Atmospheric-Boundary-Layer Height estimation using a Kalman Filter and a Frequency-Modulated Continuous-Wave Radar

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An adaptive solution based on an Extended Kalman Filter (EKF) is proposed to estimate the Atmospheric Boundary-Layer Height (ABLH) from Frequency-Modulated Continuous-Wave (FMCW) S-band weather-radar returns. The EKF estimator departs from previous works, in which the transition interface between the Mixing-Layer (ML) and the Free-Troposphere (FT) is modeled by means of an erf-like parametric function. In contrast to lidar remote sensing where aerosols give strong backscatter returns over the whole ML, clear-air radar reflectivity returns (Bragg scattering from refractive turbulence) shows strongest returns from the ML-FT interface. In addition, they are corrupted by “insect” noise (impulsive noise associated with Rayleigh scatter-

ing from insects and birds), all of which requires a specific treatment of the problem and the measurement noise for the clear-air radar case. The proposed radar-ABLH estimation method uses: (i) a first pre-processing of the reflectivity returns based on median filtering and threshold-limited decision to obtain “clean” reflectivity signal, (ii) a modified EKF with adaptive range intervals as time tracking estimator, and (iii) ad-hoc modelling of the observation noise covariance. The method has successfully been implemented in clear-air, single-layer, convective boundary layer conditions. ABLH estimates from the proposed radar-EKF method have been cross-examined with those from a collocated lidar ceilometer yielding a correlation coefficient as high as $\rho = 0.93$ (mean signal-to-noise ratio, $SNR = 18$ (linear units), at the ABLH) and in relation to the classic threshold method.

I. INTRODUCTION

The Atmospheric Boundary Layer (ABL) can be defined as that part of the troposphere that is directly influenced by the presence of the Earth's surface and responds to surface forcings with a timescale of about one hour or less [1]. Because the definition of the ABL is inherently related to the turbulence and there is not a single instrument or method to measure it directly, one must measure a proxy or driver of the turbulence, instead. Thus, remote sensing of the ABL has been tackled with different types of remote sensors and ABL height (ABLH)-retrieval methods. Examples of sensors are backscatter lidars [2][3][4], High-Resolution Doppler lidars (HRDL) [5], ceilometers [6], sodar, radar wind profilers [5], and Radio Acoustic Sounding Systems (RASS), either individually [7] or in combination [6]. Classic ABLH estimation algorithms which are applied to the temperature, relative humidity, and brightness temperature profiles measured from radiosondes [7][8] and microwave radiometers [9] include, for example Parcel Method (PM), the Bulk Richardson's number method (BR) and Multivariate Linear Regression. Derivative methods such as the Gradient Method (GM) and the Inflection Point Method (IPM), the Threshold Method (THM), the Variance Method (VM) and their variations are often applied to the measured backscattered power returns from lidars and ceilometers using atmospheric aerosols as indicators [10][11][12]. See [7] for an extensive review and [13][14][15] for studies on the convective boundary layer (CBL). The ABLH is usually retrieved with radar by applying a peak-locating algorithm [16] that finds the peak SNR in the range-corrected SNR profile measured by the wind profiler. Results are in good agreement with those obtained by applying wavelets to HRDL backscatter profiles [5].

So far, the wide majority of ABLH estimation methods such as the ones cited above

are “memoryless” algorithms. This means that the estimation of the ABLH is carried out based only on the current measured atmospheric profile (or a time-averaged spatial-smoothed version of it in order to enhance the signal-to-noise ratio (SNR) available) and independently from all other measured profiles. As a result, a major drawback of such algorithms is their lack of time continuity when providing the ABLH estimates, which can yield time discontinuities and outliers in these estimates.

The Extended Kalman Filter (EKF) [17] is an adaptive algorithm that combines past with present estimates and an “a priori” model to provide optimal time-continuous estimates. Thus, the EKF was applied in [18][19] to estimate the atmospheric optical extinction- and backscatter-coefficient profiles from backscatter lidar returns. In [20] the authors used a scalar Kalman filter to estimate the ABLH from sodar signals. Very recently, [4] has successfully applied the EKF to estimate the ABLH from backscatter lidar signals and by comparison to classic ABLH estimation methods. The filter estimates as a function of time, the ABLH, the approximate Entrainment Zone (EZ) thickness, and backscatter lidar levels within the Mixing Layer (ML) and Free Troposphere (FT). These optimal estimates provide minimum mean square error over time in a statistical sense thanks to adaptively fitting a parametric model shape function to the successive lidar measurements. Because the filter blends past estimates with the present-time measurement to yield a best present-time estimate, it enables the filter to work in low-SNR conditions without the need to degrade the temporal resolution. Although there is not a single ABLH estimation method providing ground truth, the THM [10][11][12] or derivative methods [2][3] are usually used as a reference. In the case of THM, the user-defined threshold determines the accuracy of the estimation.

Clear-air radar systems detect fluctuations of the refractive index of the atmosphere that

occur at the scale of half the wavelength of the radar [1]. In fair weather conditions, the ABL is often more humid than the FT, and centimeter-scale eddies at the interface between the mixed layer and the free atmosphere create strong returns. Within the ML there is less returned energy in spite of the strong mixing, because the humidity is more uniform.

Most Weather radars are, however, neither powerful nor sensitive enough to detect most clear-air ABL phenomena over their operating range; so only a smaller number of research radars have been used for this purpose.

In weather radar, refractive index gradients give rise to Bragg scattering [21], which is the signal component. For purpose of ABLH estimation, Rayleigh scattering due to hydrometeors and hydrometeor-like scatterers, e.g., insects and birds, is an interfering component in addition to the usual thermal noise. To remove this interference different techniques have been proposed: [16][22] based their method on elimination of spatial samples or “outliers” characterised by a SNR, speed or spectral width exceeding a predetermined standard-deviation threshold (usually 2-3 σ) computed over 1-h measurements. Out-of-threshold signal levels are discarded hence preserving information on the measured turbulence intensity from the radar signal. [23] proposed a statistical averaging technique in which, in contrast to classic Doppler-radar spectral estimators, signals from different objects are identified and separated before the average spectral estimate is made.

