Transient scour and fill: new evidence and research on its causes

The case of the Pilcomayo river

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8 **Abstract**

9 Transient scour and fill refers to the general scour and fill in a riverbed due to flood passage. It
10 was given much attention in times of Leopold and Maddock (1953) and Colby (1964) from records
11 of outstanding case studies. Field data from a station in a contracted, long reach of the large sand-
12 bed Pilcomayo River –1192 newly typed in forms– are presented, with sediment concentration in
13 the database up to 60 kg/m^3 , mostly wash load. Water surface rises and falls during a flood event,
14 whereas streambed evolves in reverse (it renders the analogy to an accordion). Streambed scouring
15 accounts for 36% of the change in flow area in floods and data reveal scouring up to 4 m from the
16 streambed elevation when flood starts. Velocity data display large hysteresis loops inexplicable by
17 unsteady flow alone. Once streambed starts to fall, river bed scour is self-sustaining. Water depth
18 gradient appears to be a crucial variable in a novel equation for transient scour and fill derived
19 from Exner equation. It turns out that flow non-uniformity is relevant to explain transient scour
20 and fill in contrast to flow unsteadiness, which is not. Quasi-steady gradually varied flow equation
21 solved along the contracted reach is the base for a model that captures well the recorded bed
22 evolution. The model explains most of the transient scour and fill by the downstream expansion,
23 but the non-uniform flow and the bed material suspended load play a role. The model fails on the
24 days of highest sediment concentrations.

25 **1. Introduction**

26 General scour usually refers to the streambed elevation change. In other words, it is the
27 scour (and, usually, the corresponding fill) that naturally occurs in a river bed, irrespective of any
28 man-made structure (Melville and Coleman, 2000). Long-term general scour, alias streambed
29 degradation, is a process through which a river can approach or recover equilibrium after a
30 disturbance [Galay, 1983; *Martín-Vide et al.*, 2010]. It usually develops along decades and should

be distinguished from short-term general scour associated with floods. “Transient scour” is used herein as a digest of the general scour and fill of the streambed during the passage of a flood.

Strikingly, such a paramount phenomenon in alluvial rivers (transient scour), with implications for river crossings and bridges, is seldom highlighted in recent literature. References in the context of Geomorphology describe transient fluctuations in streambeds of large rivers –the most important based on ancient USGS gage measurements. For instance, the San Juan, Colorado, Rio Grande and East Fork River in the United States [*Lane and Borland, 1953; Leopold and Maddock, 1953; Leopold et al., 1964, Andrews, 1979*], the Yellow River and others in China [*Lane and Borland, 1953; Wan and Wang, 1994; Wang et al., 2013*] and the Fraser River in Canada [*Hickin, 1995*]. In the context of river engineering practice, references are focused on design methods [*Bettess, 2002; Lauchlan and May, 2002; Melville, 2000*] but they often lack a clear physical meaning of the process at hand. Some advances on bed adjustments due to width variations [*Repetto et al., 2002; and Wu et al., 2011*] do not address specifically the issue of transient scour and fill. We suspect transient scour is paid little attention nowadays because past researches claimed the phenomenon had been incorrectly posed.

The empirical observations enabled *Leopold and Maddock* [1953] to stress the influence of sediment load on transient scour: “the observed changes in the stream bed resulted from changes in the sediment load brought into the reach from upstream”. Their data were obtained in cross sections with approximately constant width in spite of large discharge variations. *Lane and Borland* [1953] stated that transient scour does not occur universally –i.e. in all extension of an alluvial river– but it does in relatively narrow, primarily straight segments of mildly sloping rivers, customary characteristics of gauging stations. Indeed, to the authors’ knowledge there are no examples of transient scour in wide sections or in steep beds. The interpretation by *Leopold and*

Maddock [1953] cited above was refuted by his discussers on the same lines from the very beginning. *Colby* [1964] complained about the lack of acknowledgement on the inherent stability of sand beds elevation and noted problems in interpreting observations of scour and fill. The problems are: *a)* there are too few points to deduce an average position of the streambed at a cross section; *b)* not an average scour depth is observed but the maximum instantaneous scour depth because of the passage of bed forms and *c)* the scour and fill in a stream reach cannot be determined from observations in one or few cross sections. He used the principles of sediment continuity and bed material transport of sand to demonstrate that transient scour only exists in narrow cross sections. In the end, *Colby* [1964] may have contributed to current understanding of transient scour and fill as a mere artefact at gauging stations.

The current understanding of dynamic equilibrium at a contraction needs to be related to transient scour. Width-integrated sediment transport rate fed into a narrow reach equals the same rate transported out of the narrow reach through the wide reach downstream [*Bolla-Pitaluga et al.*, 2014]. The argument for the role of narrow cross sections as the only possible mechanism producing transient scour and fill [*Colby*, 1964], and thus considered as a minor particularity of streambeds, may have concealed other contributing factors to transient scour such as the sediment load. This paper reconsiders the topic of transient scour and fill with new data. Our data and those reported for the Yellow River [*Wang et al.*, 2013] may delineate a separate topic of rapidly varying cross sections in large alluvial rivers with high suspended sediment load (with a large share of wash load). Apart from new evidence of the phenomenon, the objective is to investigate its causes. In view of the state-of-the-art, the objective is more precisely to check if *Colby* [1964] explains the phenomenon or, otherwise, rapidly varying cross sections require new explanations such as incorporating the sediment load following *Leopold and Maddock* [1953]. The paper is divided in

two parts, empirical (sections 2-5) where data and empirical analysis are presented and analytical (sections 6-9), where new equations are sought for a physically based explanation that allows to build a predictive model.

2. Field site and methods

The Pilcomayo River flows south-eastwards from the Bolivian Andes across the Chaco plains, setting the border between Argentina and Paraguay (Figure 1a) in a very flat landscape with an average slope of $S_0 \approx 0.03\%$. The particular geomorphology of the Pilcomayo has been handled elsewhere [Martín-Vide *et al.*, 2012]. This paper shows field data from a gauging station of the river with a contributing area of 96,000 km². The station is located in a 2,500 m long, fairly straight contracted reach bounded by artificial levees with a high discharge capacity before spilling (Figure 1d). This channelized reach is stable in planform, contrary to the very dynamic upstream and downstream wider reaches, with rapid meander migration and collapse (Figure 1b). During floods, the average width for the contracted reach is $b = 160$ m and for the wide reach is $B \approx 500$ m. The streambed is composed of fine sand (median diameter $D_{50} \approx 0.2$ mm, sediment density $\rho_s \approx 2,600$ kg/m³) and the prevailing mode of transport is suspension (bedload transport is between 3% and 5% of the total sediment load transport [Amarilla and Amsler, 2005]). The riverbed at the gauging station is rapidly changing.

Since 1960, the work in this gauging station has consisted in measurements for river runoff computations with a daily frequency under normal conditions and twice a day when relevant hydrologic events occur. The main goal is the production of data for a fair distribution of water between the two lowland countries, Argentina and Paraguay. The hydrological regime is quite

98 regular –a wet season from November to April with high discharges often exceeding the 1,200
 99 m^3/s and a dry season from May to October with low discharges of around $5 \text{ m}^3/\text{s}$.

100

101 **Figure 1.** **a** Location map of the gauging station on the Pilcomayo River, named Misión La Paz
 102 (Argentina) and Pozo Hondo (Paraguay); **b** aerial satellite image of the region where the station is
 103 located (latitude -22.38°N longitude -62.52°E); **c** photograph of the station with the stadia rods and
 104 the cable way; **d** close-up view of the contracted transition reach; the station is located 500 m
 105 downstream of the inlet transition and 2,000 m upstream of the outlet transition, i.e. one-fifth and
 106 four-fifths respectively of the total length of the contracted reach (2,500 m).

