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RAPID DRAWDOWN IN SLOPES AND EMBANKMENTS

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ABSTRACT

The rapid drawdown condition arises when submerged slopes experience a rapid reduction of the external water level. Classical procedures developed to determine the flow regime within the slope and the resulting stability conditions are reviewed in the paper. They are grouped in two classes: the "stress-based" undrained approach, recommended for impervious materials, and the flow approach, which is specified for rigid pervious materials (typically a granular soil).

Field conditions often depart significantly from these simplified cases and involve materials of different permeability and compressibility arranged in a complex geometry. The drawdown problem is presented in the paper as a fully coupled flow-deformation problem for saturated/unsaturated conditions. Some fundamental concepts are first discussed in a qualitative manner and, later, explored in more detail in synthetic examples, solved under different hypothesis, including the classical approaches. Some design rules, which include a few fundamental parameters for the drawdown problem have also been solved in a rigorous manner to illustrate the limitations of simplified procedures.

A significant portion of the paper is devoted to the discussion of a comprehensive case history. In Shira earth dam pore pressures were recorded at different points inside the embankment during a controlled drawdown. Predictions of four calculation procedures (instantaneous drawdown, pure flow, coupled flow-elastic and coupled flow-elastoplastic, all of them for saturated/unsaturated conditions) are compared with measured pressure records. Only the coupled analysis provides a consistent and reasonable solution.

The role of the different soil properties in explaining the phenomena taking place during drawdown is finally discussed.

KEYWORDS: Drawdown, hydromechanical coupled analysis, pore pressure, suction, earth dams, numerical modelling,

NOTATION LIST

c'	effective cohesion, MPa
$\mathbf{D}$	stiffness tensor, MPa
e	void ratio, -
E	Young modulus, MPa
f^w	external supply of water, kg/m^{2}s
$\mathbf{h}$	constitutive vector for changes in suction, MPa
H	height of reservoir, m
H_D	drawdown drop, m
$\mathbf{j}_s$	flux of solid, kg/m^{2}s
$\mathbf{j}_w$	flux of water, kg/m^{2}s
k	permeability, m/s
K_0	coefficient of earth pressure at rest, MPa
k_{rel}	relative permeability
K_s	bulk modulus of the solid particles, MPa
k_s	parameter that controls the increase in cohesion with suction (BBM), -
k_{sat}	saturated permeability, m/s
K_w	bulk modulus of water, MPa
M	slope of critical state strength line, BBM model, -
M_{dry}	slope of critical state strength envelope for dry conditions (Rockfill model), -
M_{sat}	slope of critical state strength envelope for saturated conditions (Rockfill model), -
n	porosity, -

n_e	effective porosity, -
p	net mean stress, MPa
p'	mean effective stress, MPa
p_a	air pressure, MPa
p^c	reference stress, BBM model, MPa
p_0	parameter of water retention curve of Van Genuchten Model, MPa
p_0^*	initial mean yield stress, MPa
P_l	liquid pressure, MPa
p_w	pore water pressure, MPa
p_y	threshold yield mean stress for the onset of clastic phenomena (Rockfill model), MPa
$\mathbf{q}_w$	Darcy flux, m/s
r	parameter that establishes the minimum value of the compressibility coefficient for high values of suction (BBM), -
s	suction, MPa
s_0	initial suction, MPa
S_w	degree of saturation, -
$S_{w\,max}$	maximum degree of saturation, -
$S_{w\,min}$	minimum degree of saturation, -
t	time, days
t_{DD}	time of drawdown, days
$\mathbf{u}$	solid displacement, m
v	velocity of drawdown, m/day
α	parameter that defines the non-associativeness of plastic potential, , -
α_s	parameter to describe the rate of change of clastic compressibility with total suction (Rockfill model) -
β	parameter that controls the rate of increase in stiffness with suction, MPa^{-1}

ΔH_D	lowering of the seepage line at the interface between the dam core and upstream shell, m
$\Delta\sigma_V$	changes in vertical net stress , MPa
ε_v	volumetric strain, -
ϕ	porosity, -
ϕ'	effective frictional angle, º
γ_w	water specific weight, MN/m^3
λ	parameter of water retention curve, Van Genuchten Model, -
$\lambda(0)-\kappa$	virgin plastic compressibility for saturated conditions, -
$\lambda^i-\kappa$	virgin plastic instantaneous compressibility (Rockfill model), -
λ_0^d	virgin elastic compressibility for saturated conditions (Rockfill model), -
ν	Poisson's ratio, -
ρ_s	solid density, kg/m^3
ρ_w	water density, kg/m^3
$\boldsymbol{\sigma}$	net stress tensor, MPa
$\boldsymbol{\sigma}'$	effective stress tensor, MPa
$\sigma_x,\sigma_y,\sigma_z$	stresses in direction x, y and z

1 INTRODUCTION

The drawdown condition is a classical scenario in slope stability, which arises when totally or partially submerged slopes experience a reduction of the external water level. This is a common situation in riverbanks, subjected to changing river levels. Flooding conditions are critical in this case because river levels reach peak values and the velocity of decreasing water level tends to reach maximum values also.

Operation of dams requires changes in water level, which modify the safety factor against sliding of the upstream slope of earth dams. When the reservoir level is high, hydrostatic pressures help to stabilize the slope. A reduction of water level has two effects: a reduction of the stabilizing external hydrostatic pressure and a modification of the internal pore water pressures. The second effect has traditionally received considerable attention in dam design because it may lead to critical conditions of the slope. The subject has been approached from different perspectives, which have been largely dictated by current advances in Soil Mechanics.

Sherard et al. (1963) discuss the practical implications of rapid drawdown and a number of case histories associated with total or partial failure of the upstream slope. ICOLD (1980) and Lawrence Von Thun (1985) provide further information on drawdown-induced failures.

Current approaches to analyze drawdown are classified into two different groups: Flow methods, which should be applied in relatively pervious slopes and undrained methods, which find applications in impervious soil slopes. Methods from the first group concentrate on the solution of the flow problem in a situation that involves changes in boundary conditions and a modification of the initial free surface. These methods implicitly assume that the soil skeleton is rigid and therefore they do not consider any modification of the initial water pressure because of the change in total boundary stresses imposed by the drawdown. Methods developed to handle this problem include flow net analysis (Reinius 1954, Cedergren 1967); methods based on ad-hoc hypothesis (typically Dupuit-type of assumptions) (Brahma & Harr, 1962, Stephenson 1978); finite element analysis of flow in saturated soil (Desai, 1972, 1977, Cividini & Gioda 1984) and finite element analysis for saturated-unsaturated flow (Neumann, 1973, Hromadka & Guymon, 1980, Pauls et al, 1999).

