Air Traffic Deconfliction Using Sum Coloring

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Abstract—**This paper studies strategic conflict resolution for air traffic based on sum coloring. We consider two application scenarios: manned and unmanned air traffic, with similar targets: to improve efficiency of operations and to reduce the costs. For the Unmanned Air Vehicles Traffic Management (UTM) we consider also a payment mechanism which incentivizes the operators to share information necessary to find a socially optimal solution. We quantify the potential savings via a series of experiments, showing that our methods drastically outperform the widely used First-Come-First-Serve (FCFS) strategy.**

I. Introduction

Conflict resolution is of critical importance in aviation. Current Air Traffic Management (ATM) needs improvements to face the expected traffic growth, in particular to alleviate the air traffic controllers workload and to avoid the unnecessary costs of solving conflicts that could have been anticipated. In unmanned aviation, the issue is different: drones are expected to do numerous short flights, requiring a higher level of automation to solve conflicts.

In both ATM and UTM, aircraft operators (airlines or drone owners) submit flight plans with a planned trajectory for each flight [1]. Considering the flight plans of aircraft crossing the same airspace in the same time window, it is possible to anticipate the conflicts and to provide strategic deconfliction, with a reduction of the operators costs.

In this paper, we study how to do the conflict resolution in an optimal way. Our solution involves allocating resources to aircraft to solve the conflicts (time slots in the ATM case, flight levels in the UTM case). We model finding optimal resource allocation as a weighted sum coloring problem.

- For manned aviation, we consider a standard setting where too many airplanes want to cross a sector at the same time, leading to the sector overload. When it is detected, ground holding cannot be applied to the planes that already took off, and the congestion creates airborne delays. There may be, however, enough time to organize the crossing of the airspace in a cost-saving way. We show how to resolve the airspace overload with an optimal time slot allocation, considering the trade-offs between two criteria: total delay time and total delay cost.

- For unmanned aviation, we study the case of a centralized system that uses vertical layers to avoid conflicts between drones [2], [3]. We show how to find a solution that optimizes the social cost (the total operating cost); the agent-based strategic deconfliction is achieved with an economic mechanism for capacity management.

The rest of the paper is organized as follows: Section II reviews related work. In Sections III and IV we explain the Minimum Sum Coloring Problem (MSCP) and how optimizing the deconfliction can be modeled as MSCP. Section V describes the experimental setup. In Section VI we present our results, and in Section VII we describe the payment mechanism.

II. Related work

Currently, the European traffic is optimized by The European Network Manager Operations Centre (NMOC) [4]. When an en-route sector is overloaded, three strategies are used by the NMOC: ground holding, rerouting and metering. The goal of these strategies is to minimize the delays, the financial loss and the environmental impact of the congestion. The SESAR Undertaking seeks to improve future operations with new concepts for ATM optimization. Several strategies are explored to provide en-route deconfliction, for instance, with a better ground holding, flight level allocation and the assignment of alternative trajectories. In [5], a new ground holding approach is presented, where all potential conflicts occurring above a given flight level are solved assuming aircraft can precisely follow their planned 4D-trajectories. Adjustments in the departure times are imposed in order to avoid the conflicts; however, for busy days solving all conflicts with ground delays can be too costly. Furthermore, the authors present an approach to account for uncertainties, which unfortunately results in high delays. In [6], flight level allocation is proposed

to solve en-route conflicts; results show that the global conflict resolution workload could be alleviated by at least 20%. In [7], a combination of both ground holding and flight level allocation is presented; still, uncertainties are not taken into account. Finally, in [8], a route-slot allocation technique is used in order to minimize the number of potential conflicts between aircraft by modifying the shape of their trajectories and by shifting their departure times.

On the other hand, even though the drone industry is still in an early stage, it is already a hot topic. A lot of practical applications for drones are currently contemplated, and their presence in city is expected to grow rapidly in the future. Several risks have been assessed as a consequence of this development, such as collisions between aircraft and the risks linked to flying over people. UTM will be necessary to ensure that drones are operated with safety. According to the European ATM Master Plan [9], the deployment of drones is likely to be supported by the emergence of new business models and service providers. In this article, we focus on the case of drones in-town flights at a low altitude (under 500 ft above ground level, to be separated from manned traffic).

