Passivity-based control applied to the dynamic positioning of ships

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Abstract: A family of passivity-based controllers for dynamic positioning of ships is presented. The authors exploit the idea of shaping the energy function of the closed-loop system to obtain different formulations of the passivity-based control law using the interconnection and damping assignment-passivity-based control (IDA-PBC) methodology. A salient feature of this study is that the proposed control laws are output feedback controllers and the relative velocity measurement is not required. First, we design and analyse two static controllers which can be seen as a non-linear version of the conventional proportional-derivative (PD) controllers. In presence of unknown disturbances, these controllers do not provide the desired regulation properties. To remove this discrepancy we propose, also in the context of the IDA-PBC technique, a dynamic extension of the system and obtain two new controllers that have the desired regulation properties. These new control laws can be seen as a non-linear version of the conventional proportional-integral-derivative (PID) controllers. Simulations are included to validate the theoretical results.

1 Introduction

A dynamic positioning (DP) system is a computer controlled system which automatically maintains a vessel's position and heading by using propellers and thrusters. The computer program contains a mathematical model of the vessel which includes information pertaining to the wind and current drag of the vessel and the location of the thrusters. This knowledge, combined with the sensor information, helps the computer to calculate the required steering angle and thruster output for each thruster. The first DP system was introduced in 1960 and since then they have emerged as a popular alternative for the conventional mooring and anchoring techniques for the dynamical positioning of the ships. Over the past decades, with the revolutionary developments in microprocessor technologies and availability of fast computing machines, DP systems have become more economical and reliable.

Generally, in DP problems, only position and heading measurements are available. This leads to the use of observers to estimate the state (mainly the velocities and the bias term) which are required for feedback into the control law. This problem is studied in many papers. Some examples include the Luenberger observer used in [1], a non-linear observer designed in [2], or a passivity-based scheme considered in [3, 4].

Furthermore, the measured position and heading signals are noisy and, also, with two different frequency components. The total ship motion can be seen as a superposition of a low frequency component (because of the

wind, sea currents and thruster forces and moments) and an oscillatory term (the so-called wave-induced wave frequency (WF) motion), which represents the effect of the waves [5].

However, DP only considers the slow variations and, consequently, the motion because of the waves should be removed before it enters in the controller algorithm. Kalman filtering techniques were proposed in [6, 7] or see [8] for a recent overview. As pointed out in [3], Kalman filters require the use of a linear model, and the non-linear motion should be linearised at various operation points. To overcome this drawback, in [3, 4], a WF observer is added to compensate the wave disturbances.

Owing to the important role of the estimation and filtering process, the motion control system in the DP problem can be grouped in two basic subsystems: the observer system (or wave filter) and the controller; see Fig. 1. This paper is focused on the design of the control law assuming that the filtering and observation process are previously done.

Various controllers have been proposed to stabilise the ship to the desired position. PI controllers are often used [5]; however, more advanced techniques are applied to this problem resulting in interesting control algorithms. Backstepping design, which also includes the observer stage, are presented in [2, 9]. In [4] the whole stability of a proportional-derivative (PD)-type controller with a passive observer is proved using a separation principle argument. Recently, sampled-data control theory has also been applied to the DP problem for designing the control law [10].

Passivity-based techniques have been used in many applications. A nice feature of the passivity-based control

Fig. 1 Basic scheme of components of a ship motion-control system

design is the physical meaning of the resulting control laws and the concepts, such as storage energy or dissipation play a fundamental role in the stability analysis and performance. Stability properties, based on the Lyapunov theory, can be easily studied for the obtained closed-loop systems. In the last decade, the interconnection and damping assignment-passivity-based control (IDA-PBC) methodology has emerged as an easy and a (quasi-) step-by-step methodology to obtain passivity-based controllers; see for instance [11]. The IDA-PBC technique comes from the port-Hamiltonian formalism, which is based on shaping the energy function (called Hamiltonian function) of the system.

This paper introduces the port-Hamiltonian model of a vessel with 3 degrees of freedom in the horizontal plane. The obtained port-Hamiltonian equations are the starting point for the design of controllers using the IDA-PBC technique. The main contribution of the paper is a family of passivity-based controllers, which use the energy shaping of the closed-loop system to ensure (local/global) asymptotic stability. The resulting control laws are output-feedback and robust in front of eventual disturbances.

