus, an average increase of the load-bearing capacity over 200% might be expected when applying FRP in the current structural configuration of masonry walls.

According with the strain measurements, the FRP laminates were far from reaching their ultimate strain in the tests carried out. The maximum measured strains were below 2500 $\mu\text{m/m}$, so the tensile failure of the FRP or its debonding are not likely for the studied structural configuration. In addition, it has been proved that the maximum strain on the laminates was placed at mid-height, which agrees with the bending deformation of the walls, which were characterised by larger lateral displacements at mid-height. An example of this strain distribution might be observed in Figure 7, where the curves representing the strains of the gages SG1 and SG2, which are placed at mid-height, are larger than the measurements of SG3 and SG4, which are installed at a lower and higher level.

Moreover, it has been observed that the inclined crossed laminates had a different response depending on the position they were installed: it has been noticed that the laminates in direct contact with the masonry along all their length showed larger strains than the laminates placed crossing over these first ones. This effect might be observed in Figure 8, where the measurements of comparable (installed at equal height) strain gages are plotted for the cases of the series 3I3I. The strain gage SG3 of wall 3I3I_1 is compared with the strain gages SG2, SG4, SG6 and SG7 of this same wall. For the cases 3I3I_2 and 3I3I_3, the measurements of the strain gage SG5 are compared with the recordings of SG3 and SG4. The laminates in direct contact with the masonry were installed from the bottom-left to the up-right position, and included the gage SG3 for the wall 3I3I_1 and SG5 for 3I3I_2 and 3I3I_3 walls, whereas the laminates from the bottom-right to the up-left of the wall crossed over the previous ones. Analysing the data plotted in Figure 8, it might be noticed that the laminates which are placed over others assume an average strain of 54% of the strain measured on the laminates in direct contact with masonry. This calculation has been carried out taking into account the data from 66% of the maximum load up to the failure force because: a) there are no settling effects in this load range, b) the FRP assumes larger forces so the measured strains are more significant and c) the division of the tensile force between laminates is clearer.

Finally, it has to be remarked that the recordings obtained from the strain gages placed on horizontal laminates (see SG3 and SG4 in Figure 9) indicate that these FRP pieces are not loaded. In fact, the strain in these parts is practically zero during all the test and for all cases,

which supports the plain strain simplification hypothesis of the numerical model and implies that the installation of horizontal laminates is not required for the loading configuration.

3.2. Numerical results

The results of the numerical simulations are presented in Table 5. The third, fourth and fifth columns of this table correspond to the dimensionless values of the vertical displacement, lateral displacement at mid-height and applied force at the failure moment. The predicted load bearing capacity of each wall is shown in the sixth column, whereas the seventh column summarises the absolute value of the relative error of the calculated ultimate load in comparison with the experimental value presented in Table 4. In the last column, the average error for each series is shown.

Observing the numerical results in Table 5, it has to be highlighted that the implemented finite element analysis tends to overestimate the load-bearing capacity of the studied walls. However, the analysis of the predicted failure modes shows agreement with the experimental observations. In particular, reaching the tangential strength of the fictitious inclined contact in the masonry near the ends of the wall (see Figure 10, left) is pointed out as the collapse cause of the studied FRP-strengthened walls in agreement with the numerical results.

Finally, it has to be remarked that the definition of the contacts oriented to represent the effect of the laminates crossing over others causes a particular stress distribution in the masonry-FRP contact (see Figure 10, right). In fact, there is a slight discontinuity in the tangential stress distribution due to the bonded restraint imposed at the positions where the laminates were crossed ($1/3$ and $2/3$ of the wall's height for the 2V2H_3 case represented in Figure 10 right).

3.3. Analytical results

The results calculated with the proposed analytical approach are summarised in Table 6, which shows the load-bearing capacity, the dimensionless measure of this load-bearing capacity, the absolute value of the relative error in comparison with the experimental results and the average error for each series. It has to be highlighted that the proposed methodology is only applicable for FRP-strengthened cases, so the control ones are out of the scope of this analytical approach.

Observing the results of the analytical approach, it is significant that the predicted failure mode is always the one associated with the compressive collapse of the masonry for the studied cases. This result meets the experimental observations as no debonding processes were experimentally observed. Finally, it has to be remarked that the obtained results tend to overestimate the load-bearing capacity of the walls.

