Strategy for an efficient material non-linear multiscale analysis

Xavier Martínez [1,2,], Fermín Otero [1], Sergio Oller [1,3]*
1: International Center for Numerical Methods in Engineering (CIMNE). Gran Capità s/n, 08034 Barcelona, Spain
2: Department of Nautical Science and Engineering - UPC. Pla de Palau 18, 08003 Barcelona, Spain
3: Department of Strength of Materials and Structural Engineering. Edifici C1 Jordi Girona 1-3, 08034 Barcelona
* x.martinez@upc.edu

Abstract

As the internal microstructure of composite materials becomes more complex, multi-scale methods are gaining strength. These allow obtaining the material performance from the analysis of a representative volume element. The use of multiscale procedures is also encouraged by the increase of computational capacity. However, despite of this increase, performing a non-linear multiscale analysis of a real structure is still chimeric due to its computational cost.
This work presents a strategy to conduct non-linear multiscale analysis in an efficient way. It is based in the definition of a threshold function, which is developed specifically for each micro-model using its specific strain field. The procedure allows a large reduction of the computational cost, facilitating its use by researchers and engineers without large computational means. The validity of the proposed strategy is shown with the analysis of a pultruded cantilever beam. The results obtained prove the efficiency of the method.

Keywords: Composites, non-linear analysis, Multiscale analysis, homogenization, computational cost.

1   INTRODUCTION

When looking into recent developments made on multiscale methods, most of them are based in the analysis of specific representative volume elements (RVE), on the procedures that should be followed to connect the different scales of the problem, or on reduction methods that allow solving the problem analyzing a minimum number or RVEs. However, few work is found in which multiscale models are used to solve real structures, with a material non-linear behavior. This is because the solution of structures with multiscale methods is still too expensive to be used easily, as it is stated in Otero *et al.* 2015.

Assuming that in a real structure the material non-linear performance is found only in localized regions, in which maximum stresses are concentrated, this work proposes reducing the computational cost of solving a multiscale problem with the definition of a threshold parameter. The RVE will only be solved in those elements and load steps in which the threshold value is exceeded.

Continuum mechanics establishes the limit between linear and non-linear performance of materials using criterions that compare a given stress tensor with a threshold value (Von-Mises, Mohr-Coulomb, etc.). This approach cannot be used in a multiscale procedure because, the existing interaction between the different components, makes extremely large the number of stress combinations that will lead to material failure. To overcome this problem, the strategy proposed in this work defines a comparison function that looks, not into the maximum load that can be applied to the composite, but into the maximum level of a specific deformation value that can be applied to it. This function is specific for each single element of the simulation, as it depends on the actual deformation applied to it.

2   NON LINEAR ACTIVATION FUNCTION

The definition of a non-linear activation function is based on the fact that any given material begins its non-linear performance when a single particle of the material reaches its stress threshold. Therefore, if the material is solved a RVE, it will become non-linear when one of the materials of the RVE reaches its maximum stress value, which is defined by the constitutive model used to characterize the material.

With this assumption, the elastic energy available in the RVE can be defined as the elastic energy required by the first material that reaches its stress threshold for a given strain value. To obtain this limit elastic energy a function f_i is defined for each material of the RVE with the following expression:

$$f_i = \frac{\psi^e}{\psi^e_{\lim}} = \frac{(\sigma : \varepsilon) \cdot E}{(\sigma_{\lim})^2} \tag{1}$$

where E is the Young Modulus of the material and $\sigma_{\lim}$ is the stress threshold that can be applied to it.

The function f_i provides a value of how close is the material gauss point i of the RVE to reach the non-linear performance for a given strain value. Using the maximum f_i value of all materials in the RVE, is possible to calculate the maximum elastic energy that is available in it, before reaching its non-linear threshold. This maximum elastic energy can be calculated as:

$$\psi_{RVE}^{\lim} = \frac{\psi_{RVE}^{e}}{f_{\max}} \qquad \text{with } f_{\max} = \max\{f_i\} \qquad (2)$$

At any given load step of the analysis process it is possible to calculate the elastic energy of any given gauss point of the macro structure. Having this value, the threshold function for the gauss point in the macro scale is defined as:

$$\psi_{RVE}^{e} - \psi_{RVE}^{\lim} \leq 0 \qquad (3)$$

It is important to remark that the limit elastic energy is valid for a given RVE cell, and for a given strain value applied to this cell. If during the analysis process the strain applied to the cell varies, the threshold function must be re-calculated, in order to obtain the new elastic energy limit. To validate that the strain applied to the RVE has not changed substantially from previous load step, the following equation must be verified:

$$\left| \frac{\varepsilon_0 - \varepsilon_k}{\|\varepsilon_0\| \cdot \|\varepsilon_k\|} - 1 \right| \leq tolerance \qquad (4)$$

being ε_0 the strain used to calculate the elastic energy limit, and ε_k the strain at current step.

3 RESULTS

The proposed procedure has been used to calculate the damage evolution of a pultruded cantilever beam subjected to a punctual load in its free end. The beam is made of glass fibers embedded in a polymeric matrix. The macro-model of the beam is defined with 32 quadratic hexahedral elements, and the micro-model of the composite material is defined with 1464 linear hexahedric elements. Both materials are characterized with a damage model.

Figure 1 shows the force displacement evolution of the beam provided by the model, and the CPU time required to solve it. In it can be seen clearly that when the beam becomes non-linear, the computational time increases substantially. Figure 2 shows the damage parameter obtained in the macro-model and in the micro-model.
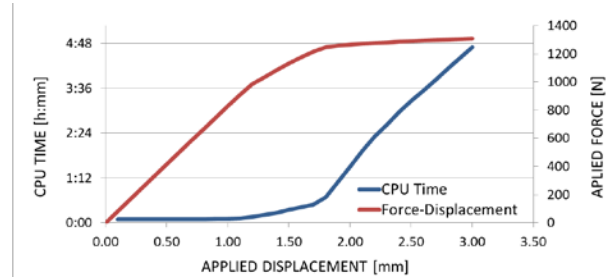

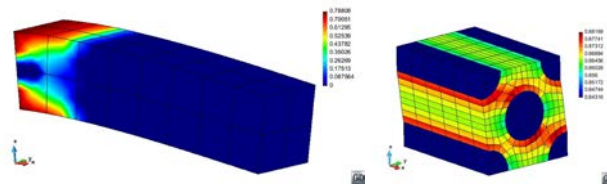**Figure 1. Force-displacement graph of the simulation and CPU time required**

Figure 2. Damage in macro model and in micro model at the last load step.

4 CONCLUSIONS

The validation example has shown that the proposed procedure can conduct non-linear simulations using a multiscale procedure. If this same problem is solved without a threshold function, the CPU time required is nearly 26 hours. This time difference proves the necessity of the developed strategy to solve these sort of problems.

5 ACKNOWLEDGEMENTS

6 REFERENCES

OTERO F., OLLER S., MARTÍNEZ X., SALOMÓN O. Numerical homogenization for composite materials analysis. Comparison with other micro mechanical formulations. *Composite Structures*, 122, pages 405-416. 2015.