The database consists of 1,192 original handwritten forms available from the Trinational Commission for the Development of the Pilcomayo River Basin (<http://www.pilcomayo.net/>). Common data are flow velocities and discharge from velocity integration, cross-section geometries and elevations (streambed and water surface). There are 734 forms completed with suspended sediment concentrations C (in kg/m^3) and the remaining 458 include water surface slopes S_s (between two stadia rods 310 m apart) (Table 1)

Table 1. Available information in number of forms by hydrological year (September to August)

year	Q , cross-section	C	S_s	Comments
72-73	308	308	-	all year round
79-80	118	-	118	from Jan to Apr (1980)
80-81	118	-	118	from Nov (1980) to Feb (1981)
81-82	222	-	222	from Oct (1981) to Apr (1982)
85-86	10	10	-	Nov (1985)
89-90	344	344	-	all year round
90-91	72	72	-	from Jan to Mar (1991)
Totals	1192	734	458	

From a cableway system, a propeller current-meter measured flow velocity and water depth at equidistant verticals (generally 10 m apart but sometimes 5 m apart) so we can obtain an adequate representation of the streambed to avoid problems *a)* too few points and *b)* maximum scour, previously mentioned by *Colby* (1964) and defined above. The six-tenths method (velocities are obtained at 0.6 of the depth from the water surface) or the three-point method (velocities are recorded at 0.2, 0.6 and 0.8 of the depth from the water surface) are typically used to calculate mean velocities at each vertical, although the float method is used in rough conditions (e.g. great

amount of debris). From simultaneous data obtained with a propeller and a float, a correction factor of 0.9 is applied to velocities acquired from a float to calculate mean flow velocities.

From the same cableway, suspended sediment was sampled with a US D-49 depth-integrating sampler of 1/8-inch diameter nozzle. Samples were obtained at three verticals (0.25, 0.5 and 0.75 of the width from the left bank). Bulk samples were taken from the river bank if the sampler was inoperative. Since 1993 suspended sediment samples have been sieved to obtain the percentage of fines (<0.062 mm).

Suspended sediment load in the Pilcomayo River is one of the largest in the world with a yearly average transport of $140 \cdot 10^6$ tons, mostly wash load from upland Andes. Although the maximum concentration of sediment in suspension in the database is as high as 60 kg/m^3 , it is far from hyperconcentrated conditions [Wan and Wang, 1994]. The database comprises both wet and dry seasons, as well as rising and falling limbs of hydrographs. Therefore, it provides exhaustive at-a-station hydraulic geometry relationships of fine-grained sand rivers with high suspended sediment concentration and is useful to illustrate the dynamics of rapidly changing cross sections. Above all, it will be devoted here to investigate the dynamics of transient scour and fill.

3. Description of field data

The dynamics of the channel cross section at the gauging station is better conveyed by an animation of the flow area development on a day-to-day basis (online supplementary material ready for download at Journal of Hydrology). Discrete measured streambed points are connected to define a polygon (i.e. channel cross section) whose upper edge is the free water surface. A fixed coordinate system and details on the time of the day of the measurements allow for a correct spatial-temporal representation. Key frames from the animation are illustrated in Figure 2. They

represent the initial stage (white), an intermediate rising stage (light grey and solid edge), the peak stage (dark grey), an intermediate falling stage (light grey and dashed line) and the final stage (black). We include a boundary area, 180 m wide and 10 m high, which is the envelope for all the cross sections in the database. The flow area moves within these boundaries, expanding and shrinking.

Figure 2. Selected cross sections from hydrologic year 1980-81 and the envelope for all database (1192 cross sections). Each cross section has the same format -colour and outline- of its centre of gravity (small circles). The inset shows the hydrograph between 1 November 1980 and 31 August 1981. The animation of the dynamics of the cross section is accessible online as supplementary material.

Initially, water free surface rises and the streambed drops. Later, this motion is present in reverse. It is conceptualized with the name of transient scour and, in addition, renders the analogy to an accordion (one hand pulling the free surface, the other hand the bed). Thus, the flood rise and fall are accompanied by scour and fill, respectively. However, there are exceptions and also the streambed rise and drop is uneven in the cross section. The deepening is not proportional to the

162 initial depth and there is a change in the outline –driving the centre of gravity with it– and a shift
163 in the thalweg. At any rate, scouring is mainly vertical and scarcely lateral (the distortion in Figure
164 2 must not be misleading). The initial and final stages are similar in the same fashion as the two
165 intermediate stages and no streambed degradation is observed at the end of the year. However, the
166 transient scour in the annual flood with a peak on 31 January 1981 amounted to more than 3 m.

167 The five largest hydrologic events in terms of peak discharge Q (hereafter simply events)
168 from the fraction of the database with measured suspended sediment concentration C are selected
169 for the analysis of transient scour (Figure 3). Events 1 and 2 and events 4 and 5 are consecutive in
170 the same high flow season of 1973 and 1991, respectively. Event 3 falls at the end of the low flow
171 season of 1985. The combination of the free surface elevation h with either the thalweg η or the
172 mean streambed elevation z distinctly depicts the accordion (with an extent described by the
173 evolution of the mean flow depth $y = h - z$). The latter (z) is preferred because it is based upon the
174 area of scour or fill between two consecutive cross section measurements defined as the net
175 scoured area ΔA_z (Figure 4).

Figure 3. Evolution of flow discharge Q (m^3/s), suspended sediment concentration C (kg/m^3), free surface elevation h (m), thalweg η (m) and mean streambed elevation z (m) for each of the events listed in the inset table, which are the largest floods in the database. There are 3 ordinate axis:

datum (h , η , z) on the right, and Q and C on the left. The US D-49 depth-integrating sampler was used in events 1, 2 and 3; samples from the river bank in events 4 and 5. The flow area A for selected days is displayed in the bottom right corner, for events 1 (a) and 3 (b). The A sequence is: grey-filled area, black full line, grey dashed line (only in event 3), and black dashed line.

Figure 4. Diagram for the definition of the mean streambed elevation z (m). If $z(t)$ is the mean streambed elevation at day t , $z(t+\Delta t) = z(t) + \Delta z$ after Δt days where $\Delta z = \Delta A_z / b_z$. The A sequence is grey-filled area after dotted line. The initial streambed elevation is defined as the free surface minus the hydraulic mean depth.

A remarkable feature in all events is a pattern of high C on the first days, events 1 and 2 standing out among the others. In particular, C peaks two days in advance of the maximum Q in event 2; and the stream rises at the same time as the maximum $C = 56.5 \text{ kg/m}^3$ is recorded at the beginning of event 1. Both features resemble the main example in *Leopold and Maddock* [1953]. Because C is weakly correlated with Q , plots C – Q reveal clockwise and counterclockwise loops, single valued and figure eight relations [Williams, 1989; Fan et al., 2012; Yang and Lee, 2018]. The fall of η is smaller (events 1 and 2), approximately the same (event 4) or bigger (events 3 and 5, η sinks nearly 4 m) than the rise of h . In general, z drop is nearly the same or smaller than that of η . Therefore, $|\Delta z| \leq |\Delta \eta|$ where Δf is the variation between two consecutive measurements, generally separated by 1 day, of any variable f .

4. Empirical analysis of the amount of transient scour

The purpose of this section is the correlation of the field data to infer the most advantageous lines of analysis. The net scoured area ΔA_z (m^2) (>0 for scour and <0 for fill) and the concurrent change in flow area ΔA (m^2) are compared in Figure 5. The two variables are selected because they are comparable quantities (ΔA_z is just a part of the ΔA) and they rely solely on the field-measured data. The plot shows the share of the streambed scour/fill ΔA_z in the change of the flow area ΔA .

Figure 5. Net scoured area ΔA_z against the change in flow area ΔA . I: still surface case; II: rigid bed case; III: trend line for $\Delta A_z - \Delta A$ (equation [1]) pairs from the selected events. Dashed lines are two hyperbolas encompassing the gross scour (up) and gross fill (down). As black squares, the selected events constitute 53 pairs $\Delta A_z - \Delta A$; as grey circles, data with $Q > 350 \text{ m}^3/\text{s}$ constitute 398 pairs $\Delta A_z - \Delta A$ in the wet season (November to April), while low discharge periods (dry season, May to October) do not exhibit significant changes in flow or scoured area and thus have been excluded from the analysis.