The second group considers only the instantaneous change in pore pressure induced by an instantaneous drawdown. This is the undrained case in which flow is not considered. Key early references for this approach are Skempton (1954), Bishop (1954) and Morgenstern (1963) and more recent work has been published by Lowe & Karafiath (1980), Baker et al. (1993) and Lane & Griffiths (2000). In a recent contribution, Berilgen (2007) uses two commercial programs for transient/flow and deformation analysis respectively and reports a sensitivity analysis involving simple slope geometry.

In dam engineering practice neither one of the two mentioned approaches can reliably approximate the field situation because compacted soils are far from being rigid and pure undrained conditions, even in the case of fairly impervious soils, are too conservative for common drawdown rates, which fall in the range 0.1 to 1 m/day.

In this paper the term "coupled" analysis refers to the joint consideration of flow and stress deformation analysis. In the general formulation applied in this paper balance equations of fluid and gas and the equilibrium equations are solved simultaneously. However, when only flow problem is solved the mechanical equations are not considered. We refer to this case as the "uncoupled" analysis. The soil is now assumed rigid. The undrained case is solved by means of the fully coupled formulation.

As an introduction to the remaining of the paper, consider, in qualitative terms, the nature of the drawdown problem in connection with Figure 1a, b.

The position of the water level MO (height H) provides the initial conditions of the slope CBO. Pore water pressures in the slope are positive below a zero pressure line ($p_w = 0$). Above this line, pore water pressures are negative and suction is defined as $s = - p_w$. A drawdown of intensity H_D takes the free water to a new level M' N' O' during a time interval t_{DD}. This change in level implies:

– A change in total stress conditions against the slope. Initial hydrostatic stresses (OAB against the slope surface; M N B C against the horizontal lower surface) change to O' A' B and M' N' B C. The stress difference is plotted in Figure 1b. The slope OB is subjected to a stress relaxation of constant intensity ($\Delta\sigma = H_D \gamma_w$) in the lower part (BO') and a linearly varying stress distribution in its upper part (O'O). The bottom horizontal surface CB experiences a uniform decrease of stress of intensity, $H_D \gamma_w$.

– A change in hydraulic boundary conditions. In its new state, water pressures against the slope are given by the hydrostatic distribution O' A' B on the slope face and by the uniform water pressure value $p_w = (H - H_D)\gamma_w$ on the horizontal lower surface.

The change in boundary total stresses result in a new stress distribution within the slope. This stress change will induce, in general, a change in pore pressure. The sign and intensity of these pore pressures depend on the constitutive (stress-strain) behaviour of the soil skeleton. An elastic soil skeleton will result in a change of pore pressure equal to the change in mean (octahedral) stress. If dilatancy (of positive or negative sign) is present, shear effects will generate additional pore water pressures. Changes in total stress-induced pore pressures are, in fact, simultaneous with the dissipation process due to the new unbalanced hydraulic boundary conditions. A transient flow will establish. In this case it is necessary to apply a fully coupled hydro-mechanical approach to take into account the simultaneous stress and flow phenomena. However, if the soil permeability is large enough, pore pressures may dissipate fast enough so that the effect of stress-induced pore pressures apparently disappears. Otherwise, in a pure “undrained” condition (high-speed of water level changes or very low permeability) changes in pore pressure will be exclusively induced by total stress changes.

It is sometimes stated that in cases of rigid materials the flow-based analysis is sufficiently accurate, implying that no stress-related changes in pore pressures are generated. It is clear that this is never the case in practice since it is required that the effective soil volumetric modulus becomes significantly higher than the water modulus. Only if the "rigid” material happens to be pervious and for a different reason, the stress coupling seems to be absent.

Consider three representative points (P_1, P_2 and P_3) of the slope sketched in Figure 1 and their expected evolution of pore pressures in qualitative terms in Figure 2a, b and c. A given time, t_{DD}, in the t axis marks the end of the drawdown operation.

A Point P_1, close to the upper part of the slope, will experience a limited change in stress due to the unloading represented in Figure 1. Therefore, no major differences should be found when comparing coupled or uncoupled analysis, even if the soil is impervious. In a pervious case, it has already been argued, no differences in practice will be found. The upper points in the slope may develop negative pore water pressures (suction).

At the other extreme of the slope, Point P_3, the slope face BO is far away. Because of the one-dimensional nature of this situation, it is well-known that pore pressures in the soil, at any depth, will follow the changing water level. However, in order to reproduce this elementary result with a computational tool, it is necessary to use a fully coupled hydro-mechanical approach or an "undrained" analysis. Otherwise, a change in water level will trigger a transient flow condition because no information on the instantaneous change in pore water pressure is available in an uncoupled model.

Predicting the behaviour of Point P_2, near the toe of the slope is more difficult. Mean and shear stresses are high and they experience significant gradients. New pore pressures generated after unloading are far from being in equilibrium among them and with respect to the new hydraulic head imposed at the boundary. In fact, in a fully coupled approach, the transient process of pore pressure dissipation has several origins. They are: the rate of water lowering (this is a boundary condition), the heterogeneous distribution of "instantaneous" pore water pressures after drawdown and the "source" or "storage" terms provided by both, the changing saturation in some parts of the domain and the deformation of the soil skeleton. Figure 2b shows that the response of Point P_2 in a coupled analysis will depend on the permeability of the soil. The problem has, however, an additional difficulty because soil stiffness, which controls the storage term associated with changes in effective stress, will also dictate the rate of the process.

Difficulties for the development of consistent, fully coupled hydro-mechanical codes for saturated/unsaturated soils, hampered by the issue of the effective stress principle and the development of consistent constitutive equations for unsaturated conditions, have probably prevented a more advanced and realistic analysis of the classical drawdown problem. This paper relies on one of the existing complete formulations in this regard. The solved cases use the finite element program CODE_BRIGHT (DIT-UPC, 2002) developed at the Department of Geotechnical Engineering and Geosciences of UPC. The code solves in a fully coupled manner thermal, mechanical and flow (air and water) problems in porous media. It may handle a variety of mechanical constitutive laws but the results presented here correspond either to elastic conditions or to elastoplastic constitutive models (BBM; Alonso et al., 1990; Rockfill model; Oldecop and Alonso, 2001). These types of models go beyond previous known attempts to analyse drawdown effects. Some relevant aspects of the formulation used in CODE_BRIGHT are briefly described in the Appendix.

Some of the qualitative descriptions offered above will be made more precise by solving the drawdown problem in a simple slope. The cases of instantaneous and progressive drawdown will be compared. Then a review of some existing rules to estimate drawdown effects on slopes will be performed. Despite the long list of developments and publications associated with drawdown analysis, almost no comparison between field measurements and calculations exists. For this reason, it was appropriate to perform an analysis of an interesting published field case (the response of Glenn Shira dam against a very rapid drawdown). Model results and measurements will be compared.