The future air transportation system comes with the challenge of establishing the "right" rules for pricing ATM services and auctioning available resources. Mechanism design is needed to drive the market, and several papers presented allocation and pricing mechanism for ATM applications [10]–[12].

The question of deconfliction via auctions arises in UTM as well [13]. In particular, limits to traffic density may be enforced in the U3 phase of U-Space deployment [9], where the dynamic capacity management may be implemented for various reasons (e.g., noise "overdose"). Here again, flight prioritization ensuring equitable allocation of airspace resources may be done via economic mechanisms.

III. MINIMUM SUM COLORING PROBLEM

Given an undirected graph $G = (V, E)$ where V is the set of vertices and $E \subset V^2$ is the set of edges, graph vertex coloring involves assigning a color to each vertex so that two adjacent vertices (linked by an edge) feature different colors. An equivalent formulation is to consider a coloring as a partition of G into subsets of vertices so that two adjacent vertices do not belong to the same subset [14]. Graph coloring has many different applications, such as in scheduling or register allocation problems, where, for instance, a set of jobs need to be assigned to time slots, assigning one slot to each job to avoid conflicts [15].

The Graph Vertex Coloring Problem is finding the minimum number of colors (or the minimum number of subsets) needed to color the graph. The Minimum Sum Coloring Problem (MSCP) is a variant of the graph
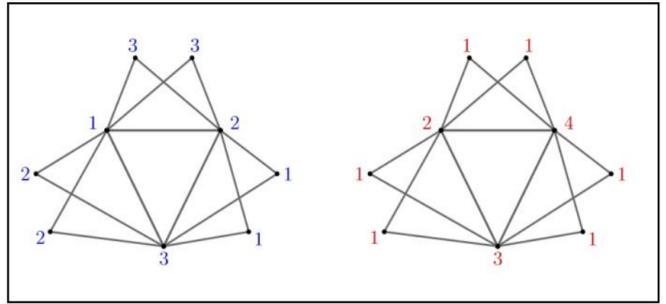

Fig. 1. Example for MSCP from Jin and Hao 2016 [16]. The left coloring uses 3 colors (which is the optimal coloring) and its sum of colors is 18. The right coloring uses one more color but its sum of colors is 15 (which is optimal)

vertex coloring, in which each color is identified with a positive integer, called the cost of the color, and the goal is to minimize the total cost (Figure 1).

Given a coloring $C : V \mapsto \mathbb{N}$, we define $S(C) = \sum_{v \in V} c_v$ as the sum of the colors assigned to the vertices ($c_v = C(v)$). The optimal sum of colors is called the *chromatic sum* of G. Both the vanilla coloring problem and the MSCP are NP-hard [17].

IV. MODELING DECONFLICTION AS WEIGHTED MSCP

In the input to our problem we have a set of planes entering an en-route sector at the same time slot, with a 4-D trajectory assigned to each plane. Two planes are in conflict if following their trajectories would lead to a loss of separation. Once we have detected the conflicts, we build a graph $G = (V, E)$ on the planes: the planes are the vertices V and conflicts between planes are represented by the edges E. For unmanned aviation, we are given trajectories of drones over a city area and create the conflict graph G in a similar way (drones are vertices and the conflicts are represented by edges). To solve the conflicts, we assign colors to the aircraft (the colors represent time slots for planes and vertical layers for drones). With this interpretation, solving conflicts becomes a graph coloring problem: vertices connected by an edge cannot be assigned the same color (otherwise we get a conflict). Furthermore, to optimize the conflict resolution, we want to find a coloring with good properties.

Note that in the standard coloring, all colors have equal value, while in our setting the users have preferences over colors. For instance, in the slot assignment, the first time slot is most desirable. In layer assignment, the layers may have different values e.g., due to different speeds a drone can attain in different layers (because of the winds or possible speed limits for layers closer to the ground imposed for safety reasons). To account for this difference we use sum coloring (instead of the usual coloring) to model optimization of conflict resolution.

