

AN ENERGY MODEL FOR THE ACOUSTIC INSULATION OF ABSORBING MATERIALS

Cristina Díaz-Cereceda^{1*}, Jordi Poblet-Puig¹ and Antonio Rodríguez-Ferran¹

¹ Laboratori de Càlcul Numèric
E.T.S. d'Enginyers de Camins, Canals i Ports de Barcelona
Universitat Politècnica de Catalunya
Jordi Girona 1, Campus Nord, E-08034 Barcelona, Spain
e-mail: {cristina.diaz-cereceda,jordi.poblet,antonio.rodriguez-ferran}@upc.edu
web: <http://www.lacan.upc.edu/>

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Abstract. *In this work an energy model for the acoustic insulation of absorbing materials is shown. This model is an extension of Statistical Energy Analysis (SEA) [1] in order to account for the effect of non-conservative connections [2, 3].*

The energy-based approach allows to solve sound insulation problems in large domains (such as those in building acoustics) in an efficient way for the whole frequency range required by regulations (50-5000 Hz). In particular, this approach is applied here for the study of the insulating behaviour of an absorbing layer (mineral wool) filling the cavity of a double wall. The absorbing layer is considered as a non-conservative connection between the two leaves of the wall.

This model is combined with detailed numerical computations to obtain the loss factors associated to the connection. With these parameters, a combined system including the transmission between rooms and double walls can be stated.

Obtained results show that absorbing layers can be modelled as non-conservative couplings and incorporated in an SEA-like system to compute the sound insulation in buildings successfully.

1 INTRODUCTION

Statistical energy analysis is widely used in building acoustics due to its low computational cost and simplicity. However, its application is restricted to domains that can be divided into reverberant subsystems and conservative couplings [4]. The effect of a dissipative connection is not taken into account in the classical SEA formulation [5].

Some efforts have been done in considering non-conservative connections with SEA by Chow and Pinnington [6], Fredö [7], Beshara and Keane [2] and Sheng et al. [8, 3]. Chow and Pinnington introduce an equivalent internal loss factor of the subsystems that models the losses in the connection. Fredö introduces radiation loss factors at the connection. Beshara and Keane also introduce a damping parameter in the connection instead of modifying the internal or coupling loss factors. Finally, Sheng et al. provide a formulation in which the effect of the non-conservative coupling is considered with an equivalent internal loss factor for the subsystems (like Chow and Pinnington) and a new coupling loss factor that accounts for transmission and damping and the same time. The two last works are restricted to mechanical problems, and are not further applied for vibroacoustic cases, or absorbing materials. Besides, the possibility of extending any of these formulations for dealing with larger SEA systems, consisting of both conservative and non-conservative connections, is not discussed.

Double walls are structural elements consisting of two leaves with an air cavity (which may be totally or partially filled with absorbing material) between them. They cheaply provide load-bearing configurations with good acoustic properties and a minimal mass. The increasing use of these elements leads to the interest in reliable models of their sound insulation. These models should cover a wide frequency range (50 to 5000 Hz) in order to evaluate the outputs defined in regulations [9, 10].

Modelling a double wall with a layer of absorbing material between the two panels can be done in different ways. Some authors have developed impedance models for these structures. They assume an infinite size for the leaves and express the transmission of pressure and vibrations with the help of transfer matrices. Geebelen et al. [11] consider layers of poroelastic materials inside and Brouard et al. [12] combine transfer matrices with interface matrices in order to take into account the interfaces between layers.

Other authors solve a one-dimensional version of the problem with the help of numerical techniques. Trochidis and Kalaroutis [13] use Fourier transforms in the propagation direction, perpendicular to the wall plane. Alba et al. [14] use this technique to calibrate material properties with the help of experimental measurements. They use iterative methods for minimising the error between the model and the experiments.

A full three-dimensional analysis can be done with the help of numerical methods [15]. However, if a more complete system is considered, with rooms and other walls, the computational cost will be unaffordable for the highest frequencies required by regulations (up to 5000 Hz) [9, 10]. In these cases, SEA is a more efficient approach, but the effect of the absorbing layer is not easy to model.