2 DRAWDOWN IN A SIMPLE SLOPE

The geometry of the slope analyzed is given in Figure 3. Some calculated results are given below for points P_A and P_B (mid-slope and slope toe respectively). Critical failures surfaces obtained in drawdown stability analysis are typically close to these two points. Two cases are considered, either an instantaneous drawdown or a drawdown at a rate often found in dam engineering applications: 0.5 m/day.

An elastic constitutive law will characterize the soil. The retention curve has been defined by means of a Van Genuchten model and the relative permeability varies with the degree of saturation following a cubic law ($k_{rel} = k_{sat} S_w^{\ 3}$). A constant saturated permeability k_{sat} = 10^{-10} m/s was also used in all calculations. This is a low value, typical of an impervious material in engineering applications.

2.1 Instantaneous drawdown

The initial water level in the slope is horizontal and it is located at the maximum level. The initial pore pressures in the soil follow a hydrostatic pattern. Drawdown is then simulated by removing instantaneously all the water in the reservoir. The water level (p_w = 0) is maintained at the level of the toe of the slope. The inclined slope surface is provided with a "seepage" hydraulic condition, which allows water flow when water pressure reaches a positive value. All the remaining boundary surfaces remain impervious.

Changes in pore water pressure developed immediately after the drawdown will be exclusively due to total stress changes. Therefore, if an uncoupled analysis is run, the pore water pressures inside the slope will maintain their initial values immediately after drawdown. In a coupled analysis, the magnitude of pore pressure changes depends on the stress – strain behaviour of the soil skeleton. In the analysis presented here several elastic soil moduli are considered (E = 10000 MPa, 1000 MPa and 100 MPa). The first case corresponds to a stiff material (a soft clayey rock, for instance). The second case is an upper limit for a very rigid compacted and low porosity material. The third case is a reasonable assumption for a well-compacted well-graded soil.

Figure 4 shows the calculated evolution of pore pressure after the instantaneous drawdown in point P_B (Figure 3). At day 1, instantaneous drawdown is simulated. In the case of uncoupled analysis, no immediate effect of the drawdown is obtained, as expected. In the coupled analysis, the instantaneous pore pressure drop depends on the compressibility of the soil skeleton.

The stiffer the soil, the more limited the stress-induced change in pore water pressure. Immediately after drawdown a dissipation process begins. The rate of pore pressure dissipation is controlled by the initial conditions after drawdown but also by the permeability and stiffness of the soil. In an uncoupled analysis the calculated dissipation rates are higher, because the implicit assumption is an infinitely rigid soil. Eventually, all cases result in the same long-term solution.

The coupled analysis leads systematically to lower water pressures than the uncoupled (pure flow) approximation during the first stages of the dissipation. This is due to the effect of the initial state after drawdown, controlled by the change in stress. However, since pressures dissipate faster the stiffer the soil, this situation changes after some time and the water pressure records may cross at some particular time, which depends on the position of the considered point in the slope. Note also that full steady state conditions were not reached at the end of the simulation period.

Figures 5 shows the pore pressure distribution along the vertical profile through P_B immediately after the instantaneous drawdown. The toe of the slope has a more complex stress distribution and this is reflected in a more irregular distribution of pore pressures after drawdown, especially in points close to the slope boundary.

For a compacted soil, typical of earth dam materials (E = 100 MPa), the uncoupled, pure flow analysis provides an extremely unrealistic answer.

2.2 Progressive drawdown (*v* = 0.5 m/day)

Conditions of the analysis remain unchanged except for the drawdown rate. Figure 6 provides the calculated pore pressures in point P_A. The pure flow analysis leads to high pore pressures, if compared with the more accurate coupled case, during a first stage. Later, the higher rate of pore pressure dissipation implied by the flow model leads to pressures lower than the calculated values for the coupled case. In the latter case the effect of soil stiffness can be seen although it has a relatively minor influence for the range of moduli considered. If the soil permeability is increased, the differences between coupled and uncoupled analysis reduce and eventually they provide the same answer because the high dissipation rates mask the stress-induced response of pore pressure change. However, it is by no means easy to decide "a priori" which is the threshold permeability which justifies the use of an uncoupled analysis.

3 SOME DESIGN RULES REVISITED

Let us consider now the case of relatively pervious materials. Mechanical coupling in these cases is not relevant for the reasons mentioned before and the common recommendation is to base the analysis on the determination of flow nets by means of numerical, analytical or graphical procedures. However, the drawdown implies that an initially saturated soil becomes progressively unsaturated. The distribution of pore water pressures in the slope depends now on some key properties of the unsaturated soil, and, in particular, on the water retention characteristics.

Some approximate procedures have been proposed to estimate the pore pressures in a slope during drawdown. Figure 7 illustrates one case, in connection with the stability analysis of earth dams (US Corps of Engineers, 1970). The idea of the chart is to facilitate a procedure to locate the position of the free surface after drawdown. This is achieved by providing the lowering of the seepage line at the interface between the impervious dam core and the upstream shell (ΔH_D).

This distance is a function of the total drawdown drop (H_D), the soil permeability, k, the velocity of drawdown, v, the effective – or "drained"- porosity, n_e, and the slope geometry, given by the slope angle, β .

With the purpose of showing the effect of correctly modelling the saturated-unsaturated transition, a few cases directly inspired in the geometry and conditions considered in this design plot have been calculated. The cases run correspond to the three points marked in Figure 7 for the slope angle $b = \cot(\beta) = 1.8$. A set of soil properties, matching the conditions of these three points are given in Table 1.

Drawdown velocity was fixed at 0.5 m/day. A common soil porosity n = 0.3 was also selected. Three values of saturated soil permeability, k_{sat} = 5x10^{-8} m/s, k_{sat} = 10^{-6} m/s and k_{sat} = 10^{-4} m/s correspond to a relatively impervious shell (typically a mixture of gravel, sand, silt and some clay), a partially draining material (typically a compacted well graded mixture) and a free draining material (typically a gravely sand). The effective porosity for these three cases is indicated in Table1. The n_e values selected reflect the type of soil associated with the three cases analyzed. The additional soil property, not considered in Figure 7, is the water retention of the soil. The effect of a reasonable variation of this property was investigated. To do so, a Van Genuchten representation of the water retention curve is selected. By changing parameter p_0, associated with the air entry value, different soil retention capabilities are simulated. The second parameter of the retention curve, λ was kept constant at the value given in the table. All the calculations have been performed in a coupled mode, using an elastic soil modulus E = 100 MPa.

Figures 8 to 10 indicate the calculated distribution of water pressures, below the saturation line, for the extreme cases analyzed. These plots provide the possibility of calculating ΔH_D and the range of calculated values has been indicated in the caption of each figure. These values are also plotted on Figure 7.