Two different energy functions are proposed in this paper. We start by illustrating the methodology using a quadratic and a trigonometric Hamiltonian function and recover a simple static controller, which guarantees asymptotic stability. The energy shaping, based on a trigonometric function, improves the heading control. These controllers do not produce the desired regulation properties in presence of unknown disturbances. Consequently, in order to achieve the desired performance, a dynamic extension is proposed, and it results in a control law that can be interpreted as a non-linear version of the conventional PID controller. A salient feature of the proposed controllers is that they do not require the relative velocity measures and, thanks to a dynamic extension, they also ensure a good regulation behaviour even in presence of disturbances or unknown (or non-estimated) terms.

The presentation of the contents of this paper is as follows: Section 1 is reserved for the introduction. It explains some of the basic details of the problem under consideration and recalls some existing works on this subject. In Section 2, the port-Hamiltonian framework is introduced and an overview of the IDA-PBC methodology for the design of passivity based controllers is given. Section 3 contains the details of the ship model that we study in this paper and its port-Hamiltonian form is derived. In Section 4, a static controller based on the IDA-PBC methodology is obtained. Two variants, a quadratic and a trigonometric, of this controller are then analysed. The limitations of the static controller are studied in Section 5. Then, a dynamic extension is proposed to revamp the static controller for an improved performance. Section 6 contains the simulation results to give a qualitative measure of

the performance of the proposed controllers and, finally, Section 7 consists of the concluding remarks.

2 Hamiltonian-based control

2.1 Port-Hamiltonian modelling

A large class of physical systems of interest in control applications can be modelled in the general form of port-Hamiltonian systems (PHS) [12]. PHS generalise the Hamiltonian formalism of classical mechanics to physical systems connected in a power-preserving way and encodes the detailed energy transfer and storage in the system, and is thus suitable for the control schemes based on the IDA-PBC.

A PHS can be written, in an implicit form, as

$$\dot{\mathbf{x}} = (\mathcal{J} - \mathcal{R})\partial H(\mathbf{x}) + g(\mathbf{x})\mathbf{u} \quad (1)$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state (or Hamiltonian variables) vector, $\mathcal{J}(\mathbf{x}) \in \mathbb{R}^{n \times n}$ is the interconnection (skew-symmetric, $\mathcal{J} = -\mathcal{J}^T$) matrix, $\mathcal{R}(\mathbf{x}) \in \mathbb{R}^{n \times n}$ is the dissipation matrix (which is symmetric positive semidefinite, $\mathcal{R} = \mathcal{R}^T \geq 0$), $g(\mathbf{x}) \in \mathbb{R}^{n \times m}$ is the external connection matrix, $\mathbf{u} \in \mathbb{R}^m$ is the control input vector, and $H(\mathbf{x})$ is the Hamiltonian (or energy) function. The $\partial_{\mathbf{x}}$ (or ∂ , if no confusion arises) operator defines the gradient of a function of $\mathbf{x}$, and in what follows we will take it as a column vector.

The so-called passive output, $\mathbf{y} \in \mathbb{R}^m$, is given by

$$\mathbf{y} = g^T(\mathbf{x})\partial H(\mathbf{x}) \quad (2)$$

and the product $\mathbf{u}^T \mathbf{y}$ has, usually, unity of power.

2.2 IDA-PBC technique

The IDA-PBC, [11], is a technique for designing controllers based on the port-Hamiltonian framework. It uses the passive stability properties to ensure the convergence of the system to the desired fixed point.

The main idea behind the IDA-PBC is to define a new closed-loop (or target) system with a Hamiltonian structure. The design problem summarises into finding a control law such that the system behaves as

$$\dot{\mathbf{x}} = (\mathcal{J}_d - \mathcal{R}_d)\partial H_d \quad (3)$$

where $\mathcal{J}_d(\mathbf{x}) = -\mathcal{J}_d^T(\mathbf{x})$, $\mathcal{R}_d(\mathbf{x}) = \mathcal{R}_d^T(\mathbf{x}) \geq 0$ and $H_d(\mathbf{x})$ has a minimum at the desired regulation point $\mathbf{x}^d$, $\mathbf{x}^d = \arg \min(H_d(\mathbf{x}))$. The stability of this system can be easily proved by using H_d as a Lyapunov function ($\dot{H}_d(\mathbf{x}) = -(\partial H_d)^T \mathcal{R}_d \partial H_d \leq 0$, see for instance, [11, 13] for a detailed discussion).