Here, an SEA-like model for non-conservative couplings is proposed. This technique is combined with numerical simulations to deal with real life problems. In particular, it is applied to modelling multilayered walls consisting of plasterboards and layers of absorbing material. The absorbing layers are considered as non-conservative couplings between the wall leaves. The SEA-like approach is combined with classical SEA for modelling systems with both conservative and non-conservative couplings.

2 SEA EXTENSION FOR NON-CONSERVATIVE COUPLINGS

An SEA-like formulation is proposed here for the case of two subsystems connected through a non-conservative coupling. This formulation is derived for the case of a point connection and assumed to apply for any non-conservative coupling if the frequency-dependent coupling loss factors are estimated from numerical simulations.

The effect of a point connection between two SEA subsystems may be studied with the equivalent circuit approach [16]. This technique is used by Hopkins [17] to compute the coupling loss factor caused by a spring connecting two leaves. In general, for any point device connecting two leaves, the global system may be represented as a circuit like that of Figure 1, where Y_1 and Y_2 are the point mobilities of leaves 1 and 2 respectively and Y_c is the mobility of the connection.

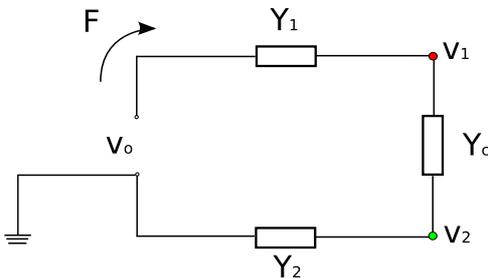


Figure 1. Circuit equivalent to a double wall.

Mechanics	Electrics
Force F	Intensity I
Velocity v	Potential V
Admittance (point mobility Y)	Impedance Z

Table 1. Mechanical-electrical analogy.

The mechanical–electrical analogy is described in Table 1 and the assumptions of the analysis are:

- Leaf 1 has an external excitation and leaf 2 has none.

- v_0 is the propagating bending wave velocity of leaf 1 far from the point connection.
- Any point of the unexcited leaf that is far enough from the connection point has a negligible velocity compared to v_0 .
- v_1 and v_2 are the velocities at the connecting point of leaves 1 and 2 respectively.

By analogy with the electrical circuit, the power entering the connection (on the closest side to leaf 1) can be expressed as [17]

$$\Pi_{12}^{(1)} = \frac{1}{2} \frac{\text{Re}\{Y_2 + Y_c\} |v_0|^2}{|Y_1 + Y_2 + Y_c|^2} \quad (1)$$

and the power leaving the connection (on the leaf 2 side) as

$$\Pi_{12}^{(2)} = \frac{1}{2} \frac{\text{Re}\{Y_2\} |v_0|^2}{|Y_1 + Y_2 + Y_c|^2}. \quad (2)$$

Therefore, the power dissipated at the connection is

$$\Pi_{12}^{\text{diss}} = \frac{1}{2} \frac{\text{Re}\{Y_c\} |v_0|^2}{|Y_1 + Y_2 + Y_c|^2}. \quad (3)$$

If the connection is conservative, the power leaving subsystem 1 enters subsystem 2 without losses, that is $\Pi_{12}^{\text{diss}} = 0$: the value of the mobility is purely imaginary. For the particular case where the connection is a spring, the value of the mobility is $Y_c = i\omega/K$ where K is the spring stiffness, $i = \sqrt{-1}$, $\omega = 2\pi f$ and f is the vibration frequency.

If the coupling, on the contrary, has a dissipating behaviour, part of the power leaving subsystem 1 is transmitted to subsystem 2 and the rest is dissipated at the connection. The mobility Y_c in this case is a complex number with a non-zero real part. For instance, in the particular case of a set of spring and dashpot shown in Figure 2, $Y_c = 1/(C + K/i\omega)$ and therefore $\text{Re}\{Y_c\} \neq 0$. Some power is dissipated at the connection.

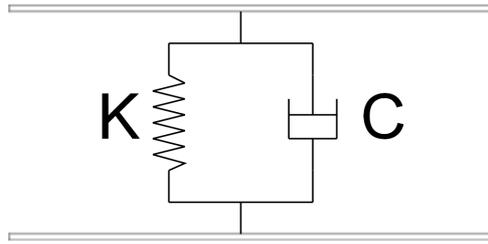


Figure 2. Equivalent circuit for a double wall.