At first sight, the results in Figures 8-10 a and b may look contrary to expectations, since the height of the phreatic surface decreases when the air entry value increases. This is a result valid for the particular permeability selected when comparing the effect of alternative water retention curves. Therefore, it makes sense only when the range of water retention curves analyzed is limited since all of them should provide essentially the same saturated permeability. The calculated result is better explained if one considers also the distribution of degree of saturation within the slope. In Figure 11, the degree of saturation along a vertical profile in the middle of the slope is represented for the two extreme cases having a common saturated permeability k_{sat} = 10^{-6} m/s (Case 2).

For a given soil permeability, the amount of water to be drained during drawdown is similar for both cases. Above the phreatic line (s > 0), if p_0 is low, even for low suction (close to the value of

p_0) the degree of saturation decreases significantly (this is determined by the retention curve) and the amount of drained water from the unsaturated zone is higher. In the other case (higher p_0), the zone above the phreatic line is almost saturated (Figure 11) although pore water pressures remain negative. Then the phreatic line may reach a lower elevation for the same amount of drained water. Therefore, if p_0 decreases, the phreatic line ($p_w = s = 0$) remains at higher elevation (the saturated zone of the slope is larger).

If only positive pore water pressures are considered in stability calculations, higher p_0 may lead to higher safety factors against slope failure than the case of a lower air entry value. For the particular case of $k_{sat} = 10^{-6}$ m/s again, the safety factor calculated by means of a Morgenstern-Price method against an imposed failure surface through the middle of the slope has been calculated. A Mohr-Coulomb failure criterion (strength parameters ϕ'=28º and c'=0) has been considered. For the case of p_0 = 0.2 MPa, a safety factor equal to 1.35 is obtained. If p_0 is reduced to 0.007 MPa, the calculated safety factor is 1.48. However, this conclusion may change if a more comprehensive description of soil strength, valid for saturated and unsaturated conditions is introduced in the analysis, a subject that is outside the purpose of this paper.

The results obtained have been included in Figure 7. The largest discrepancies with the Manual recommendations are obtained for low values of the index P_D (more impervious materials, always with respect to drawdown velocity). Recommendations are too conservative in these cases. The fact to be stressed is that the set of parameters included in the design procedure implied in Figure 7 is incomplete, even if couplings effects are disregarded. Only in the case of very pervious materials, drawdown predictions of the chart reproduced in Figure 7 seem to be accurate.

4 COUPLED ELASTO-PLASTIC ANALYSIS OF DRAWDOWN

All the coupled analyses reported so far describe the soil by means of an elastic constitutive law. In principle, drawdown leads to a reduction in mean stress. However, the particular geometry of the problem and the nonuniformity of applied boundary stresses may result in significant shearing. In addition, the progressive reduction in pore water pressures implies a parallel increase in effective confining stresses. If yielding conditions are reached, plastic deformations will take place and additional local sources of water will develop. They will modify the pore pressure response of the slope.

In order to show some aspects of the elastoplastic response of the soil during drawdown, the dam geometry analyzed in the previous section was considered again. Dam materials (core and shell) are now simulated by means of elastoplastic models. To facilitate the selection of parameters and to reproduce, as much as possible, a real situation, the mathematical description of the two materials involved were borrowed from previous work by the authors on Beliche dam. The shell material is equivalent to the "inner rockfill" of Beliche whereas the clay core of the example analyzed here reproduces also Beliche's core. The shell will be described by a "rockfill model" presented in Oldecop and Alonso (2001) and Alonso et al (2005). The clay core is described by means of the BBM (Alonso et al, 1990; Appendix). CODE_BRIGHT handles both models. Material parameters were derived from the backanalysis of large-scale laboratory tests and are given in Tables 2 and 3.

The analysis performed reproduces construction, impoundment and drawdown stages. Figure 12 a and b shows the stress-suction path followed by a representative point located inside of the upstream rockfill shoulder. For the simulation of dam construction, the weight of the whole dam is applied, in a single stage, in a ramp manner. A low initial isotropic yield stress, p_0^*, is assumed for the compacted materials. Therefore, the weight load applied induces immediately the yielding of the dam. Plastic deformations will accumulate during the construction stage (step 0-1 in Figure 12 a and b). During construction, suction decreases due to the reduction of porosity (from s = 0.5 MPa, initial value, to the calculated value, s = 0.36 MPa).

During the impoundment step 1-2, total stresses and pore pressures change. Because of saturation a compressive strain (collapse) develops in the rockfill and additional irreversible volumetric deformation are accumulated. The final size of the yield envelope is determined by the isotropic yield stress reached at zero suction. Path 1-2 essentially implies an elastic unloading in the deviatoric plane. Mean and deviatoric stresses reduce simultaneously, following a path parallel to the initial construction path. Water pressures change from negative values (soil under suction) to positive ones. Note also that the strength parameter (M) is not constant during this path. In fact strength envelopes in the rockfill model depend on the current suction and they are defined in terms of two extreme values (M_{dry} and M_{sat}) given in Table 2.

During drawdown (at a velocity of 0.5 m/s) point P experiences a sudden reversal in its stress path (Figure 12). Both mean and the deviatoric stresses increase again simultaneously and follow a path parallel to the initial construction path (Figure 12a). The shape of this path depends on the permeability and the compressibility of the material. In the case represented in Figure 12 a and b, when the end of the drawdown is close (Point 3), the current yield surface is reached and new plastic deformations take place. The plastic reduction of the porosity will release some water, which will be dissipated at the expense of an increase in pore water pressure. In the case analyzed this is a minor effect because yielding at the final drawdown stage is very limited. The next discussion on a case record (Shira dam) will provide additional insight into these phenomena.

5 GLEN SHIRA DAM CASE HISTORY

Glen Shira Lower Dam is part of a pumping storage scheme in Northern Scotland. The reservoir was expected to experience fast drawdown rates and this situation prompted the field experience reported by Paton and Semple (1961). Probably this is one of the best-documented case histories concerning the effect of drawdown on earth dams. The maximum cross section of the dam is presented in Figure 13. The 16 m high embankment has a centered thin reinforced concrete wall. The homogeneous embankment is made of compacted moraine soil. A rockfill shell covers the upstream slope of the compacted moraine to increase stability. Published grain size distributions of the moraine soil indicate a well-graded material having a maximum size of 15 cm. Plasticity is not reported for this soil. It was apparently compacted wet of optimum at an average water content w =

15%. The attained average dry density was 19.8 kN/m^3, which is a relatively high value for a granular mixture. A friction angle $\phi' = 36^{\circ}$ is reported.

For the rockfill a porosity of $n = 0.4$, a dry density of 16.7 kN/ m^3 and a friction angle $\phi' = 45^{\circ}$ are mentioned in the paper.

Five porous stone piezometer disks, previously calibrated against mercury columns, were located in the places shown in Figure 13. They were connected to Bourdon gauges through thin polyethylene tubing. The authors conclude in their paper that the possibility of instrumental error are *"of minor order and can be neglected"*.