4. Comparison and Discussion

Regarding the experimental results, it is observed that placing three vertical laminates might increase the load-bearing capacity of the walls in comparison with the cases with two laminates. This fact would agree with a stiffer response of the walls strengthened with more quantity of fibre in the vertical direction. However, the observed failure mode, which is controlled by the masonry response, suggests that the influence of the FRP pattern might be

not significant for the studied cases. In addition, the effectiveness of including horizontal laminates is not clear because for one comparison case (2V2H versus 2V0H) there is an increase of the load-bearing capacity, but for another case (3V5H versus 3V0H) the capacity of the walls decrease when including the horizontal strengthening. These particularly confronting results together with the evidence that the horizontal laminates did not bear significant loads (according with the measurements of the strain gages) makes it suitable to suggest avoiding the installation of horizontal FRP laminates if the aim of the intervention is strengthening brick masonry walls against second order effects due to eccentric compressive loads. Similarly, the cases 3I3I can be compared with 3V0H specimens because in most of the cross sections of the 3I3I cases there are 4 laminates, and half of them (2) are working at 54% of the strain of the other two, as the experimental strain measurements suggest. Thus, from a practical point of view, it might be interpreted that the 3I3I case is equivalent to the 3V0H case in relation to the effective FRP. Taking this fact into account and comparing the load-bearing capacity of the cases 3I3I with the reference cases 3V0H, it might be concluded that placing the laminates in the vertical direction is more effective. Hence, it is not recommendable to install inclined FRP laminates if the purpose of the strengthening system is to enhance the performance against second order effects derived from an eccentric compressive loading configuration. Finally, the experimental evidences are not concluding about the influence of the different strengthening patterns on the load-bearing capacity.

Nevertheless, from a practical point of view it might be recommendable to reduce the contribution of the inclined laminates by a $\frac{3}{4}$ factor. This factor is obtained from considering two laminates (in direct contact with the masonry) working at a reference level and another two laminates (crossing over the first ones) working at approximately halve level (54%). Thus, the average working level of the fourth laminates considered together should be reduced to 77% of the reference level, which has been approximated to 75% or $\frac{3}{4}$ factor used in the formulation presented before.

Comparing the FRP strengthened cases with comparable unreinforced masonry walls (see [25]), it can be observed that the scattering of the load-bearing capacity of the strengthened walls (6.7%) is far less than the scattering of comparable unreinforced cases (22.1%). Both scattering levels are within a typical scattering range of 30% for the results of tests on masonry structures. Thus, the analysed strengthening technique might be a suitable way to uniform the response of the masonry structures and thus, to increase the confidence of the practitioners and reduce the safety factors associated with this material. In addition, it has been observed (Figure 6) that the maximum lateral displacement associated with the collapse load is similar for all strengthened cases and the control unreinforced walls. However, the vertical displacement associated with the collapse is far lower for the unreinforced ones. Thus, these evidences might indicate that the failure of the studied walls is always associated with a lateral bending deformation and the corresponding stress state in the masonry. Applying an FRP strengthening solution contributes to enlarge the force required to reach this critical lateral deformation. Hence, the FRP basically contributes to make the lateral response of the wall stiffer. Finally, it is important to remark that the load bearing capacity increase is significant for all studied strengthening patterns, so the FRP is effective at strengthening brick masonry walls subjected to eccentric compressive loads.

The main contribution of the numerical model is supporting the experimental evidence that the collapse load is almost the same for all considered strengthening patterns. This result meets the fact that all walls fail due to the masonry compressive/shear collapse. No failure mode related with the FRP was observed nor predicted. This numerical result helps to understand and analyse the experimental evidences, which were not concluding about the influence of the strengthening patterns. According with the numerical results, considering different FRP patterns do not influence the load-bearing capacity of the studied walls.

In addition, it has to be noticed that the simulated vertical descending displacement is comparable, but slightly higher, to the experimental one. In contrast, the model is not so accurate at predicting the lateral displacements at mid-height, which are larger than the experimentally measured ones. However, the predicted failure mode corresponds to the experimental observations and the load-bearing capacity is predicted with an average error of 25.1%, which is within the typical scattering of experiments on masonry structures (around 30%). In addition, the implemented numerical model tends to overestimate the load-bearing capacity of the FRP-strengthened wall. Taking all this information together, it seems likely that the used definition of the applied FRP might be stiffer than the real solution in the vertical direction but more flexible than the used material in the lateral orientation. This effect might be explained by the contribution of the adhesive layers, which were not considered in the model and were thicker than proposed by the producers due to applicability limitations as mentioned in section 2.1, combined with the possible underestimation of the Young's modulus of the masonry. In this line, placing a thicker layer of adhesive might have contributed to limit the experimental lateral deformation of the tested walls because it means introducing an additional material stiffer than masonry.