Specifically, we assume that assigning a color to the aircraft $v \in V$ creates a cost for the operator of v. We

identify the colors with their costs, so that c_v is the cost incurred by v in the coloring C. In particular, in the time slot assignment, c_v will represent the delay. Minimizing the total delay amounts to solving the MSCP for G, i.e., minimizing:

$$TotalDelay = \sum_{v \in V} c_v \qquad (1)$$

Furthermore, different users have different value of time (e.g., due to the number of passengers and/or fuel consumption rate). To take this into account, we consider the weighted version of MSCP, where each aircraft v has a weight w_v. In the weighted version, the objective is to minimize the *weighted sum of colors*:

$$TotalCost = \sum_{v \in V} c_v w_v \qquad (2)$$

We use the same formula for the total cost of the layer assignment in UTM, with c_v representing the cost of the layer assigned to the drone v and w_v representing the weight (value) given by the drone operator to the flight.

V. EXPERIMENTAL SETUP

For manned aviation we used traffic files from EU-ROCONTROL's Demand Data Repository (DDR2) [18]. For UTM, we used simulated flights over a metropolitan area. More details are given in the following sections.

A. ATM Trajectory Data

From the historical data we obtained the information about the planes that flew inside the LOCCAOI sector (above Germany) during one hour (Figure 2). To simulate the congestion (i.e., the situation when the planes want to use the airspace at the same time) we forced all planes to enter the sector at the same time, delaying or advancing the time in the flight plans accordingly. Then, we detected the conflicts between the aircraft trajectories and built the graph G.

B. Weights for Manned Aviation

The financial values of time were used as weights for the sum coloring. These values were obtained from [19], where values for the cost of delay for European airlines are computed for several aircraft models. Strategic and tactical costs of delay are considered in the document, and also 3 scenarios: low, base and high. In our case, we used the strategic cost of delay and the base scenario. As all the aircraft are in the en-route phase, the fuel costs will represent the major source of cost. Note that only 15 aircraft models are considered in the reference document; for those aircraft for which no data is provided the cost of delay of an equivalent aircraft in terms of dimensions and performance (and included in [19]) is used. Figure 3 shows a table with the en-route strategic costs for the 3 scenarios and for the 15 aircraft models.
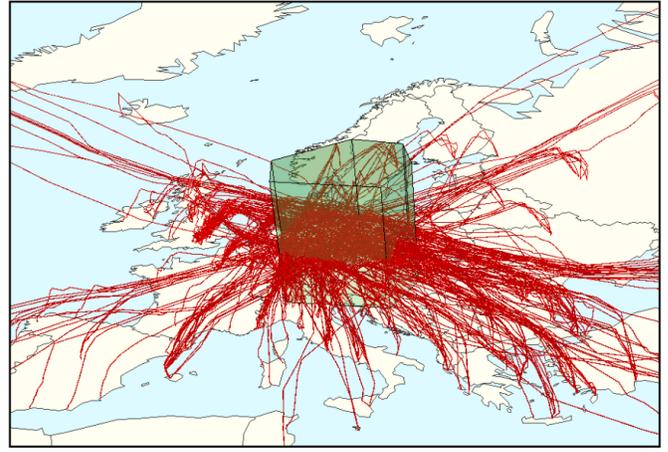

Fig. 2. Traffic sample flying inside LOCCAOI sector visualized with NEST

Aircraft	Low scenario	Base scenario	High scenario
B733	2 280	3 200	4 370
B734	2 360	3 290	4 540
B735	2 130	2 950	4 040
B738	2 590	3 650	5 330
B752	3 110	4 210	5 640
B763	4 600	6 230	9 070
B744	8 890	10 950	14 030
A319	2 400	3 420	4 840
A320	2 480	3 490	5 090
A321	2 950	4 130	5 770
AT43	580	900	1 350
AT72	830	1 270	1 890
DH8D	1 110	1 630	2 420
E190	1 920	2 750	3 950
A332	5 330	7 220	10 470

Fig. 3. En-route total strategic costs in euros per hour [19]

C. Drone Data

We used the Cal model [20] to simulate drone flights over Norrköping municipality in Sweden. The model is stochastic: the drones have randomly generated departure and arrival, with a higher probability to be located in more densely populated area. For our experiments, we simulated the traffic over one hour. We varied the number of drones from 100 to 5000. We assigned random weights to the drones assuming that the majority will have small weights (where delay does not create high cost), while some have very high weights (for which punctuality is critical).