No significant pore water pressures were recorded during construction. Positive pore pressures were measured only after reservoir filling

A total water level drawdown of 9.1 meters in four days was applied to Glen Shira dam. This maximum drawdown was imposed in four stages of rapid (7.2 m/day) water lowering followed by short periods of constant water level. Details of changing water level in the reservoir and the measured pore water pressures are indicated in the set of figures prepared to analyze this case.

Measured pore pressures have been compared with calculated values in Figures 14 to 18. The following hypotheses, ordered in the sense of increasing complexity, were made to perform calculations:

1. A pure flow analysis for saturated/unsaturated conditions that follows the changing hydraulic boundary conditions actually applied to the upstream slope. Table 5 provides the hydraulic parameters used in calculations. These parameters are common to the remaining analyses described below.

2. An instantaneous drawdown of the maximum intensity, followed by pore water pressure dissipation. This is a coupled analysis, which attempts to reproduce the classical hypothesis behind the undrained methods, briefly described in the introduction of the paper. The procedure does not correspond strictly to Bishop's

method because in the analyses reported here the correct change in total stresses is actually applied. The soil was simulated as an elastic material. (Properties are given below, in Table 4).

3. A coupled analysis (saturated/unsaturated), following the applied upstream changes in hydrostatic pore pressures. The soil is considered elastic (properties are given in Table 4).
4. A coupled analysis (saturated/unsaturated) following the applied upstream changes in hydrostatic pore pressures. The soil is considered elasto-plastic following the BBM model, Alonso et al (1990) (properties are given in Table 4). The elastic parameters of this model are taken from the previous elastic model.

The case of Shira dam is especially interesting because the permeability of the compacted moraine fill (around 10^{-8} m/s; see below) is an intermediate value between impervious clay and a free draining material. One may wonder to what extent the classical hypothesis for drawdown analysis (undrained or pure flow) approximates the actual behaviour. This aspect will be discussed later.

The following ideas have guided the selection of parameters. The elastic (unloading-reloading) elastic moduli of compacted moraine and rockfill are typical of a stiff soil. In fact, well graded granular mixtures become rather stiff when compacted. The virgin compressibility, $\lambda(0)-\kappa$, is approximately one order of magnitude higher than the elastic compressibility. Parameters r and β controls the shape of the yield LC curve of BBM. The moraine soil is assumed to gain limited stiffness as suction increases (parameter r). Also, the increase in stiffness with suction is fast for relatively low values of suction and remains fairly constant thereafter (parameter β). The slope of the critical state strength line reflects the friction angles provided in the paper. Zero cohesion is assumed throughout the analysis, irrespective of suction (parameter k_s). A small reference stress (p^c) is assumed. Associated flow conditions were assumed in both materials (parameter α =1). Rockfill properties were assumed to be similar to the compacted moraine, except for the higher friction angle.

The dam was built in a single step. A more detailed representation of dam construction plays a minor role in the analysis of drawdown. The following “as compacted” initial suction and saturated yield stress were imposed: $s_0 = 0.01$ MPa and $p_o^* = 0.01$ MPa. Given the low value of p_o^*, which reflects the isotropic yield state after compaction, dam conditions at the end of construction correspond to a normally consolidated state. The dam was then impounded until steady state conditions were reached. The presence of the impervious concrete membrane results in a simple initial state: all points upstream of the concrete wall maintain hydrostatic water pressure conditions. This initial state correspond to day 5 in the plots presented later.

The information given in the original paper provided data to approximate hydraulic parameters. Two saturated values of permeability are mentioned for compacted specimens in the laboratory (1.6 x 10^{-8} m/s, when compacted at optimum water content and 1.6 x 10^{-7} m/s when compacted wet of optimum). However, the dry densities reached in the field (19.8 kN/m^3) are higher than the optimum laboratory B.S. compaction (19.3 kN/m^3) and this leads to a reduction in permeability. A saturated permeability value $k_{sat} = 1.6 \times 10^{-8}$ m/s was therefore selected for field conditions.

Water retention properties for the moraine were derived following a simplified procedure, which makes use of the grain size distribution. Since the moraine soil is a granular material, capillary effects will dominate the water retention properties. On the other hand, pore size distributions may be approximated if grain size distributions are known. An example is given, for a beach sand, in Alonso and Romero (2003). The idea is that the pore size distribution follows the shape of the grain size distribution. However, the pore diameter is a fraction of the equivalent grain size. In the sand reported by Alonso and Romero (2003) this fraction is approximately 0.25. It is probably lower in a well-graded material although this ratio was accepted to derive the pore size distribution from the known average value of the grading curve for the moraine soil. The next step is to use Laplace equation to derive the suction emptying a given pore size. This leads immediately to the water retention curve. The Van Genuchten expression fitted to the derived water retention curve corresponds to parameters (see also Table 5): $p_0 = 0.05$ MPa and $\lambda = 0.2$. The rockfill retention curve was approximated with a significantly lower air entry value (lower p_0) and an increased facility to desaturate (higher λ) when suction is applied. Finally, a cubic law, in terms of the degree of saturation, defined the relative permeability.

The known history of the final stages of reservoir filling and drawdown history of the reservoir levels is indicated in Figures 14 to 18.

These five figures include a comparison between the calculated evolution of pore pressures and the corresponding measurements of the five piezometers. The analysis corresponds to case 3 of the list of four cases described above: a coupled flow-elastic deformation for saturated/unsaturated conditions. The agreement is satisfactory. The pattern of recorded pore pressures and the smoothing effect introduced by the soil stiffness and permeability (specially noteworthy in piezometers 1 and 2) are well captured by the model. A better agreement between measurements and calculations probably requires the consideration of certain field heterogeneity in permeability and/or soil stiffness.

Paton and Semple (1961) plotted also contours of piezometric head during drawdown. Two examples are given in Figures 19b and 20b. They correspond to drawdown drops of 4.85 and 8.8 m. The reservoir level reaches 9.15 and 5.2 m respectively (with respect to the zero reference level which in this paper is placed at the dam base: point 0 in Figures 19 and 20). The authors used the data recorded on the five piezometers to interpolate the curves shown in the figure. They made the hypothesis of a zero water pressure at the shell-rockfill interphase. The computed distribution of heads inside the dam shell, for the same amount of drawdown, is also plotted in Figures 19a and 20a. The agreement is quite acceptable, although some discrepancies exist, which, in part could be attributed to the limited accuracy of the interpolation made.

There was also an interest in comparing the performance of the different methods of analysis (1. to 4.) listed above. Figures 21 to 25 illustrate this comparison. Consider first the hypothesis of instantaneous drawdown (9.5 m of water level drawdown, instantaneously). The calculated pressure drop is indicated in the figures by means of a vertical bar. A (coupled) dissipation process is then calculated and the progressive decay in pore pressures is also plotted. If compared with the actual pore pressures measured at the end of the real drawdown period, the hypothesis of instantaneous drawdown leads obviously to an extremely pessimistic and unrealistic situation. (The end point of the instantaneous drawdown at t = 9 days is to be compared with the pore pressure recorded at the end of the drawdown period at t = 12.4 days).