Data located in the first/third quadrant of Figure 5 are consistent with what may be expected during the rising/falling limb of the hydrograph if the transient scour analogy to an accordion is to

be accepted. There are some measurements in the second and fourth quadrants but the majority are located near the origin in the graph, where both ΔA and ΔA_z are small. The identity line $\Delta A_z = \Delta A$ (line I, still-surface) denotes an increase/decrease in ΔA at the expense of an increase/decrease of only ΔA_z , and therefore h stands still. The x-axis $\Delta A_z = 0$ (line II, rigid-streambed) describes a stationary streambed despite the increase/decrease in ΔA that would occur at the expense of a rise/fall of only h . The data are predominantly in the sector delimited by both line I (still-surface) and line II (rigid-streambed). The relationship ΔA_z – ΔA by simple linear regression (line III) for data with $Q > 350 \text{ m}^3/\text{s}$ is

$$\Delta A_z = 0.36 \cdot \Delta A \quad [1]$$

Only the regression coefficient is significant and the R-squared is 0.75. On average, the net scoured area accounts for the 36% of the change in flow area.

The two components of ΔA_z , the gross scour and the gross fill, are represented by its envelope in Figure 5. They reveal the unevenness in the cross section variations. The bigger/smaller ΔA is, the closer to the fitting equation the gross scour/fill gets. In other words, data scatter concentrates around low absolute values of ΔA_z and ΔA . If we dismiss data located in the second and fourth quadrants (24% of the data with $Q > 350 \text{ m}^3/\text{s}$) from the analysis since they violate the accordion hypothesis the simple linear regression model results in $\Delta A_z = 0.39 \cdot \Delta A$ and an R-squared of 0.86.

Because a design hydrograph is usually an input to estimate general scour, a simple linear regression of ΔA_z against ΔQ instead of ΔA would be more appealing as a predicting tool. Unfortunately, such a regression lessens R-squared to 0.59 as it involves flow characteristics of the alluvial channel (i.e. whatever the relation ΔA – ΔQ or stage-discharge relation is), which are

dealt with next. Furthermore, a simple linear regression for the relationship $\Delta A_z - \Delta(C \cdot Q)$ minimizes the R-squared to 0.32 corroborating the weak correlation between Q and C .

5. Brief analysis of field data

In this section, a basic analysis of resistance to flow, hysteresis and sediment transport is provided. The Manning coefficient n ($\text{s/m}^{1/3}$), worked as $n = v^{-1} \cdot R_h^{2/3} \cdot S_s^{1/2}$ where v is the mean flow velocity (m/s), R_h is the hydraulic radius (m) and S_s is the surface slope (-), is restrained between 0.011 and 0.025, with most data ranging from 0.014 to 0.021 (Figure 6). Measured S_s are restrained between 0.02% and 0.04% similar to bed slope S_0 , with a trend to have lower S_s for higher Q . The relationship $n-Q$ is modelled by simple linear regression for data with $Q > 350 \text{ m}^3/\text{s}$ and, at 95% confidence level, the relationship $n-Q$ is not significant for data with $Q > 350 \text{ m}^3/\text{s}$. Thus, $n = 0.018$ can be regarded as a constant.

Figure 6. Manning roughness coefficient n against discharge Q . The trend line (dashed line) only considers data with measured surface slope S_s and $Q > 350 \text{ m}^3/\text{s}$.

However, for the selected events, n is computed with $S_s = S_0 \approx 0.03\%$ and there is not a random scatter around the mean 0.018 (Figure 6). The lower threshold $n = 0.011$ agrees with a

grain roughness of 0.2 mm in the Strickler formula $n = D_{50}^{1/6}/21$. We may in principle attribute higher n values to bedforms and to the effect of high C . In the context of high concentration C of fine sediment, only small bedforms are expected [Best, 2005; Fredsoe and Deigaard, 1992; Wan, 1985]. In addition, Julien [2002] predicts washed-out dunes for n between 0.014 to 0.025 for C ranging from 1 to 4 kg/m³ (i.e. C one order of magnitude smaller than in the events in Figure 3). In conclusion, transient scour measured in the Pilcomayo should not be linked to passing bedforms, not only because field measurements are not maximum but average scour, but because low n values do not warrant their presence.

Field data reveal loop rating curves (typically in the curve $h-Q$). Observed hysteresis is more pronounced in the plot $v-Q$, especially in events 3 and 5 (Figure 7a, concomitant with a great transient scour). The rising stage exhibits higher v than the falling stage, thus generating a clockwise loop. The plot $C-y$ (Figure 7b) also displays clockwise loops with higher/lower concentrations during rising/falling limbs, respectively. Then, at a given Q , there is a decrease of C concomitant with a decrease in v and an increase in y , although it does not necessarily imply a cause-effect relation. This statement agrees with Leopold and Maddock [1953] if a constant width b is assumed. The significance of the variation of b with time is studied in following sections.

Figure 7. Broad hysteresis for events 3 and 5. **a** $v-Q$; **b** $C-y$; and **c** $b/y-Q$, dealt with later.

Regarding sediment transport, *Amarilla and Amsler* [2005] determined that the *Engelund and Hansen* [1967] equation (EH) is satisfactory to characterize the bed material transport of the Pilcomayo by comparing the outcome from several equations with 10 years of data. The sediment load was separated into sand (>0.062 mm and 11% of the total load in average) and fine sediment (<0.062 mm and 89% of the total load in average); sand was assigned to bed material load and fine sediment to wash load. Engelund-Hansen equation reads: $q_t = K \cdot v^2 \cdot \tau^{3/2}$ where q_t is unit bed material transport, v is velocity, τ is shear stress and K is the constant $0.05 \cdot g^{-2} \cdot \rho^{3/2} \cdot R^{-2} D_{50}^{-1}$, where $g=9.81$ m/s², ρ is water density, R is $(\rho_s - \rho)/\rho$ with ρ_s sediment density and D_{50} is median grain size.

6. Analytical approach: mass conservation

After new evidence on transient scour and fill, the analytical research on its causes starts. We extend the empirical equation [1] (obtained from daily measurements) to a continuous change in time by replacing $dA = b \cdot dy$ and $dA_z = -b_z \cdot dz$ to obtain:

$$\frac{dz}{dt} = -0.36 \cdot \frac{b}{b_z} \frac{dy}{dt} \quad [2]$$

where b is the surface width and b_z is the width of the streambed affected by two consecutive measurements (Figure 4). Note that the negative sign in [2] indicates that bed scoring ($dz/dt < 0$) occurs when water depth increases –accordion analogy. Equation [2] is site-specific because b/b_z depends on the cross-section shape and the coefficient 0.36 is only valid for this site. As such, it should replicate the streambed evolution of the river. The imperfect fitting of Equation [2] for $b/b_z=1$ (see Figure 8 later on) announces the difficulty of scour and fill prediction in the selected events. Indeed, the large scatter for $Q > 350$ m³/s in Figure 5 anticipates the disagreement in Figure 8. For events 3, 4 and 5 the bed scours more than the replicated (then, a ratio b/b_z higher than 1

could slightly improve the results). The fitting is especially poor in the first days of events 1 and 2 (and, to a lesser extent, first days of event 5) where channel bed aggrades contrary to the replicated.