In this paper, we propose time-adaptive ABLH estimation from FMCW S-band radar returns using an EKF under conditions of clear-air, single-layer, convective boundary-layer atmosphere. There are at least two differential elements worth mentioning as compared to [4], in which the EKF is applied to backscatter lidar returns. First is the application of the EKF to the radar case itself. While in the lidar case, and under similar single-layer convective boundary layer conditions, the ML-FT transition interface was well modelled by means of an

erf-like function, in the radar case, these atmospheric regions as well as the EZ in between are largely distorted by the presence of impulsive noise peaks due to insect-and-birds echoes (Fig. 1a and Fig. 3) [21][24]. As a result, traditional ABLH derivative methods, for which such peaks are largely amplified by the derivative operator, are insufficient to deal with raw radar returns. Second is the elimination of interfering Rayleigh noise component (birds and insects), which involves a specific pre-processing for the radar case and appropriate modeling of the residual noise as observation-noise to the filter.

This paper is organized as follows: Section II revisits the concept of reflectivity, and Bragg and Rayleigh scattering in the context of the weather-radar equation. Sect. III develops the EKF adaptive ABLH detection method with emphasis on (i) the pre-processing steps used to remove the contribution of Rayleigh scatterers (insects and birds), and (ii) adaptation of the radar EKF to the radar case. Sect. IV illustrates a real-case 1.5-h tracking scene where the radar EKF estimates are cross-examined with those from a lidar ceilometer as ground truth. Finally, Sect. V gives concluding remarks.

II. WEATHER-RADAR FOUNDATIONS

FMCW radars rely on the same radar equation as pulsed radars with the exception that the transmitted power is no longer pulsed but continuous-wave frequency modulated, instead [25]. Hereinafter, whenever the term “radar” is used, it will refer to “FMCW radar”, unless otherwise indicated.

The monostatic radar equation (i.e., with collocated emission and receiver antennas) can be expressed as

$$P_r = \frac{P_t}{4\pi R^2} G_t \frac{1}{4\pi R^2} \eta V \left(\frac{\lambda^2}{4\pi} G_r \right), \quad (1)$$

104 where P_r [W] is the receiver power, P_t [W] is the transmitted power, R [m] is the range along
 105 the radar line of sight (LOS), G_t [] and G_r [] are the transmitter and receiver antenna gains,
 106 respectively, λ [m] is the radar wavelength, and η [m²/m³] is the volume reflectivity within
 107 the sampling volume V . The sampling volume, $V = \Delta R \Delta S$, is defined as the product of the
 108 radar range resolution, ΔR , times the “beam area” of the receiving antenna at the range R ,
 109 $\Delta S = R^2 \Delta \theta \Delta \phi$, with $\Delta \theta$, $\Delta \phi$ the angular E- and H-plane HPBW (Half Power BandWidth)
 110 of the antenna pattern.

When *clouds and precipitation* are considered (scatterers’ diameter, e.g., a raindrop, much smaller than the radar wavelength, $D \ll \lambda$) the prevailing scattering mechanism is Rayleigh, in which the volume reflectivity is given by

$$\eta = \frac{\pi^5}{\lambda^4} |K|^2 Z, \quad (2)$$

111 In Eq. 2, $|K|^2$ is a factor depending on the dielectric constant of the scattering medium
 112 ($K = (\varepsilon - 1)/(\varepsilon + 2)$, $\varepsilon = 0.934 - j0.011$ for water at 0°C and $\lambda = 10$ cm) and Z is the
 113 radar reflectivity factor.

114 Insects are generally not good “tracers” of the ABL [21]. In [26], a dual-polarized S-band
 115 radar was used to evaluate insects as a tracer of the ABL motion. It was inferred that insects
 116 were reorienting themselves in response to air motion, to avoid temperatures less than 10-15
 117 deg. Therefore, insects are not passive tracers and cannot be used to estimate the ABLH.
 118 Moreover, insects are effective tracers of horizontal wind velocities during summer daylight
 119 hours (Wilson et al. [22][27]). Birds have their own velocity of movement (10-20 [ms⁻¹])
 120 and they can be treated like insects [28].

121 In *clear-air conditions*, Bragg and Rayleigh scattering (the latter mostly due to birds and
 122 insects) are the prevailing scattering mechanisms. Bragg scattering at the radar frequencies
 123 of interest (S and C bands [25][29]) is due to very strong gradients and random fluctuations

of the refractive index associated with discontinuities and/or turbulences of the atmosphere.

For Bragg scattering, η is commonly related to the refractive index structure function parameter, C_n^2 , [25][30] by

$$\eta = 0.38 C_n^2 \lambda^{-1/3}. \quad (3)$$

In terms of frequency, Bragg scattering composes a substantial part of the backscatter for frequencies below 3 GHz [31][24], while Rayleigh scattering tends to dominate for higher frequencies [21].

III. ABLH ESTIMATION

A. Radar reflectivity pre-processing

The pre-processing steps are aimed at outputting “clean” time-height profiles of the Bragg component of radar reflectivity by removing the insect’s interference in the radar reflectivity measurement, so that the ABLH can be estimated from an almost “clean” Bragg scattering atmosphere (Fig. 1f). Towards this aim, the pre-processing methodology outlined in [21] is analytically formulated and implemented.