The power balances of the two leaves are

$$\Pi_1^{\text{in}} = \Pi_1^{\text{diss}} + \Pi_{12}^{(1)} \quad (4)$$

for the excited leaf and

$$\Pi_2^{\text{diss}} = \Pi_{12}^{(2)} \quad (5)$$

for the unexcited one respectively, where Π_1^{in} is the power injected to leaf 1, $\Pi_i^{\text{diss}} = \eta_{ii}\omega\langle E_i \rangle$ is the power dissipated at leaf i and η_{ii} is the internal loss factor of leaf i .

Assuming that $\langle E_1 \rangle = M_1\langle v_{\text{rms}}^2 \rangle$, where M_1 is the mass of leaf 1 and $\langle v_{\text{rms}}^2 \rangle = |v_o|^2/2$, the power dissipated at the connection can be expressed as

$$\Pi_{12}^{\text{diss}} = \omega\gamma_{12}\langle E_1 \rangle \quad (6)$$

and the power transmitted to subsystem 2 as

$$\Pi_{12}^{(2)} = \omega\eta_{12}\langle E_1 \rangle. \quad (7)$$

Therefore, the power entering the connection is $\Pi_{12}^{(1)} = \omega(\eta_{12} + \gamma_{12})\langle E_1 \rangle$.

In equations (6) and (7), two new parameters have been introduced. On the one hand, a factor governing the amount of power dissipated at the connection: the Non-conservative Coupling Loss Factor (NCLF)

$$\gamma_{ij} = \frac{\text{Re}\{Y_c\}}{\omega M_i |Y_i + Y_j + Y_c|^2}, \quad i \neq j. \quad (8)$$

On the other hand, a factor governing the amount of power reaching the unexcited leaf: the Conservative Coupling Loss Factor (CCLF)

$$\eta_{ij} = \frac{\text{Re}\{Y_j\}}{\omega M_i |Y_i + Y_j + Y_c|^2}, \quad i \neq j. \quad (9)$$

The power balances of equations (4) and (5) can be rewritten in terms of the averaged energies for the non-conservative coupling as

$$\begin{aligned} \Pi_1^{\text{in}}/\omega &= \eta_{11}\langle E_1 \rangle + (\eta_{12} + \gamma_{12})\langle E_1 \rangle \\ \eta_{22}\langle E_2 \rangle &= \eta_{12}\langle E_1 \rangle. \end{aligned} \quad (10)$$

Following the same procedure in a more general case, with excitations on both subsystems, the global system yields

$$\begin{aligned} \Pi_1^{\text{in}}/\omega &= (\eta_{11} + \eta_{12} + \gamma_{12})\langle E_1 \rangle - \eta_{21}\langle E_2 \rangle \\ \Pi_2^{\text{in}}/\omega &= -\eta_{12}\langle E_1 \rangle + (\eta_{22} + \eta_{21} + \gamma_{21})\langle E_2 \rangle \end{aligned} \quad (11)$$

which can be written in matrix form as

$$\omega \begin{bmatrix} \eta_{11} + \eta_{12} + \gamma_{12} & -\eta_{21} \\ -\eta_{12} & \eta_{22} + \eta_{21} + \gamma_{21} \end{bmatrix} \begin{Bmatrix} \langle E_1 \rangle \\ \langle E_2 \rangle \end{Bmatrix} = \begin{Bmatrix} \Pi_1^{\text{in}} \\ \Pi_2^{\text{in}} \end{Bmatrix}. \quad (12)$$

The effect of the non-conservative joint leads to an SEA-like system with two new factors in the diagonal: the non-conservative coupling loss factors γ_{12} and γ_{21} . If these factors are equal to zero, the conservative case is recovered. However, if the coupling dissipates energy, they are different from zero and factors η_{12} and η_{21} change with respect to the conservative case.