The design procedure reduces to finding matrices $\mathcal{J}_d(\mathbf{x})$ and $\mathcal{R}_d(\mathbf{x})$ and a desired closed-loop energy function $H_d(\mathbf{x})$, which solve the so-called matching equation

$$(\mathcal{J} - \mathcal{R})\partial H + g\mathbf{u} = (\mathcal{J}_d - \mathcal{R}_d)\partial H_d \quad (4)$$

Then, the control law becomes

$$\mathbf{u} = (g^T g)^{-1} g^T ((\mathcal{J}_d - \mathcal{R}_d)\partial H_d - (\mathcal{J} - \mathcal{R})\partial H) \quad (5)$$

A drawback of the IDA-PBC controllers is that they are, in general, not able to reject disturbances. To remove this discrepancy of the control design usually a dynamic

extension of the system is done to obtain an integral action on the output error. Extension of the closed-loop dynamics in the IDA-PBC framework can be done, in a natural way, only for passive outputs, [13]. A completely different problem addresses for non-passive outputs (or higher relative degree one outputs). In this case, a Hamiltonian-based controller with an integral action can be obtained via a change of variables [14].

Let us consider that the $\mathbf{x}_o \in \mathbb{R}^r$ are the higher relative degree one (or non-passive) outputs. The main idea is to introduce a new variable $\mathbf{z}_e \in \mathbb{R}^r$, which is used to enforce the equilibrium point of the closed-loop system to the desired one, and a change of variables $\mathbf{z} = f(\mathbf{x}, \mathbf{z}_e) \in \mathbb{R}^{n-r}$ to cast the target system in a Hamiltonian structure as follows

$$\begin{bmatrix} \dot{\mathbf{x}}_o \\ \dot{\mathbf{z}} \\ \dot{\mathbf{z}}_e \end{bmatrix} = \begin{bmatrix} J_o - R_o & J_{zo} - R_{zo} & J_e \\ -J_{zo}^T - R_{zo}^T & J_z - R_z & O_3 \\ -J_e^T & O_3 & O_3 \end{bmatrix} \partial H_{de} \quad (6)$$

The power-preserving interconnection structure of the proposed target system is defined by, $J_o = -J_o^T \in \mathbb{R}^{r \times r}$, $J_z = -J_z^T \in \mathbb{R}^{(n-r) \times (n-r)}$, $J_{zo} \in \mathbb{R}^{r \times (n-r)}$ and $J_e \in \mathbb{R}^{r \times r}$. Dissipation is given by $R_o = R_o^T \in \mathbb{R}^{r \times r}$, $R_z = R_z^T \in \mathbb{R}^{(n-r) \times (n-r)}$, and $R_{zo} \in \mathbb{R}^{r \times (n-r)}$. Finally, the Hamiltonian function, H_{de} takes the form

$$H_{de} = H_d(\mathbf{x}_o, \mathbf{z}) + \frac{1}{2} \mathbf{z}_e^T K_e \mathbf{z}_e \quad (7)$$

where H_d has a minimum at the desired regulation point $(\mathbf{x}_o^d, f(\mathbf{x}_o^d, 0), 0)$.

Following the idea of the IDA-PBC technique, the stability is guaranteed if the dissipative matrix is positive semidefinite, that is, $R_o \geq 0$, $R_z > 0$ and $R_o - R_{zo} R_z^{-1} R_{zo}^T \geq 0$.

The key point of the PHS structure in (6) is that, the existence of a minimum of H_{de} in $\mathbf{x}_o^d$, implies $\partial_{\mathbf{x}_o} H_d|_{\mathbf{x}_o^d} = 0$ which, evaluated in the $\mathbf{z}_e$ dynamics,

$$\dot{\mathbf{z}}_e = -J_e^T \partial_{\mathbf{x}_o} H_d \quad (8)$$

ensures that $\mathbf{x}_o^d$ is an equilibrium point.