An equation akin to [2] seems the ideal predicting tool, yet with the coefficient varying with the relevant variables instead of being a site-specific constant, i.e. 0.36 for the Pilcomayo. The differential equation of sediment mass conservation is used to obtain [3] (details on its derivation and the proof that the wash load does not change the streambed elevation are given in Appendix 1). Equation [3] is written as per unit width to be applied to the narrow reach of uniform width, while the wide channel downstream will set its boundary condition (for a general cross-section see Appendix 1). It reads:

$$\frac{\partial z}{\partial t} \cdot (1 - \lambda) = -\frac{\partial}{\partial t}(\alpha \cdot C \cdot y) - \frac{\partial q_t}{\partial x} \approx -\frac{\partial q_t}{\partial x} \quad [3]$$

where q_t is the bed material transport (bedload and suspended load excluding wash load), α the fraction of bed material in the total depth-averaged, volumetric suspended sediment concentration C ($\alpha = 0.11$ after *Amarilla and Amsler* [2005], see §5), and λ a constant bed porosity (assumed 0.4). With measured data $q_t = \alpha \cdot C \cdot v \cdot y$. Unfortunately for the ideal equation, the local term (time variation) of the right-hand side of [3] only amounts to 0.2% of the total variation with our data, far from the 36% on average according to [1]. *Cui et al.* [2005] also argue against the importance of the local term. Therefore, the spatial term (convective derivative) on the right-hand side of [3] is dominant in explaining how bed elevation changes in time. To sum up, the depth variation at the gauging station $\partial y / \partial t$ seems not to be a good indicator of the capacity of a flood to scour the bed, contrary to [2], when the mass conservation in differential form is used. The counter-example of an increasing depth in stagnant water is telling to reject an equation akin to [2].

7. Spatial variation versus unsteadiness

The role of the spatial variation in the transient scour can be further investigated with simple differential analysis. First, the bed material transport q_t in [3] can be equalled to a sediment transport capacity equation. The EH formula, that suits to the Pilcomayo as stated above, is chosen. By using the Manning equation and the water mass conservation equation, $\partial y/\partial t + \partial q/\partial x = 0$, the spatial derivative of EH results:

$$\frac{\partial q_t}{\partial x} = -5 \cdot \frac{q_t}{q} \cdot \left(\frac{\partial y}{\partial t} + 1.1 \cdot v \cdot \frac{\partial y}{\partial x} \right) \quad [4]$$

Other monomial sediment transport formulas and resistance equations produce similar derivatives (e.g. the coefficient 5 would be 3 with the Meyer-Peter and Müller bed load equation, and the coefficient 1.1 would be 1 with the Chézy equation). More importantly, the local term in eq. [4] is again much smaller in absolute value than the spatial term and thus it can be neglected: we can estimate an average $v = 2.3$ m/s for the selected events in our database, while $\partial y/\partial x \approx \pm 0.01\%$, so that the term $1.1 \cdot v \cdot \partial y/\partial x$ amounts to ± 22 m/day, much larger than $\partial y/\partial t$ (the maximum is 4 m/day). Therefore, the Exner equation is expressed as:

$$\frac{\partial z}{\partial t} \cdot (1 - \lambda) = 5.5 \cdot \frac{q_t}{q} \cdot v \frac{\partial y}{\partial x} \quad [5]$$

Equation [5] retains the dominant component in transient scour, discarding the two minor local terms $\partial/\partial t$ in [3]-[4], and can be adapted to other mild rivers by varying sediment transport capacity and resistance formulas that lead to coefficient 5.5. Ultimately, non-uniformity represented by the crucial spatial derivative $\partial y/\partial x$, unlike flow unsteadiness, appears relevant to the transient scour.

The differential analysis can be further useful. Appendix 2 proves that the unsteady flow term is not able to explain the amount of transient scour that occurs simultaneously within a flood

hydrograph. Therefore, there must be other triggering processes of the scour. Let us try to provide a physical explanation of transient scour based on the hysteretic behaviour registered in the stream gauge (Figure 7). Note in this regards that, as pointed out in Appendix 2, $\partial y/\partial x$ is precisely the term that accounts for the extent of the loops observed in Figure 7 [Henderson, 1966]. To this purpose, the analysis of the loop rating curves (y — Q or v — Q curves) is revealing. Real data plotted in these curves exhibit a large magnification of v and $\partial y/\partial x$, which control transient scour through eq. [5], with respect to the analytical solution of the unsteady flow over a fixed bed. As said before, this suggests a triggering factor, other than flow unsteadiness, responsible for these large magnifications and large transient scour. One concurrent factor is the large suspended loads exhibiting hysteresis loops, as well (Figure 7b), but another factor is the channel width change between the gauging station and the wider regions upstream and downstream. The analysis of the second factor is what follows next.

8. The role of the two channel widths

The gauging station is located in a long, contracted reach ($b = 160$ m) bounded upstream and downstream by two wider reaches ($B \approx 500$ m). Straub [1935] and Laursen [1958], as reported by Vanoni [1975] and Raudkivi [1990], studied long-term, equilibrium scour in a long transition under constant discharge. Under such steady flow, width-integrated sediment transport through the channel width (B) should be constant, i.e. $\partial(B \cdot q_t)/\partial x = 0$. The authors found that the semi-empirical expression $B \cdot y^k = \text{constant}$, where k is a function of the excess of shear stress with respect to the critical one, is the long-term, steady solution that allows uniform flow to exist in both the contracted reach (its normal depth called y_{nb}) and the wide reaches (normal depth y_{nB}). Note that

since the exponent k is ≥ 1 the scoured bed area is less than the flow area lost by the contraction.

Based on their solution, the formula for the long-term scour at the contracted reach is

$$\delta z = -\left(\left(\frac{B}{b}\right)^{1/k} - 1\right) \cdot y_{nB} \quad [6]$$

where δz means the difference of bed elevation between reaches. Equation [6] can be extended to

a continuum time, by assuming constant widths and non-changing bed elevation in the wide reach

$$\frac{dz}{dt} = -\left(\left(\frac{B}{b}\right)^{1/k} - 1\right) \cdot \frac{dy_{nB}}{dt} \quad [7]$$

To some extent, Eq. [7] is a reach-specific equation since it encompasses long distances. It resembles the site-specific equation [2] in the sense that the transient scour is related to temporal changes of one water depth, namely the normal depth at the wide reach. Eq. [7] with $k = 1.5$ (valid for high sediment transport, *Vanoni* [1975]) is applied by computing the normal depth y_{nB} for each daily discharge for the selected events. The results together with those of eq. [2] are plotted in Figure 8. Note that Eq. [7] yields predictions closer to the data, although similar to eq. [2]. The fitting is still poor in the first days of events 1 and 2.

It may be surprising that the reach-specific equation [7] replicates the transient scour as well as the site-specific relation does (despite the latter is obtained by regression of a large amount of data). Indeed, this reach-specific equation is a semi-empirical formula for long-term scour under steady flow, which does not involve any variable of the gauging station but only the regional characteristics b , B and y_{nB} . The reader may even feel that the simple equation [7] supersedes real data. Indeed, it is understandable to question both the need of field measurements and the complexities of Exner and flow equations (Appendix 1 and 2): what are they for?

Figure 8. Mean streambed elevation z : measured, eq. [2] for $b/b_z = 1$ and eq. [7]. For clarity, all events start from a relative elevation of 0 m.

Contrary to what has been concluded in §6, it should be noted that time variation of one water depth $\partial y_{nB}/\partial t$ does seem to be a good indicator of the capacity of a flood for scouring. Furthermore, from Figure 8 we can infer that width changes seem to trigger transient scour and that one day seems a long term regarding transient scour. Next section tries to reconcile the semi-empiricism of eq. [7] with flow and Exner equations.

Regarding the time variation of the width at the gauging station itself, the coefficient 5.5 in eq. [5] should be replaced by $(5.5 - m)$ where $0 < m < 1$ (Appendix 1). Figure 7c shows the quotient b/y as function of discharge Q for two events. The higher Q the smaller b/y , i.e. the increase in b is smaller than the rise of y when Q grows. Data loop because b is larger during the rising than during the falling limb for the same Q and y .

