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Crack growth analysis and fatigue life estimation in the piston rod of a Kaplan hydro turbine

X. Liu¹ | A. Presas² | Y. Luo² | Z. Wang²

¹Huaneng Clean Energy Research Institute, Lab Building A, CHNG Innovation Base, Beijing, China

²Department of Energy and Power Engineering, Tsinghua University, Beijing, China

Correspondence

Zhengwei Wang, Department of Energy and Power Engineering, Tsinghua University, Beijing, China.
Email: wzw@tsinghua.edu.cn

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Abstract

Linear elastic fracture mechanics (LEFM) approaches are seldom used to analyze fatigue problems in hydraulic turbomachinery, where extremely simplified fatigue models are normally considered. In 1 of the largest Kaplan hydraulic turbines in the world, the piston rod, which is an important load transfer component, was found broken twice at the same location. This event, which leads to extremely high direct repair costs and indirect costs, is a very useful case for research of fatigue problems in hydraulic turbines by means of LEFM. For this purpose, the crack growth progress in the piston rod is simulated with numerical LEFM techniques. A probabilistic model for the initial flaw size is considered. A reliability criterion, consisting in the probability of not having a critical crack or a fracture in the piston rod after a specific service time, is defined. Based on this model, the influence of initial crack models and operating services of the machine on the reliability of the unit is discussed. The calculated crack path and predicted number of loading cycles as a function of the initial crack size agree with the actual fracture section and operating time to fracture.

KEYWORDS

crack growth, fatigue life prediction, hydraulic turbine, LEFM, piston rod, reliability

1 | INTRODUCTION

Hydropower is a mature technology and has been used for more than 1 century for electricity generation and storage. During many decades, hydraulic turbines were used for a base generation, working most of the time close to their design conditions. Nevertheless, with the

massive entrance of renewable energy sources, such as wind or solar, which are extremely unpredictable and volatile, the electricity grid needs from flexible generating units, such as hydraulic turbines. In this context, the output power of these units has to be adjusted so that the total electricity generation in the grid matches the electricity demand at every instant. As a consequence,

NOMENCLATURE: a , Crack length; C , m , Constants in Paris law; CFD, Computational fluid dynamics; DRM, Design Reference Mission; F , Cumulative distribution function; f , Geometrical function for the crack tip; FEM, Finite element method; $GK1$, $GK2$, Operating conditions; J , Contour path integral; K , Stress intensity factor; K_I , K_{II} , K_{III} , Stress intensity factor in modes I, II, and III; LEFM, Linear elastic fracture mechanics; M , Contour path integral; m_{ij} , N_{ij} , Quantities derived from the compliance tensor; N , Number of cycles; n_i , Vector normal to contour path; R , Reliability; r , Radial coordinate; SIF, Stress intensity factor; T_f , Time to fracture; t , Random variable of time; u_{ij} , Local displacement vector; W , Strain energy density; ΔK_{MM} , Stress intensity factor range in mixed mode situation; ϵ_{ij} , Local strain; θ , Angular coordinate; μ , β , Parameters for the Gumbell distribution; σ_{ij} , Local stress

hydraulic turbines have to work at many different operating conditions, far from its best efficiency point and withstanding multiple transients.

These unfavourable operating conditions can lead to excessive vibration, and, in some cases, fatigue cracks have been found in hydropower unit components.¹⁻⁴ The consequences of these cracks are not normally a safety issue (excepting the catastrophic failure of Sayano-Sushenskaya⁵), but they cause direct reparation costs and high indirect costs because of the loss in the electricity production. For this reason, fatigue problems in hydraulic turbine units have to be carefully addressed and considered.

Although in other fields the crack growth analysis by means of LEFM is widely used and developed,⁶⁻⁸ many recent reports in hydropower consider fatigue problems by means of the rainflow-counting algorithm of the dynamic stress and the Miner rule.⁹⁻¹² This procedure allows us to determine the relative damage of every operating condition of the unit, but it is not suitable for accurate predictions of the remaining life. Furthermore, initial flaws and defects in the structure, which is a fundamental input factor in crack growth analysis, are not taken into account in the aforementioned references. Therefore, these estimations are not useful to define inspection intervals and estimated reparation costs for the power plant operators.^{13,14} Some researchers in hydropower, such as Gagnon et al,^{13,14} have used probabilistic models applied to the Kitagawa diagram¹⁵ (with the correction proposed by El Haddad¹⁶). This diagram considers a threshold that comes from linear elastic fracture mechanics (LEFM) theory and a threshold that is the fatigue limit. The idea of those papers is to define reliability as the probability of not exceeding 1 of these thresholds. Although the probabilistic models shown in that study are well presented and discussed, real experimental cases were not discussed. Furthermore, the crack propagation behaviour, ie, crack growth rate, cannot be considered with the aforementioned methods.

For this purpose, more advanced approaches based on LEFM which include the use of computational finite element method (FEM) are necessary.¹⁷⁻²⁵ The idea is to analyze the local displacements around the crack tip. Three different loading modes (I or tensile, II or in-plane shear, III or out-of-plane shear) are defined to evaluate the crack tip behaviour. The determination of the stress intensity factors (SIFs or K_i) associated to each of these modes is 1 of the objectives as they can predict the stress state near the crack tip and have been successfully used as a failure criterion in many cases. Nevertheless, the behaviour of the SIFs at the local point of the crack tip under some complex circumstances is only partially known.¹⁷ Recent studies have used numerical models

with extremely fine meshes to show complicated 3-dimensional effects and induced loading modes (for example, K_{III} inducing K_{II}) in thin plate structures. According to the recent state-of-the-art review of Pook et al,¹⁷ the understanding of the stress field in the vicinity of the crack tip is incomplete and unlikely to improve significantly in the foreseeable future.

