**IABSE SYMPOSIUM
MADRID 2014**

*Engineering for Progress,
Nature and People*

REPORT

PUBLISHER

IABSE

C/o ETH Hönggerberg
CH-8093 Zurich, Switzerland

Tel. +41-44-633 2647

Fax. +41-44-633 1241

Email: secretariat@iabse.org

Web: www.iabse.org

ISBN – 978-3-85748-134-5

Printed in Spain

Scientific Committee

- Martinez-Cutillas, Antonio – Chair, Spain
Goicolea Ruigomez, José M. -
Vice-Chair, Spain
Camara Casado, Alfredo – Secretary, Spain
Anderson, John, USA
Bartlett, F. Michael, Canada
Beier, Marcos, Brasil
Boegle, Annette, Germany
Briseghella, Bruno, Italy
Brühwiler, Eugen, Switzerland
Bucur, Carmen, Romania
Campos e Matos, Jose, Portugal
Cerdeiríña, Roberto, Spain
Chrimes, Mike, UK
de Boer, Ane, The Netherlands
Dritsos, Stephanos, Greece
Furuta, Hitoshi, Japan
G. Pulido, M. Dolores, Spain
Giarlelis, Christos, Greece
Gómez Hermoso, Jesús, Spain
Guo, Tong, China
Huang, Dongzhou, USA
Huang, Hongwei, China
Janjic, Dorian, Austria
Kim, Ho-Kyung, Korea
Kirk, Martin, UK
Lampropoulos, Andreas, UK
Limsuwan, Ekasit, Thailand
Lozano Galant, José Antonio, Spain
Lu, Xinzhen, China
Malaga-chuquitaype, Christian, UK
McCall, Joanne, Canada
McGormley, Jonathan, US
Montens, Serge, France
Morgenthal, Guido, Germany
Nakamura, Shunichi, Japan
Oliveira-Santos, Luis, Portugal
Orcesi, Andre, France
Palmisano, Fabrizio, Italy
Payá Zaforteza, Ignacio, Spain
Pelke, Eberhard, Germany
Ratay, Robert, USA
Revilla Angulo, Roberto, Spain
Romo, José, Spain
Ruan, Xin, China
Ruiz Teran, Ana Maria, Spain
Rus Jenni, Laurent, Spain
Saad, Fathy, Egypt
Santana, Guillermo, Costa Rica
Schlaich, Mike, Germany
Siebert, Geralt, Germany
Stroetmann, Richard, Germany
Subbarao, Harshavardhan, India
Teng, Jin-Guang, China
Traykova, Marina Doncheva, Bulgaria
Turmo Cordeque, José, Spain
Vejrum, Tina, Denmark
Vergoossen, Rob, The Netherlands

Collaborators of the Scientific Committee

- Nguyen Gia, Khanh - Assistant
Secretary, Spain
Dan, Sorin, Romania
Hernández Montes, Enrique, Spain
Luna, Javier, Spain
Zanaica, Luca, Italy
Peters, Tom F., Switzerland

Reinforced Concrete Predimensioning to Enhance Optimization

Marc SANABRA
Assistant Professor
UPC – Barcelona Tech
Barcelona, Spain
marc.sanabra@upc.edu

Marc Sanabra, born 1978, received his architect degree from the Univ. Politecnica de Catalunya (UPC), Spain. He worked for BOMA structural consultants, Barcelona, before becoming Professor at the UPC. His main area of research is serviceability of prestressed concrete structures.

Andrew SCANLON
Full Professor
Penn State University
State College, PA, US
AISCE@engr.psu.edu

Andrew Scanlon received his B. Sc degree from the University of Glasgow and his Ph. D from the University of Alberta. His research interests focus on reinforced and prestressed concrete structures.

Summary

Due to the nature of the material, reinforced concrete elements normally need a complex computation process to find efficient sections. Recommendations of some main codes face the problem by offering very simple non-transparent all purpose predimensioning criteria. This paper demonstrates how transparency and accuracy are not in contradiction with simple predimensioning methods. This opens the possibility to make it easier to reach optimization of reinforced concrete elements.

Keywords: Reinforced concrete; predimensioning; optimization; span-to-depth ratio; beams; slabs.