3 Ship model

A useful model describing the dynamics of a surface ship sailing in a horizontal plane having 3 degrees of freedom, is given in [5], and it can be written as the following non-linear system

$$\begin{bmatrix} \dot{\boldsymbol{\eta}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} O_3 & J(\psi) \\ O_3 & -M^{-1}D \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta} \\ \mathbf{v} \end{bmatrix} + \begin{bmatrix} O_3 \\ M^{-1} \end{bmatrix} \boldsymbol{\tau} + \begin{bmatrix} O_3 \\ M^{-1}J^T(\psi) \end{bmatrix} \mathbf{b} \quad (9)$$

where $\boldsymbol{\eta} = [x \ y \ \psi]^T$ is the position coordinate vector in the Earth-fixed reference frame, $\mathbf{v} = [u \ v \ r]^T = [\dot{x}_r \ \dot{y}_r \ \dot{\psi}_r]^T$ is the relative vessel-frame velocity coordinate vector, $\boldsymbol{\tau} = [\tau_u \ \tau_v \ \tau_r]^T$ is the vector describing the forces and the torque in vessel-fixed reference frame provided by the propulsion system of the ship acting in the surge, sway and yaw directions, respectively, and

$$J(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is the coordinate transformation matrix, which relates the Earth-fixed frame to the relative-frame of reference. A

Fig. 2 Description of the Earth-fixed and the vessel-fixed frames of reference

description of the two frames of reference is given in Fig. 2. The D and M matrices are given by

$$D = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{23} & d_{33} \end{bmatrix}, \quad M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{23} & m_{33} \end{bmatrix}$$

which are positive definite, $D = D^T > 0$ and $M = M^T > 0$, and O_3 is a 3×3 zero matrix. The environmental disturbances because of the sea currents, waves, and wind are represented by $\mathbf{b} = [b_1 \ b_2 \ b_3]^T \in \mathbb{R}^3$ in the Earth-fixed reference frame. This bias term is constant in the Earth-fixed reference frame, under assumption of constant or slowly varying currents.

Along this paper, it is assumed that these nature effects (some times called as bias forces and moments), which can also be modelled as a first-order Markov process [5], are either known or an estimate of the bias vector is available. Adding a note on the output of the system, we consider that the measurement system gives us noise free position and orientation measurements and that the WF components from the measured output are filtered or estimated. Hence, in this paper, we skip the dynamics of the bias and the WF components.

The main goal in the dynamic positioning problem is to stabilise the ship in a given $\boldsymbol{\eta}$ -coordinate. Without loss of generality, our objective is to design an appropriate control law $\boldsymbol{\tau}$, which stabilises the system to the origin $(x, y, \psi) = (0, 0, 0)$. Additionally, as the measurement of the relative velocity vector is not available, the control law should be independent of $\mathbf{v}$, and must be able to reject unknown disturbances or uncertainties.

We can write the system described in (9) in a PHS form (1) by using as a state $\mathbf{x}^T = [\mathbf{q}^T, \mathbf{p}^T] \in \mathbb{R}^6$, where $\mathbf{q} = [q_1 \ q_2 \ q_3]^T \in \mathbb{R}^3$ represents the Earth-fixed position and heading, and the momentum $\mathbf{p} = [p_1 \ p_2 \ p_3]^T \in \mathbb{R}^3$, is defined as $\mathbf{p} = M\mathbf{v}$. Substituting $\boldsymbol{\eta} = \mathbf{q}$ and $\mathbf{v} = M^{-1}\mathbf{p}$ in (9), we obtain the following system

$$\dot{\mathbf{x}} = (\mathcal{J}(q_3) - \mathcal{R})\partial H + g_{\boldsymbol{\tau}}\boldsymbol{\tau} + g_b(q_3)\mathbf{b} \quad (10)$$