9. A physically-based explanation

Once flow unsteadiness is proved negligible (§7), an explanation on the grounds of the quasi-steady gradually-varied flow equations on an erodible bed in the contracted reach with a boundary condition at the expanded reach downstream is tested. The reach is assumed prismatic, i.e. all cross-sections downstream are equal to the one at the gauging station all time. There is no alternative for this bold assumption since the field work and bed survey was limited, as customarily, to one gauging station. Appendix 3 explains the rationale of this physically-based model. It is concluded that scour is expected during the rising limb (a drawdown profile occurs) but fill during the falling limb (a backwater profile). If discharge remains constant, channel bed is bound to approach an equilibrium condition, in which the difference in bed elevations between the contracted and the wide reaches equals the difference in uniform flow depths between them, because uniform flow occurs throughout. This is the assumption supporting eqs. [6]- [7]. In this sense, the physically-based model should tend asymptotically to eq. [7] under a constant discharge.

How much time the model needs to do so is the last issue to reconcile. Appendix 4 provides a demonstration that eq. [6] can be derived from steady gradually-varied flow, so that [7] express the spatial flow variation—the main cause of transient scour—in another way. Thus, [6] is semi-theoretical rather than semi-empirical, and the good performance of [7] to replicate real data becomes more comprehensible.

The approximate eq. [3] and the analytical solution of the gradually-varied flow by the Bresse function [*Henderson*, 1966] are preferred in the physically-based model. The tabulated Bresse function retains the essence of the spatial variation in a wide prismatic channel, while avoiding numerical models too skilled for such a simplified, hypothetical geometry. Once a flow solver (the Bresse function) is included in the model, bed elevation changes at the gauging station are calculated by using the integral approach of eq. [3]. This approach does not lose resolution nor accuracy with respect to the differential approach and has the advantage of highlighting the reach length that experiences transient scour, and thus it indirectly involves a morphodynamic time scale. Equation [3] is integrated with respect to space x and time t in a control volume of length L within the prismatic narrow reach ($L < 2,000$ m) and then extended to the elapsed time between two measurements Δt

$$\Delta z \cdot L \cdot (1 - \lambda) + \int_t^{t+\Delta t} (q_t(t)_L - q_t(t)_0) dt = 0 \quad [8]$$

where $q_t(t)_0$ and $q_t(t)_L$ are the bed material transport at the gauging station ($x = 0$) and at a distance $x = L$ downstream in the narrow reach, respectively. The control volume length L , not known in principle, must be commensurate with the distance travelled by the bed wave generating transient scour during the integration time ($\Delta t \approx 1$ day). Note that the ratio $L/\Delta t$ is the celerity of the bed

414 wave (or contains the inverse of the morphodynamic time scale). As data are only available at $x =$
 415 0 m, the time integral in eq. [8] is rewritten as

$$\int_t^{t+\Delta t} (q_t(t)_L - q_t(t)_0) dt = (\beta - 1) \cdot \int_t^{t+\Delta t} q_t(t)_0 dt \quad [9]$$

416 where β encompasses the gradually-varied flow within distance L and is defined as the ratio
 417 between bed material transport capacities at $x = L$ m and $x = 0$ m. It is derived from the tabulated
 418 Bresse function. The trapezoidal rule is applied to the time integral of q_t and, finally, eq. [9] is
 419 substituted into eq. [8] to obtain:

$$\Delta z \cdot (1 - \lambda) = -\frac{\Delta t}{L} \cdot (\beta - 1) \cdot \frac{q_t(t)_0 + q_t(t + \Delta t)_0}{2} \quad [10]$$

$$\beta = \frac{1}{2} \cdot \left(\frac{q_t(t)_L}{q_t(t)_0} + \frac{q_t(t + \Delta t)_L}{q_t(t + \Delta t)_0} \right) \quad [11]$$

420 where β has been averaged between two measurements one day apart ($\Delta t \approx 1$). The solution of eq.
 421 [10] is implicit and spatial derivatives are fully avoided. Except for β and the length L controlling
 422 the speed of the bed wave, all other quantities in eq. [10] are variables present in the database.
 423 Only to be consistent with the Bresse function solver, the EH formula is used in eq. [11] for both
 424 $q_t(-)_L$ and $q_t(-)_0$ at any time.

425 The parameter β is a function of L and the differences in bed elevations (δz) and normal
 426 depths (δy_n) between the contracted and wide reaches. In addition, total energy losses are neglected
 427 between the narrow and the wide sections at the outlet of the contracted reach. For successive
 428 steps, δy_n changes according to Q so the comparison between δz and δy_n induces a drawdown ($\beta >$
 429 1, the flow accelerates) or a backwater ($\beta < 1$, the flow slows down) (§Appendix 3). The streambed
 430 change Δz is assumed completed at each Δt , and thus a volume $\Delta z \cdot L$ is scoured (or filled), and δz

is updated every Δt . Equation [10] is a robust convergence model for the database. As it converges to an equilibrium scour for a constant Q , δz at the beginning of the computation (day 1) is obtained by using eq. [10] with the event's first day Q but extended for a long time (i.e. long-term scour).

10. Results and discussion of scour and fill

The integration length L is selected after a sensitivity analysis. Figure 9 shows the evolution of z for event 3 modelled with eq. [10] (with measured q_t and $\alpha = 0.11$, see §6) and a range of L between 50 m and 1,950 m ($< 2,000$ m separating the gauging station from the downstream transition). Note that the smaller the L the less dynamic the transient scour and vice versa.

Figure 9. Integration length sensitivity analysis for the event 3 (L ranges between 50 m and 1950 m) by using equation [10] with measured q_t ($\alpha = 0.11$). Black dashed-line represents the actual streambed evolution.

Modelled z for all selected events is obtained from eq. [10] with $L = 1,950$ m in three ways shown in Figure 10. If measured data are used, q_t is computed with $\alpha = 0.11$ and $\alpha = 0.33$, and is called replication; if no measured data are used, the EH formula is used to obtain q_t and is called prediction. The increase in bed material load in the measured suspension load by using $\alpha = 0.33$

447 instead of the average $\alpha = 0.11$, means that part of the assumed wash load is no longer wash load
 448 but interacts with the bed, and thus enhances the dynamics of scour and fill.

449

Figure 10. Mean streambed elevation z : measured and modelled with equation [10]. The integration length is $L=1950$ m. For clarity, all events and model results start from a relative elevation of 0 m.

Discussion of the reach- and site-specific (Fig.8) and physically based model (Fig.10) follows:

1. The physically based model traces fairly well the streambed evolution, capturing the order of magnitude and the sign (i.e. the analogy to an accordion) of the transient scour and fill. However, modelled scour is smaller than measured in events 4 and 5, slightly smaller in event 3, and sometimes larger in events 1 and 2. Although this may be deemed a poor performance for a model, a number of assumptions and simplifications (e.g. flow, prismatic geometry and control volume) led to equation [10].

2. Bed evolution is more dynamic for $\alpha = 0.33$ than in the other cases. It gets closer to the measured data in events 3, 4 and 5, yet farther in events 1 and 2. It follows that the definition of wash load based upon the separation of sand and finer sediment (at size 0.062 mm) may not be satisfactory for the phenomena of transient scour and fill. Obviously, parameter α provides a means of tuning the model results, which would be arbitrary without a deeper knowledge of the physics of suspended sediment transport.

3. Both prediction and replication results have the same relative success. The choice of an appropriate sediment transport formula (EH in this research) may be credited for this success.

4. Differences between sediment supply and transport at-capacity conditions do not explain the disagreement between model results and data. Instead, they are the cause of the little difference between replication and prediction. Although the so-called non-equilibrium conditions (supply $\neq$

capacity) entail scour and fill, the observed shortage of sediment load in event 5 makes little difference in the results (see Figure A1c).

5. Results from equations [2] and [7] are very similar (Figure 8) although the latter exaggerates the scour displayed by the former, especially in event 3. Again, scour is smaller than data in events 4 and 5 and the performance is poor in events 1 and 2. Overall, equation [7] is not worse than equation [2] as a replication of the streambed data.