It is also interesting to compare the results of the fully coupled analysis of the instantaneous drawdown with the approximated method of analysis suggested by Skempton/Bishop. Table 6 shows the comparison. The change in vertical stress ($\Delta\sigma_V$) has two contributions: the change in free water elevation above a given point and the decrease in total specific weight of the rockfill material covering the moraine shell. An effective saturated porosity of 0.3, after drainage, was assumed to calculate the drop in total specific weight. Bishop hypothesis leads systematically to a higher pore pressure drop than the more accurate analysis. This is specially the case for the piezometers located deep inside the fill. Discrepancies are due to the simplified stress distribution assumed in the approximate method.

Consider now the opposite calculation method: a pure flow analysis. In this case, Figures 21 to 25 indicate that the predicted pore pressures are the lowest ones if compared with the remaining methods of analysis. Calculated water pressures follow closely the history of reservoir levels. The "damping" effect associated with soil compressibility is absent. When the water level is increased, at the end of the drawdown test, the pure flow analysis indicates, against the observed behaviour, a fast recovery of pore pressures within the embankment.

Coupled analyses are closer to actual measurements. This is true in absolute terms but also in the trends observed when boundary conditions (changes in reservoir level) are modified.

Construction of Shira Dam leaves most of the embankment under normally consolidated conditions. This is a consequence of the low initial yield stress, p_o^*, adopted in the analysis. p_o^* is related to the energy of compaction, but a detailed discussion of this topic is outside the limits of this paper. Granular materials, and certainly rockfill, tend to yield under low stresses after compaction. Therefore, the accumulation of layers over a given point will induce plastic straining. The stress paths in points relatively away from the slope surfaces follow K_0 – type of conditions. Figure 26 a and b indicates the stress path of points located in the position of Piezometers 1 and 3, respectively. Plotted in the figure are also the yield surfaces at the end of construction. The maximum size of the yield surface corresponds to these construction stages. Once the dam is completed, reservoir impoundment leads to a reversal of the stress path, which enters into the elastic zone. Drawdown leads to a new sharp reversal in the stress path and the increase in deviatoric stresses. However, the end of the drawdown path remains inside the elastic locus in the two cases represented in Figure 26 a and b. The possibility of inducing additional plastic straining during drawdown depends on the geometry of the dam cross section and on the constitutive behaviour of the materials involved. Shira dam has a stable geometry because of the low upstream slope (3 to 1) and shear stresses inside the dam are relatively small. In addition, the granular shell material has a high friction angle (35º). However, under different circumstances, plastic straining may develop during drawdown, and, in this case, pore pressures will probably increase because the yield point, located in the “wet” (compression) side of the yield locus (see Figure 26 a and b) implies that additional local sources of local are available for dissipation. Note also the differences in calculated stress paths for piezometers 1 and 3 during drawdown. Piezometer 3 is located deep inside the embankment, at a high elevation and therefore pore pressure changes are small: the effective mean stress remains constant and the stress path moves vertically upwards. However, the change in deviatoric stresses is also small and the final stress point is far from reaching critical state conditions. Piezometer 3, on the contrary, is close to the upstream shell, at a lower elevation. Changes in pore pressure and deviatoric stress are large in this position and the stress path moves approximately parallel to the initial construction path and approaches yielding conditions in compression.

There is, however, an additional effect, which leads to a different drawdown behaviour when comparing elastic and elastoplastic modelling approaches. If permeability is made dependent on void ratio, the construction of the dam will lead to lower values of permeability (distributed in a heterogeneous manner). If the dam compacted material yields during construction, plastic volumetric compaction will add to the elastic strains. In addition, collapse phenomena upon impounding will reduce further the porosity. These effect has been also explored in the case of Shira dam. Permeability was made dependent on void ratio, *e*, following a Kozeny type of relationship (permeability depends on $e^3/(1+e)$). The calculated records of pore pressure evolution during drawdown are also shown in Figures 21 to 25. The reduction in permeability, if compared with the coupled elastic case, leads to a systematic increase in pore pressures. The agreement with measurements is now better in some piezometers (1, 3 and 4).

The conclusion, for the particular embankment material of Shira dam and its overall geometry and design, is that the classical methods of analysis are far from explaining the recorded behaviour. The "instantaneous" or undrained method is conservative, but very unrealistic. A fully coupled analysis of the instantaneous drawdown results in higher pressure drops than the classical Bishop proposal. At the opposite extreme, the pure flow analysis leads to a systematic and unsafe underestimation of fill pressures during drawdown. Coupled analysis captures well the actual measurements. In the case of Shira dam, plastification during drawdown was probably nonexistent, and the simpler elastic approach provides a good approximation to recorded pore water pressures. However, the full elastoplastic simulation offers a better understanding of the phenomena taking place during construction and impounding. This is shown in the stress paths calculated, in the occurrence of yielding during construction, and in the effect of permeability reduction on the drawdown response.

6 CONCLUSIONS

Pore water pressures in an initially submerged slope and later subjected to drawdown depend on several soil parameters and "external" conditions: soil permeability (saturated and unsaturated), soil water retention properties, mechanical soil constitutive behaviour, rate of water level lowering and boundary conditions. The paper stresses that a proper consideration of these aspects is only possible if a fully coupled flow – mechanical analysis, valid for saturated and unsaturated conditions is employed. A review of the literature on the subject reveals that the published procedures refer usually to limiting cases (impervious or rigid materials), which prevent often its use in real problems and make it difficult to judge the degree of conservatism -if any- introduced.

Leaving apart for the moment the issue of the transition from saturated to unsaturated conditions which takes place during drawdown, there are two fundamental mechanisms controlling the resulting pore water pressure: the change in pore pressure induced by boundary changes in stress and the new flow regime generated. Both of them require a coupled analysis for a proper interpretation and consistency of results. In particular, pure flow models are unable to consider the initial changes in pore pressure associated with stress unloading. The intensity of pore pressure changes induced by a stress modification is controlled by the soil mechanical constitutive equation. In a simplified situation, under elastic hypothesis for the soil skeleton, the pore pressure depends on the ratio of soil bulk stiffness and water compression modulus. In most situations, this ratio is small and the influence of soil stiffness is negligible. This implies a maximum response of the saturated material to stress changes. Without this coupling, the initial pore pressures do not change during fast unloading (as an illustration, pure flow models are unable to detect that all points in the porous media instantaneously feel a change in water level in a large submerged area).