1. Introduction

Reinforced concrete flexural elements are far from easy to optimize. Unlike timber or steel beams, efficient reinforced concrete members can only be found by iterating. This mainly has two reasons: on the one hand, concrete has a high weight-to-elastic-modulus ratio, such that self-weight of the structural element is not negligible when compared to superimposed loads; on the other hand, concrete has a low ratio of tension-strength-to-compression-strength, such that under flexural actions elements undergo cracking, leading to variable section properties along their main axis. This causes difficulties in estimating the modulus of inertia of the several cross sections, and thus of the whole member.

As a consequence, designing efficient reinforced concrete elements is done by a relatively long process. First, a certain height of the element is chosen to enable strength check and find the amount of reinforcement required. Then, deflection is computed to see if it matches allowable values. These two steps must be repeated until the desired efficient result is reached. The longer the iteration sequence is, the better the optimization. Alternatively, the most accurate is the first step in the sequence the shorter the iteration may be to get a good optimization.

2. Recent trends in predimensioning

2.1 Codes recommendations and everyday practice

At present, the most usual practice consists in:

- a) Predimensioning the height of the section using simple predimensioning criteria such as $L/\text{constant}$ 'based on prior experience' given by codes (ACI [1], EC [2]).
- b) Checking if deflection is bigger than allowed, based on calculation, sometimes using FEM software.
- c) If the computed deflection is larger than the allowed value, the height is increased until the deflection criterion is satisfied.

This short iteration process would be good enough if the predimensioning criteria would be such that it implicitly led to an effective deflection control matching allowed deflection limits. However, this is almost never true if predimensioning criteria used are those described in the most influential standards: ACI and EC.

Recommendations given in these and similar codes have a number of features that favour inaccurate design: a) The designer is given the option to avoid deflection check, as the simple $L/\text{constant}$ criteria are deemed to satisfy deflection control criteria; b) the range of spans and loads for which these simple provisions can be used are not specified; c) in general, the criteria lack transparency and 'prior experience' is the only support to their validity. Due to that, it is impossible to know to which extent these criteria do take into account some of the factors that most influence the control of deflections: load intensity and history, allowable deflection, concrete properties, section shape, etc.

To date, these and other concerns have been discussed by a number of authors [3] [4] [5], and even some codes [2] [6] are replacing or complementing these predimensioning methods with new ones that intend to be more accurate. Rangan in 1982 [4] suggested setting out the problem of predimensioning based on elastic deflection equation and Branson's concept of effective moment of inertia [7]. Two main lines of research have issued from Rangan's original proposal: On the one hand, the idea has been incorporated in the Australian code [6] with input from Gilbert; on the other hand, Scanlon and co-workers have proposed modifications and simplifications to the work of Rangan and Gilbert. The main results of this second line are summarized and developed further in this paper.

2.2 Fundamentals of the Rangan-Scanlon Method

Rangan's proposal is to transform the elastic formula for deflection of uniformly loaded elements by putting the effective moment of inertia (I_e) as a function of the gross moment of inertia (I_g). For cracked concrete I is I_e (Branson), and Rangan expresses it as $I_e = \alpha \cdot I_g$.

$$\Delta = \frac{\kappa}{384} \cdot \frac{W \cdot l^4}{E \cdot (\alpha I_g)} \quad (1)$$

Rangan's equation was originally written as a function of effective depth (d). However, for more simplicity Scanlon and Choi [8] prefer to express it as a function of the height (h), as it shows in (1). As I_g is $bh^3/12$ for solid rectangular sections, equation (1) can be rewritten in the form (2) and finally in terms of the span-to-depth ratio as a function of allowable deflection (Δ_{allow}) (3). As the allowable deflection is very often expressed as a function of the span $\Delta_{allow} = (l/ct)_{allow} = l/ct_{\Delta,allow}$ it can be rewritten as equation (4).

$$\Delta_{allow} = \frac{\kappa}{32} \cdot \frac{W \cdot l}{E \cdot \alpha \cdot b} \cdot \left(\frac{l}{h}\right)^3 \quad (2) \rightarrow \frac{l}{h} = \sqrt[3]{\frac{\Delta_{allow}}{l} \cdot \frac{32 \cdot \alpha \cdot b \cdot E}{\kappa \cdot W}} \quad (3) \rightarrow \frac{l}{h} = \sqrt[3]{\frac{\Delta_{allow}}{l} \cdot \frac{32 \cdot \alpha \cdot b \cdot E}{\kappa \cdot W}} \quad (4)$$