The pre-processing block diagram is shown in Fig. 2 and the successive pre-processing steps are shown in Fig. 1, each panel (a-f) corresponding to one step. In Fig. 1a, the time-height measured reflectivity image, $\eta_{raw,1}$, is a $M \times N$ matrix consisting of M time profiles of N range samples each. The spatial resolution, ΔR , is 5 m/sample and the temporal resolution, Δt , 16 s/sample. Further measurement details are given in Sect. IV.

Impulsive noise due to insects can be virtually removed by using a 7×7 -sample median filter applied to the time-height radar reflectivity, $\eta_{raw,1}$, of Fig. 1a (blue trace in Fig. 3). Though the optimum window size of the median filter depends on the density of insects,

Figure 1. Radar reflectivity pre-processing case example (Boulder, CO., August 16, 2007, 14:41:36 UTC (08:41:36 LT) to 15:44:43 UTC, time records t_{100} to t_{335}). (a) Raw reflectivity image, $\eta_{raw,1}$. Isolated red dots visible from approximately 200–450 m in height correspond to “insect noise”. Vertical line around 800 s is dead time where the radar is not measuring (data backup). (b) Median-filtered image, η_{med} , by using a 7×7 -sample filter applied to (a). (c) Residual error, ε_{rs} , computed as image (a) minus image (b). (d) Black&white mask, η_{th} , referred to as “insect echo image”, Eq. 5. The figure represents η_{th} in inverted black-and-white colour-map form, so that 1 (“insect”) is coded as black and 0 (“no insect”) is coded as white. (e) Bragg scattering image, $\eta_{raw,2}$, composed after masking (a) with mask (d), Eq. 6. Note that insects have been replaced by voids. (f) Clean reflectivity image, η_{clean} , composed by “filling” these voids in image (e) with the median-filtered values in (b).

Figure 2. Radar reflectivity pre-processing block diagram. Letters in brackets refer to the different processing stages in Fig. 1 panel. Refer to Eq. 7 for the mask equation

by experiment a 7×7 -sample window yields the best trade-off between temporal/spatial resolution and impulse-noise cancellation [21]. The filter serves to remove isolated impulsive echoes that occupy less than half of window size. The resulting median-filtered image, η_{med} (Fig. 1b and red trace in Fig. 3), is subtracted from the original one to yield a *differential* image (Fig. 1c and magenta trace in Fig. 3),

$$\varepsilon_{rs} = \eta_{raw,1} - \eta_{med}. \quad (4)$$

According to [21], a 1-dB threshold level is applied to the differential image of Eq. 4 above. Pixels equal to or above this threshold are identified as “impulsive noise” (i.e., insect pixel) and hence they are reset to zero. Pixels below are assumed to be “signal component” (i.e., Bragg scattering) and are retained. Formally,

$$\eta_{th} = \begin{cases} 1 & \text{if } \varepsilon_{rs} \geq 1 \text{ dB} \\ 0 & \text{if } \varepsilon_{rs} < 1 \text{ dB} \end{cases} \quad (5)$$

140 The 1-dB threshold equals two standard deviations above the average echo power for an
 141 averaged profile of 100 samples and assumes a probability of false alarm (i.e., identifying a

Figure 3. Radar reflectivity as a function of range (height AGL). Time profile, $t_{120}=1917$ s (14:46:58 UTC) in Fig. 1. (Blue) Raw reflectivity, $\eta_{raw,1}^{120}$ (compare with Fig. 1a). (Red) Median-filtered reflectivity, η_{med}^{120} (compare with Fig. 1b). (Magenta) Residual error, ε_{rs}^{120} (compare with Fig. 1c). (Horizontal dashed black line) 1-dB threshold level. Black dots superimposed to $\eta_{raw,1}^{120}$ indicate ranges where the residual error exceeds the 1-dB threshold of Eq. 5 and hence an “insect” or interfering Rayleigh scatterer is detected. (Dashed green) Clean reflectivity, η_{clean}^{120} (compare with Fig. 1f). (Thick black) Erf-like model profile fitted to the clean reflectivity, $\eta_{raw,1}^{120}$. R_1 and R_2 indicate initial and end data-processing ranges of the EKF, respectively, at time t_{120} . R'_1 and R'_2 indicate the approximate start and end ranges of the erf transition.

142 pixel as containing an insect when it does not) of less than 2.5%.

This filtered and thresholded image, η_{th} , is referred to as the “insect-echo” image. Because η_{th} is of Boolean type, it is represented as a black-and-white image in Fig. 1d (dots in Fig. 3). In Fig. 1d, η_{th} is represented by using an inverted black-and-white colour-map, that is, with 1 (“insect”) coded as black and 0 (“no insect”) coded as white. By this means, this black-and-white figure can be better interpreted as a black-and-transparent mask that once visually superimposed to the raw reflectivity image, $\eta_{raw,1}$ of Fig. 1a, it enables to see the raw-reflectivity part of the image that is free from insects. Mathematically, when the mask η_{th} is applied to $\eta_{raw,1}$, the free-of-insects Bragg scattering image, $\eta_{raw,2}$, is obtained as

$$\eta_{raw,2} = \eta_{raw,1} \overline{\eta_{th}}. \quad (6)$$

The Bragg scattering image above is noted $\eta_{raw,2}$ because it is still a raw image with insects replaced by voids (zeroes). In Fig. 1e, these voids correspond to the above-1-dB-threshold pixels identified in Eq. 5.