VI. RESULTS

This section reports the results of experimenting with the above model on manned and unmanned traffic. In particular, we compare our results with First Come First Served (FCFS) deconfliction. We used an integer

programming (IP) formulation of MSCP, specifically a weighted version of the "compact formulation" [21]. This formulation introduces decision variables X_{vc} associated with each vertex v and each color $c \in \mathbb{N}$:

$$min \sum_{v \in V} \sum_{c \in \mathbb{N}} w_v X_{vc} c \tag{3}$$

$$\sum_{c \in \mathbb{N}} X_{vc} = 1 \quad \forall v \in V \tag{4}$$

$$X_{ic} + X_{jc} \leq 1 \quad \forall (i,j) \in E, \forall c \in \mathbb{N} \tag{5}$$

$$X_{vc} \in \{0,1\} \quad \forall v \in V, \forall c \in \mathbb{N} \tag{6}$$

We solved our IPs using Gurobi on a server with two Intel(R) Xeon(R) Gold 6132 2.60GHz CPU nodes, 64 RAM and 2.59 TB temporary disk space.

For ATM, we ran experiments on two test cases:

- *ATM Case 1*: on June 2nd, 2018 from 12am to 1pm, with $V = 450$ planes (vertices) and $E = 1399$ conflicts (edges), and
- *ATM Case 2*: on June 6th, 2018 from 12am to 1pm with $V = 462$ and $E = 770$.

For UTM, we also considered two test cases:

- *UTM Case 1*: with $V = 800$ drones and $E = 663$ edges, and
- *UTM Case 2*: with $V = 1000$ and $E = 770$.

In all cases finding the optimal IP solution in one instance completed within few minutes.

As discussed above, for the manned aviation the allocation has two objectives – minimizing the total delay and minimizing the total cost. The delay minimization is achieved by setting all weights to 1, while with specific weight for each vertex we obtain the operating costs (which we obtained from [19]; see Section V-B). Note that the delay and the cost values are in arbitrary units, and only their ratios between the aircraft is relevant.

We computed the Pareto frontier with respect to our two objectives (Pareto frontier consists of *non-dominated* solutions, i.e., those for which one objective cannot be improved without deteriorating the other). Figures 4 and 5 show the results for the two ATM test cases.

We also computed the values for the two objectives (for the same graphs) for 20 random orders and FCFS time slot attribution. In the first test case, FCFS leads to average delay of 1423.9 (minimum found FCFS delay was 1389) and an average total cost of 4959.5 (minimum found FCFS total cost was 4712). In the second test case an average delay of 1416 (with a minimum value of 1366) and an average total cost of 4450.8 (with a minimum value of 4256.2) were obtained. For both cases, both the delay and the cost are worse than every point of the Pareto frontier (out of bounds of the axes to plot on the figures).
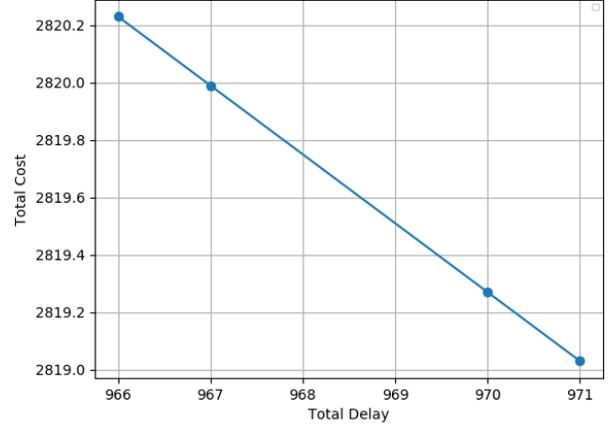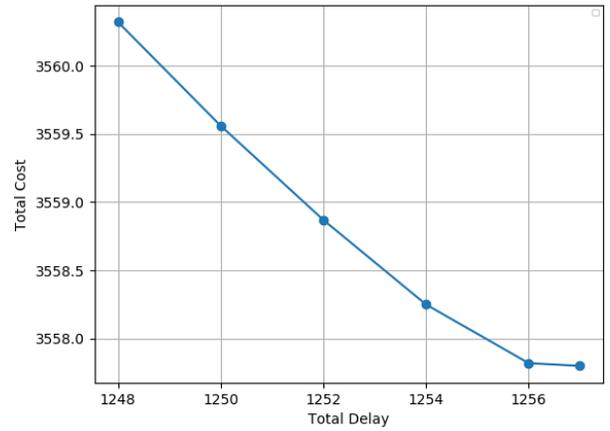