Using expression (4) the optimal span-to-depth ratio can be found. However, there are two reasons why this equation is not linear to use: the two same reasons mentioned above for the non-linearity in concrete elements' optimization. On the one hand, a variation of h modifies substantially W , especially when it comes to flat elements (slabs, plates, etc.). This makes iteration mandatory. On the other hand, α depends on h and the reinforcement ratio. To solve this, Rangan gave two equations to find α mainly depending in the reinforcement ratio [4]. Scanlon and Choi suggested in 1999 that a constant value of α might be useful for one-way slab predimensioning. A value of 0,4 was suggested at that time. Even if α is not really a constant, this simplification implies little error as $l/h = f(x, y, \dots) \cdot \alpha^{1/3}$. In 2006, Lee and Scanlon changed α to 0,52 [9], when they suggested the idea of a constant α could be extended to any sort of slab and beam. Notice this change in the criteria is apparently important, a +30% increase in α , but it only means a +6% increase in the height (h) computed by this method.

Expression (4) is applicable to any desired allowed deflection, as long as this limit is written in the form $\Delta_{allow} = l/ct_{\Delta,allow}$, which is the most common in code provisions. This gives the expression versatility for project specific criteria.

However, for an important number of structural elements incremental deflection after the construction of non-structural elements is the most restrictive criterion. Taking this into account, equation (4) can be rewritten as follows for one-way elements:

$$\frac{l_n}{h} = \sqrt[3]{\frac{1}{ct_{\Delta inc, allow}} \cdot \frac{32 \cdot \alpha \cdot b \cdot E}{\kappa \cdot (\lambda W_S + W_{L(added)}}} \quad (5)$$

Where:

λ long-time deflection multiplier

W_S sustained load [= $W_{SW} + W_{SD} + \gamma \cdot W_L$]

W_{SW} self-weight

W_{SD} superimposed dead load

$\gamma \cdot W_L$ sustained portion of live load

$W_{L(added)}$ variable portion of live load [= $(1 - \gamma) \cdot W_L$]

It is important to notice that computing incremental deflection is only possible by knowing load history, or by making a reasonable supposition of this load history. In 1985 Graham and Scanlon [10] suggested a standard load history, considering the structure has to endure the most critical loads when it is under construction, as it bears the self weight of the floors above it due to propping. Using this criterion, α can be computed in order to use properly equation (5)

3. Formulae of the Short Rangan-Scanlon Method

3.1 Formula for plates and slabs

In 1985, Gilbert [11] suggested to add several coefficients to the one-way expression to make it good for two-way slabs. Taking this into account and using the crossing beam method [12] to compute deflection in flat plates and slabs, Lee and Scanlon suggested in 2006 [9] the next two variations of the equation (5), where α is always taken as a constant (0,52). For edge-supported slabs use (6), for column-supported flat plates use (7):

$$\frac{l_n}{h} = \beta_{ES} \cdot \sqrt[3]{\frac{1}{ct_{\Delta inc, allow}} \cdot \frac{32 \cdot \alpha \cdot E}{\kappa \cdot k_{AR} \cdot (\lambda W_S + W_{L(added)}}} \quad (6)$$

Where:

β_{ES} aspect ratio (= l_L/l_S)

$k_{AR} = 0,2 + 0,4 \cdot \beta_{ES}$ (for $1 \leq \beta_{ES} \leq 2$) and $k_{AR} = 1,0$ (for $\beta_{ES} < 2$) [For more refined values of k_{AR} see [9] and [13]]

l_L long span

l_S short span

$$\frac{l_n}{h} = \sqrt[3]{\frac{1}{ct_{\Delta inc, allow}} \cdot \frac{32 \cdot \alpha \cdot k_{DP} \cdot E}{\kappa \cdot k_{SS} \cdot (\lambda W_S + W_{L(added)}}} \quad (7)$$

Where:

k_{DP} (=1,35) factor to account for extra stiffness due to drop panels with a thickness equal to $1,25 \cdot h$

k_{SS} (=1.35) slab system factor. It accounts for an increase in moment intensity on column strips

The equations of 2006 were originally written in a slightly different way, but they are put here in a way that makes them consistent with this paper. The main modifications made here are: the concept of $ct_{\Delta, allow}$ is introduced; loads (W) must be introduced here as surface loads; and the parameter b (width) is eliminated. The two last changes are made because the crossing beam method is used here supposing the “beams” are slabs with a unit width. Thanks to that, the expressions are independent from the units system one may use.