Because of the aforementioned complexities, many researches propose the use of energy approach methods in a control volume around the tip. The stress energy density (SED) has been used in recent works^{18,19,21} to characterize the crack tip stress field under antiplane loading, which can include coupled effects in thin structures.¹⁹ Another well-used technique, in general 3-dimensional structures with complex loading mode situations, is the calculation of the SIFs, by means of an energetic contour path integral around the crack tip (usually J-integral or M-integral). Once the SIFs are obtained, they are introduced in a crack propagation law, such as the Paris law²⁶ or variations of it²⁷ Recent studies have also shown that it is also possible to predict the crack growth direction in complex cases.²⁸⁻³⁰ According to the recent research of Highsmith,²⁸ at the present, there are no single formulations available to predict crack growth direction for all cases based on the SIF, and the most appropriated one has to be selected depending on the particular load case.

This paper analyzes 1 of the largest running Kaplan hydraulic turbine in the world (located in China). The piston rod, which is an important load transfer component, was found broken twice at the same location. Therefore, to validate the procedure used, besides the basic operating data of the unit, 2 service periods until the fracture was reached are used. Furthermore, the real crack path is also inspected and analyzed. This case was firstly analyzed by Wang et al.³¹ In that study, a complete computational fluid dynamic (CFD) model was used to calculate the loads and dynamic stresses on the unit for 2 differentiated operating conditions. Luo et al³² improved the mechanical model and transmission of loads to the piston rod. Finally, as a preliminary fatigue research, Liu et al³³ used the local strain approach and a crack growth model to estimate the fatigue life of the piston rod. That study, which serves as a base for the present paper, shows the numerical model and performs a calculation of the fatigue life in a simplified condition without analyzing the sensitivity of the different parameters. That study also shows that local strain and local stress approaches systematically fail when trying to predict fatigue life.

This paper proves the feasibility of using crack growth analysis in hydropower. For this purpose, a numerical model based on the SIFs determined by

means of the M-integral, that considers the crack growth rate and path, is used. Taking advantage of the analysis of the real fracture, the real crack path and the simulated one are compared obtaining good agreement. With respect to the previous study, the numerical model is improved, and the effects and sensitivity varying the operating conditions and flaw size are discussed. As in other studies, a probabilistic distribution model for the size of the initial flaw, which is usually 1 of the critical and most difficult parameters to be determined, is newly introduced. In this way, a discussion on the reliability parameter, defined as the probability of not having reached the fracture after a specific service time, is presented. Experimental observations of 2 previous failures perfectly agree and justify the use of this LEFM-based model to perform future fatigue life estimations.

2 | NUMERICAL MODEL

The objective of the crack propagation approach is to estimate the crack growth path and the number of loading cycles necessary to reach a fracture failure. Alternatively, the failure can be defined as reaching a critical crack size, after the crack rapidly grows. It is assumed that initially a small crack or flaw exists in the material. In the previous study,³³ local strain and local stress methods were also discussed, showing that they were not able to realistically predict the number of cycles. Therefore, in the present paper, the crack growth method is explained and developed.

2.1 | General formulation

The crack growth life calculation method is developed using LEFM. Linear elastic fracture mechanics may be applied when the nonlinear deformation zone of material (plastic zone) constitutes a very small material fraction near the crack tip.²⁷ In such cases, the relationship between the local stresses at the crack tip σ_{ij} , and the SIF K , is defined according to Equation 1:

$$\sigma_{ij}(r, \theta) = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) \quad (1)$$

Stress intensity factor K is a fundamental parameter of crack growth analysis. In Equation 1, r and θ define the cylindrical coordinates, with origin on the crack tip (see Figure 1) and $f_{ij}(\theta)$ is a function that depends on load and geometry of the considered case.

The effective calculation of SIF for a general geometry and loading state is a complex task, which usually requires the use of FEM. The fatigue crack growth rate can usually be described by various empirical relationships such as the Paris equation,²⁶ or modified versions

FIGURE 1 Coordinates of the crack [Colour figure can be viewed at wileyonlinelibrary.com]

of them.³⁴ Equation 2 shows the classical Paris equation for crack growth analysis:

$$\frac{da}{dN} = C(\Delta K)^m = C(K_{MAX} - K_{MIN})^m \text{ for } \Delta K_{th} \leq \Delta K \leq \Delta K_{IC} \quad (2)$$

This equation, which is developed for simple cracks and loading situations, relates the propagation rate of the crack $\frac{da}{dN}$ with the SIF, where a is the crack size, N is the number of loading cycles, ΔK or $(K_{MAX} - K_{MIN})$ is the stress intensity variation in the σ cycle range, and C and m are parameters that depend on the material. In this paper, steel BS4360 grade 43 is selected ($C = 1.834 \times 10^{-13}$ and $m = 3$).³⁵

For generic cases, the crack can present complex shapes that can be cyclically loaded, and the crack front suffers from nonproportional mixed modes.²⁷ Then, the Paris law (Equation 2) has to be modified by other equivalent expressions adapted to the corresponding loading situation.³⁶⁻³⁸ In such cases, the SIF ΔK_{MM} is introduced to replace the conventional ΔK , which can be determined by calculating the M -integral.

2.2 | M-integral and ΔK_{MM}

The M -integral (also called the interaction integral) was first presented by Yau et al³⁹ as a means to extract the SIFs for the 3 fracture modes from the global energy release rate. Wang et al⁴⁰ developed the M -integral for mixed mode problems in monoclinic and orthotropic materials. Wawrzynek et al²⁰ and Banks-Sills et al⁴¹ summarized the methods for calculating SIFs in predominantly anisotropic materials through the M -integral. In this paper, the approach followed by Wawrzynek et al²⁰ to obtain the SIF factors by means of the M -integral is followed.

J and M -integrals are contour path integrals around the crack tip. The J -integral was firstly developed by Rice,⁴² and it can be expressed as follows:

$$J = \int_{\Gamma} \left(W n_i - T_i \frac{\partial u_i}{\partial x} \right) ds \quad (3)$$

where W is the strain energy density, Γ is a contour path around the crack tip that starts and ends in the crack surface, n_i is the outer normal vector to the integration path, $T_i = \sigma_{ij} \times n_j$, and $W = \frac{1}{2} \sigma_{ij} \varepsilon_{ij}$ (strain energy density), where σ_{ij} and ε_{ij} are respectively the local stresses and strains. Finally, u_i is the local displacement vector.