The significant error of the numerical model might be related with the definition of the materials. In particular, the compressive behaviour of the masonry and the fracture energies of masonry, FRP and interfaces between them might be defined in a more realistic way to model the experimental response of the studied structures with more accuracy. Regarding the compressive response of the masonry, two improvements are possible: implementing a parabolic stress-strain compressive response and using a more representative value of the Young's module. However, the first improvement will be against the aim of keeping the model definition as simple as possible. The possible underestimation of the Young's modulus of the masonry used in the numerical model might explain the error in the lateral deformation. In addition, higher values of the Young's modulus of the masonry would result in lower lateral deformations and a better adjustment of the experimental data. In contrast, increasing the Young's modulus of the masonry will derive in a larger load-bearing capacity, accentuating the current overestimation trend of the model, so other variables seem to be necessarily changed. About the fracture energies, it seems necessary to develop analytical approaches to determine each fracture energy as function of the corresponding strength among other possible parameters. This improvement will provide a scientifically justified alternative to the first approach of using of the same relationship for all fracture phenomena, which has been adopted herein to avoid providing arbitrary values. In fact, the used definition of the fracture energies might explain a great deal of the obtained error and improving it would help to reach more accurate results.

Another simplification which might have contributed to the error of the numerical model is the uniform linear distribution of the mechanical properties of the discrete strengthening system (laminates) along the width of the wall to allow the bidimensional hypothesis. This fact might be related with the overestimation of the load-bearing capacity of the walls because the used simplification assumes a uniform stress distribution on the interfaces although it is known that, for example, the tangential stress distribution is not uniform and there are sections with minor shear strength than the assumed one.

The analysis of the results of the numerical model included in Figure 10(left) show that the simulated walls fail when the tangential strength is reached in half of the length of the inclined contact, supporting the fact that the masonry was the critical material which controls the failure of the modelled structures. However, the resulting non-linear tangential stress distribution in the contacts plotted in Figure 10(left) supports the idea that it is necessary to improve the calculation of the equivalent properties for the bidimensional models. The results plotted in Figure 10(right) indicate that the restraining effect of the horizontal laminates might lead to increasing the tangential stress of the masonry-FRP contact in punctual areas, which might cause adherence problems in the simulated model. Thus, installing horizontal crossing laminates to avoid the debonding phenomena might be counter-productive because the tangential stress distribution of the vertical ones is modified increasing the possibility of a punctual sliding failure that starts a global failure of the laminate adherence. However, this ultimate effect has not been experimentally observed nor numerically predicted.

Comparing the results of the finite element analysis among them, it is noticed that restraining the masonry-FRP contacts to represent the presence of horizontal laminates causes a load-bearing capacity decrease (comparing 2V2H with 2V0H and 3V5H with 3V0H). This is in line with the previous comment. Thus, according with the model, and for the proposed structural configuration and loading pattern, including horizontal FRP laminates is not effective. In contrast, the model reproduces the effect of installing three vertical laminates (3V0H) in comparison with the cases with two laminates (2V0H). A minor load-bearing capacity increase is predicted when adding vertical laminates as experimentally observed. Finally, it has to be remarked that the average load bearing capacity predicted by the model for the cases 3I3I and 3V0H are practically the same. This fact supports the hypothesis of assuming a reduction factor (%) for the FRP properties associated with inclined laminates in the described configuration.