3.2 Formula for beams

Based on the work of Lee and Scanlon in 2006 here is presented an expression for beams. In this expression, for a better uniformity with equations (6) and (7), loads are kept superficial. Parameter

b (width of the beam) is recovered here by using the parameter k_B :

$$\frac{l_n}{h} = \sqrt[3]{\frac{1}{ct_{\text{allow}} \cdot \kappa \cdot k_B \cdot (\lambda W_s + W_{L(\text{add})})} \cdot \frac{32 \cdot \alpha \cdot E}{}} \quad (8)$$

Where:

$k_B (=L_{\text{trib}}/b)$ load width factor

L_{trib} tributary width

k_B is a good way to see how narrow or wide is a beam. A very narrow beam has a $k_B=30$, a more conventional narrow beam has it around 20, and a very wide beam has it around 6 or 7. A $k_B=1$ is for one-way slabs. For support strips of column supported plates $k_B=1.35=k_{SS}$ (notice how these makes equations (7) and (8) fully compatible). As it will be shown below, this transition from narrow beams to slabs has a tight correlation with reinforcement ratio of elements.

When using expression (8) it is important to notice that the self weight of the dropped part of the beam must be divided by L_{trib} when including it as a part of W_s . This gives an idea on how little is the influence of the self weight of the beam in these calculations.

4. The Long Rangan-Scanlon Method

As we have already seen, in 1999 Scanlon and Choi found that α could be taken as a constant. This opened the way to the Short Rangan-Scanlon method, which makes it easy, only after a short iteration, to find optimal predimensionings for concrete elements. Scanlon and Choi were able to do so only after an extensive parametric study, performed using spreadsheets. Those were able to compute for the variation in α , which depends mainly on the height and on the reinforcement ratio of the element. A slightly retouched version of that iteration process is described here:

Step 1: Choose a height (h) to account for self weight. (To start with, an h from L/h ACI provisions may be a good one)

Step 2: Find the required area of steel for strength. (If reinforcement ratio is ever bigger than allowed by the code, go back to step 1 to increase h)

Step 3: Compute I_e to find α .

Step 4: In order to find h , use α into the appropriate equation (6) to (8), depending on the sort of element.

Step 5: Check if h computed in step 4 is equal to h in step 1 (or within tolerance).

Step 6: Repeat steps 1 to 5 until 5 is true.

5. The issue of tension strength of concrete

Table 1: Tension strength criteria

Author of the criterion	Tension strength (f_r) [h in m ; f_{ck} and f_r in N/mm^2]
ACI	$0,625 \cdot f_{ck}^{1/2}$
Scanlon and Murray	$0,333 \cdot f_{ck}^{1/2}$
EC	$(0,48-0,3 \cdot h) \cdot f_{ck}^{2/3} \geq 0,3 \cdot f_{ck}^{2/3}$

When it comes to compute deflection of cracked elements, the criterion selected to account for tension strength of concrete has a strong influence on the results. However, very different opinions have been established on the matter. Three of those criteria are showed in Table 1.

ACI specifies a constant and quite high value of the tension strength (f_r). Scanlon and Murray [14] keep the idea of a constant value but do reduce ACI's recommendation

to account for the effect of shrinkage restraint. Finally, the EC links the value of f_r to the height of the element: the lesser the height, the bigger the value of f_r .

These three criteria have been used for computations in the following parametric study to check for the influence of f_r in the results.

6. Parametric study performed using the Long Rangan-Scanlon Method

6.1 Study on column-supported flat plates

The next plots show the values obtained for l_n/h , α and ρ using the Long Rangan-Scanlon method on column-supported plates (eq. 7). Only the case of fixed-fixed two-way continuous plates bearing partitions has been studied, as it is considered to be representative enough.

$\rho = A_s/(b \cdot d)$ is the amount of inferior reinforcement at midspan.

Fig. 1: span-to-depth ratio (l_n/h) and α factor for column-supported flat plates

Fig. 2: Reinforcement ratio (ρ) for plates

From the observation of these plots, the next partial conclusions can be set forth:

- The span-to-depth ratio (l_n/h) of continuous plates shows a considerable dispersion of values depending on several factors. It is strongly influenced by span, load and tension strength of concrete (f_r). The bigger the load or the span, and the smaller the f_r , the smaller the l_n/h .
- The stiffness factor (α) of continuous plates is very influenced by the tension strength of concrete (f_r), somewhat influenced by load and not influenced at all by span (EC is an exception to that). The bigger the f_r , the bigger the α .
- Reinforcement ratio at midspan in continuous plates is very influenced by f_r , but very little influenced by the load and the span (EC shows a little influence of the span). The bigger the f_r , the bigger the α .