6. The discrepancy between equation [2] and equation [7] in event 3 and not as much in other events may be attributed to their dates (1973, 1985 and 1991). The Pilcomayo is changing over time [Martín-Vide *et al.*, 2012]. Planform changes in downstream reaches modify the stage-discharge relationship that affects y_{nB} . Moreover, y_{nB} is not allowed to change over time in the model. If the wide region downstream aggrades while the contracted region scours, backwater is promoted in the contracted reach, and thus a more rapid recovery from scour. Event 5, on the contrary, clearly displays how the model is struggling to recover from scour.

7. The success of equation [7] in replicating field data highlights the change in width from b to B as the main cause and triggering factor of the transient scour and fill. The widening downstream explains most of the transient scour and fill within the contracted reach. At low flow, the two channel widths are probably similar but, as the flow rises, they may start to differ and thus to trigger bed scouring. This point fully supports the assertion by Colby [1964].

8. Equations [2] and [7] fail when C is maximum (first days of events 1 and 2) and when C peaks in advance of the maximum Q (event 2 and to a certain extent the first days of event 5) because they produce large scours whereas the actual bed slightly fills or scours. The physically based model improves results, and so it is postulated that large sediment loads are another cause of transient scour and fill, superimposed on the expansion downstream. This second cause is in

accordance with *Leopold and Maddock* [1953] as quoted in the Introduction. Examples of the Yellow river in the hyperconcentrated flow domain [Wan and Wang, 1994] show large bed fills under huge sediment loads, in an almost chaotic way. Sediment loads in the Pilcomayo, although much lower than those in the Yellow river, may be on the verge of a similar behaviour in events 1 and 2, different from current understanding of scour and fill according to *Colby* [1964].

9. As well as the good performance of [7], the ability of the system to approach rapidly any scour or fill stage was also striking. One day is a long enough time for the bed to adjust to changes in flow. This suggests that the morphodynamic time scale of bed change is similar to the time scale at which the flow is changing. Figure 9 and Figure 10 prove that the contracted reach length is commensurate with the distance travelled by the bed wave of the transient scour and fill in one day. The celerity of such a scour or fill wave (of a magnitude of meters) is then about 2 km/day. This figure is similar to 3.6 km/day obtained from the approximation of the celerity of a bed disturbance in the differential analysis of the flow equations with Exner, under all the assumptions of shallow water flow [De Vries, 1975, Garde and Ranga Raju, 1985]. For the selected events, it is computed as $a \cdot v \cdot q_t / [q \cdot (1 - \lambda) \cdot (1 - Fr^2)]$, where $a = 5$ is the exponent in the EH function ($q_t \propto v^a$), $Fr \approx 0.3$ is the Froude number, $q_t/q \approx 0.02$ and $v \approx 2.3$ m/s.

10. The results of the physically-based model and eq. [7] are comparable, apart from events 1 and 2 where the model performs better. This is discouraging because flow profile, sediment transport and time, included in the physically-based model, are not necessary to predict transient scour and fill, actually. The expansion from b to B appears to override all other factors. The similarities between these results not only confirm the support to *Colby* [1964] but also mean that the expansion is necessarily acting through the intermediate mechanisms of flow profile, sediment transport and time.

11. Conclusions

New field data showcasing rapidly varying cross sections in a long contracted sand-bed reach from a gauging station draining 96,000 km² in the Pilcomayo River is presented. The maximum recorded suspended sediment load and streambed scour in one day are 60 kg/m³ and ~4 m, respectively. The objective is to check whether the often-narrow gauging stations explain the transient scour and fill (as asserted by *Colby* [1964], reinforced by *Vanoni* [1975] and currently understood) or the changes in sediment supply are the cause of the observed scour and fill (as asserted by *Leopold and Maddock* [1953]).

Exner equation, elaborating on its convective term, leads to equation [5] for transient scour and fill, which can be generalized to other rivers. Water depth gradients $\partial y/\partial x$ during a flood are crucial in the phenomenon. Indeed, large differences in $\partial y/\partial x$ between the rising and falling limb are observed in the data, thus displaying large hysteresis loops. On the contrary, the analytical solution in unsteady flow produces a negligible transient scour yet having the correct sign. Then, the unsteadiness cannot explain the transient scour, in spite of occurring simultaneously.

Once river bed starts to fall, scouring is self-sustaining in the sense that the change in $\partial y/\partial x$ and the scour or fill explain each other by the flow equations. The remaining question is the triggering factor that starts either the scour or the increase in $\partial y/\partial x$. Incidentally, suspended sediment concentration data loop simultaneously with flow variables, as described by *Leopold and Maddock* [1953]. However, this does not imply a cause-effect relation. In fact, the self-sustaining scour does not need suspended sediment load in the explanation.

The quasi-steady gradually varied flow in a long contraction (with a boundary condition downstream) coupled with the sediment mass conservation in a control volume leads to equation [10]. This model is consistent with the equilibrium scour in long transitions. The increase/decrease

in discharge determines a drawdown/backwater profile, which in turn determines bed elevation changes, i.e. scour/fill. The term $(\beta-1)$, multiplied by the total bed material load, has a different sign for drawdown and backwater conditions and is null in uniform flow conditions. Non-uniformity, not pure unsteadiness, is relevant to transient scour.

The results prove that the expansion downstream of the long, contracted reach explains most of the transient scour and fill through non-uniform flow and bed material suspended load. However, as this fails on days of highest concentrations ($>40 \text{ kg/m}^3$) when the physically-based model results are better, it is postulated that large sediment loads are another cause of transient scour and fill, superimposed on the expansion downstream. It should be noted that equilibrium scour in long transitions (equation [7]) fully supports *Colby* [1964].

Lastly, *Leopold and Maddock* [1953] raised coincidental observations to causal statements. Although their observations are confirmed by our new data, we adhere to the refutation by *Colby* [1964] with new arguments. Narrow sites cause transient scour, even in long transitions such as in the Pilcomayo. This is intimately connected with flow and sediment transport (as eq. [10] accounts for, unlike eq. [7]). The large sediment load as another cause of transient scour and fill is not a vindication of the main contents of *Leopold and Maddock* [1953]. This postulate cannot be more precise at this point except for: a) it does not have to do with the non-equilibrium conditions (supply $\neq$ capacity), and b) it does not have to do with the total suspended load partitioned between wash load (not interacting with the bed) and bed material load.

Appendix 1: The use of the sediment load conservation (Exner) equation

In rivers carrying a load coming from the streambed (bed material load), the mass conservation equations for each transport mode, bedload and suspended load, per unit width read respectively [Parker, 2004; Paola and Voller, 2005]

$$\frac{\partial z}{\partial t} + \frac{\partial q_b}{\partial x} = D - E \quad [A1]$$

$$\frac{\partial (C_b \cdot y)}{\partial t} + \frac{\partial q_s}{\partial x} = E - D \quad [A2]$$

with $z = z \cdot (1 - \lambda)$ the streambed elevation, positive upwards, embodying a constant void ratio (λ), q_b and q_s (m^2/s) the unit bedload and suspended load, C_b (m^3/m^3) the depth-averaged bed material suspended concentration, E the entrainment of bed material into suspension and D the deposition of suspended sediment. The sum of [A1] and [A2] is

$$\frac{\partial z}{\partial t} + \frac{\partial (C_b \cdot y)}{\partial t} + \frac{\partial q_t}{\partial x} = 0 \quad [A3]$$

with $q_t = q_b + q_s$ the total bed material transport (i.e. excluding the wash load). Wash load is the load of very fine material controlled by the basin, carried in suspension at the same velocity as the water and not interacting with the streambed at a reach scale ($D = 0$, $E = 0$). The total sediment transport, including wash load, is $q_T = q_t + W \cdot q$, where W (m^3/m^3) is the wash load concentration and q (m^2/s) the unit discharge. If wash load is added, a new total suspended sediment concentration C replaces C_b and total load q_T replaces bed material load q_t . A case of interest is if the sediment load consists entirely of wash load ($C = W$), reducing [A1] [A2] to

$$\frac{\partial z}{\partial t} = 0 \quad \text{and} \quad \frac{\partial (W \cdot y)}{\partial t} + \frac{\partial (W \cdot q)}{\partial x} = 0 \quad [A4]$$

