Nondiffractive-nondiffusive beams in complex crystals

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The study of spatial dispersion of two-dimensional complex crystals, with periodic modulations of both gain-loss and the refractive index, reveals simultaneous nondiffractive-nondiffusive light propagation. Narrow light beams and light patterns propagate without dispersion while amplified through the complex material. We determine and explore nondiffractive-nondiffusive regimes for collinear and noncollinear propagation.

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I. INTRODUCTION

Spatial dispersion of light beams is a fundamental phenomenon limiting the performance of many linear and nonlinear optical devices. Spatial dispersion or, equivalently, diffraction, not only is an intrinsic property of light beams but also is common to any kind of wave, since all beams display broadening and distortion as they propagate. The narrower the beam is, the more pronounced the diffractive broadening is. As recently found, however, the diffractive broadening can be suppressed by externally imposing periodic spatial modulations on the embedding medium [1–7]. Artificial structured materials presenting a spatial modulation of the refraction index on the wavelength scale, known as photonic crystals (PhCs), provide the ability to tailor diffraction of propagating eigenmodes. For particular symmetries and amplitudes of the modulation of the refractive index, narrow beams can propagate through these crystals without any diffractive broadening, in the so-called self-collimation regime [1–6]. More recently, similar beam propagation effects have been proposed in gain-loss modulated (GLM) materials. Such materials display a homogeneous refractive index; however, the gain or the losses are periodically modulated on the wavelength scale [7–9], which introduces an anisotropy to propagating waves. An angular dependence of both the phase velocity and gain arises, leading to diffractive and diffusive effects, respectively. GLM materials allow, therefore, tailoring both beam diffraction and beam diffusion; although both effects cannot be managed independently as being mutually related. On the one hand, GLM materials support nontrivial spatial effects associated with diffusion management, such as filtering and beam shaping, and on the other hand, for particular parameters, GLM materials support diffractive effects such as a zero diffractive regime, self-collimation [10]. However, the absence of diffraction is generally accompanied by an anisotropic gain, which causes a diffusive broadening of the beam; i.e., the central angular components are more amplified than components at larger angles [8]. More generally, gain-loss and refractive index modulations can occur together in a given material. In this direction, some authors have recently considered one-dimensional complex crystals with balanced real and imaginary modulations, phase-shifted $\pi/2$, which result in real eigenvalues in non-Hermitian systems, the so-called $\mathcal{PT}$ symmetry [11–15].

Actually, in real materials, gain-loss and the refractive index are not independent due to the noninstantaneous and causal response of the medium, as governed by the Kramers-Kronig relation. For instance, introducing a weak modulation on a given actual material results in either in-phase or antiphase loss and index modulations, depending on frequency. Here, we consider two-dimensional complex potentials with in-phase or antiphased modulations between gain and the refraction index, resulting in gain and index modulated materials, i.e., complex crystals. These materials allow the independent management of diffraction and diffusion of light beams. An important practical consequence is the possibility of simultaneous elimination of beam diffraction and beam diffusion by considering just weak and smooth spatial modulations, allowing an efficient amplification of narrow beams without any shape distortion. The search for such nondiffractive-nondiffusive beam propagation regimes in complex crystals is the topic of the present article.

The characteristics of the spatial modulation required for the simultaneous suppression of diffraction and diffusion are analytically explored and located in a parameter space. Numerical simulations based on a paraxial propagation model show collinear beams propagating without any distortion in complex crystals while being amplified; which is in good agreement with the analytically determined spatial dispersion diagrams. Following a similar procedure, noncollinear amplification of narrow beams, which maintain a constant profile, is also obtained in complex crystals.

II. ANALYTICAL MODEL

We consider a two-dimensional spatially periodic lattice described by the complex refractive index function: $n(x, z) = \Delta n \cos(q_{\perp} x) \cos(q_{\parallel} z)$, where $\Delta n = \Delta n_{\text{Re}} + i \Delta n_{\text{Im}}$ represents the amplitudes of both the refractive index and gain modulations and where $(\pm q_{\perp}, \pm q_{\parallel})$, the reciprocal lattice vectors of the modulation, are of the same spatial scale as the wavelength of the incident monochromatic light with $k_0 = 2\pi/\lambda$. The normalized paraxial propagation equation reads

$$\frac{\partial A}{\partial z} = i \frac{\partial^2 A}{\partial x^2} + i V(x, z) A, \quad (1)$$

where $A(x, z)$ is the complex slowly varying envelope of the electromagnetic field and where the transverse and longitudinal coordinates have been scaled: x in terms of $q_{\perp}^{-1}$ and z in terms of $2k_0/q_{\perp}^2$ [the wave vector's components k_x and k_z are correspondingly scaled to $q_{\perp}$ and $q_{\perp}^2/(2k_0)$]. The normalized potential $V(x, z) = 4m \cos(x) \cos(Qz)$ thus depends on the dimensionless geometrical parameter $Q = 2k_0 q_{\parallel}/q_{\perp}^2$ and on the complex-valued normalized modulation amplitude $m = \Delta n k_0^2/(4q_{\perp}^2)$. In this way, the material modulation is totally determined by the normalized pair of parameters (Q, m) , within the limits of the paraxial approximation.