with the following interconnection and damping matrices

$$\mathcal{J}(q_3) = \begin{bmatrix} O_3 & J(q_3) \\ -J^T(q_3) & O_3 \end{bmatrix}, \quad \mathcal{R} = \begin{bmatrix} O_3 & O_3 \\ O_3 & D \end{bmatrix} \quad (11)$$

the external connection matrices

$$g_{\boldsymbol{\tau}} = \begin{bmatrix} O_3 \\ I_3 \end{bmatrix}, \quad g_b(q_3) = \begin{bmatrix} O_3 \\ J^T(q_3) \end{bmatrix} \quad (12)$$

and the Hamiltonian function given by

$$H = \frac{1}{2} \mathbf{p}^T M^{-1} \mathbf{p}. \quad (13)$$

Note that the Hamiltonian function contains only a kinetic energy term, associated with the momentum variable. A potential energy, artificially added by the controller will play a key role to stabilise the ship in the desired position. From (2), we observe that the passive output for the system (10) is the velocity vector, which does not correspond to the actual output of the system, the position and the orientation. This is an important consideration for the control design, especially for the dynamic IDA-PBC control design in Section 5.

4 Static IDA-PBC controllers

A family of static feedback controllers can be obtained via the IDA-PBC methodology. In the design process, a nominal case is considered (i.e. where the disturbances are assumed to be completely known), and then the stability against an unknown disturbance vector is analysed.

As presented in Section 2, the control laws are obtained from matching the dynamical system (10) with the target dynamics (3). To solve this, the desired interconnection matrix is fixed as in (10), that is

$$\mathcal{J}_d = \begin{bmatrix} O_3 & J(q_3) \\ -J^T(q_3) & O_3 \end{bmatrix} \quad (14)$$

the dissipation matrix set as

$$\mathcal{R}_d = \begin{bmatrix} O_3 & O_3 \\ O_3 & R_p \end{bmatrix} \quad (15)$$

where $R_p \in \mathbb{R}^{3 \times 3}$ is a symmetric positive definite matrix ($R_p = R_p^T > 0$), and the closed-loop energy function, $H_d(\mathbf{x})$, is shaped as

$$H_d(\mathbf{q}, \mathbf{p}) = \Psi(\mathbf{q}) + \frac{1}{2} \mathbf{p}^T M^{-1} \mathbf{p} \quad (16)$$

where $\Psi(\mathbf{q})$ has a minimum at the origin, that is, $\partial_q \Psi|_{q=0} = 0$ and $\partial_q^2 \Psi|_{q=0} > 0$. This implies that the desired energy function has the minimum at the desired stabilising point $(\mathbf{q}^*, \mathbf{p}^*) = (0, 0)$. From a physical point of view, the controller adds some potential energy, in the $\mathbf{q}$ coordinates, with respect to the original Hamiltonian function (13).

From the resulting matching equation (4), equality corresponding to the first row is automatically satisfied while, from the second equality, we obtain the following control law

$$\boldsymbol{\tau} = -J^T(q_3)(\partial_q \Psi + \mathbf{b}) - (R_p - D)M^{-1} \mathbf{p} \quad (17)$$

From the $\mathbf{q}$ dynamics in (10), we obtain $\mathbf{p} = MJ^T(q_3)\dot{\mathbf{q}}$, and defining $K_D := R_p - D$, the state feedback algorithm (17) takes the form

$$\boldsymbol{\tau} = -J^T(q_3)\partial_q \Psi - K_D J^T(q_3)\dot{\mathbf{q}} - J^T(q_3)\mathbf{b} \quad (18)$$

which can be seen as a non-linear output feedback PD controller with a feedforward term, $-J^T(q_3)\mathbf{b}$. Note that with the choice $R_p = D$, the controller simplifies because $K_D = 0$. But, we keep the general result because, as we point out in the simulations, by increasing the dissipation of the system

improves considerably the performance of the closed-loop system.

Proposition 1: Consider the dynamical system (10) in a closed-loop with the control law (18), where the origin $\mathbf{q} = 0$ is a local minimum of $\Psi(\mathbf{q})$, and that the bias vector $\mathbf{b}$ and the D matrix are known. Then, the desired regulation point, $(\mathbf{q}^*, \mathbf{p}^*) = (0, 0)$ is locally asymptotically stable. Furthermore, if $\mathbf{q} = 0$ is the global minimum of $\Psi(\mathbf{q})$, then $(\mathbf{q}^*, \mathbf{p}^*) = (0, 0)$ is globally asymptotically stable.