3 ESTIMATION OF COUPLING LOSS FACTORS

The values of η_{ij} and γ_{ij} , defined in Equations (8) and (9) respectively for a point connection, cannot be computed with analytical expressions for most of the real-life dissipative connections. Therefore, a large vibroacoustic problem with many subsystems and non-conservative connections may be solved by computing these parameters from the numerical simulation of smaller parts of the problem. Once these parameters are known, the global system can be solved with the SEA-like approach.

Parameters η_{ij} and γ_{ij} of a non-conservative coupling between two subsystems can be obtained from numerical simulations of the energy transmission between the two subsystems. These simulations should include the dissipative behaviour of the coupling. For a given excitation, both the input powers Π_i^{in} and the averaged energies of the leaves $\langle E_i \rangle$ can be computed solving the vibroacoustic problem numerically. Assuming that η_{ii} is known for every subsystem, the rest of the SEA parameters can be isolated from the power balances of the system.

If the two subsystems have different properties (which is the most common case), the structure is not symmetric and there are four parameters to compute: η_{12} , η_{21} , γ_{12} and γ_{21} .

To obtain them all, the SEA formulation of two mutually independent problems is required, in a similar way as the approach used by [18] for conservative connections. These two problems correspond to the system behaviour for two different excitations: one on subsystem 1 and the other on subsystem 2. For each different excitation, the averaged energies of the subsystems are computed numerically. If these energies are replaced in the SEA-like formulation of each problem (12), a 4×4 linear system can be solved to obtain the four parameters desired

$$\begin{pmatrix} \langle E_1 \rangle & -\langle E_2 \rangle & \langle E_1 \rangle & 0 \\ \langle \hat{E}_1 \rangle & -\langle \hat{E}_2 \rangle & \langle \hat{E}_1 \rangle & 0 \\ -\langle E_1 \rangle & \langle E_2 \rangle & 0 & \langle E_2 \rangle \\ \langle -\hat{E}_1 \rangle & \langle \hat{E}_2 \rangle & 0 & \langle \hat{E}_2 \rangle \end{pmatrix} \begin{pmatrix} \eta_{12} \\ \eta_{21} \\ \gamma_{12} \\ \gamma_{21} \end{pmatrix} = \begin{pmatrix} \Pi_1^{\text{in}}/\omega - \eta_{11}\langle E_1 \rangle \\ -\eta_{11}\langle \hat{E}_1 \rangle \\ -\eta_{22}\langle E_2 \rangle \\ \Pi_2^{\text{in}}/\omega - \eta_{22}\langle \hat{E}_2 \rangle \end{pmatrix}. \quad (13)$$

In system (13), $\langle E_i \rangle$ and $\langle \hat{E}_i \rangle$ are the averaged energies of subsystem i for excitations applied to subsystems 1 and 2 respectively.

Analytical solution of the linear system (13) leads to

$$\eta_{12} = \frac{\Pi_2^{\text{in}}\langle E_2 \rangle}{\omega \left(\langle E_1 \rangle \langle \hat{E}_2 \rangle - \langle E_2 \rangle \langle \hat{E}_1 \rangle \right)} \quad (14)$$

and

$$\gamma_{12} = \frac{\Pi_1^{\text{in}}\langle \hat{E}_2 \rangle - \Pi_2^{\text{in}}\langle E_2 \rangle}{\omega \left(\langle E_1 \rangle \langle \hat{E}_2 \rangle - \langle E_2 \rangle \langle \hat{E}_1 \rangle \right)} - \eta_{11} \quad (15)$$

(and similar expressions for η_{21} and γ_{21}). Equation (14) is a robust way of computing the conservative coupling loss factor η_{12} . Equation (15), on the other hand, is unreliable

for $\gamma_{12} \ll \eta_{11}$, because it involves the subtraction of two similar numbers in such a case. Indeed, the subtraction of two similar quantities is a dangerous operation, prone to amplify any errors in the numerical estimation of the energies and powers, see [19].