574 Operating and introducing the water conservation equation $\partial y/\partial t + \partial q/\partial x = 0$

$$\frac{dW}{dt} = \frac{\partial W}{\partial t} + v \frac{\partial W}{\partial x} = 0 \quad [A5]$$

575 where d/dt is the total derivative. Equation [A5] describes mathematically the wash load as a
 576 conservative matter (no interaction with the bed) carried by water in pure convection (its diffusion
 577 is not considered). Consequently, the transit of such a load through the river does not change the
 578 streambed elevation. For the general case:

$$\frac{\partial z}{\partial t} + \frac{\partial(C \cdot y)}{\partial t} + \frac{\partial q_t}{\partial x} + \frac{\partial(W \cdot q)}{\partial x} = 0 \quad [A6]$$

579 where C includes sediment from the streambed and the basin (it is the actual measured quantity).
 580 By introducing equation [A5] into [A6]:

$$\frac{\partial z}{\partial t} + \frac{\partial[y \cdot (C - W)]}{\partial t} + \frac{\partial q_t}{\partial x} = 0 \quad [A7]$$

581 A closure linking C and W is convenient because it reduces one unknown, improving its predictive
 582 capability. In a similar fashion to *Yang and Simões* [2005], $W = (1-\alpha) \cdot C$ and $\alpha < 1$ expresses the
 583 bed material fraction as a part of the total sediment concentration in suspension, so that

$$\frac{\partial z}{\partial t} = - \frac{\partial(\alpha \cdot C \cdot y)}{\partial t} - \frac{\partial q_t}{\partial x} \quad [A8]$$

584 [A8] is the fundamental equation for the transient scour as a function of the sediment load variation
 585 (eq. [3] in the main text). It is purely theoretical and based upon mass conservation only.

586 For a general cross-section (not per unit width), the differential of the elevation dz is
 587 replaced by $B_c \cdot dz$, where B_c is the active bed width (constant with time), q_t is replaced by Q_t and
 588 C is the total area-averaged suspended concentration. The equation equivalent to [A7] becomes

$$B_c \cdot \frac{\partial z}{\partial t} + \frac{\partial[A \cdot (C - W)]}{\partial t} + \frac{\partial Q_t}{\partial x} = 0 \quad [\text{A9}]$$

589 where A is the cross-section area. Provided that q_t is constant across the river so that $Q_t = B_c \cdot q_t$,

$$\frac{\partial Q_t}{\partial x} = B_c \cdot \frac{\partial q_t}{\partial x} + q_t \cdot \frac{\partial B_c}{\partial x} \quad [\text{A10}]$$

590 The equivalent to equation [A8] with the W - C closure, neglecting local terms $\partial/\partial t$ and introducing

591 Eq.[4] of the main text is

$$\frac{\partial z}{\partial t} = -\frac{1}{B_c} \cdot \frac{\partial(\alpha \cdot C \cdot A)}{\partial t} - \frac{\partial q_t}{\partial x} - \frac{q_t}{B_c} \frac{\partial B_c}{\partial x} \approx 5.5 \cdot \frac{q_t}{q} \cdot v \cdot \frac{\partial y}{\partial x} - \frac{q_t}{B_c} \frac{\partial B_c}{\partial x} \quad [\text{A11}]$$

592 If the river is prismatic, $\partial B_c/\partial x = (dB_c/dy) \cdot (\partial y/\partial x)$, so that combining with [A11]:

$$\frac{\partial z}{\partial t} = \frac{q_t}{q} \cdot v \cdot \frac{\partial y}{\partial x} \cdot \left(5.5 - \frac{y}{B_c} \frac{dB_c}{dy}\right) \quad [\text{A12}]$$

593 If the cross-section shape is a monomial expression $B_c = y^m$ ($0 < m < 1$ for concave), finally

$$\frac{\partial z}{\partial t} = \frac{q_t}{q} \cdot v \cdot \frac{\partial y}{\partial x} \cdot (5.5 - m) \quad [\text{A13}]$$

594 **Appendix 2: Flow unsteadiness and transient scour**

595 During the rising limb of a flood $\partial y/\partial x < 0$ and the situation is reverse during the falling

596 limb. The sign of $\partial z/\partial t$ concurs with the sign of $\partial y/\partial x$ (i.e. the analogy to an accordion), according

597 to eq. [5]. The curves y - Q in mild slope channels show a loop of hysteresis, i.e. discharge Q is not

598 a unique function of y but also, among other terms, of $\partial y/\partial x$. In a diffusive wave for example, the

599 ratio of the actual Q to the uniform flow discharge equals to $(1 - S_0^{-1} \cdot \partial y/\partial x)^{1/2}$ [Henderson, 1966].

600 This function of $\partial y/\partial x$ represents the extent of the loop. In our data, the hysteresis (Figure 7a) and

the transient scour (Figure 3) are concomitant, and a large hysteresis comes with a large transient scour. Are, therefore, hysteresis and transient scour linked by a cause-and-effect relationship?

The mass and momentum equations in unsteady shallow water flow are:

$$\begin{array}{c} \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \\ \dots\dots\dots \\ \frac{1}{A} \cdot \frac{\partial Q}{\partial t} + \frac{1}{A} \cdot \frac{\partial}{\partial x} \left(\frac{Q^2}{A} \right) + g \cdot \frac{\partial y}{\partial x} - g \cdot (S_o - S_f) = 0 \end{array} \quad \text{[A14]}$$

The diffusive wave [Jain, 2001] is an approximation that only keeps those terms under the grey dashed lines of [A14]. The routing of the flood in a diffusive wave follows a diffusion equation. The equation needs an initial wave at the upstream end. If the initial wave is a sine curve, an analytical solution of the diffusive wave is:

$$y(x, t) = y_0 + \frac{M}{2} - \frac{M}{2} \cdot \exp \left[-D \cdot t \left(\frac{2\pi}{L} \right)^2 \right] \cdot \cos \left[\frac{2\pi}{L} \cdot (x - c \cdot t) \right] \quad \text{[A15]}$$

where y_0 is the depth at the start of the wave and initial time ($x = 0, t = 0$), M (m) is the base-to-peak amplitude at $t=0$ (i.e. the growth in depth of the wave from the base) and L (m) is the wavelength. The wave celerity c and the diffusion coefficient D are assumed constant and defined at $t = 0$ by averaging within one wavelength as $c = (5/3) \cdot v$ (assuming Manning coefficient is constant) and $D = 3 \cdot c \cdot y / (10 \cdot S_f)$, the bed slope $S_o = 0.03\%$ is constant and the friction slope is $S_f = S_o - \partial y / \partial x$ according to a [A14]. The analytical routing of this flood wave, over a potentially infinite fixed bed, shows a decaying wave amplitude in space and time.

Figure A1 contrasts this analytical solution with the values in the database. Events 2 and 5 are selected because they resemble the sine wave (Figure A1a). The hysteresis is extremely narrow

617 in the analytical solution but very broad in the data (v - Q clockwise in Figure A1e and
 618 anticlockwise y - Q in Figure A1d). Thus, in the analytical solution $\partial y/\partial x$ hardly changes between
 619 the rising and falling limb, whereas the change in the actual data $\partial y/\partial x$ is very big. In other
 620 words, the actual gradients $\partial y/\partial x$ are much exaggerated with respect to the analytical ones,
 621 almost imperceptible. Similarly, for the actual velocities v . Foreseeably, the change in streambed
 622 elevation computed with eq. [5] after introducing the analytical results of $\partial y/\partial x$ therein, is only
 623 ~1% of the scour in the database (Figure A1b, in which the analytical transient scour is multiplied
 624 by 70). In the end, apart from the correct sign, the flow unsteadiness explains very little (~1%) of
 625 the transient scour $\partial z/\partial t$ because the change in gradient $\partial y/\partial x$ is almost negligible. Numerical
 626 experiments with Saint-Venant and Exner equations using a bed load predictor over a plane,
 627 erodible bed express similar conclusions concerning the negligibility of transient scour due to
 628 unsteady flow [Medina and Bateman, 2010].