Hoenig⁴³ derived the relationship between the J -integral and the SIFs. This relation is expressed as follows:

$$J = -\frac{1}{2} \left(K_I \operatorname{Im} \left[m_{2i} N_{ij}^{-1} K_j \right] + K_{II} \operatorname{Im} \left[m_{1i} N_{ij}^{-1} K_j \right] + K_{III} \operatorname{Im} \left[m_{3i} N_{ij}^{-1} K_j \right] \right) \quad (4)$$

K_I , K_{II} , and K_{III} are respectively the SIF associated to pure modes of traction (perpendicular to the plane of the crack), in-plane shear and out-of-plane shear; Im is the imaginary component of the bracketed quantities with a summation being applied to repeated indices; the parameters m_{ij} and matrix N_{ij} are derived from the compliance tensor S_{ijkl} , where $\varepsilon_{ij} = S_{ijkl} \times \sigma_{kl}$ (additional details are provided in ref.⁴³).

For mode I crack problems, the J -integral is a powerful and capable method for obtaining the SIFs by means of its standard version⁴⁴ or a path-independent J_1^* -integral.⁴⁵ For mixed-mode crack problems, some researchers have evaluated mixed-mode SIFs by means of the path-independent J_k^* -integral.^{46,47} Results of their research indicated that the accuracy of the J_k^* -integral is usually better suited to mode I (which involves J_1^*) than to mixed-mode crack problems (which involve J_1^* , J_2^*). The reason for this is that the numerical treatment of crack faces, which is needed to ensure path independence of J_2^* , may lead to loss of accuracy.⁴⁷ Moreover, both the theoretical formulation and numerical implementation for J_1^* are much simpler than for J_2^* .²⁴ Therefore, the M -integral, which is based solely on the standard J -integral, is a favourable method for evaluating mixed-mode SIFs.^{39,40} With the M -integral, mixed-mode SIFs can be determined along 3D cracks in homogeneous materials,^{48,49} 3D biomaterial interface cracks,^{50,51} and through cracks.⁵²

To obtain the M -integral, 2 solutions are required and superposed. This is possible because of the assumption of a linear elastic material. The superposition of these 2 generic solutions is expressed as follows:

$$\begin{aligned} \sigma_{ij} &= \sigma_{ij(1)} + \sigma_{ij(2)} \\ \varepsilon_{ij} &= \varepsilon_{ij(1)} + \varepsilon_{ij(2)} \\ u_{ij} &= u_{i(1)} + u_{i(2)} \\ K_{ij} &= K_{i(1)} + K_{i(2)} \end{aligned} \quad (5)$$

The solution (1) in Equation 5 is usually the real state of the crack problem determined by means of FEM, which is usually a mixed-mode situation, and (2)

is an auxiliary state (fictitious). Substituting the expressions of Equation 5 in Equation 3, the following expression is obtained:

$$J = J^{(1)} + J^{(2)} + M^{(1,2)} \quad (6)$$

where $J^{(1)}$ and $J^{(2)}$ are the respective forms of J with the solutions (1) and (2). The M -integral is then an interaction integral between the 2 solutions, and its expression is:

$$M^{(1,2)} = \int_{\Gamma} \left(W^{(1,2)} n_1 - T_{i(1)} \frac{\partial u_{i(2)}}{\partial x_1} - T_{i(2)} \frac{\partial u_{i(1)}}{\partial x_1} \right) ds \quad (7)$$

Here, only $W^{(1,2)}$ needs a specification $W^{(1,2)} = \sigma_{ij}^{(1)} \varepsilon_{ij}^{(2)} = \sigma_{ij}^{(2)} \varepsilon_{ij}^{(1)}$. In the context of FEM method, it is much favourable to calculate this integral as a volume integral instead of a path integral. Expressions for this volume integral can be found in ref.²⁰

In a way similar to that by Hoenig,⁴³ the relationship between the M -integral and the SIFs can be obtained as is shown in Equation 8.²⁰

$$M = \frac{1}{2} \times \left[\begin{aligned} &K_I^{(1)} \operatorname{Im} \left(m_{2i} N_{ij}^{-1} K_j^{(2)} \right) + K_I^{(2)} \operatorname{Im} \left(m_{2i} N_{ij}^{-1} K_j^{(1)} \right) \\ &+ K_{II}^{(1)} \operatorname{Im} \left(m_{1i} N_{ij}^{-1} K_j^{(2)} \right) + K_{II}^{(2)} \operatorname{Im} \left(m_{1i} N_{ij}^{-1} K_j^{(1)} \right) \\ &+ K_{III}^{(1)} \operatorname{Im} \left(m_{3i} N_{ij}^{-1} K_j^{(2)} \right) + K_{III}^{(2)} \operatorname{Im} \left(m_{3i} N_{ij}^{-1} K_j^{(1)} \right) \end{aligned} \right] \quad (8)$$

Usually the different favourable auxiliary solutions (2) are imposed to simplify the expression above. In fact, the objective of all this procedure is to obtain $K_{I(1)}$, $K_{II(1)}$, $K_{III(1)}$, which are the SIF factors in the real mixed-mode state. These factors should not depend on the auxiliary solution (2) selected. Therefore, usually 3 different solutions (2a), (2b), (2c), which represent fictitious pure mode situations (2a), (2b), (2c), which represent fictitious pure mode situations K_I , K_{II} , K_{III} are selected to easily obtain $K_{I(1)}$, $K_{II(1)}$, $K_{III(1)}$ in the mixed mode situation.²⁰

Finally, after the SIF factors in the mixed mode situations are obtained with the aforementioned procedure, the well-adopted formula proposed by Tanaka³⁸ to calculate ΔK_{MM} is used.

$$\Delta K_{MM} = ((\Delta K_I)^m + q(\Delta K_{II})^m)^{1/m} \quad (9)$$

where ΔK_I and ΔK_{II} are obtained from the maximum variation of the SIF in the whole cycle. m and q are weighting factors, which are suggested to be $m, q = 2$ or $m = 4, q = 4.8$.²⁷