Proof: Replacing (18) in (10), we obtain a PHS in the form (3) with (14) and (15). Using the Hamiltonian function (16) as a Lyapunov function, we obtain

$$\dot{H}_d = -\mathbf{p}^T (M^{-1})^T R_p M^{-1} \mathbf{p} \leq 0 \quad (19)$$

and invoking LaSalle's invariance principle, asymptotic stability is proved. See further details in Section 3.4.3 of [15]. $\square$

In (18), we have a rather general expression for the control law depending on $\Psi(\mathbf{q})$. What follows are two special cases of the control law depending upon two different energy shapings.

4.1 Quadratic energy shaping

The simplest function with a global minimum has a quadratic form: $\Psi(\mathbf{q}) = (1/2)\mathbf{q}^T K \mathbf{q}$, where $K = K^T > 0$ is a gain matrix. It is easy to see that $\mathbf{q} = 0$ is a minimum of $\Psi(\mathbf{q})$ and the desired energy function (16) becomes

$$H_{d1}(\mathbf{q}, \mathbf{p}) = \frac{1}{2} \mathbf{q}^T K \mathbf{q} + \frac{1}{2} \mathbf{p}^T M^{-1} \mathbf{p} \quad (20)$$

which implies, from (18), the following control law

$$\boldsymbol{\tau} = -J^T(q_3)(K \mathbf{q} + \mathbf{b}) - K_D J^T(q_3)\dot{\mathbf{q}} \quad (21)$$

Using Proposition 1, we can conclude that the closed-loop system (10) with (21), is globally asymptotically stable.

4.2 Trigonometric energy shaping

Inspired by the energy function of a pendulum, we propose to shape the desired Hamiltonian containing a trigonometric function with the form

$$\Psi(\mathbf{q}) = \frac{1}{2} \mathbf{q}_{12}^T C_{12} \mathbf{q}_{12} + c_3(1 - \cos q_3) \quad (22)$$

where $\mathbf{q}_{12} = [q_1 \ q_2]^T$, $C_{12} = \text{diag}\{c_1, c_2\}$ and $c_1, c_2, c_3 > 0$. This function, contains multiple local minimums at $\mathbf{q} = (0, 0, a2\pi)$, where $a \in \mathbb{Z}$. With this choice the desired energy function (16) becomes

$$H_{d2}(\mathbf{q}, \mathbf{p}) = \frac{1}{2} \mathbf{q}_{12}^T C_{12} \mathbf{q}_{12} + c_3(1 - \cos q_3) + \frac{1}{2} \mathbf{p}^T M^{-1} \mathbf{p} \quad (23)$$

Fig. 3 illustrates the difference between the two proposed energy shapings (20) and (23).

The main motivation for this kind of energy shaping is that, for certain applications where there are no constraints (for instance, links with external objects), stabilisation in $q_3 = 0$ or $q_3 = 2\pi$ is exactly the same. Fig. 4, shows a possible scenario, where the path for stabilising in $q_3 = 2\pi$ is shorter than stabilising in $q_3 = 0$.

Fig. 3 Comparison between the quadratic and the trigonometric functions, H_{d1} and H_{d2} , respectively, in the coordinates q_1 and q_3

Fig. 4 Possible scenario when it is advantageous to use the trigonometric energy function instead of the quadratic energy function

The Hamiltonian (23) with the target closed-loop system defined by (14) and (15), implies that the control law (18) takes the final form as

$$\tau = -J^T(q_3)C \begin{bmatrix} q_1 \\ q_2 \\ \sin q_3 \end{bmatrix} - K_D J^T(q_3) \dot{q} - J^T(q_3) \mathbf{b} \quad (24)$$

where $C = \text{diag}\{c_1, c_2, c_3\}$ is a positive definite gain matrix.

As in the previous subsection using Proposition 1, we can conclude that the closed-loop system (10) with (24), is (locally) asymptotically stable.