Due to the unreliability of Equation (15) for $\gamma_{12} \ll \eta_{11}$, an alternative formulation is proposed. If the power exchanged between the subsystems and the connection is computed on both sides of the connection, it can be expressed in terms of the energies of the subsystems for the two types of excitations as

$$\begin{aligned} \Pi_{12}^{(1)}/\omega &= (\eta_{12} + \gamma_{12}) \langle E_1 \rangle - \eta_{21} \langle E_2 \rangle \\ \Pi_{12}^{(2)}/\omega &= \eta_{12} \langle E_1 \rangle - (\eta_{21} + \gamma_{21}) \langle E_2 \rangle \end{aligned} \quad (16)$$

and

$$\begin{aligned} \widehat{\Pi}_{12}^{(2)}/\omega &= (\eta_{21} + \gamma_{21}) \langle \widehat{E}_2 \rangle - \eta_{12} \langle \widehat{E}_1 \rangle \\ \widehat{\Pi}_{12}^{(1)}/\omega &= \eta_{21} \langle \widehat{E}_2 \rangle - (\eta_{12} + \gamma_{12}) \langle \widehat{E}_1 \rangle. \end{aligned} \quad (17)$$

Subtracting the two equations of each system and rearranging them, the following system results:

$$\begin{pmatrix} \langle E_1 \rangle & \langle E_2 \rangle \\ \langle \widehat{E}_1 \rangle & \langle \widehat{E}_2 \rangle \end{pmatrix} \begin{Bmatrix} \gamma_{12} \\ \gamma_{21} \end{Bmatrix} = \frac{1}{\omega} \begin{Bmatrix} \Pi_{12}^{(1)} - \Pi_{12}^{(2)} \\ \widehat{\Pi}_{12}^{(1)} - \widehat{\Pi}_{12}^{(2)} \end{Bmatrix}. \quad (18)$$

Analytical solution of the linear system (18) leads to

$$\gamma_{12} = \frac{\left(\Pi_{12}^{(1)} - \Pi_{12}^{(2)} \right) \langle \widehat{E}_2 \rangle - \left(\widehat{\Pi}_{12}^{(1)} - \widehat{\Pi}_{12}^{(2)} \right) \langle E_2 \rangle}{\omega \left(\langle E_1 \rangle \langle \widehat{E}_2 \rangle - \langle E_2 \rangle \langle \widehat{E}_1 \rangle \right)} \quad (19)$$

(and a similar expression for γ_{21}). As shown in [19], Equation (19) is only unreliable if the coupling transmission is much higher than the dissipation ($\Pi_{12}^{(2)} \gg \Pi_{12}^{(1)} - \Pi_{12}^{(2)}$). However, in that case the connection may be considered conservative and the classical SEA formulation should be used, assuming $\gamma_{12} = 0$. Therefore, Equation (19) is more robust than Equation (15) because its reliability does not depend on the properties of the subsystems. The main disadvantage of system (18) is the extra postprocessing effort required for computing $\Pi_{12}^{(i)}$ and $\widehat{\Pi}_{12}^{(i)}$ from the results of the numerical simulation. However, it is the most advisable way to compute γ_{12} and γ_{21} .

If the two leaves are identical, the problem is symmetric and then $\eta_{12} = \eta_{21}$ and $\gamma_{12} = \gamma_{21}$. For this case, only one simulation is required, and the systems to be solved are simpler.

4 RESULTS

4.1 Modelling absorbing layers

Lightweight double walls consist of two thin plates (or leaves) separated by a cavity (see Figure 3). This cavity may be filled with an absorbing material.

If the cavity between the leaves is only full of air, a study done in [20] shows that it can be considered as a connection between the two subsystems (leaves), if the value of

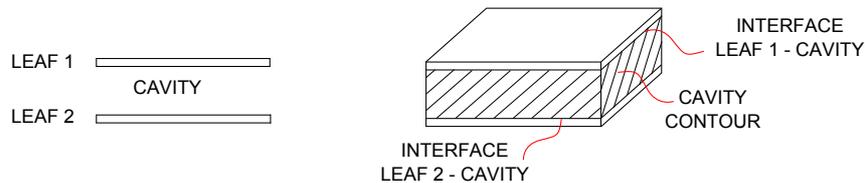


Figure 3. Parts of a double wall.

the coupling loss factor associated to that connection is estimated at each frequency band from the energies of the leaves. These energies are computed solving the vibroacoustic problem numerically. In this way, both the resonant and non-resonant transmission are taken into account.