2.3 | Crack growth direction

There are presently no single formulations available to predict crack direction for all cases. Despite this, it is

possible to predict crack trajectories based on many criteria: maximum tensile stress, maximum energy release rate, principle of local symmetry, and minimum strain energy density. Recently, Highsmith²⁸ presented an extensive work on the crack growth direction prediction. He contrasted the empirical models with experimental test pieces and proposed a best-practice approach. This is based on the maximization of the function $\Delta\bar{k}_i$ that depends on the kink angle θ , which indicates the crack growth direction in the actual state.

$$\begin{aligned}\Delta\bar{k}_i &= \Delta k_i^w \cdot k_{i,\max}^{1-w} \\ k_1 &= \cos\frac{\theta}{2} \left(K_I \cos^2\frac{\theta}{2} - \frac{3}{2} K_{II} \sin\theta \right) \text{ and} \\ k_2 &= \frac{1}{2} \cos\frac{\theta}{2} (K_I \sin\theta + K_{II} (3 \cos\theta - 1))\end{aligned}\quad (10)$$

Here, the expressions for k_1 and k_2 are obtained, with the previous calculated SIFs and θ as a variable. w is a fitting parameter (between 0 and 1). Highsmith²⁸ suggested $w = 0.5$ for most of situations.

With the SIF in the mixed mode situation, the crack growth rate (Equation 2) and direction (Equation 10) can be predicted, and the crack path as a function of cycles can be iteratively calculated.

3 | ANALYZED CASE

3.1 | Hydraulic turbomachine and crack description

A Kaplan turbine unit was put into operation in 1995. The diameter of the turbine is 8 m, with 6 blades. The

rated discharge and power is respectively 467.6 m³/s and 204 MW. In February 2000, this unit was discovered to have been experiencing severe vibrations. The oil level in the oil collection groove then rapidly dropped. Examination revealed that the central axis of the piston rod had broken at the joint of the nut with the crosshead, and the broken part had fallen into the cone.³¹ Because of this, the plant opted to change from the nut structure to a retainer ring structure. The structural strength of the new design seemed to be sufficient; however, 5 years later, the retainer ring structure had also broken at the same location (Figure 2). A comprehensive investigation F2 conducted in 2010³² showed that the fractures occurred at a location of stress concentration. While stress concentrations are a factor that likely contributed to the piston rod's fracture, it must be brought into question as to whether this was the sole cause of failure. The 2010 investigation was unable to answer this because life estimation was not considered. In 2014,⁵³ we firstly used fatigue analysis applied to this case. It was shown that local strain approach, without considering flaws or initial defects, was not able to predict such a failure in a short time. A first attempt to use crack growth propagation analysis with LFEM was also made showing that results with this method were more accurate.

Following that research, in this paper, we have refined the numerical simulation model, by refining the mesh on the crack tip and adding mesh midpoints. Furthermore, the effect of assuming different cycle lengths and different operating conditions has been discussed. Finally, the initial crack size is considered in a probabilistic matter, so that a term called “reliability” has been introduced.

FIGURE 2 Schematic diagram of the broken piston rod in the Kaplan turbine [Colour figure can be viewed at wileyonlinelibrary.com]

3.2 | FEM model

The diameter of the piston rod is 670 mm. The original material of the piston rod was hammered steel 20SiMn. Because of a lack of accurate fatigue-related information, the material selected for analysis here is structural steel BS4360 Grade 43D which has similar material properties T1 and performance as illustrated in Table 1.

According to the well-known Goodman relation, both materials will have a similar fatigue behaviour as the admissible alternating and mean stresses depend on the tensile ultimate strength, which is the same for both materials, and the fatigue strength, which is approximately 0.5 times the tensile ultimate strength for steels. Furthermore, according to recent work of Strzelecki et al,⁵⁴ the S-N curve can be approached based on these 2 coefficients and the tensile yield strength, which is very similar for both materials, as seen on Table 1.

The determination of realistic loads in hydraulic machines is a complicated task and in this case has been deeply analyzed.^{31,32} Wang et al³¹ performed a CFD simulation using the unsteady Reynolds-averaged Navier Stokes (RANS) equations with the SST $k-\omega$ turbulence

TABLE 1 Material properties of the steel selected

Material Property	20SiMn	BS4360
Density	7850 kg/m ³	7850 kg/m ³
Young's modulus E	2.07 GPa	2.07 GPa
Poisson's ratio	0.3	0.3
Tensile yield strength	275 MPa	281 MPa
Tensile ultimate strength	490 MPa	490 MPa
Constant C in Paris' law	-	1.834×10^{-13}
Constant m in Paris' law	-	3.0

model. The final mesh had 730 000 nodes. With this work, the excitation characteristic of the flow was determined. Then, Luo et al³² introduced the actual FEM model (FEM model with 43 000 nodes). To describe the load transmission from the turbine blades (in contact with the flow) to the critical section in the piston rod, multibody system analysis with nonlinear contact technology was used. The whole system was subjected to gravity and centrifugal acceleration because of the rotational motion. Through the multibody system simulation, an accurate stress-strain distribution was obtained for all the operating conditions analyzed and for a transient time of 3 rotating periods of the Kaplan unit. Both studies were performed with the software ANSYS[®]. Details of the CFD and FEM models can be found in references.^{31,32}

The crack growth simulation is based on LEFM with element methods. Departing from the stress field obtained with the FEM model and considering that a crack exists (Figure 3), the SIFs have been obtained by means of the F3 code FRANC3D,²⁰ which is based on the M -integral (see Section 2). For every crack size shown in Figure 6, F6 the model has been updated and remeshed and the SIFs have been recalculated. Based on the work of Highsmith²⁸ (Equation 10), the local growth direction of the crack has been iteratively obtained for every crack size.

3.3 | Operating conditions selected for the present study

Different operating conditions of the unit have been analyzed in the previous related studies.³¹⁻³³ For the present study, 2 different well-differentiated operating conditions are selected. GK1 is small blade angle and guide vane opening, while GK2 is large blade angle and guide vane opening near the optimum operating point. Therefore,

FIGURE 3 Initial flaw considered with 4-mm width and 2-mm depth [Colour figure can be viewed at wileyonlinelibrary.com]

this condition is close to the smoothest operation of the machine. The operating parameters of the 2 selected conditions are listed in Table 2.