The next step is to replace them with the median-filtered values of Fig. 1b, which are the interpolated values for the voids using the available data. Thus, the clean reflectivity signal (Fig. 1f) or equivalently, the green trace in Fig. 3, which is due only to Bragg scattering, is constructed as

$$\eta_{clean} = \eta_{raw,2} + \eta_{med} \eta_{th}. \quad (7)$$

If Eq. 6 is substituted into Eq. 7,

$$\eta_{clean} = \eta_{raw,1} \overline{\eta_{th}} + \eta_{med} \eta_{th}, \quad (8)$$

η_{th} can be seen as a binary digital selector (0/1) so that any pixel identified as “insect” ($\eta_{th} = 1$) is replaced by its corresponding median filtered one and any other pixel identified as “no-insect” ($\eta_{th} = 0$) retains the measured reflectivity, $\eta_{raw,1}$. The outcome of this pre-processing is that impulsive noise due to insects becomes smoothed out in the clean reflectivity image and without loss of the original spatial/temporal resolution of the measured data, $\eta_{raw,1}$, for those pixels not corrupted with insects (for corrupted pixels the image resolution is approximately degraded by a factor 7, the median-filter window size).

B. ABLH estimation using an EKF

After the pre-processing steps carried out in Sect. III A, the ABLH is now estimated from the clean reflectivity image, η_{clean} (Fig. 1f) using an Extended Kalman Filter (EKF). In what follows, η_{clean} is noted η to simplify notation.

The formulation of the EKF is essentially the same as the one proposed in [4] for the

lidar case with the exception that now backscatter returns have been replaced by the radar reflectivity ones. Extension of the EKF to the FMCW-radar case is summarised next. Noise modelling, as a key distinguishing feature of its application to the radar case, is discussed in Sect. III C.

The EKF is a recursive adaptive filter that uses a parameterisation of the ABL – the so-called *state vector*, x_k – to estimate the ABLH and accessory ABL-related parameters at each succeeding discrete time, t_k . The filter uses two models: (i) the measurement model and (ii) the state-vector model.

Measurement model.– Following [32], the ML-FT interface is modelled by means of an erf-like function, which is parameterised as [4]

$$h(R; R_{bl}, a, A, c) = \frac{A}{2} \left\{ 1 - \operatorname{erf} \left[\frac{a}{\sqrt{2}} (R - R_{bl}) \right] \right\} + c, \quad R \in [R_1, R_2], \quad (9)$$

and where the state vector is defined by the column vector,

$$x_k = [R_{bl,k}, a_k, A_k, c_k]^T, \quad (10)$$

with subscript k a reminder of discrete time t_k . In Eq. 9 and 10 above, R stands to the range (height), usually in the form of a N -sample discrete vector, $[R_1, R_2]$ is the inversion range, $R_{bl,k}$ stands for the ABLH at time t_k , a_k is a scaling factor related to the entrainment zone (EZ) transition thickness ($2.77a^{-1}$) at time t_k , A_k is the transition amplitude of the radar reflectivity profile (equivalently, the difference between ML and FT reflectivity values), and c_k is an offset term modelling the FT reflectivity or noise level at the end of the inversion range. An example of this erf-like behaviour is depicted in Fig. 3 (thick black trace).

The measurement model is formulated as

$$z_k = h(x_k) + v_k, \quad (11)$$

173 where z_k is the pre-processed observation vector or clean radar reflectivity, η_{clean} (dB), h is
 174 the ABL transition model of Eq. 9, and v_k is the observation noise at time t_k . The latter
 175 merges into a single body both measurement noise and modelling errors by means of its
 176 associated noise covariance matrix $R_k = E[v_k v_k^T]$ (see Sect. III C).

In the Kalman filter recursive cycle, the observation model of Eq. 11 is linearised around
 the “a priori” state-vector estimate, $\hat{x}_k^-$ (i.e., prior to assimilating the present measurement
 at time t_k), in the form of a $N \times 4$ -Jacobian or “sensitivity” matrix [17],

$$H_k(R; x) = \left[\frac{\delta h(R)}{\delta R_{bl}}, \frac{\delta h(R)}{\delta a}, \frac{\delta h(R)}{\delta A}, \frac{\delta h(R)}{\delta c} \right] \bigg|_{x=\hat{x}_k^-}. \quad (12)$$

177 In Eq. 12 each column is the first-order derivative of Eq. 9 with respect to components 1-4
 178 of the state vector, x_k , and each row corresponds to a discrete range R_i , $i = 1..N$.
 179 Because of the different slopes or sensitivities of the erf function in the range interval $[R_1, R_2]$,
 180 the first two derivatives ($\frac{\delta h(R)}{\delta R_{bl}}$ and $\frac{\delta h(R)}{\delta a}$) are computed in the inner range interval, $[R'_1, R'_2]$,
 181 where the idealised erf-transition structure occurs (see Fig. 3). The second two derivatives
 182 ($\frac{\delta h(R)}{\delta A}$ and $\frac{\delta h(R)}{\delta c}$) are computed in the outer range intervals, $[R_1, R'_1] \cup [R'_2, R_2]$, where the erf-
 183 model is nearly constant (“plateau” intervals). Selection of ranges R'_1 and R'_2 is not critical,
 184 the key requirement being that they must define inner and outer range intervals containing
 185 erf-transition and erf-plateau characteristics, respectively, as described above.

In the present implementation of the EKF, boundaries R'_1 and R'_2 are allowed to adap-
 tively change with time. Though this is not a requirement, this is of computational advantage
 in instances where the ABLH, $R_{bl,k}$, may substantially change from its initialisation value
 during the time frame under study or when the time frame to be tracked by the EKF is long
 (e.g., several hours). As an example of the present implementation of the EKF, R'_1 and R'_2
 (inner part of the erf-like model where the function is more abrupt), and R_1 and R_2 (outer
 part of the erf-like model, “plateau” ranges) change adaptively with the estimated ABLH,

Figure 4. Temporal evolution of EKF adaptive-range boundaries, R_1 and R_2 (inner-range boundaries, refer to Fig. 3), and R'_1 and R'_2 (outer-range boundaries) according to Eqs. 13-14. R_1 (black solid line) and R_2 (grey solid) are the starting and end points of the inversion range, respectively. R'_1 (black dashed line) and R'_2 (grey dashed) represent start and end ranges of the erf transition. Filled symbols indicate the initialization for these four boundaries. W_0 , W_1 , and W_2 are the instantaneous widths of the inner range interval $[R'_1, R'_2]$, and outer intervals $[R_1, R'_1]$, $[R'_2, R_2]$, respectively (Eq. 13)