6.2 Study on solid narrow and wide rectangular beams

The next plots show the values obtained for l_n/h , α and ρ using the Long Rangan-Scanlon method on narrow and wide beams (eq. 8). Only the case of fixed-fixed continuous beams bearing partitions has been studied.

Fig. 3: span-to-depth ratio (l_n/h) and α factor for narrow beams

Fig. 4: Reinforcement ratio (ρ) for narrow beams

almost irrelevant. The bigger the load and the span, the bigger the α .

- The amount of reinforcement (ρ) is very influenced by the increase of load. Span shows no influence, and tension strength has only a certain influence in the case of EC. The bigger the load, the bigger the amount of reinforcement.

Fig. 5: span-to-depth ratio (l_n/h) and α factor for wide beams

Fig. 6: Reinforcement ratio (ρ) for wide beams

influenced by load and tension strength (f_r), while span is a less important factor. The bigger the tension strength and the load, the bigger the α .

- The amount of reinforcement (ρ) is mainly controlled by the load, but the tension strength shows also a certain influence. Span shows no influence at all. The bigger the load and the tension strength of concrete, the bigger the amount of reinforcement.

6.3. Evaluation of the results

Observing the plots, it is easy to see a very different behaviour for narrow beams and plates, while wide beams are in a transition zone between those two extremes.

From the observation of the three plots of narrow beams ($k_B=25$), the next partial conclusions can be set forth:

- The span-to-depth ratio (l_n/h) is almost constant and independent from all the studied parameters. Only the amount of load has a small influence in the result. For lower loads, l_n/h increases a bit.
- The stiffness factor (α) is strongly influenced by load. Span has also a minor influence, and tension strength is

Observing now the three plots of wide beams ($k_B=7$), the next partial conclusions can be set forth:

- Wide beams behave in general like narrow beams, but in some aspects they show similarities to plates.
- The span-to-depth ratio (l_n/h) is almost constant, but span and load -specially the least- show a certain influence. The smaller the load and the span, the bigger the span-to-depth ratio (l_n/h).

- The stiffness factor (α) is moderately influenced by load and tension strength (f_r), while span is a less important factor. The bigger the tension strength and the load, the bigger the α .

Fig. 7: Schematic plot of the values of α depending on the sort of structural element

flatter zone of the plot (almost constant value of α), while beams fall in the steep zone at higher reinforcement ratios.

The behaviour seen here for continuous elements has been found to be similar for other boundary conditions, history loads and deflection limit. Even it is not shown here, the authors consider the conclusions found here are independent of boundary conditions, history loads and deflection limit.

6.3 L/h method for beams and Short Rangan-Scanlon method for flat slabs

After these observations it is obvious that for an optimal design of plates, the L/h method loses precision in favour of the Rangan-Scanlon method. For everyday use the Short method with a constant value of α may be the best. However, the Long one is the most suited to find the proper value of α to be used in each situation.

When it comes to narrow beams, the Short Rangan-Scanlon method may be of little interest for everyday design, as the L/h (or l_n/h) method is much more convenient as well as accurate. However, the Long Rangan-Scanlon method has clearly an application when it comes to find the proper values of l_n/h to be used for beams in each situation.

For wide beams, both methods may be accurate enough when used properly. However, if a certain precision is aimed, both methods may require the use of tables including the variation of their main parameter (α or l_n/h). That definitely makes more efficient the l_n/h method for wide beams.

Based on the Long Rangan-Scanlon Method, the existing tables of provisions of span-to-depth ratios included in the codes may be replaced by other much more complete tables that take into account the main parameters influencing deflection control: total amount of load, load history, deflection limit, strength of materials, etc. These more extensive tables may even not be included in the codes, just as steel profiles catalogues are not necessarily included in steel construction codes.

7. Conclusions

The Rangan-Scanlon method has been clarified by making a distinction between the Long and the Short method, and by rewriting the equations as shown. The new equations are clearer and independent from the units system one may use. Also, the concept of $k_B (=L_{trib}/b)$ has been introduced to simplify the study of beams depending on their width.