The analytical treatment is based on a harmonic expansion in terms of the periodicity of the modulation profile; the field amplitude is then written as

$$A(x, z) = e^{ik_x x} \sum_{l,p} a_{l,p} e^{-i[(1-p)x + (l+p)Qz]}. \quad (2)$$

Inserting the harmonic field expansion of Eq. (2) into Eq. (1) provides a system of equations for the considered harmonics. A similar treatment has been used to study self-collimation in PhCs and in GLM materials [7–9]. Note however that for the present case of complex crystals, the m coefficient is complex and the coupling between the harmonic components can be both reactive and dissipative. After truncation to the three most relevant harmonics, $(l, p) = (0, 0), (1, 0)$, and $(0, 1)$, the principal spatial effects can be deduced from the system of coupled equations:

$$\begin{aligned} \frac{da_{0,0}}{dz} &= -ik_x^2 a_{0,0} + m(a_{1,0} + a_{0,1}), \\ \frac{da_{1,0}}{dz} &= -i(k_x + 1)^2 a_{1,0} + iQa_{1,0} + ma_{0,0}, \\ \frac{da_{0,1}}{dz} &= -i(k_x - 1)^2 a_{0,1} + iQa_{0,1} + ma_{0,0}. \end{aligned} \quad (3)$$

When m is real-valued, i.e., for PhCs, the reactive coupling pushes eigenvalues apart near the resonances between coupled modes, opening band gaps. The on-axis modes ($k_x \approx 0$) are at resonance for $Q \approx 1$, as follows from Eq. (3). Moreover, all eigenvalues are real-valued and provide the propagation wave number of the Bloch mode. In the case of imaginary m , i.e., for GLM materials, the dissipative coupling creates complex-conjugated eigenvalues in a particular parameter range near resonances. Complex crystals generally provide complex but not conjugated eigenvalues; thus, the real parts of eigenvalues unlock and band gaps appear. The curvature of the real part of the dispersion curve determines diffraction. The imaginary part corresponds to the wave amplification or decay during propagation and its curvature describes the light diffusion. Imposing that the real or the imaginary part of the second derivative of the spatial dispersion relations equals zero for a range of transverse modes, we obtain the conditions corresponding to either nondiffractive-nondiffusive regimes, respectively. For PhCs, only nondiffractive propagation can be achieved, since m_{Im} equals 0 see [Fig. 1(a)]. In this case the corresponding real spatial dispersion exhibits a flat segment, as shown in Fig. 1(b). Figure 1(c) depicts the (Q, m_{Im}) parameter space for pure GLM materials; hence with no index contrast, $m_{\text{Re}} = 0$. The nondiffractive condition is fulfilled on the parabolas $Q = 1 + m_{\text{Im}}^2/2$ and $Q = 1 - m_{\text{Im}}^2$, which

FIG. 1. (Color online) Spatial dispersion of the three-modes description. (a) The solid curves provide the nondiffraction conditions in the (Q, m_{Re}) parameter space for PhCs. (b) Corresponding spatial dispersion relation for nondiffractive propagation for $m = 0.2$ and $Q = 0.51$. (c) Solid (dotted) lines provide the nondiffraction (nondiffusion) conditions in the (Q, m_{Im}) parameter space for GLM materials. (d, e) Corresponding spatial dispersion relations of a nondiffractive-nondiffusive propagation for $m = 0.2i$ and $Q = 1.02/1.4$. Solid (dashed) curves represent the real (imaginary) part of the wave number.

corresponds to the situation depicted in Fig. 1(d), which also exhibits a flat segment of the real part of the spatial dispersion. Moreover, the lines $Q = 1 \pm 2m_{\text{Im}}$ in Fig. 1(d) correspond to the nondiffusive propagation accounted for by the flat segments in the imaginary part of the dispersion relation. Hence, to simultaneously accomplish both conditions, nondiffusion and nondiffraction, complex crystals are required.