Permeability and soil stiffness controls coupled flow. The uncoupled analysis implicitly assumes a rigid soil and therefore it leads to a maximum dissipation rate. Both effects (the initial change in pore pressure and the subsequent dissipation) should be jointly considered for a better understanding of the evolution of pore pressures. In addition, the rate of change of boundary conditions is a key information to interpret the results. No simple rules can be given to estimate the pore pressures in the slope. This is even more certain if due consideration is given to the unsaturated flow regime. In this regard, some design rules for earth dam stability calculations, which provide the position of the phreatic surface in relatively "free draining" materials, have been reviewed with the help of the fully coupled, complete formulation used in this paper. An interesting result is that, other parameters of the problem being equal, the average height of the phreatic line increases as the air entry value of the water retention curve decreases. This is a paradoxical result at first sight, but it may be explained if one considers the amount of drained water induced by the drawdown. In addition, the position of the phreatic line does not provide enough information to calculate safety factors against slope failure if due consideration is given to the strength for positive suctions, above the zero-suction surface. Therefore, methods for drawdown analysis, which concentrate on the determination of the position of the phreatic line, using formulations for saturated flow, may lead to significant errors. The evaluation of the design chart for dams subjected to drawdown, performed in one of the sections of this paper, is a good example.

The elastoplastic analysis performed on a synthetic example (an earth and rockfill dam whose parameters correspond to a real case: Beliche dam) has provided additional information on the stress paths that develop inside the dam. Points inside the embankment, except for shallow positions, follow a K_0 – type of stress path during construction. Impounding and drawdown imply strong stress reversals. Drawdown, in particular, is characterized by a parallel increase in effective mean stress and deviatoric stress. Yield conditions may be approached although it is believed that the drawdown paths tend to remain in the elastic domain.

A well documented case history (Shira dam) was analyzed to provide further insight into the drawdown problem. The case is very interesting because the soil involved (a compacted moraine) has an intermediate permeability between impervious clays and free draining granular materials. It should be added that materials with this intermediate permeability are very common in dam engineering. Therefore, the two classical procedures to analyze drawdown effects (undrained analysis for clays and pure flow for granular materials) will meet difficulties. In fact, these two methods proved quite unrealistic when compared with actual records of pore water pressures in different points of the dam. In particular, the pure flow (uncoupled) analysis leads to faster dissipation of pore pressures and this is an unsafe result in terms of stability calculations. The fully coupled analysis (elastic or elastoplastic) provides consistent results.

The elastoplastic analysis allows a proper consideration of the entire history of dam construction, impoundment and drawdown. Since embankment dams experience significant yielding during construction, this is an important consideration. It has been shown also that the reduction in permeability associated with material volumetric compression has a significant effect on the subsequent drawdown behaviour: it leads to higher pore water pressures being maintained inside the slope.

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APPENDIX A. COUPLED FLOW-DEFORMATION FORMULATION FOR SATURATED/ UNSATURATED CONDITIONS

This appendix summarizes the balance equations required for coupled flow-deformation for saturated and unsaturated conditions.

In what follows, it will be considered that the state variables (unknowns) are: solid displacements, ***u*** (three spatial directions) and liquid pressure, P_l. Balance of momentum for the medium as a whole is reduced to the equation of stress equilibrium together with a mechanical constitutive model which relates stresses with strains. Strains are defined in terms of displacements. Small strains and small strain rates for solid deformation are assumed. Advective terms due to solid displacement are neglected once the formulation is written in terms of material derivatives (in fact, material derivatives are approximated as eulerian time derivatives). In this way, volumetric strain is properly considered.

The governing equations for non-isothermal multiphase flow of water and gas through porous deformable saline media have been presented by Olivella et al. (1994). A detailed derivation is given there, and only a description of the reduced formulation for hydro-mechanical problems is presented in this appendix.

Mass balance of solid present in the medium is written as:

$$\frac{\partial}{\partial t}\left(\rho_s\left(1-\phi\right)\right)+\nabla\cdot\left(\mathbf{j}_s\right)=0 \tag{A1}$$

where ρ_s is the mass of solid per unit volume of solid and $\mathbf{j}_s$ is the flux of solid. From this equation, an expression for porosity variation can be obtained if the flux of solid is written as the velocity of the solid multiplied by volumetric fraction occupied by the solid phase and the density, i.e. $\mathbf{j}_s=\rho_s\left(1-\phi\right)\frac{d\mathbf{u}}{dt}$:

$$\frac{D_s\phi}{Dt}=\frac{(1-\phi)}{\rho_s}\frac{D_s\rho_s}{Dt}+(1-\phi)\nabla\cdot\frac{d\mathbf{u}}{dt} \tag{A2}$$

The material derivative with respect to the solid is defined as:

$$\frac{D_s(\bullet)}{Dt}=\frac{\partial}{\partial t}+\frac{d\mathbf{u}}{dt}\cdot\nabla(\bullet) \tag{A3}$$

Equation (A2) expresses the variation of porosity caused by volumetric deformation and solid density variation.

In the formulation required for the analyses in this paper, water component and liquid phase are the same. The total mass balance of water is expressed as:

$$\frac{\partial}{\partial t}\left(\rho_w S_w \phi\right)+\nabla\cdot\left(\mathbf{j}_w\right)=f^w \tag{A4}$$

Where S_w is the degree of saturation of water, ρ_w is the water density, $\mathbf{j}_w$ is the flux of water, and f^w is an external supply of water. Water flux is a combination of a Darcy flux and an advection caused by the solid motion:

$$\frac{\partial}{\partial t}\left(\rho_w S_w \phi\right)+\nabla\cdot\left(\rho_w\mathbf{q}_w+\rho_w\phi S_w\frac{d\mathbf{u}}{dt}\right)=f^w \tag{A5}$$

The use of the material derivative leads to:

$$\phi\frac{D_s\left(\rho_w S_w\right)}{Dt}+\rho_w S_w\frac{D_s\phi}{Dt}+\rho_w S_w\phi\nabla\cdot\left(\frac{d\mathbf{u}}{dt}\right)+\nabla\cdot\left(\rho_w\mathbf{q}_w\right)=f^w \tag{A6}$$

The mass balance of solid is introduced in the mass balance of water to obtain, after some algebra:

$$\frac{\phi S_w}{\rho_w}\frac{D_s\rho_w}{Dt}+\phi\frac{D_s S_w}{Dt}+S_w\frac{(1-\phi)}{\rho_s}\frac{D_s\rho_s}{Dt}+S_w\nabla\cdot\left(\frac{d\mathbf{u}}{dt}\right)+\frac{1}{\rho_w}\nabla\cdot\left(\rho_w\mathbf{q}_w\right)=\frac{1}{\rho_w}f^w \tag{A7}$$

This equation has four storage terms, related to:

I. Water compressibility since $\frac{1}{\rho_w}\frac{d\rho_w}{dp_w}=\frac{1}{K_w}$ is the volumetric compressibility of water

II. Retention curve storativity since $\frac{dS_w}{dp_w}$ is obtained from the retention curve.

III. Solid compressibility since $\frac{1}{\rho_s}\frac{d\rho_s}{dp}=\frac{1}{K_s}$ is the compressibility of the soil particles.