According to the operating data of the unit, given by the operator plant, and previous analysis,³¹⁻³³ these 2 conditions approximately represent the operation of the machine for a wide part of its operating time. It is important to notice that the developed procedure in this paper could be used for an accurate future prediction by using a more detailed planned operating service or DRM¹⁰ including more operating conditions.

3.4 | Initial flaws and SIF calculation

For the crack growth analysis, an initial crack, defect, or flaw has to be assumed. In this case, because the piston rod had failed 2 times at the same section, an initial small flaw is considered at that section (Figures 2 and 3). Because of the large diameter of the piston rod (670 mm), it is reasonable to think that initial flaws can exist. Requirements for a correct assessment of the crack growth model are an accurate calculation of the SIF factors and the correct prediction of the crack growth direction. Furthermore, an accurate definition of the time of the operating conditions leads to more accurate results regarding the number of loading cycles needed to produce the crack.

As is mentioned in Section 2, the calculation of the SIFs is based on a numerical integration around the crack front.²⁰ Therefore, a sensitivity analysis to decide the number of circumferential divisions and mesh contours is performed considering an initial flaw of 2-mm depth at the root of the fillet. The flaw is assumed to be semielliptical, perpendicular to the rod axis and with a width of 4 mm (Figure 3). The selection of this particular geometry, which will be discussed in the following

TABLE 2 Operating parameters for small and large blade angles

Operating Condition	Head (m)	Power (MW)	Guide Vane Opening (%)	Blade Angle (°)
GK1	58.5	100	37.3	6.6
GK2	58.5	210	64.0	18.0

TABLE 3 Mesh sensitivity analysis of the crack front at 2 mm and condition GK1

	Mesh Contours	Circumferential Divisions	SIF ΔK_I (MPa m ^{0.5})	SIF ΔK_{II} (MPa m ^{0.5})
2	8	138.56	3.25
	4	8	137.46	2
	6 ^a	8 ^a	137.95	3.4
	6	16	137.95	3.4

a-Mesh contours
b-Circumferential divisions

section, has been made according to the past real fractures and according to the load characteristic.

Quadratic hexahedron elements were adopted in the 2 retainer rings, the crosshead, and piston rod except for the crack region. In the crack region, quadratic wedge elements were adopted in the inner ring around the crack front, while quadratic hexahedron elements were used in the outer rings. In the other parts of the crack region, quadratic tetrahedron elements were used. The estimated SIFs with different number of circumferential and mesh contours around the crack tip are shown in Table 3 (condition GK1).

For the main SIF K_I , all of the configurations show a very similar value. For the SIF K_{II} , which is much less important in magnitude, there is a difference in the second configuration tried. As is seen in this table, there is no difference between the number of circumferential divisions, if 6 mesh contours are selected. Therefore, the third configuration is selected for the rest of the study.

4 | DYNAMIC STRESS AND SIF AS A FUNCTION OF THE CRACK LENGTH

4.1 | Initial crack and crack growth analysis

The initial crack has been selected based on realistic initial crack sizes and the inspection of the 2 fractures occurred. Although several initial orientations have been simulated, the loads (Table 3), which are clearly dominated by the mode K_I , grow the predicted crack in approximately the same direction, when maximizing the function presented in Section 2.2 (Equation 10). Furthermore, for the size, it has been considered that if the initial defect or flaw would have been enough large, this should have been detected during the installation of the piston rod, and therefore, the defect would have been repaired or the piece replaced. It has been found that the assumption of an initial semielliptic flaw with an average length around 1.5 to 2 mm, which is also a realistic size according to many researches and standards,^{13,14,55} and a width of 4 mm (Figure 3) qualitatively approximates the observed crack path (Figure 4). This figure and the supplementary *Video* provided show how the simulated

FIGURE 4 Simulated crack and real crack [Colour figure can be viewed at wileyonlinelibrary.com]

crack is propagated. From the critical section, the crack propagates in an oblique direction until it reaches the lower section and the fracture occurs. This occurs for a crack front path of approximately 300 mm.

The crack shape obtained in Figure 4 is approximately the same when using the induced loads at GK1 or GK2 (without considering the thresholds of Equation 2). Although the SIF levels are much larger in GK1 (Figure 5), the growth direction predicted according to Highsmith²⁸ is almost identical in both conditions because of the geometry and load characteristics (stress concentration region at the root of the fillet and same direction loads).

4.2 | Influence of the initial flaw on the dynamic stress

Dynamic stress graphs assuming respectively no initial crack and 2-mm crack are shown in Figure 5. The point

FIGURE 5 Dynamic stress time domain graph in GK1 and GK2. Comparison of no-crack and 2-mm crack [Colour figure can be viewed at wileyonlinelibrary.com]

of maximum stress is located at the root of the fillet, at the contact between the piston rod, and the retainer ring for the no-crack scenario.³¹ This point agrees with the 2 cases of fracture experienced in the piston rod.

The influence when assuming an initial crack is significant. For GK1, the mean stress increases from 113.6 to 676.6 MPa, while the alternate stress grows from 71.2 to 430.8 MPa. For GK2, the mean stress grows from 74.5 to 439.4 MPa while the dynamic stress changes from 17.7 to 103.6 MPa.

4.3 | Influence of the crack length on the SIFs

The SIF K_{MM} (Equation 9) is calculated based on the SIFs K_I and K_{II} numerically obtained with the M-integral (Section 2). Note that K_{MM} is a function in the time domain as the stresses are dynamic. Three rotation cycles of the turbine are considered here (Figure 5). ΔK_{MM} is calculated then as $[\max(K_{MM}(t)) - \min(K_{MM}(t))]$ and shown in Figure 6 as a function of the crack length that follows the simulated path (Figure 4), considering GK1 and GK2 separately.