$R_{bl,k}$, but are constrained to constant range interval widths, W_i , $i = 0..2$, which are preset by the user (refer to Fig. 3),

$$R'_{2,k} - R'_{1,k} = W_0, \quad R'_{1,k} - R_{1,k} = W_1, \quad R_{2,k} - R'_{2,k} = W_2, \quad \forall k. \quad (13)$$

The recursive procedure (illustrated in Fig. 4) to adaptively update the boundary ranges ensures that R'_1 and R'_2 are always centered around the estimated ABLH, $R_{bl,k}$, via the recursive step,

$$R'_{1,k+1} = R_{bl,k} - W_0/2, \quad R'_{2,k+1} = R_{bl,k} + W_0/2. \quad (14)$$

186 R_1 and R_2 are updated accordingly by using Eq. 13 above.

State-vector model.- This model formulates a random transition model for the state vector from time t_k to time t_{k+1} of the form,

$$x_{k+1} = \Phi_k x_k + w_k. \quad (15)$$

From [4], a Gauss-Markov random model with $\Phi_k = I$, I the identity matrix, has been found a simple and convenient model. The state-vector model requires three “a priori” inputs provided by the user: *i)* an initial guess of the state vector to be estimated, $\hat{x}_0^-$, *ii)* an estimate of the atmospheric state-noise covariance matrix, $Q_k = E[w_k w_k^T]$, and *iii)* an estimate of the initial “a priori” state-vector error covariance matrix, $P_0^- = E[e_0^- e_0^{-T}]$, where $e_0^- = x_0 - \hat{x}_0^-$ is the “a priori” error between the atmospheric state vector, x_0 (unknown), and the initial guess, $\hat{x}_0^-$. The state-noise covariance matrix is aimed at statistically modelling the atmospheric fluctuations or variability in the state-vector components, which should be formulated in terms of assumed variances and correlations among them [18].

In the lidar case, a diagonal matrix, $Q_k = \text{diag}[\sigma_{R_{bl}}^2, \sigma_a^2, \sigma_A^2, \sigma_c^2]$, with standard deviations proportional to the state-vector initial guess $(\sigma_{R_{bl}}, \sigma_a, \sigma_A, \sigma_c) = \mu_Q (R_{bl,0}, a_0, A_0, c_0)$ via a factor μ_Q has been found to express successfully this concept in a simple form and, by experiment, to be a convenient extension of the lidar model to the radar case presented here. In short form,

$$\sigma_Q = \mu_Q \hat{x}_0^-, \quad \sigma_Q = (\sigma_{R_{bl}}, \sigma_a, \sigma_A, \sigma_c). \quad (16)$$

For example, if the ABLH at the filter start-up time, t_0 , is initialised with $R_{bl,0} = 2000$ m and $\mu_Q = 0.1$, this means, after Eq. 17, that 3- σ fluctuations in the ABLH are expected to be of roughly ± 600 m.

The “a priori” state-vector error covariance matrix is also expressed in diagonal form, $P_0^- = \text{diag}[\sigma_{e,R_{bl}}^2, \sigma_{e,a}^2, \sigma_{e,A}^2, \sigma_{e,c}^2]$, where $\sigma_{e,X}$ is the user’s “a priori” error on the state-vector components, $X = R_{bl}, a, A, c$. Or, equivalently to Eq. 16, it can be formulated,

$$\sigma_P = \mu_P \hat{x}_0^-, \quad \sigma_P = (\sigma_{e,R_{bl}}, \sigma_{e,a}, \sigma_{e,A}, \sigma_{e,c}), \quad (17)$$

where σ_P denotes $\sigma_{P_0^-}$ and μ_P is the “a priori” state-vector covariance matrix factor.

At each successive iteration of the EKF, a new reflectivity measurement z_k is assimilated and (i) a new state vector $\hat{x}_k$, (ii) a new “a posteriori” (i.e., after assimilating the current measurement from time t_k) error covariance matrix, P_k , and (iii) a new Kalman gain, K_k (the “projection” gain) are estimated. With this information, the filter can correct its projection trajectory and enhance its current estimation of the state vector parameters, $\hat{x}_k$ and, more specifically, of the ABLH.

C. Treatment of the observation noise

The noise covariance matrix R_k at a time t_k is defined as the covariance of the observation noise vector v_k . This covariance matrix merges into a single body both measurement noise and modelling errors, and it is aimed at informing the filter of the quality of the measurement observables, z_k , at each successive t_k . Formally,

$$R_k = E[v_k v_k^T], \quad (18)$$

where $E[\]$ is the expectancy operator over the ensemble of noise realizations, and v_k is the N -component noise vector, i.e., associated to height ranges, R_i , $i = 1..N$.

Under the assumption of “clean” radar reflectivity measurements corrupted with white Gaussian additive noise, R_k takes the form of the diagonal matrix,

$R_k = \text{diag}[\sigma_n^2(R_1), \sigma_n^2(R_2), \dots, \sigma_n^2(R_N)]|_{t=t_k}$, where each element along the diagonal is the noise variance, $\sigma_n^2(R_i)$, $i = 1..N$. A major difficulty impairing estimation of the noise covariance matrix at each successive time t_k is that only a single noise realization $\nu_k = [\nu(R_1), \nu(R_2), \dots, \nu(R_N)]|_{t=t_k}$ is available at each measurement, not an ensemble of realizations from which to compute Eq. 19.