Using this new approach, a parametric study was performed for several continuous (fixed-fixed) reinforced concrete elements (plates, narrow beams, wide beams) from which the following conclusions are presented:

- Rigor and transparency in predimensioning can be achieved using the methods presented.
- The Short Rangan-Scanlon method may be very useful to optimize plates and slabs, as the L/h method is not able to cope with the large amount of variables involved in the problem.

In narrow beams, the amount of reinforcement increases rapidly as loads or span do grow. The same behaviour can be found for the factor α of these beams, as reinforcement and α are tightly linked. Surprisingly, no matter how big is the growth in ρ and α , the span-to-depth ratio (l_n/h) keeps constant.

A reversed behaviour can be seen in plates: ρ and α are almost constant while l_n/h tends to reduce as load and span grow.

Wide beams show a behaviour similar to narrow beams, but they are less reinforced and their span-to-depth ratios (l_n/h) are not exactly constant, as they tend to diminish smoothly for bigger loads and spans.

These behaviour can be seen in a synthetic way in figure 7: plates generally fall in the

However, it must be taken into consideration that the election for the criterion chosen to establish the tension strength of reinforced concrete may have a strong influence in the computation of the constant values of the stiffness factor $\alpha (=I_e/I_g)$.

- c) The l/h (or l_n/h) method is by far the most efficient method to predimension beams.
- d) However, both the l/h method and the Short Rangan-Scanlon method should be complemented with design aids such as tables including the different values of their main parameters (l_n/h and α , respectively), depending on the main variables that control deflection: span, deflection limit, loads history, boundary conditions, etc. The values included in those tables may well be found using the Long Rangan-Scanlon method.

8. References

- [1] ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318R-08) and Commentary", American Concrete Institute, Farmington Hills, Michigan, 2008, p. 473.
- [2] European Committee for Standardization, "EN 1992:2004 Eurocode 2: Design of concrete structures Part 1-1: General rules and rules for buildings", European Committee for Standardization, Brussels, 2010, p. 225.
- [3] GROSSMAN J.S., "Simplified Computations for Effective Moment of Inertia, I_e and Minimum Thickness to Avoid Deflection Computations", ACI Journal, *Proceedings*, Vol. 78, No. 6, 1981, pp. 423-439.
- [4] RANGAN B.V., "Control of Beam Deflections by Allowable Span-to-Depth Ratios", ACI Journal, *Proceedings*, Vol. 79, No. 5, 1982, pp. 372-377.
- [5] BONDY K.B., "ACI Code Deflection Requirements – Time for a Change?", *Serviceability of Concrete*, SP-225, F. Barth, R. Frosch, H. Nassif and A. Scanlon, eds., American Concrete Institute, Farmington Hills, Michigan, pp. 133-145.
- [6] AS 3600-2001, "Australian Standard for Concrete Structures", Standards Australia, Sydney, 2001, p. 175.
- [7] BRANSON D.E., "Deformation of Concrete Structures", McGraw-Hill, New York, 1977, p. 546.
- [8] SCANLON A., CHOI B.-S., "Evaluation of ACI 318 Minimum Thickness Requirements for One-Way Slabs", *ACI Structural Journal*, Vol. 96, No. 4, 1999, pp. 616-621.
- [9] LEE Y. H., SCANLON A., "Unified Span-to-Depth Equation for Nonprestressed Concrete Beams and Slabs", *ACI Structural Journal*, Vol. 103, No. 1, 2006, pp. 142-148.
- [10] GRAHAM C.J., SCANLON A., "Long-Time Multipliers for Estimating Two-Way Slab Deflections", ACI Journal, *Proceedings*, Vol. 83, No. 6, 1986, pp. 899-908.
- [11] GILBERT R.I., "Deflection Control of Slabs Using Allowable Span to Depth Ratios", ACI Journal, *Proceedings*, Vol. 82, No. 1, 1985, pp. 67-72.
- [12] ACI Committee 435, "Control of Deflection in Concrete Structures (ACI 435R-95)", American Concrete Institute, Farmington Hills, Michigan, 1995, p. 77.
- [13] TIMOSHENKO S., WOINIWSKY-KRIEGER S., "Theory of Plates and Shells", McGraw-Hill, 1959, p. 580.
- [14] SCANLON A., MURRAY D.W., "Practical Calculation of Two-Way Slab Deflections", *Concrete International*, Vol. 4, No. 11, 1982, pp. 43-50.