Fig. 4. Pareto frontier for *ATM Case 1*

Fig. 5. Pareto frontier for *ATM Case 2*

For UTM, we considered only one objective (the total cost of the deconfliction), which was 1788 and 1791 in *UTM Case 1* and *UTM Case 2* respectively. For 20 random orders, FCFS leads to average costs at 8830.55 (min=3701) and 9430.85 (min=3661) respectively, which confirms sum coloring approach outperforms FCFS is all tested cases.

VII. PAYMENT MECHANISM DESIGN FOR UTM

In this section we study design of a payment mechanism for conflict resolution in UTM. A market-based economic mechanism specifies how to allocate the resources and how much every user has to pay for the allocation. One important property of a mechanism is *Social Optimality*: the resource allocation should maximize benefit to the society. In our case, this amounts to finding the optimal solution to the weighted MSCP (as described above). The difficulty here is that in order to compute the

optimum, we need to know the value of time (W_V) for each drone v. However, if we ask for this value directly, the drone operator could give a value higher than reality to be favored by the algorithm. A *truthful* (or *Incentive Compatible*) mechanism enforces the payments that incentivize the users to report w_v truthfully (see [11] for application of truthful mechanisms in ATM). The other two mechanism properties which may be desirable in applications are *Individual Rationality* (each user benefits from participating in the market) and *Budget Balance* (the resource owner's net profit is 0).

A classical result in economics implies that no mechanism can possess the four properties at the same time [22], and thus at least one of them has to be left out. In our case, we forgo the Budget Balance, and use the *Vickrey–Clarke–Groves* (VCG) *mechanism* to determine the payments for the users (VCG is essentially the only socially optimal truthful individually rational mechanism [23]). In VCG, the payments of user v is the "harm" that the user brings to the society, i.e., the difference between the social welfare of the others in the presence of v and the social welfare they could have gotten if v were not in the society:

$$p_v = \sum_{j \in V \setminus \{v\}} c_j^v w_j - \sum_{j \in V \setminus \{v\}} c_j^{V \setminus \{v\}} w_j \qquad (7)$$

, where c_v^V is the color allocated to v by the VCG mechanism when deconflicting the drones in the set V. With this payment, the total amount of money invested to make the flight for v is $p_v + w_v c_v$. Note that Budget Balance is not met with this mechanism because all drone operators are paying to the mechanism. If one were to reach budget balance, drones surrendering good layers to others would have to receive reimbursement (be paid by the mechanism); this, however, could create problems, such as incentivizing people to fly drones with the only purpose of selling their slot.

Figure 6 shows the mean cost of a flight as a function of the number of drones flying during one hour. Each point on the figure is the average of 5 runs of the traffic simulation, with r = 100m as the separation loss distance. The weighted MSCP was solved with a greedy algorithm (the last instances are too big to compute the exact solution) on a computer with an Inter(R) Core(TM) i7-4720HQ 2.60GHz CPU nodes and 8 Go RAM. One immediate conclusion is that the denser the traffic, the more expensive drone flying gets. Thus, VCG gives a means of capacity regulation, as some flights may become discouraged from flying by the payments.