Since an air cavity acts as a conservative connection between the two leaves of the double wall, a cavity filled with absorbing material is considered in this work as a non-conservative coupling connecting the two leaves. The coupling loss factors η_{ij} and γ_{ij} for this type of connection are obtained as described in Section 3.

In order to study the influence of the flow resistivity on a real-life problem, the SEA-like approach suggested in this work is used to simulate the sound reduction index between two rooms separated by a double wall filled with absorbing material. Five double walls are simulated: four walls filled with absorbing materials of different flow resistivities and another one without absorbing material inside (air cavity). The properties of the leaves are shown on Table 2. The rooms have dimensions 2 m \times 3 m \times 5 m and the cavity between the walls is 70 mm thick.

Variable	Symbol	Value
Young's modulus	E	$2.5 \times 10^9 \text{ N m}^{-2}$
Density	ρ	692.3 kg m^{-3}
Poisson's ratio	ν	0.3
Loss factor	η	3%
Plate size, x direction	L_x	2.4 m
Plate size, y direction	L_y	2.4 m
Thickness	h	0.013 m

Table 2. Properties of the GN plasterboard plates.

For the simulation, the system is divided into four SEA subsystems: sending room, leaf 1, leaf 2 and receiving room. The absorbing material is considered as a non-conservative connection between subsystems 2 and 3.

The internal loss factors of subsystems 2 and 3 are the loss factors of the leaves ($\eta_{ii} = \eta = 0.03$) and the internal loss factors of subsystems 1 and 4 are computed as

$$\eta_{ii} = \frac{cS_{\text{cav}}\alpha}{8\Pi fV_{\text{cav}}}, \quad (20)$$

where S_{cav} is the surface of the room boundary, $\alpha = 0.1$ is the absorption coefficient at that boundary and V_{cav} is the volume of the room. The excitation is a sound source in one of the rooms (subsystem 1).

To obtain all the parameters η_{ij} and γ_{ij} required by the SEA-like approach, three small deterministic problems have been solved. On the one hand, the double wall itself has been simulated, in order to obtain the values of η_{ij} and γ_{ij} between the two leaves. The simulation has been performed with a combination of modal analysis for the leaves and Finite Layer Method for the cavity, as described in [15]. The absorbing material is modelled as an equivalent fluid with the Delany-Bazley-Miki approach [21]. On the other hand, the coupling loss factors between each leaf and its adjacent room have been computed. To do so, two numerical simulations of systems consisting of a room in contact with a leaf are performed. One of them has a sound source on the room and is used to obtain η_{12} and η_{21} . The excitation of the other simulation is a pressure wave impinging on the leaf. That one is used to compute η_{34} and η_{43} . In these cases, the vibroacoustic problem has been solved with modal analysis.

In Figure 4 the effect of the flow resistivity on the sound reduction index between the two rooms is analysed. The insulating effect of filling the cavity with an absorbing material is remarkable. However, different values of the flow resistivity only provide different values of the sound reduction index for high frequencies. This behaviour was also reported by Stani et al. [22]. Increasing the resistivity causes an increment both on the density and the wavenumber of the equivalent fluid. An increment of the fluid density causes a better propagation of the waves through it. On the other hand, a larger wavenumber results in a lower transmission of the waves through the fluid. At low frequencies the increment of the wavenumber compensates the increment of the density and the sound insulation does not change with the resistivity. However, at high frequencies the effect of the wavenumber dominates that of the density and larger resistivities cause more insulating behaviours. The drop in the insulation at 2500 Hz is associated to the coincidence frequency of the leaves.

4.2 Triple walls

The potential applications of the technique presented here are further illustrated by computing the sound reduction index through a triple wall. This wall consists of a leaf-cavity-leaf-cavity-leaf configuration, where the leaves are identical to those in Table 2. The two cavities are 70 mm thick each and both of them are filled with an absorbing material of resistivity $\sigma = 8000 \text{ N s m}^{-4}$. A 5×5 SEA-like system is solved at each frequency, using the same internal and coupling loss factors of the double wall. The sound reduction index through this triple wall is compared with that of the classical double wall on Figure 5.