There is a way out if the ergodicity principle [33] is assumed to compute the noise covari-

ance statistics over uniformly spaced range intervals, which is equivalent to replacing the time ensemble by the spatial ensemble. The method is based on subdividing the measurement range R_i , $i = 1..N$ into uniform-length intervals, $I_p, p = 1, \dots, P$ ($P=20$ in Fig. 5), where the variance is to be estimated at time t_k . Therefore, the instantaneous noise covariance matrix estimate is computed as

$$\hat{R}_k = \text{diag}[\sigma_n^2(I_1), \sigma_n^2(I_2), \dots, \sigma_n^2(I_P)]|_{t=t_k}, \quad p = 1..P, \quad (19)$$

where $\sigma_n^2(I_i)$ is the piece-wise spatial noise variance computed over the range interval I_p :
 $[R_{(p-1)\Delta+1} \dots R_{p\Delta}]$, $\Delta = \frac{N}{P}$; $p = 1..P$. Reba et al. [34][35] have successfully applied piece-wise estimation methods to assess the signal-to-noise ratio from elastic backscatter lidar signals.

The ergodicity principle for stationary random processes assumes that the variance computed over random samples of a given realization of the process is equal to the variance computed over the ensemble of realizations. To test the validity of the ergodicity hypothesis applied to the radar case of Fig. 1, Fig. 5 compares the time-averaged piece-wise *spatial variance* of the noise, $\bar{\sigma}_n^2(I_p)$ (green thick line), with the range-dependent *temporal variance* (blue thick line), $\sigma_n^2(R_i) = E[v_k(R_i) v_k^T(R_i)]$, $i = 1..N$. While $\sigma_n^2(I_p)|_{t_k}$ is computed “on-line” by the EKF, $\sigma_n^2(R_i) = E[v_k(R_i) v_k^T(R_i)]$ must be computed “off-line” as it requires to have the whole set of measurements available.

The fact that both temporal and spatial variances are approximately coincident in Fig. 5 except for the first interval I_1 validates the ergodicity hypothesis previously assumed. The discrepancy in interval I_1 is due to the parallax of the radar antennas below 50m in height and to ground clutter effects blinding the radar at low heights. The reader will also notice that Fig. 5 displays a height range up to 1500m (in contrast to the usual 500m m in Fig. 1) to show the strong returns from “insect” noise above 400m, where the atmosphere is expected

Figure 5. Spatial and temporal variances of the observation noise (case example of Fig. 1). $N = 300$ range cells (spatial resolution, 5 m), $P = 20$ range intervals, $\Delta = 15$ samples/interval. 15 measurement records, (time resolution, 16s). (Grey lines) Instantaneous piece-wise *spatial noise variance*, $\sigma_n^2(I_p)|_{t=t_k}$, $p = 1..P$ computed along the 20 range intervals, I_p . There are 15 variance estimates (horizontal lines) per range interval. Each variance estimate is associated to a time realization, t_k , of the reflectivity, η_{clean} . (Green thick line) Time-averaged piece-wise spatial variance, $\bar{\sigma}_n^2(I_p)$. Computed as the time average of these 15 spatial variance estimates in each range interval, I_p . (Blue thick line) Range-dependent *temporal variance*, $\sigma_n^2(R_i) = E[v_k(R_i) v_k^T(R_i)]$, $i = 1..N$. Computed as the time ensemble of the 15 radar reflectivity realizations. (Black thick dashed line) *Piece-wise temporal variance*. Average of the blue thick line over each range interval, I_p , for visual comparison.

to be clear. Strong peaks appear around 500 and 700m in height along with a large amount of scattered residual peaks from 400m up.

IV. DISCUSSION

The case presented here and already introduced in Fig. 1 was taken on August 16th, 2007, 14:15:01 UTC (08:15:01 LT) to 15:44:43 UTC when the University of Massachusetts deployed an S-band FMCW radar (11cm wavelength, 250W, 3-deg bandwidth) along with a Vaisala CL-31 lidar ceilometer (910-nm wavelength, 1.2- μ J energy, 100-ns pulse width, 10-kHz repetition rate, coaxial laser-telescope arrangement, 96-mm aperture) at NOAA's Boulder Atmospheric Observatory (BAO) in Erie, Colorado. Because within the ML the

aerosol mixture can be considered homogeneous, causing strong optical lidar backscatter returns, the ceilometer instrument was taken as reference ground truth.

The ceilometer was co-located with the radar to monitor the ABL and cloud cover with both instruments pointed vertically [36]. The radar was configured to operate with $\Delta R_{radar}^{raw} = 5$ m spatial resolution. The temporal resolution was $\Delta t_{raw}^{radar} = 1$ s. The ceilometer was operated with $\Delta R_{raw}^{ceil} = 10$ m spatial resolution and $\Delta t_{raw}^{ceil} = 16$ s temporal resolution. Because of the different temporal resolutions between both instruments, radar measurements have been time averaged in blocks of 16 s (Fig. 1a) to yield the same data temporal resolution as that of the ceilometer (the so-called, “clean data” time resolution, $\Delta t = \Delta t_{clean}^{radar} = \Delta t_{clean}^{ceil} = 16$ s). A further advantage of this averaging has been to increase the signal-to-noise ratio (SNR) and hence to comparatively minimise the impact of synchronous interferences (weak horizontal lines in Fig. 1d) with the emission of radar pulses.

The pre-processing steps described in Sect. III A do not significantly degrade the spatial/temporal resolutions above as evidenced by the fact that the masking procedure does not seriously blur Fig. 1f when compared to Fig. 1a. This is because the 7×7 median filter is only applied to a comparatively small population of pixels occupied by insects. As a result, clean spatial and temporal resolutions, $\Delta R_{radar} = 5$ m and $\Delta R_{ceil} = 10$ m and $\Delta t = 16$ s can respectively be assumed.