III. COLLINEAR REGIME

In complex crystals a modulation of gain and the refractive index allows simultaneous control over both the real part and the imaginary part of the wave number. Hence, as pointed out above, particular parameters may provide modes with a positive gain, i.e., with a negative imaginary part of the wave number, and a flat curvature in both the real spatial dispersion (k_z^{Re}, k_x)—determining diffraction—and in the imaginary dispersion (k_z^{Im}, k_x)—determining diffusion. Figures 2(a) and 2(b) show the complex dispersion relations for the three-mode expansion of Eq. (3), for particular complex crystals, with in-phase index and gain modulations. Note that the most amplified mode, the dominating mode—the one with the most negative imaginary eigenvalue [Fig. 2(b)]—displays a homogeneous amplification around $k_x = 0$ while simultaneously exhibiting a flat spatial dispersion [see Fig. 2(a)]. Hence, as shown in direct numerical simulations of Eq. (1) [see Fig. 2(c)], a narrow beam incident on-axis on such a structure does not suffer from diffractive or diffusive broadening while propagating and being amplified. The normalized amplification coefficient is $\alpha = 1.1$ for this particular case. The half-maximum beam

FIG. 2. (Color online) Real part (a) and imaginary part (b) of the spatial dispersion curve for the parameters: $m = 0.51 + 0.31i$ and $Q = 0.2$. The solid curve corresponds to the nondiffractive-nondiffusive mode for $k_x = 0$. (c) Beam intensity profile in propagation along and beyond the same modulated structure, with a normalized length of 500. The solid (yellow) line corresponds to the beam half-maximum propagated in free space. (d) Beam intensity profile for propagation within a nondiffractive-nondiffusive structure with only transverse modulation: $Q = 0, m = 0.47 + 0.52i$. As intensity increases exponentially, it is normalized at each propagation step in plots (c) and (d).

intensity profile in free space propagation is also provided for comparison in Fig. 2(c). As an example of a possible real device, this particular configuration would correspond to a broad area semiconductor amplifier with a refractive index of $n = 3$, a Henry factor of 2, and transverse and longitudinal periodicities of 8 and 250 μm , respectively. The considered m value corresponds to a complex modulation obtained by a nonhomogeneous pumping with a refractive index modulation amplitude of 5×10^{-4} .

We note that a similar effect can be achieved in complex crystals with only transverse gain-loss and index modulation, $Q = 0$, as shown in Fig. 2(d). This particular case can be thought of as a generalization of the well-known self-guiding or self-collimation phenomena. At the end of the modulated media, entering the homogeneous space, beams split into diffracted components, with propagation angles determined by the transverse modulation period. It is important to note that, for $Q \neq 0$, the central beam carrying most of the energy still propagates with only a slight broadening for some distance after exiting the modulated structure see [Fig. 2(c)].

Next, we locate semianalytically the nondiffractive-nondiffusion conditions for the three-mode expansion model. As shown in Fig. 3(a), we obtain that, in the parameter space, such a condition bifurcates from the purely nondiffractive line for materials with only index modulation, PhCs, which lay on the $m_{\text{Im}} = 0$ plane. Figures 3(b) and 3(c) show the real and imaginary parts of the dispersion relations along the nondiffractive-nondiffusive curves of Fig. 3(a). The minimum beam width supporting propagation without diffraction and diffusion is determined by the width of such plateaus of the spatial dispersions. The real part of the plateau is narrower than the imaginary part for small Q values and wider for large Q

FIG. 3. (Color online) (a) Nondiffractive-nondiffusive conditions in the parameter space (solid curve) and purely nondiffractive condition for PhCs (dashed curve). Plateaus for the real parts (b) and the imaginary parts (c) of the wave numbers corresponding to the nondiffractive-nondiffusive curves for GLM materials.

values. This means that sufficiently narrow input beams will propagate predominantly in a diffractive (diffusive) way for small (large) Q values [see Fig. 3(c)].

The situations described in Figs. 2(a)–2(c) correspond to a middle point in the curve of Fig. 3(a).

IV. NONCOLLINEAR REGIME

For larger values of Q , as shown above, on-axis nondiffractive-nondiffusive propagation is no longer possible. However, interesting effects can be found for off-axis, i.e., for $k_x \neq 0$, noncollinear propagation. In contrast to the collinear nondiffractive-nondiffusive propagation, the amplification maxima are located off-axis. When diffraction is zero, precisely in the directions of maximal amplification, the amplification of beams becomes highly directional in propagation along the crystal; see the direct integration of Eq. (1) in Fig. 4(a).