In relation with the analytical results, it has to be noticed that the calculated load bearing capacity is approximately the same for all the considered strengthening patterns. It has to be reminded that the failure mode predicted by the analytical approach was always the compressive collapse of the masonry and the results obtained by applying this methodology indicated that the FRP laminates were barely loaded in comparison with their strength. Because of this, modifying the strengthening pattern has little influence on the results of the analytical approach, which focuses its sensibility on the compressive response of the masonry and the geometric characteristics of the walls. Hence, the evidences seem to indicate that analysing cases with less amount of strengthening laminates would lead to open the possibility of studying the sensibility of the proposed calculation method on the different strengthening patterns. The average error of the analytical approach, if compared with the experimental results, is 10.1%. Thus, the proposed calculation tool is accurate at predicting the load bearing

capacity of FRP-strengthened brick masonry walls subjected to eccentric compressive loads. This fact is more evident if taking into account that the comparison experimental results have a scattering of 6.7%. However, a few particular cases are not perfectly represented, like walls 3V5H_3 and 3I3I_2. Nevertheless, these two cases also show significant experimental differences with the other cases of the corresponding series. Thus, the analytical model correctly calculates the load-bearing capacity of the FRP-strengthened walls and it only presents greater error for the cases which also showed a different experimental response respect to the other two walls of the series.

Comparing the results of the analytical approach and the numerical simulation, it has to be highlighted that the analytical approach achieves better accuracy (10.1% of error) at predicting the load bearing capacity of the studied cases than the numerical model (25.1% of error). However, the finite element analysis is useful at analysing the failure mode, the stress distribution or the vertical displacement, which cannot be obtained with the analytical approach.

5. Conclusions

An experimental campaign on real-size FRP-strengthened brick masonry walls subjected to eccentric compressive load up to failure has been presented and analysed. Taking into account the evidences it can be concluded that installing the FRP on current brick masonry requires thicker adhesive layers than recommended by the producers, which has economic implications and might contribute to a stiffer lateral response. In addition, it has been observed that all the FRP-strengthened walls collapsed due to a compressive/shear failure mode located in the masonry near the ends of the wall. Moreover, the FRP laminates have not debonded from the masonry in any studied case. Thus, the masonry fails before the FRP's debonding process is activated, suggesting an over strengthening of the studied structures. Finally, the analysis of the obtained experimental data, taking into account the geometric differences mentioned in the text, has brought two significant conclusions: the application of FRP strengthening systems allows increasing the load-bearing capacity of the walls over 200%, and the vertical direction is the most effective orientation of the laminates when strengthening against second order bending effects. In this case, the horizontal laminates are not useful and only a partial contribution of the inclined laminates can be taken into account.

The objective of implementing a numerical model, which was capable of predicting the load-bearing capacity of FRP-strengthened walls subjected to eccentric compressive load at a low computational cost, has been achieved. In addition, this model accurately predicts the experimentally observed failure mode and the small influence of the strengthening pattern on the resistance of the analysed cases. Regarding the deformational response of the numerical results, it can be concluded that the predicted lateral deformation might be larger than the experimental one due to the stiffening effect of the thick layer of epoxy adhesive but it might be mostly due to the possible underestimation of the value of the Young's modulus of the masonry. Because of this, further experimental research to study the response of the FRP-masonry interface might be necessary to obtain more accurate values of the corresponding input parameters (fracture energies). In fact, it might be concluded that a significant part of the error of the numerical model might be due to the approximate and maybe controversial

definition of the fracture energies. In addition, a parametric analysis will provide more information for the assessment of the implemented methodology and performing three-dimensional analyses might expand the investigation opportunities in relation with the definition of the interfaces and its influence on the structural response of FRP-strengthened masonry walls.

The analytical approach developed to calculate the load-bearing capacity of FRP-strengthened walls subjected to an eccentric load achieves accurate results, even better than the numerical model, at predicting the load-bearing capacity of the analysed structural cases. Because of this, it might be used for design applications.

To sum up, it can be concluded that the research presented herein contributes to enhance the knowledge about the FRP-strengthening of brick masonry walls and the numerical and analytical calculation of this structural case. In particular, this research has covered the existing gap around a specific loading configuration (eccentric compressive load) that leads to develop second order bending effects.