Fig. 6 shows radar and ceilometer *observables* in color-plot form for the whole observation period. The ABLH estimated by the ceilometer EKF and the radar EKF is superimposed. Ceilometer EKF implementation is described in [4] and filter initialization parameters are summarised in Table I. The blue bands and the bottom of the radar and ceilometer plots of Fig. 6 are due to the different starting measurement ranges of these instruments.

Radar-EKF observation-range parameters, R_1 , and R_2 , and R'_1 and R'_2 have been ini-

Figure 6. FMCW-radar and ceilometer observables to the EKF along with ABLH estimates. (Boulder, CO., August 16, 2007, 14:15:01 UTC (08:15:01 LT) to 15:44:43 UTC). *a*) (Magenta dots) ABLH estimated from radar reflectivity measurements and the radar EKF. (White dots) Comparison with the classic THM. *b*) Validation using Vaisala CL-31 lidar ceilometer. (Magenta dots) ABLH from the ceilometer EKF. Dotted lines in time intervals 1600 to 2000, and 4500 to 4900 s delimit “thermals” (see discussion).

267 tialized from rough visual inspection of the first reflectivity profiles of Fig. 6, which are
 268 detailed in Fig. 7a-c, along with the guidelines of Sect. IIIB. Thus, R'_1 and R'_2 are a pair
 269 of ranges representative of the erf-falling transient (red trace in Fig. 7c and red-to-blue
 270 decay in Fig. 6a) resulting in an interval width, $W_0 = 100$ m (Eq. 13). Once R'_1 and R'_2
 271 are set, R_1 and R_2 define $[R_1, R'_1]$ and $[R'_2, R_2]$ “plateau” intervals of approximate widths,
 272 $W_1 = 25$ m and $W_2 = 75$ m, respectively (red trace in Fig. 7c; red and blue shades around

Table I. Initialization EKF parameters for the radar and ceilometer.

Parameter	Symbol	Radar	Ceilometer
Observation-range parameters			
Observation start range	R_1	100 m	76 m
Observation end range	R_2	300 m	316 m
Erf-transition start range	R'_1	125 m	116 m
Erf-transition end range	R'_2	225 m	216 m
State-vector parameters			
Initial ABLH	R_{bl}	130 m	136 m
EZ-scaling factor	a	$2.77 \times 10^4 \text{ km}^{-1}$	$2.77 \times 10^4 \text{ km}^{-1}$
Reflectivity profile / Backscatter-coefficient transition amplitude	A	20 dB	$25 \text{ km}^{-1} \text{sr}^{-1}$
FT-reflectivity / Molecular-backscatter background	c	10 dB	$15 \text{ km}^{-1} \text{sr}^{-1}$
Covariance matrix parameters			
State-vector covariance-matrix factor	μ_Q	0.1	0.1
A priori-error covariance-matrix factor	μ_P	0.3	0.1

273 the ML-FT transition in Fig. 6a). *State-vector parameters* have also been initialised from
 274 these first profiles. In Fig. 7a, the mean measured reflectivity (black thick line) in the ML
 275 range $[R_1, R'_1]$ is approximately in the FT range $[R'_2, R_2]$ is 10 dB, which corresponds to a
 276 transition amplitude of 20 dB as initialization value for parameter A , and a 10-dB initial-
 277 ization figure for c . The ABLH is close to 130m (initialisation for R_{bl}), and the mean EZ

thickness ($2.77a^{-1} \approx 100$ m) is used to initialise the scaling parameter a . Factors μ_Q and μ_P in Table I assume $1-\sigma$ fluctuations/a-priori uncertainties in the atmospheric state vector of 10% and 30%, respectively. For example, with the initialization $R_{bl,0}=130$ m and $\mu_P=0.3$, the user expresses an “a priori” uncertainty at $1-\sigma$ of roughly ± 40 m. Similar reasoning can be applied to EKF-ceilometer parameters.

Performance.- Fig. 7b,e shows the instantaneous reflectivity profiles or “observables” estimated by the EKF. Notice that, in spite of the distorted shape of the reflectivity profile, the erf-like estimates fitted to the observables by the filter (thin lines) stick to Eq. 9 model with a center point, R_{bl} , varying with time. The green thick line is the mean profile of the EKF-estimated reflectivity profiles at each successive t_k and follows the erf model. For visual comparison the mean measured reflectivity plotted in Fig. 7a,d (black thick line) is reproduced in Fig. 7b,e also in black thick line. The green thick line is the mean profile of the EKF-estimated profile at each successive t_k and follows the erf model.

In Fig. 7c,f the ABLH estimated from the radar EKF is compared with the well-known threshold method (THM) with excellent agreement. In the classic THM [10][11][12], the instantaneous ABLH is determined as the height associated to a user-defined threshold reflectivity of 15 dB, which is approximately the mean value between the peak-high (30 dB) and peak-low (0 dB) reflectivity levels in the ML interval and FT interval of the first measured profiles. As discussed in the literature, the main disadvantage of the classic THM is the difficulty to assess a consistent threshold because noise spikes would largely change it from one observable to the other.