The parameter sets fulfilling this condition are located around $Q \approx 2$ in a wide region of the parameter space, as follows from the three-mode model, in good agreement with the paraxial simulations, as shown in Fig. 4(b). In these regimes, noncollinear nondiffractive beams hardly suffer from diffusive broadening while being amplified. Beyond the crystal output, they bifurcate into several beams in the homogeneous space indicating the contribution of higher harmonics. Hence, the three-mode expansion [Figs. 4(c) and 4(d)] may not be sufficient in this case and a deeper analysis is required that takes into account at least five modes in the field expansion, namely, $(l, p) = (0, 0), (1, 0), (0, 1), (2, 0),$ and $(0, 2)$ [Figs. 4(e) and 4(f)]. The resultant coupled equation system is analogous to Eq. (3); however it contains five equations, corresponding to the five considered modes. The resonance condition between the modes $(0, 0)$ and $(\pm 1, 0)$ is $k_x = \mp(Q - 1)/2$, while between $(\pm 1, 0)$ and $(\pm 2, 0)$ it is $k_x = \mp(Q - 3)/2$. When the frequencies of two modes approach and almost lock, the imaginary part of the corresponding wave number grows, as shown in Figs. 4(d) and 4(f). For $Q = 1.7$, these two

FIG. 4. (Color online) (a) Beam propagation along and beyond a nondiffractive structure with $m = 0.12 + 0.24i$ and $Q = 1.7$. The vertical white line indicates the end of the complex crystal. The intensity in plot (a) is normalized as it increases exponentially with z . (b) $Q(m)$ values for crystals supporting noncollinear nondiffractive beams. Real part [plots (c) and (e)] and imaginary part [plots (d) and (f)] of the spatial dispersion diagram for the same noncollinear nondiffractive structure as derived from the three- and five-mode analysis.

resonances are located at opposite sides of the k_x axis ($k_x = \mp 0.35$ and $k_x = \pm 0.65$), providing two different amplification maxima that correspond to different modes [see solid curves in Fig. 4(f)]. This allows a symmetric noncollinear, nondiffractive propagation regime, where each beam is composed of the interaction of two modes that are amplified inside the structure and split into two beyond the complex crystal, i.e., when propagating in free space [see Fig. 4(a)].

On the contrary, for $Q > 3$ the resonance condition between modes $(0,0)$ and $(\pm 1,0)$ and between modes $(\pm 1,0)$ and $(\pm 2,0)$ are on the same side of the k_x axis. This allows the combination of two amplification maxima in the imaginary part of the wave vector for the same mode. Therefore, achieving nondiffractive regimes in noncollinear propagation. While the coupling of modes is now more complex, straight segments arise in the dispersion curves at given frequency ranges also showing reduced diffusion. Figures 5(a)–5(c) correspond to a noncollinear nondiffractive-nondiffusive regime for $Q = 4$ with harmonic resonances at $k_x = \mp 0.5$ and $k_x = \mp 1.5$

FIG. 5. (Color online) (a) Patterned beam propagation along a nondiffractive-nondiffusive complex crystal with $m = 0.5 + 0.5i$ and $Q = 4$ (blue) and through free space (light purple). The intensity is normalized as it increases exponentially with z . (b) Initial transverse four-bar beam profile and (c) final profiles with and without the crystal. Real part (a) and imaginary part (b) of the spatial dispersion diagram for the noncollinear nondiffractive-nondiffusive propagation as derived from the five-mode analysis.

integrated from Eq. (1). The lack of dispersion of the solid dark straight mode in Figs. 5(d) and 5(e) enables the non-diffracting propagation of any pattern along the structure [Fig. 5(a)].

V. CONCLUSION

We have proposed nondiffractive-nondiffusive amplification of narrow beams in materials with a periodic modulation of gain and the refractive index, the complex crystals. We have shown that while both PhCs and GLM materials offer the possibility to eliminate diffraction, the modulation of gain-loss enables one also to tailor the diffusion. As a consequence, complex crystals with modulations of gain-loss and the refraction index allow one to tailor and completely suppress both diffraction and diffusion simultaneously. Indeed, we have analytically found the parameters that satisfy conditions for both collinear and noncollinear nondiffractive-nondiffusive propagation in complex crystals. Such predictions are in accordance with numerical simulations performed under a paraxial approximation.

Finally, we note that the parameters used here could be realized in actual systems such as semiconductor materials. The relations between gain and index modulations considered in this article are compatible with most semiconductors having an α factor between 1 and 5. Therefore, these results could be implemented in optical amplifiers based on semiconductors.

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