Figure A1. Result of the simulation for the events 2 (in grey) and 5 (in black) as diffusive waves defined and routed theoretically: **a)** y compared with a theoretical sine wave as depth hydrographs; **b)** measured z compared with the analytical solution, the latter multiplied by a factor of 70 just for clarity; **c)** q_t measured ($\alpha = 0.11$) compared with the result from the EH formula; **d)** loop rating curve $y-Q$; **e)** loop rating curve $v-Q$. A constant width of 125 m is assumed in the analytical solution (otherwise expressed as per unit width).

Finally, q_t computed with the EH formula for the analytical wave is compared with the actual bed material load from the database, assuming the ratio of bed material transport to total sediment transport is constant $\alpha = 0.11$ (Figure A1c). On event 2, the actual sediment transport is

markedly larger than the computed one in the rising limb, but the difference vanishes in the falling limb; on event 5 the trend is alike for the first days, yet with a large actual shortage around the peak. Sediment supply and transport capacity (EH) are not equal.

Appendix 3: Gradually-varied flow and transient scour

Let us consider eq. [A14] with the local terms neglected therein. Only those terms above black dotted lines are kept. The actual flow is well portrayed by subsequent steady stages of constant Q (i.e. a quasi-steady flow). The dynamic equation, then, stands for the gradually-varied flow in the form of backwater (M1) and drawdown (M2) surface profiles in subcritical flow. A boundary condition downstream applies. The 2,500 m long contracted reach, assumed prismatic with the gauging station in it, is a long transition because the flow is controlled by friction and slope, apart from the downstream depth control. Moreover, since all changes (either scour or fill) take part of this prismatic geometry, it follows that river bed is assumed straight and parallel to the initial bed throughout the contracted reach (a single bed slope S_0 thus occurs in space and time).

First, a rigid bed is considered and the downstream condition is the normal depth in the wide (B) region y_{nB} . Water depth in the contracted reach (width b) will approach its own normal depth y_{nb} asymptotically in the upstream direction. Since $b < B$, the difference between normal depths $\delta y_n = y_{nb} - y_{nB}$ is $\delta y_n > 0$ at any Q (δ stands for a spatial difference). Also, at different daily stages of Q separated by ΔQ (Δ stands for 1 day difference), changes in normal depths verify $|\Delta y_{nb}| > |\Delta y_{nB}|$, simply because $b < B$ again. The two surface profiles M1 and M2 allow for an initial difference δz in the streambed elevation between wide and contracted reaches (the sign $\delta z > 0$ means the narrow is lower, Figure A2). A drawdown occurs if $\delta z < \delta y_n$ and a backwater if $\delta z > \delta y_n$. In case of drawdown, flow accelerates ($\partial v / \partial x > 0$) leading to an increase in bed shear stress ($\partial \tau / \partial x > 0$) and therefore in sediment transport capacity ($\partial q_b / \partial x > 0$). The situation is reverse in case of

backwater. According to Exner (see eq. [A8]), channel bed scours in case of a drawdown curve ($\partial z/\partial t < 0$), whereas it is filled in case a backwater curve ($\partial z/\partial t > 0$) occurs.

Figure A2. Water surface profiles and bed profiles in a long transition on an erodible bed, following *Straub* [1935] and *Laursen* [1958]. **b** Plan view: the gauging station is located 500 m downstream of the inlet and 2,000 m upstream of the outlet, $b = 160$ m, and $B \approx 500$ m. **a** Side view: general scour δz is smaller than $\delta y_n = y_{nb} - y_{nB}$ (dark grey dotted lines), larger than δy_n (light grey dashed lines), or equal than δy_n (black full lines). Changes at the transitions (inlet and outlet) are omitted in the drawing. Although following *Lane's balance analogy* (Martín-Vide, 2006) there is a slope reduction in the contracted reach, S_0 occurs throughout space and time and uniform flow is achieved in the wide and narrow sections if $\delta z = \delta y_n$, as stated in the text.

The erodible bed behaviour can be approached on the same lines. Whenever the surface profile prevails as a drawdown curve in a stage of Q , river bed in the contracted reach will be scoured so the difference δz increases and approaches δy_n . The opposite response occurs when a

backwater curve prevails: channel bed will be filled, decreasing δz to approach δy_n . Then, the quasi-steady flow is approached similarly. The drawdown curve is bound to prevail if discharge rises, $\Delta Q > 0$, because δy_n increases (i.e. $\Delta y_{nb} > \Delta y_{nB}$). Conversely, the backwater curve is bound to prevail if discharge falls $\Delta Q < 0$ because δy_n decreases (i.e. $|\Delta y_{nb}| > |\Delta y_{nB}|$). To sum up, scour and fill are expected during the rising and falling limbs, respectively so the analogy to an accordion is depicted. If $\Delta Q = 0$ river bed is bound to approach an equilibrium condition with $\delta z = \delta y_n$ and uniform flow prevails throughout (the flow depth is y_{nb} in the narrow reach and y_{nB} in the wide).

In the quasi-steady approach of daily changes ΔQ , actual velocities are higher than uniform velocities and $|\partial y / \partial x| > S_0$ in case of a drawdown curve, which occurs if $\Delta Q > 0$. If the water surface profile is a backwater curve, occurring when $\Delta Q < 0$, the situation is reverse, i.e. actual velocities are lower than uniform velocities and $|\partial y / \partial x| < S_0$. This property is intrinsic of gradually varied flow. Thus, depending on whether δz is bigger or smaller than δy_n , the scour or fill produces a change in $\partial y / \partial x$ and v so that it feeds itself back into the transient scour, which, according to Equation [5], is proportional to the product $v \cdot \partial y / \partial x$. In other words, the transient scour is “self-sustaining”, i.e. the exaggeration of v and $\partial y / \partial x$ observed in the real hysteresis loops can be explained by transient scour itself and vice versa in a sort of feedback.

Appendix 4: Derivation of Vanoni equation from gradually-varied flow

It is well-known that the gradually-varied flow equation ([A14] above black dots) can also be obtained by the derivation of the Bernoulli equation, i.e. by developing the following derivative: $d/dx (z + y + v^2 / 2g) = S_f$. To reconcile the semi-empirical eq. [7] with the gradually-varied flow, consider the application of the Bernoulli equation to a finite distance L between the narrow and the wide regions (Figure A3):

$$z + y + \frac{v^2}{2 \cdot g} = Z + Y + \frac{V^2}{2 \cdot g} + S_f \cdot L \quad [A16]$$

Figure A3. Sketch for application of Bernoulli equation between the narrow and wide reach.

Assume $S_f = S_0$ throughout, set water conservation as $byv = BYV$ and combine it with $b \cdot y^{1.5} = B \cdot Y^{1.5}$ [Vanoni, 1975] $k = 1.5$, to get $v^2/y = V^2/Y$ and note that $z - (Z + S_0 \cdot L) = \delta z$, so

$$\begin{aligned} \delta z &= Y - y + \frac{v^2}{2 \cdot g} \cdot \left(\frac{Y}{y} - 1 \right) = \dots = (Y - y) \cdot \left(1 + \frac{\text{Fr}^2}{2} \right) = \dots \\ &= - \left(1 + \frac{\text{Fr}^2}{2} \right) \cdot \left(\left(\frac{B}{b} \right)^{1/1.5} - 1 \right) \cdot Y \end{aligned} \quad [A17]$$

where Fr is the Froude number anywhere. Eq. [A17] is the same as eq. [6] with $k=1.5$ and Y the normal depth in the wide region, except for the factor $(1 + \text{Fr}^2/2)$ very slightly larger than 1. Therefore, eq. [6] can be derived from the same principle used in the physically-based model and the model is consistent with equation [7].

Acknowledgments and Data

The data for this paper is available online as supporting information.

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