Figure 7 shows the cost of the flight against the FCFS strategy. Note that even though FCFS does not involve any payments (the cost of the flight is only $w_v c_v$), the FCFS costs are visibly higher than in the VCG mechanism (where the cost is $p_v + w_v c_v$).
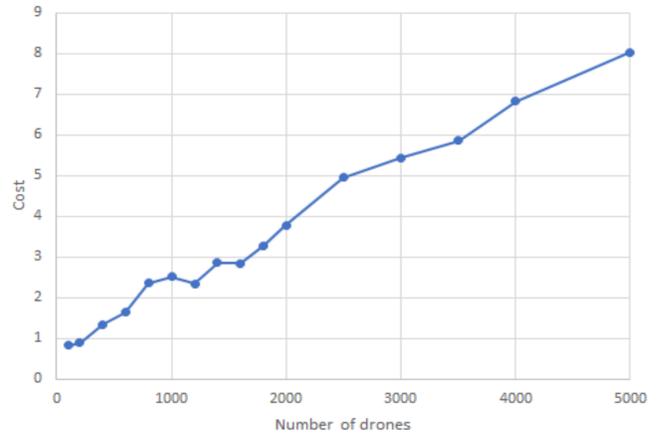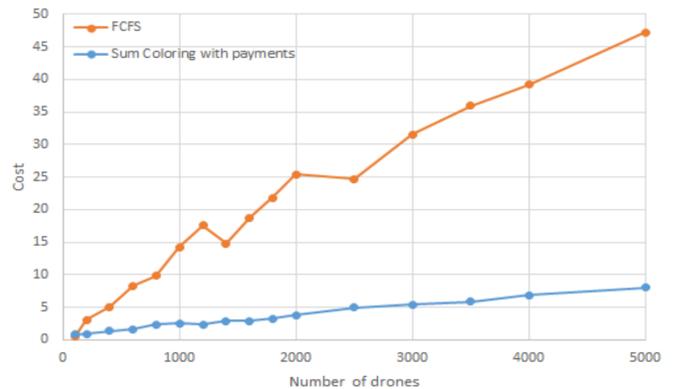

Fig. 6. Average cost of a drone flight

Fig. 7. Average cost of a drone flight under VCG and FCFS

VIII. Conclusions

We explored the use of sum coloring for strategic deconfliction in ATM and UTM. For the former, we considered two goals (minimizing total delay and total cost) and computed Pareto optimal solutions with respect to the two objective functions. For the latter we focused on only one objective (minimizing the total cost) and studied the performance of agent-based truthful economic mechanism. Both in ATM and UTM our solutions work remarkably better than the FCFS allocation.

One limitation of our approach is that in ATM practice several other aspects should be taken into account: giving a higher priority to planes in an emergency situation or low on fuel, favoring ground holding for planes still on the ground, considering the rerouting possibilities depending on the sector, etc. For UTM, our formulation permits the use of an unlimited number of layers. Even though our socially optimal solution tends to have a nearly-minimum number of colors, in reality there will be an upper bound on the number of available layers, depending on the airspace division, drones altimetry capabilities, noise concerns, etc.

Another limitation of our methods is that they assume

to solve all the conflicts. However, real-time adjustments would be probably necessary because of trajectories uncertainties and unexpected events (such as wind or other meteorological conditions, systems failure, etc.). Our goal is to reduce the conflicts as much as possible and to decrease operating costs.

Last but not least, the computing time to get the payments in the VCG mechanism grows with the number of drones involved, which can become an issue if the instances are too big. As a remedy, a sliding window and a division of the airspace into sectors would probably be required. Future work could take this into account and divide the airspace into sectors to improve the robustness of the deconfliction. Our allocation method could also be combined with other conflict resolution schemes to make it more efficient.

ACKNOWLEDGEMENTS

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