As expected, the sound insulation increases by adding new layers to the wall. Moreover,

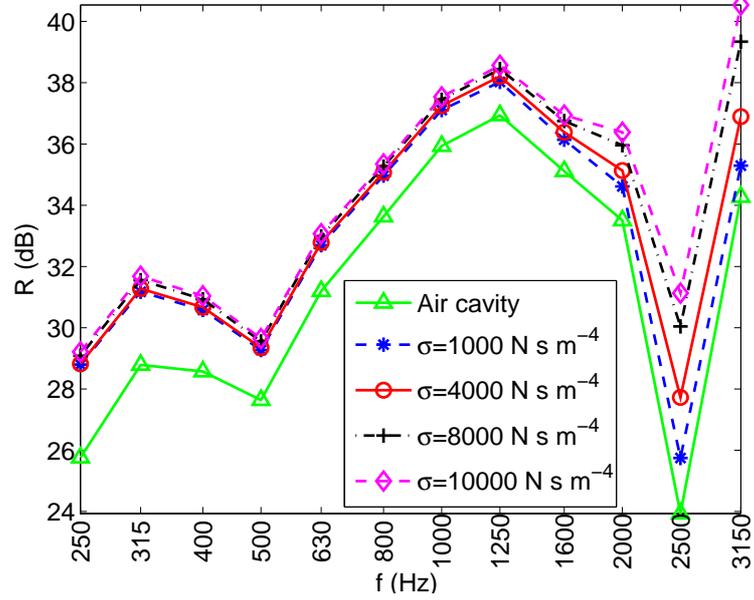


Figure 4. Effect of the flow resistivity on the sound reduction index.

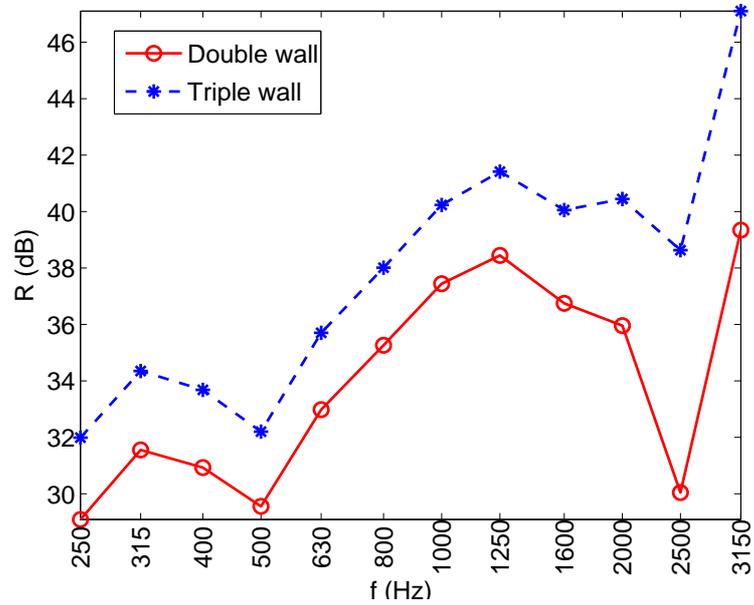


Figure 5. Effect of a second absorbing layer on the sound reduction index.

the drop in the insulation at 2500 Hz is considerably reduced by the addition of the extra layers.

5 CONCLUSIONS

The main conclusions drawn from this work are the following:

- Non-conservative connections can be taken into account in a SEA-like formulation with two types of coupling loss factors: the conservative (η_{ij}) and the non-conservative (γ_{ij}) coupling loss factors.
- Factors η_{ij} and γ_{ij} required for modelling non-conservative couplings can be computed with numerical simulations of a system consisting of 2 subsystems. Attention must be paid to the error propagation in these computations. Once they are computed, these factors can be used to solve larger problems with SEA.
- The influence of the flow resistivity of the absorbing material filling a plasterboard double wall is only relevant for high frequencies. At low frequencies, the behaviour is different if the cavity is empty or filled with absorbing material, but the resistivity of the material does not affect the sound insulation.
- The addition of new layers of plasterboard and absorbing material reduces considerably the sound transmission through the wall and dampens the effect of the coincidence frequency of the plasterboards.

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