Communication Items

“Erratum to: A Closed-Form Expression for the Uncertainty in Odometry Position Estimate of an Autonomous Vehicle”

Josep M. Mirats Tur, Carlos Albores-Borja, and José Luis Gordillo

Abstract—The uncertainty of the odometry position estimate for a given vehicle can be obtained from the definition of the covariance matrix by using its kinematic model as well as its sensory system data. This communication item extends a previous work in [1] by giving the appropriate guidelines so as to compute the cross-covariance terms between the previous position estimate and the actual increments of position. This is necessary in general because the orientation errors on the robot’s previous position will affect the calculation of the actual increments of position.

Index Terms—Localization, navigation, Positioning uncertainty.

In a previous work [1], an expression for the uncertainty in the odometry position estimate of a mobile vehicle using the covariance definition was obtained. This covariance matrix was derived from the vehicle’s kinematic model, and, for each time step, the total covariance was calculated under the assumption that the cross covariance between the previous position and the actual increment of position was zero. However, this hypothesis does not hold in general because the orientation errors in the robot’s previous position will affect the calculations of the actual increment of position. Some considerations are outlined with respect to the mentioned issue in this paper.

Summarizing the concepts given in [1], suppose that for time¹ $t - 1$, the position of the vehicle and its associated uncertainty are given as

$$\hat{P}_t = [\hat{x}_t \quad \hat{y}_t \quad \hat{\theta}_t]^T$$

where

$$\hat{P}_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{t-1} \\ \hat{y}_{t-1} \\ \hat{\theta}_{t-1} \end{bmatrix} + \begin{bmatrix} \Delta \hat{x}_t \\ \Delta \hat{y}_t \\ \Delta \hat{\theta}_t \end{bmatrix}$$

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J. M. Mirats Tur is with the Institut de Robòtica Industrial, Consejo Superior de Investigaciones Científicas (UPC-CSIC), C/Llorens i Artigas 4–6, Barcelona 08028, Spain (e-mail: jmirats@iri.upc.edu).

C. Albores-Borja and J. L. Gordillo are with the Center for Intelligent Systems (CSI), Tecnológico de Monterrey, Monterrey N.L. 64849, Mexico (e-mail: carlos.albores@itesm.mx; jlgordillo@itesm.mx).

¹We are using $t - 1$, for $t - \Delta t$, $t - 2$ for $t - 2\Delta t$, and so on.

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$$= \begin{bmatrix} \hat{x}_{t-1} + \Delta \hat{d}_t \cos \left(\hat{\theta}_{t-1} + \frac{\Delta \hat{\theta}_t}{2} \right) \\ \hat{y}_{t-1} + \Delta \hat{d}_t \sin \left(\hat{\theta}_{t-1} + \frac{\Delta \hat{\theta}_t}{2} \right) \\ \hat{\theta}_{t-1} + \frac{\Delta \hat{\theta}_t}{L} \end{bmatrix}.$$

The variables $\Delta \hat{d}_t$, $\hat{\phi}_t$, which are the displacement and steering measurements that the automated vehicle actually senses, are described as: $\Delta \hat{d}_t = \Delta d_t + \varepsilon_{\Delta d_t}$ and $\hat{\phi}_t = \phi_t + \varepsilon_{\phi_t}$, with white independent Gaussian probability distributions for their errors. Uncertainty of the position estimate is then given by the covariance matrix $\text{Cov}(\hat{P}_t)$, which can be further decomposed into the expression

$$\begin{aligned} \text{Cov}(\hat{P}_t) &= \text{Cov}(\hat{P}_{t-1}) + \text{Cov}(\Delta \hat{P}_t) + \text{Cov}(\hat{P}_{t-1}, \Delta \hat{P}_t^T) \\ &\quad + \text{Cov}(\Delta \hat{P}_t, \hat{P}_{t-1}^T). \end{aligned}$$

Dealing in depth with the previously presented formulation, we found that additional considerations had to be taken into account. First, an error associated to θ_{t-1} , $\hat{\theta}_{t-1} = \theta_{t-1} + \varepsilon_{\theta_{t-1}}$ needs to be considered in the formulations. The error $\varepsilon_{\theta_{t-1}}$ is assumed to be Gaussian in order to facilitate posterior mathematical derivations. Expressions for trigonometric functions of a Gaussian variable can be found in [2]. Second, in the previous presented formulation, the term $\mathbf{E}[\tan(\phi_t + \varepsilon_{\phi_t})]$, was simply bounded to the worst case $\varepsilon_{\phi_t \max}$, using physical considerations. A more accurate approximation to $\tan(\phi_t + \varepsilon_{\phi_t})$ is proposed, given by

$$\begin{aligned} \tan \hat{\phi}_t &= \tan(\phi_t + \varepsilon_{\phi_t}) \\ &= \frac{\tan \phi_t + \tan \varepsilon_{\phi_t}}{1 - \tan \phi_t \tan \varepsilon_{\phi_t}} \\ &= \tan \phi_t + \frac{\tan \varepsilon_{\phi_t} \sec^2 \phi_t}{1 - \tan \phi_t \tan \varepsilon_{\phi_t}} \\ \tan \hat{\phi}_t &\approx \tan \phi_t + \tan \varepsilon_{\phi_t} \sec^2 \phi_t. \end{aligned}$$

Finally, and most importantly, the formulation developed in [1] considered the errors of the previous position $\hat{P}_{t-1}$, and the actual increments of position $\Delta \hat{P}_t$, to be independent. However, as the angle (and its error) of the previous position is considered in the computation of both $\Delta \hat{x}_t$ and $\Delta \hat{y}_t$, there exists a cross covariance between $\hat{P}_{t-1}$ and $\Delta \hat{P}_t$. This causes the considered error model to not completely capture the behavior of the position uncertainty. However, it is beyond the scope of this paper to propose and describe a method to account for the cross-covariance terms; it should be left for future papers.

REFERENCES

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