IV. Soil skeleton compressibility since the divergence of solid velocity can be transformed into $\nabla\cdot\left(\frac{d\mathbf{u}}{dt}\right)=\frac{d}{dt}\left(\nabla\cdot\mathbf{u}\right)=\frac{d\varepsilon_v}{dt}$, and $\frac{d\varepsilon_v}{dt}=\frac{d\varepsilon_v\left(\boldsymbol{\sigma}',p_w\right)}{dt}=\frac{d\varepsilon_v}{d\boldsymbol{\sigma}}\frac{d\boldsymbol{\sigma}}{dt}+\frac{d\varepsilon_v}{dp_w}\frac{dp_w}{dt}$ is the volumetric strain rate that should be calculated with a corresponding constitutive model for the soil. The mechanical model may include effective or net stress terms (volumetric or deviatoric) or suction terms. Effective or net stress has to be considered here as the total stress minus the water pressure or the air pressure, respectively, for saturated or unsaturated conditions. The final terms are left as a function of total stress.

The relative importance of the different terms depends on the conditions of the soil. For instance, for saturated conditions the second term disappears. When the compressibility of the skeleton is large, the compressibility of the particles is negligible. The compressibility of the water may be negligible in some cases but it is not possible to neglect it in general for hard soils.

The final objective is to find the unknowns from the governing equations. Therefore, the dependent variables will have to be related to the unknowns in some way. Doing this in the last equation leads:

$$\begin{gathered}\frac{\phi S_w}{K_w}\frac{dp_w}{dt}+\phi\frac{dS_w}{dp_w}\frac{dp_w}{dt}+S_w\left(\frac{\partial\varepsilon_v}{\partial\boldsymbol{\sigma}}\frac{d\boldsymbol{\sigma}}{dt}+\frac{\partial\varepsilon_v}{\partial p_w}\frac{dp_w}{dt}\right)+\frac{1}{\rho_w}\nabla\cdot\left(\rho_w\mathbf{q}_w\right)=0\\ \frac{\phi S_w}{K_w}\frac{dp_w}{dt}+\phi\frac{dS_w}{dp_w}\frac{dp_w}{dt}+S_w\frac{\partial\varepsilon_v}{\partial\boldsymbol{\sigma}}\frac{d\boldsymbol{\sigma}}{dt}+S_w\frac{\partial\varepsilon_v}{\partial p_w}\frac{dp_w}{dt}=-\frac{1}{\rho_w}\nabla\cdot\left(\rho_w\mathbf{q}_w\right)\end{gathered}\tag{A8}$$

where the compressibility of the solid particles has been neglected and the source/sink is assumed to be 0. The material derivatives have been approximated as eulerian.

This equation permits to calculate the pressure development for a soil subjected to changes in total stress in the following way:

$$dp_w = \frac{-S_w \frac{\partial \varepsilon_v}{\partial \boldsymbol{\sigma}} d\boldsymbol{\sigma} - \frac{dt}{\rho_w} \nabla \cdot \left(\rho_w \mathbf{q}_w \right)}{\frac{\phi S_w}{K_w} + \phi \frac{dS_w}{dp_w} + S_w \frac{\partial \varepsilon_v}{\partial p_w}} \tag{A9}$$

Deformation is assumed negative in compression from these equations, and stress is also negative in compression. This implies that d**σ** is negative in compression (loading) and produces positive pressure increments. Note that, the general stress tensor is maintained because volumetric deformations can be caused by any stress variation (not only isotropic), and that depends on the response of the soil. For instance, dilatancy is a volumetric expansion induced by shear.

In equation A9, the volumetric deformation derivatives $\frac{\partial \varepsilon_v}{\partial \boldsymbol{\sigma}}$ and $\frac{\partial \varepsilon_v}{\partial p_w}$ should be calculated with and appropriate constitutive model. These are volumetric deformation terms and can be obtained from a model for unsaturated soils such as the elastoplastic model BBM (Alonso et al, 1990). A general equation, including the effect of effective or net stresses and the suction, is written as:

$$\begin{aligned}
d\boldsymbol{\sigma}' &= \mathbf{D} d\boldsymbol{\varepsilon} + \mathbf{h} ds \\
d\boldsymbol{\varepsilon} &= \mathbf{D}^{-1} d\boldsymbol{\sigma}' - \mathbf{D}^{-1} \mathbf{h} ds \\
d\varepsilon_v &= \mathbf{m}^t d\boldsymbol{\varepsilon} = \mathbf{m}^t \mathbf{D}^{-1} d\boldsymbol{\sigma}' - \mathbf{m}^t \mathbf{D}^{-1} \mathbf{h} ds \\
\mathbf{m}^t &= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}
\end{aligned} \tag{A10}$$

Where suction can be defined as $s = \max\left(p_a - p_w, 0\right)$ and effective or net stress as $\boldsymbol{\sigma}' = \boldsymbol{\sigma} + \max\left(p_a, p_w\right)$. This is valid for saturated and unsaturated conditions, and considers stresses in compression as negatives. The model parameters are included in **D** which is the stiffness tensor (6x6) or constitutive matrix for changes in net or effective stress and **h** which is the constitutive vector for changes in suction. Both are nonlinear functions.

Note that, the derivatives of volumetric deformation needed in A9 can be obtained in the following way $\frac{\partial \varepsilon_v}{\partial \boldsymbol{\sigma}} = \frac{\partial \varepsilon_v}{\partial \boldsymbol{\sigma}'}\frac{\partial \boldsymbol{\sigma}'}{\partial \boldsymbol{\sigma}}$ and $\frac{\partial \varepsilon_v}{\partial p_w} = \frac{\partial \varepsilon_v}{\partial \boldsymbol{\sigma}'}\frac{\partial \boldsymbol{\sigma}'}{\partial p_w} + \frac{\partial \varepsilon_v}{\partial s}\frac{\partial s}{\partial p_w}$. By comparison with equation (A10), the following terms are obtained: $\frac{\partial \varepsilon_v}{\partial \boldsymbol{\sigma}'} = \mathbf{m}^t\mathbf{D}^{-1}$ and $\frac{\partial \varepsilon_v}{\partial s} = -\mathbf{m}^t\mathbf{D}^{-1}\mathbf{h}$.

The nonlinear elastic part of the BBM model, gives the following volumetric deformation:

$$d\varepsilon_v = \frac{\kappa}{1+e}\frac{dp'}{p'} + \frac{\kappa_s}{1+e}\frac{ds}{s+0.1} \qquad \text{(A11)}$$

Where, e is void ratio, κ and κ_s are material parameters, p' is the mean net or effective stress which is defined as $p' = \left(\sigma_x + \sigma_y + \sigma_z\right)/3 + \max\left(p_a, p_w\right)$, and s is suction.