Figure 6 shows the evolution of ΔK_{MM} without considering the growth rate equation, ie, this value is plotted

FIGURE 6 Evolution of the SIF with the crack length for GK1 condition and GK2 condition [Colour figure can be viewed at wileyonlinelibrary.com]

assuming that a determinate crack size exists. This does not necessary mean that the crack growth really occurs if we consider the threshold values in the general Paris law (Equation 2). This threshold SIF ΔK_{th} is taken from the report⁵⁶ and the British standard (BS 7910: 2013+A1:2015). In this research, this value is considered to be $\Delta K_{th} = 3 \text{ N mm}^{0.5}$ based on the material and actual stress ratio level.

machine, shows whether an actual state contributes or not to the crack propagation. As is seen in Figure 7A, with 2-mm crack, only GK1 contributes to the crack growth (Equation 2), because it has some cycles where the SIF overcomes the threshold. Those cycles N_i where $\Delta K_{MM-i} > \Delta K_{th}$ are the considered ones in the Paris law (Equation 2). It is seen that in this condition, running the machine at GK2 condition will not propagate the crack (Figure 7B) as all $\Delta K_{MM-i} < \Delta K_{th}$.

When a very large crack exists, such as 200 mm, the SIF for GK1 condition is very large, and almost all the cycles of the time signal contribute to a propagation of the crack, as is shown in Figure 8. Furthermore, it can

FIGURE 7 Analysis of the SIF at an initial stage of the crack (2 mm). A, For GK1. B, For GK2 [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 8 Analysis of the SIF at an advanced stage of the crack (200 mm). A, For GK1. B, For GK2 [Colour figure can be viewed at wileyonlinelibrary.com]

be seen that even the smoothest condition GK2 will propagate the crack, once this crack length exists.

This is a very interesting conclusion that cannot be drawn with the classical fatigue methods (such as the Miner rule combined with S-N curve). This information is also very useful for plant operators, because it indicates that once a certain size of the crack exists, it will irretrievably grow even if the machine is used in the smoothest condition possible (GK2). In contrast, before reaching this point, one can extend the operating service of the machine by using it close to the best efficiency point (GK2). The determination of this critical size for GK2 will be discussed in the following section.

5 | FATIGUE LIFE ESTIMATION BASED ON A PROBABILISTIC INITIAL FLAW SIZE

In this section, we introduce some simplified assumptions on the operating conditions of the machine and assume a probabilistic distribution of the initial flaw, which is the most critical parameter, ie, the accuracy of the remaining life prediction highly depends on the correct determination of this parameter, according to the studies of Gagnon.^{13,14}

5.1 | Simulated operating service

Based on the operating data of the power plant, we make some assumptions that permit to describe the past service of the machine with enough accuracy and in a simplified way. For future predictions, a detailed long-term operating service or Design Reference Mission¹⁰ could be introduced in the model, for more accurate calculations.

We do not consider the contribution of the transients on the crack growth, because for this unit, they are relatively smooth and, compared to other units, the number of start/stops is small. This unit works part of the time close to the best efficiency point. In this condition, the stresses are not very high and the contribution to the crack growth is negligible for small cracks (Figure 7). Part of the time, the machine works with small guide vane opening angles (GK1) to regulate the electrical grid. From previous studies, it has been checked that GK1 is 1 of the most damaging conditions in stresses^{31,32} and, therefore, it will be considered as representative for the machine working with small guide vane angles. Furthermore, the average operating service of the unit was 45%, ie, from every 100 hours, the unit worked 45 hours and was stopped for 55 hours.

Based on these considerations and the operating data given by the plant owner, as a reference condition, it is

assumed that the machine was working 50% of the time at GK1 and 50% of the time at GK2. Finally, the effect of the cycle length of the sequence (GK1-GK2) and relative time of GK1 and GK2 will be also discussed.

To calculate the crack length as a function of service time t_s , the expressions below, derived from the Paris law (Equation 2), have been used. As the crack growth depends on ΔK_{MM} , it is important to emphasize that this variable has to be taken as a function of the actual crack length and the operating condition (Figure 6). Given 1 value of ΔK_{MM} , the rainflow algorithm applied to the corresponding simulated time signal (as represented in Figure 7) permits to determine which cycles N_i , where the SIFs $\Delta K_{MM-i} > \Delta K_{th}$, contribute to the crack growth. These N_i cycles are applied during 1 simulated time ($t_{sim} = 1.7$ seconds in this case). The crack size increase, during t_{sim} , is expressed as:

$$\Delta a_{tsim}(a, GK_j) = \sum_i C \Delta K_{MM-i}(a, GK_j)^m N_i(a, GK_j) \quad (11)$$

$\Delta a_{tsim}(a, GK_j)$ is a function of the crack size (before t_{sim} starts) and the operating condition. Finally, the actual crack size can be expressed as a function of the service time of the unit t_s :

$$a(t_s) = a_{ts=0} + \sum_{n=0}^{n_{ts}} \Delta a_{tsim}(a, GK_j) \text{ and } n_{ts} = \text{int}\left(\frac{t_s}{t_{sim}}\right) \quad (12)$$

$a_{ts=0}$ corresponds to the initial crack. Every n step corresponds to 1 simulated cycle t_{sim} at GK1 or GK2. Equation 12 has to be iteratively applied, and in every n step, the variable a has to be updated. This equation can be sequentially used, changing the condition GK, according to the real operating sequence of the machine during its service time.

5.2 | Fracture crack and critical crack length

As discussed before, the 2 operating conditions lead to a very similar crack path, and therefore, the simulated crack path in Figure 4, which approximately coincides with the experimental fracture observed, is considered. The fracture crack occurs when the length reaches 300 mm (Figure 4). Using the Paris law, considering the thresholds (Equation 12), the crack growth and time to fracture can be separately predicted for GK1 and GK2 (Figure 9).