Finally, Fig. 8 compares the time evolution of the ABLH retrieved by the radar and ceilometer EKFs, again with excellent agreement. To quantify it, Fig. 8b plots ABLH estimates in scatter-plot form. Radar and ceilometer ABLHs exhibit linear correlation with

Figure 7. Comparison between measured and EKF-estimated reflectivity profiles in two different time intervals: Initialization (t_1 to t_{40} , 14:15:01 to 14:25:29 UTC, 1-628 s) and tracking-interval example (t_{240} to t_{280} , 15:19:12 to 15:29:56 UTC, 3851-4495 s in Fig. 1 and Fig. 6). (a) (Thin color lines) Profiles of the *measured clean reflectivity* as function of height or EKF “observables”, $\eta^{(1-40)}$. (Black thick line) Mean clean reflectivity profile, $\bar{\eta}^{(1-40)}$. (b) (Thin color lines) Instantaneous *EKF-estimated reflectivity* profiles in response to (a). (Green thick line) Mean EKF-estimated reflectivity profile. (Black thick line) Mean clean reflectivity profile, same as (a). (c) Comparison with the classical THM (detail for time t_{28}): (Black line) Measured clean reflectivity profile, η^{28} . (Red line) EKF-estimated reflectivity. ABLH estimated with two different methods: (magenta dot) EKF, (blue dot) classic THM. Vertical blue vertical lines indicate, from left to right, adaptive ranges R_1 , R'_1 , R'_2 and R_2 . (d) – (e) Same as (a) – (b) for time t_{240} to t_{280} . (f) Same as (c) for time $t_{250}=4013$ s (15:21:54 UTC).

negligible bias and correlation coefficient as high as $\rho = 0.93$ (determination coefficient, $\rho^2 = 0.87$, [33]). The narrow departure from ideal linear correlation is attributed to the

Figure 8. Comparison between radar and ceilometer ABLH estimates (see 6). *a*) ABLH estimates as a function of time: (Solid black) Radar EKF estimates. (Solid grey) Ceilometer EKF estimates. *b*) Scatter plot relating the ABLH estimated by the radar EKF (horizontal axis) with the ceilometer one (“ground truth”, vertical axis). (Black dashed line) Regression line. Labels indicate regression-line and correlation coefficients.

fact that both instruments measure different physical quantities as proxies of the ABLH, which results in a mismatch on the detection of the thermal boundaries, and, to a lesser extent, to their different temporal and spatial raw resolutions. Thus, following [1], while the ceilometer measures aerosol backscatter lidar returns that typically show diameters of thermals decreasing with height, the Doppler sodar reflectivity (and by extension Doppler radar in this work) usually shows constant or increasing diameters [37][38].

As discussed in [4], in time intervals where the SNR is low (typically, $SNR \approx 5$ at the ABLH or $SNR \approx 1$ at the maximum range) classic methods cease to correctly estimate the ABLH. This is the case of the THM in, for example, the time interval 800-1450 s of Fig. 6a. If ABLH-radar estimates for both the EKF and the THM are compared in this time interval by using similar scatter-plot methodology as in Fig. 8, a determination coefficient as low as $\rho^2=0.35$ and a regression slope of 0.68 are obtained. For comparison, the mean SNR over the whole time frame of Fig. 6 is $SNR = 18$ (linear units), which can be considered a medium-high SNR scene.

V. CONCLUSIONS

A Kalman filter (EKF) has successfully been applied to adaptively estimate and time track the ABLH from FMCW S-band radar returns under single-layer, convective boundary layer conditions. Application of the adaptive EKF to the radar case relies on three important aspects: (i) an ad-hoc processing of the radar reflectivity signal, (ii) formulation of the Kalman filter itself, and (iii) treatment of noise.

Radar pre-processing of the reflectivity signal has been formulated in Eqs. (9)-(13) and Fig. 2 block diagram and it is aimed at removing impulsive noise, mainly due to insects. This pre-processing yields a “clean” Bragg scattering atmosphere showing a well-defined ML-FT transition.

Formulation of the Kalman filter relies on a parametric erf-like model used to model the ML-FT interface (the so-called “observation” model) and a simple Gauss-Markov transition model for the state vector (state-vector model). The formulation departs from a previous application of the adaptive EKF (and in comparison with non-adaptive morphological classic methods) to the problem of ABLH estimation from aerosol backscatter lidar returns [4]. Its

extension to the radar case, for which the returns are physically due to refractive index turbulence and not to aerosols, continues to enable application of the erf-like model but, in contrast to the lidar case, radar returns in the ML and FT “plateau” intervals ($[R_1, R'_1]$ and $[R'_2, R_2]$, respectively) depart more significantly from the idealised erf profile (Fig. 3). Besides, “insect noise” represents an additional source of shape distortion. These departures from the idealised erf-model can be merged into a single body by treating them as “modelling noise” into the noise covariance matrix, which is updated at each successive discrete time, t_k . Because there is only one observable available at each successive discrete time, the noise covariance matrix cannot be estimated from the time ensemble of measurements but from the noise spatial statistics along the observation height, instead. This ergodicity assumption has also been validated by experiment (Fig. 5).

State-vector and a-priori error covariance matrices have been modelled as simple diagonal matrices with initial values proportional to the state vector (μ factor in Eqs.(21)-(22)). The relatively large variability of the ABLH in the time frame under study has been solved by using adaptive ranges for the EKF observation model (range boundaries $[R_1, R_2]$ and $[R'_1, R'_2]$).

Though neither instrument is necessarily “perfect” ground truth, the time period analysed of developing ABL shows that both radar and lidar instruments compare fairly well and satisfactorily validates the erf model used. Thus, Radar-ABLH estimates have been validated from a collocated Vaisala CL-31 ceilometer in both cases using an EKF implementation yielding a correlation coefficient, $\rho = 0.93$, and a regression-line slope of 0.97 with 0.01 bias for the case example discussed. Because of the relatively high SNR at the ABLH (mean $SNR = 18$ (linear units)), ABLH height estimates from the classic THM also become in good agreement with those obtained from the EKF. However, in time intervals

where the SNR is “low” ($SNR \approx 5$ (linear units) or lower at the ABLH) the THM ceases to correctly estimate the ABLH, which underlines the advantage of the EKF not only for its time tracking capability but also for its ability to operate in low-SNR scenarios.

As future research work, one must also acknowledge the convenience of evaluating alternative ABL models, which could also become appropriate or more appropriate for the radar case and perhaps improve the already high correlation coefficients obtained here. However, this is at the expense of re-formulating the filter’s measurement and state-vector models.

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