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List of tables

Table 1. Geometric and mechanical properties of the considered cases

Formatat: anglès (EUA)

Table 2. Mechanical properties of the masonry

Formatat: anglès (EUA)

Table 3. Modified mechanical properties of the FRP and FRP-masonry interface due to the bidimensional simplification

Formatat: anglès (EUA)

Table 4. Experimental results of the tests on FRP-strengthened brick masonry walls under eccentric compressive load

Formatat: anglès (EUA)

Table 5. Numerical results of the simulations

Formatat: anglès (EUA)

Table 6. Analytical results of the calculation of the load-bearing capacity of FRP-strengthened brick masonry walls

Formatat: anglès (EUA)

Series	Wall	e_0 (mm)	H_{ef} (mm)	t (mm)	b (mm)	$\#_{v,FRP}$	$\#_{h,FRP}$	$\#_{i,FRP}$	α (°)	F_{max}^{EXP} (kN)
2V0H	1	29.5	1,798	126.5	837					523.4
	2	31.9	1,815	125.9	840	2	0	0	---	548.4
	3	29.8	1,798	126.1	838					554.4
3V0H	1	29.7	1,811	126.4	829					598.3
	2	31.2	1,814	126.3	833	3	0	0	---	601.0
	3	29.4	1,824	126.3	837					751.0
2V2H	1	29.4	1,788	126.1	843					604.5
	2	32.7	1,792	124.4	835	2	2	0	---	600.8
	3	30.1	1,797	124.7	827					618.0
3V5H	1	28.7	1,777	123.8	833					545.3
	2	27.2	1,784	124.7	830	3	5	0	---	524.2
	3	27.6	1,761	124.5	831					453.0
3I3I	1	26.8	1,830	124.3	826					643.5
	2	28.8	1,831	124.6	831	0	0	4	61	451.9
	3	27.4	1,812	123.8	827					569.7
Control	W#13	31.8	1,660	134.0	900	---	---	---	---	83.8
	W#15	29.0	1,845	134.0	895					236.7

Table 1. Geometric and mechanical properties of the considered cases

	f_m (MPa)	f_x (MPa)	G_f^I (N·m)	f_s (MPa)	G_f^{II} (N·m)	ρ (kg/m ³)	E_{m0} (MPa)
FRP strengthened walls	21.3	0.36	13	1.5	55	1732	2560
W#13 & W#15	10.8	0.36	13	1.8	66	1732	780

Table 2. Mechanical properties of the masonry

	f_{tFRP}^* (MPa)	f_{bFRP}^* (MPa)	G_{fint}^* (N·m)	f_{sFRP}^* (MPa)	G_{fint}^{*II} (N·m)	ρ_{FRP}^* (kg/m ³)	E_{FRP}^* (MPa)
Original FRP ([^])	3100	5.40	200	1.67	60.0	160.0	170000
2V0H and 2V2H cases	595	1.04	38	0.32	11.7	30.7	32650
3V0H and 3V5H cases	893	1.56	57	0.48	17.6	46.1	48980
3I3I cases	1021	1.78	65	0.55	20.2	70.3	56000

([^]) The properties of the first row are not modified and correspond to the ones provided by the manufacturer or are directly calculated from these by using (Eq.1).

Table 3. Modified mechanical properties of the FRP and FRP-masonry interface due to the bidimensional simplification

Series	Wall	$v_{max}/H_{ef}(\cdot 10^{-3})$	h_{max}/t (%)	ϕ_{max} (%)	F_{max}^{EXP} (kN)	Scattering (%)	Load-bearing increase (%)
2V0H	1	5.7	8.6	27.3	523.4	2.3	227
	2	3.7	9.6	28.6	548.4		242
	3	4.8	8.5	28.9	554.4		246
3V0H	1	4.9	11.8	31.5	598.3	10.4	273
	2	5.1	12.8	31.5	601.0		275
	3	1.2	9.3	39.2	751.0		369
2V2H	1	5.5	12.9	31.3	604.5	1.1	277
	2	5.0	11.6	31.9	600.8		275
	3	5.6	9.8	33.1	618.0		286
3V5H	1	4.3	7.7	29.1	545.3	7.2	240
	2	4.4	6.9	27.9	524.2		227
	3	4.4	9.1	24.1	453.0		183
3I3I	1	4.4	10.4	34.5	643.5	12.4	302
	2	5.1	5.5	24.0	451.9		182
	3	4.9	9.8	30.7	569.7		255
Control	W#13	2.2	9.4	6.5	83.8	47.7	---
	W#15	3.2	10.1	18.4	236.7		---

Table 4. Experimental results of the tests on FRP-strengthened brick masonry walls under eccentric compressive load