For GK1, the initial crack size that defines the threshold is about 1.12 mm. For this size of crack and this operating condition, the maximum cycle in the rainflow diagram (Figure 7) tends to $\Delta K_{th} = 3 \text{ N mm}^{0.5}$ (ΔK_{th} selected in Section 4.3). Therefore, smaller crack sizes will not grow the

FIGURE 9 Crack propagation for GK1 (A) and GK2 (B) departing from threshold condition [Colour figure can be viewed at wileyonlinelibrary.com]

crack. The evolution from this threshold crack size until the fracture is shown in Figure 9A (GK1 condition). The total service days necessary to propagate the crack from the threshold to fracture is approximately 500 service days. If a larger initial crack is considered (2 mm), the time to fracture will be obviously shorter as seen in this figure.

For GK2 (Figure 9B) which is the smoothest condition, only if a very large crack exists (183 mm), $\Delta K > \Delta K_{th}$ and then a further propagation will occur. Shorter and realistic initial cracks (such as 5 mm) will not propagate if the machine ideally works continuously in this condition (Figure 9B). Consequently, if the machine would only work in this condition from the beginning, no fracture would occur on the piston rod. Nevertheless, such a large crack size can exist after many cycles of the unit working at GK1.

This threshold point is very informative, as it indicates that, after this crack size is reached, even running the machine close to its smoothest condition (GK2), the

crack will irretrievably grow. In the context of this problem related to hydropower, where units are used in an extensive operating range for regulation, we think that it is meaningful to define the critical size as precisely the threshold value for the smoothest condition (GK2 in this paper). This value indicates that, below this critical size of the crack, the operation of the unit can be extended (theoretically until infinite life) if it works close to the smoothest condition. Therefore, this criterion of failure is very useful for the plant operator to design the plan operation.

We consider now the simulated service of the machine which is a combination of GK1 and GK2. As is mentioned before, based on the operating data, it is assumed that the ratio of every condition (GK1 and GK2) in 1 cycle is 50%, and the initial crack is the threshold crack (Figure 10), which is obviously the same as for GK1. The length of this cycle can be very different depending on the hydropower unit considered. In some

FIGURE 10 Combined GK1 and GK2. Influence of cycle length [Colour figure can be viewed at wileyonlinelibrary.com]

type of units such as some Francis turbines, this cycle can last 1 year because the available head changes in a long term.⁵⁷ In other units, such as pump turbines or Kaplan turbines (analyzed case in this paper), these cycles are usually shorter, as the available head is more constant during the year.

Figure 10 shows the 2 extremes, considering in all the cases a relative time of 50% for GK1 and GK2. The shortest cycle possible (which is absolutely unrealistic for a hydraulic turbine) is to consider that the machine is running few seconds in GK1 and few seconds in GK2 (cycles corresponding to Figure 7). The difference between considering this type of cycles and 1 cycle which consists in 1 day at GK1 and 1 day at GK2 is almost negligible as seen in Figure 10.

Then, the longest cycle of 1 year (165 service days) is considered. In this case, the influence of starting the cycle with GK2 or GK1 is important, because if it starts with GK2, the crack will not grow during the first part of the year but then, in the second part, will grow faster than for the shorter cycles. Starting the service with GK1, the opposite behaviour will occur. It is important to see that, after 1 year, the accumulated damage is the same for all the cases and therefore they converge in a single point (Figure 10). For this reason, the expected time to fracture can vary around 10% as is seen in Figure 10 if extreme long cycles are considered. In the present case (Kaplan unit), the cycles are much shorter than 1 year, and the operating conditions can change in a very short term of days or hours to regulate the electrical grid. Therefore, the approach of “short cycles” will be considered as a realistic one.

5.3 | Probabilistic distribution of the crack growth

The initial crack size or defect is 1 of the most critical parameters (life prediction accuracy is very sensitive to this parameter), and unfortunately, it is difficult to determine it with precision.^{13,14} Specific methods, that can be used on-site (during the installation), can detect a minimum size of 1.5 mm, although smaller sizes can exist. Some standards refer 3 mm as a size where the flaw can be detected with high confidence level.⁵⁸ Larger cracks may be easier detected and therefore repaired before the service of the machine starts.

To introduce this uncertainty in the model, we assume a probabilistic function based on the previous considerations and past studies. One of the most common probabilistic distributions used is the Gumbel distribution^{13,14,55,59,60} which is also used here. The Gumbel distribution is an extreme value distribution that is generally used to model the distribution of the maximum

related to a number of samples. Therefore, considering that in the large critical section of the piston rod many small and not detectable defects can exist, this function represents the uncertainty for the largest initial defect, which presumable will start to grow and lead to the final fracture. This distribution can be considered here as a transition between the 2 other extreme value distributions Weibull (upper limit of the maximum crack size) and Fréchet (lower limit of the maximum crack size). The Gumbel cumulative distribution function is expressed as Equation 13:

$$F(a) = P(A < a) = e^{-e^{-\left(\frac{a-\mu}{\beta}\right)}} \quad (13)$$

where $F(a) = P(A < a)$ determines the probability that the random variable A has values below a , μ is the mean value, and β represents the variability. Gagnon et al⁶¹ adopted $\mu = 1.5$ mm and $\beta = 0.5$ mm. The adoption of these parameters seems to be reasonable and realistic, as it includes many not observable sizes (below 1.5 mm) and excludes almost all physically impossible cases below 0 mm (see the cumulate distribution function in Figure 11). Furthermore, it excludes most of the cases with sizes larger than 3 mm, where the crack can be more easily detected and then repaired.