<i>Series</i>	<i>Wall</i>	$v_{max}/H_{ef} (\cdot 10^{-3})$	h_{max}/t (%)	ϕ_{max} (%)	$F_{max}^{SIM} (kN)$	<i>Abs. rel. err.</i> (%)	<i>Av. err.</i> (%)
2V0H	1	5.2	30.0	33.6	763.4	45.8	26.9
	2	5.0	29.7	31.1	707.0	28.9	
	3	3.1	16.0	25.9	587.6	6.0	
3V0H	1	4.8	26.1	34.0	761.8	27.3	18.8
	2	4.6	24.7	33.1	745.7	24.1	
	3	4.3	23.2	31.5	713.4	5.0	
2V2H	1	3.3	17.2	26.8	610.3	1.0	8.1
	2	3.6	20.3	24.5	545.0	9.3	
	3	4.8	28.4	31.9	703.9	13.9	
3V5H	1	4.6	24.9	32.7	722.1	32.4	36.5
	2	3.6	18.5	30.7	681.7	30.0	
	3	3.7	19.4	30.0	665.6	46.9	
3I3I	1	4.2	22.1	33.8	744.0	15.6	35.4
	2	4.3	22.8	32.6	722.0	59.8	
	3	4.4	23.2	34.0	746.0	31.0	
Control	W#13	1.5	7.3	6.8	87.8	4.8	33.9
	W#15	1.5	8.9	6.9	87.7	63.0	

Table 5. Numerical results of the simulations

<i>Series</i>	<i>Wall</i>	ϕ_{max} (%)	$F_{max}^{AN} (kN)$	<i>Abs. rel. error</i> (%)	<i>Av. error</i> (%)
2V0H	1	25.8	582.6	11.3	7.0
	2	25.6	576.2	5.1	
	3	25.7	579.5	4.5	
3V0H	1	26.5	591.7	1.1	7.9
	2	26.4	592.2	1.5	
	3	26.3	593.2	21.0	
2V2H	1	25.9	586.8	2.9	6.1
	2	25.5	564.0	6.1	
	3	25.5	560.0	9.4	
3V5H	1	26.3	578.6	6.1	15.7
	2	26.4	582.6	11.1	
	3	26.7	588.0	29.8	
3I3I	1	26.5	579.8	9.9	13.7
	2	26.5	584.7	29.4	
	3	26.6	580.0	1.8	

Table 6. Analytical results of the calculation of the load-bearing capacity of FRP-strengthened brick masonry walls

List of Figures

Figure 1. FRP strengthening patterns and position of the strain gages

Formatat: anglès (EUA)

Figure 2. Failure mode of the wall 3V5H_3

Formatat: anglès (EUA)

Figure 3. Test setup for the experimental campaign and geometric simplification for the numerical model

Formatat: anglès (EUA)

Figure 4. Mesh and position of the contacts for the numerical model

Formatat: anglès (EUA)

Figure 5. Proposed procedure for the analytical calculation

Formatat: anglès (EUA)

Figure 6. Lateral displacement of the walls at mid-height versus the applied force

Formatat: anglès (EUA)

Figure 7. Strain measurements at the FRP laminates of the wall 2V0H_1

Formatat: anglès (EUA)

Figure 8. Strain measurements at the FRP laminates of the walls strengthened with an inclined pattern, 3I3I series

Formatat: anglès (EUA)

Figure 9. Strain measurements at the FRP laminates of the wall 2V2H_3

Formatat: anglès (EUA)

Figure 10. Tangential stress distribution on the inclined fictitious contact of the wall 3V0H_3 (left) and tangential stress distribution on the contact between the masonry and the FRP for the wall 2V2H_3 (right)

Formatat: anglès (EUA)

Figure 1. FRP strengthening patterns and position of the strain gages

Figure 2. Failure mode of the wall 3V5H_3

Figure 3. Test setup for the experimental campaign and geometric simplification for the numerical model

Figure 4. Mesh and sketch of the position of the contacts for the numerical model

Figure 5. Proposed procedure for the analytical calculation

Figure 6. Lateral displacement of the walls at mid-height versus the applied force

Figure 7. Strain measurements at the FRP laminates of the wall 2V0H_1

Figure 8. Strain measurements at the FRP laminates of the walls strengthened with an inclined pattern, 3I3I series

Figure 9. Strain measurements at the FRP laminates of the wall 2V2H_3

Figure 10. Tangential stress distribution on the inclined fictitious contact of the wall 3V0H_3 (left) and tangential stress distribution on the contact between the masonry and the FRP for the wall 2V2H_3 (right)