5.4 | Life estimation and reliability as a function of the initial flaw and operating service

As the initial crack is assumed to follow a random variable (Gumbel), the predicted time to fracture will also follow a random process but with a known probabilistic model. Therefore, a reliability function (t), function of time, is introduced here and defined as the probability to not have reached the fracture after a service time t of

FIGURE 11 Cumulate distribution function for the initial crack size (Gumbel). $\mu = 1.5$ mm and $\beta = 0.5$ mm [Colour figure can be viewed at wileyonlinelibrary.com]

the unit (according to the operating plan considered). $R(t)$ can be also alternatively used with the event “critical crack” (as defined in Section 5.2) instead of fracture. This is equivalent as considering the probability that the random time to failure, T_f , occurs in a larger time than t . This can be expressed as follows:

$$R(t) = P(T_f \geq t) \quad (13)$$

It is also assumed that, given an initial crack size, the time to failure is deterministically obtained according to the Paris law sequentially used, considering a determinate operating service. Therefore:

$$T_f = g(A) \quad (14)$$

where g is the function followed that can be determined with the procedure explained in this paper. Because the cumulative probability function of A is known, then:

$$R(t) = P(T_f \geq t) = P(g(A) \geq t) = P(A \leq g^{-1}(t)) \quad (15)$$

In the last equality, it is assumed that the function $g(A)$ is monotonically descendent, as for a longer initial crack, less time to failure is expected. By definition of Equation 11

$$R(t) = F(g^{-1}(t)) \quad (16)$$

Then, the parameter of reliability can be represented as a function of service time considering the failure as the occurrence of the fracture in the piston rod. Alternatively, it can be defined as the occurrence of reaching a critical size. The reliability of the unit adopting the aforementioned operating service and initial crack distribution **F12s** shown in Figure 12.

Two interesting limits are observed in this figure. Considering only the reliability function to fracture (black line in Figure 12), for times smaller than 900 service days, the reliability tends to 100%. This means that no fracture is

FIGURE 12 Estimated reliability of the piston rod. Reliability to fracture and to critical crack [Colour figure can be viewed at wileyonlinelibrary.com]

expected before 900 days. This is because even the largest lengths of initial crack considered, which are around 3.5 mm (less than 1% will be larger than 3.5 mm according to Figure 11), need 900 days to reach the fracture. The other interesting limit is after 975 days (black line), where the reliability remains constant and approximately 10% until infinite time. This reliability function has this behaviour because of the threshold limits, ie, for initial cracks smaller than 1.12 mm, no fracture is expected (or infinite lifetime). According to the distribution model (Figure 11), approximately about 10% of the initial cracks do not reach the threshold level and, therefore, this corresponds to the asymptotic limit of the reliability. Between the first limit (100% of reliability) and the second (about 10%), the reliability is greatly reduced, which means that most of the fracture times will occur in this time interval (between 900 and 975 hours). For the critical crack size, the asymptotic limits are the same, but, as the time to reach this point will be shorter than for fracture, the reliability curve is shifted to the left.

As was pointed by Gagnon,¹³ the model assumptions for the initial flaw are 1 of the critical points, and still more research is needed, to estimate initial crack sizes more precisely. To illustrate this, the model with $\mu = 1.5$ mm and a model with $\mu = 2$ mm and same variability are compared in Figure 13. In this increased flaw **F13** model, only very few initial cracks will be below 1.12 mm (less than 1%), and therefore, the reliability limit in the right part of the graph is almost 0%. The drop in the reliability also occurs in a shorter time as, in average, larger initial cracks sizes are expected.

The 2 fractures occurred in the past, which both occurred after the 5th service year, are represented with an uncertainty level in Figure 13. This uncertainty exists, because it is not possible to accurately determine the service day with the operating data available. As is seen in this figure, both failures occurred between the left and the right limits (black curve), where most of the predicted

FIGURE 13 Comparison of original initial flaw model and larger initial flaw [Colour figure can be viewed at wileyonlinelibrary.com]

time services to fracture according to the model will be located. The increased flaw model could be also reasonable for these 2 failures, although the original model covers a higher probabilistic range.

Finally, 1 of the goals of the model presented in this paper is to predict the reliability of the unit in future cases, based on possible operating services. The intended operating plan or Design Reference Mission¹⁰ could be introduced here to have accurate predictions of the reliability. This could help operators to balance use costs of the machine at out-of-design conditions (regulation), which will give short-term benefits, or use costs with a more conservative plan (more time close to the best efficiency point), which will give less short-term benefits but unit will last longer.

F14 As is shown in Figure 14, when the machine is used more time at part load (GK1 condition), the reliability decreases before. Using the machine only at GK2 will not cause a failure of the piston rod, as is seen in this figure.

6 | CONCLUSIONS

In this paper, the analysis of a real fatigue fracture in the critical section of a piston rod, in 1 of the biggest Kaplan hydraulic turbomachine in the world, has been analyzed. This fracture occurred twice in a much shorter time than expected and represented extremely high direct and indirect costs. Compared to other engineering fields, fatigue failures in hydropower are much less common, and therefore, it is usually not possible to validate theoretical and numerical fatigue models.

A simulation model for the crack growth analysis and service time until failure has been used. The model is based on LEFM and assumes a probabilistic distribution for the initial crack size. Dynamic stresses have been obtained by means of previous CFD and FEM simulation

models of the machine in real operating conditions. The SIFs have been calculated with a numerical evaluation of the *M*-integral. A reliability parameter, which quantifies the probability of having the rod without failure after a determinate service time, has been defined.

The simulated crack shape predicted, using a maximum stress criterion, qualitatively agree with the 2 real fracture cracks occurred in the past. The analysis of the reliability parameter shows that the 2 past service times to fracture could have been estimated by the present model. It has been also shown that assuming thresholds in the crack growth law implies that, in 10% of the cases, the fracture will never occur (infinite lifetime). Even in the most unfavourable initial crack possible, the piston rod will last at least 850 service days (5 years of real operating time).

Regarding the operating conditions and regulation of the machine, it has been shown that the reliability of the piston rod tends to 100% if the machine works in the smoothest condition or close to the best efficiency point. Working the machine with small guide vane angles (part load) will drastically reduce the useful life of the piston rod.

The methodology proposed here, which has been rarely used in the context of hydropower, seems to be a promising way, as an analysis of the reliability parameter can be used to evaluate economic and safety costs of using the machine far away from its smoothest condition and help to design the intended operated plan of the unit. Finally, this model can be improved introducing stochastic components in the SIF's determination and improving the actual knowledge and evaluation methods for initial flaws.

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ORCID

Z. Wang <http://orcid.org/0000-0002-1803-9718>

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FIGURE 14 Estimated reliability of the piston rod for different operating plans considering GK1 and GK2 [Colour figure can be viewed at wileyonlinelibrary.com]

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

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