5 Dynamic IDA-PBC controllers

5.1 Motivating problem

The control law (18) assumes that $\mathbf{b}$ is known and that (10) perfectly models the ship motion. In practical situations, the ship model only represents a simple dynamics of the actual system and furthermore the bias vector, $\mathbf{b}$, has to be estimated. Let us study the influence of unmodelled behaviours, wrong estimations or, in general case, the presence of disturbances.

In order to analyse the performance of the proposed passivity-based controller, we study the closed-loop system

with the perturbed system

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} O_3 & J(q_3) \\ -J^T(q_3) & -D \end{bmatrix} \partial H + \begin{bmatrix} O_3 \\ I_3 \end{bmatrix} \tau + \begin{bmatrix} O_3 \\ J^T(q_3) \end{bmatrix} \mathbf{b} + \begin{bmatrix} O_3 \\ I_3 \end{bmatrix} \Delta \quad (25)$$

where $\Delta \in \mathbb{R}^3$ is a vector, which represents the disturbances. As a first approximation, and only to motivate the use of the extended dynamics, we consider this vector as a constant.

The system (25) in a closed-loop with (18) results in the following system

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} O_3 & J(q_3) \\ -J^T(q_3) & -R_p \end{bmatrix} \partial H_d + \begin{bmatrix} O_3 \\ I_3 \end{bmatrix} \Delta \quad (26)$$

which has an equilibrium point which satisfies $\partial \Psi|_{q=q^*} = J(q_3^*)\Delta$ and $\mathbf{p}^* = 0$. It implies that in presence of disturbances, the closed-loop system (25) with (18) has a different equilibria than $(0, 0)$. However, stability should be also further analysed.

Moreover, in the special case where the disturbances are also affected by a $J^T(q_3)$, given by

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} O_3 & J(q_3) \\ -J^T(q_3) & -D \end{bmatrix} \partial H + \begin{bmatrix} O_3 \\ I_3 \end{bmatrix} \tau + \begin{bmatrix} O_3 \\ J^T(q_3) \end{bmatrix} \mathbf{b} + \begin{bmatrix} O_3 \\ J^T(q_3) \end{bmatrix} \Delta \quad (27)$$

it can be seen that some stability properties still remain. With the control law (21) global asymptotic stability can be proved by shifting the Hamiltonian function to the new equilibrium point, that is, with

$$\tilde{H}_{d1} = \frac{1}{2}(\mathbf{q} - \mathbf{q}^*)^T K(\mathbf{q} - \mathbf{q}^*) + \frac{1}{2}\mathbf{p}^T M^{-1} \mathbf{p} \quad (28)$$

as a Lyapunov function, where $\mathbf{q}^* = K^{-1}\Delta$.

Similarly, the closed-loop with the control law derived from the trigonometric energy shaping (24) has the new equilibria in

$$\mathbf{q}^* = \left[\frac{\Delta_1}{c_1}, \frac{\Delta_2}{c_2}, \arcsin\left(\frac{\Delta_3}{c_3}\right) \right]^T \quad (29)$$

and $\mathbf{p}^* = 0$. Local asymptotic stability of this new set of equilibria can be proved with the Hamiltonian function

$$\tilde{H}_{d2}(\mathbf{q}, \mathbf{p}) = \frac{1}{2}(\mathbf{q}_{12} - \mathbf{q}_{12}^*)^T C_{12}(\mathbf{q}_{12} - \mathbf{q}_{12}^*) + c_3 \left(1 - \cos q_3 - q_3 \frac{\Delta_3}{c_3} \right) + \frac{1}{2}\mathbf{p}^T M^{-1} \mathbf{p} \quad (30)$$

which has local minima if $(\Delta_3/c_3) < 1$.

Summarising, the presence of unknown disturbances results in a bad positioning of the ship. Although, in case (27), a high gain in the K and C matrices (in controllers (21) and (24), respectively) implies stabilisation close to the desired equilibrium point but it may not be a rational strategy because of practical limitations of the propulsion units. This discrepancy in the desired performance of the control law motivates the use of a dynamic extension in order to achieve the stabilisation at the desired point.

5.2 Target-extended system

A dynamic extension for a non-passive output, maintaining the port-Hamiltonian structure, is possible by means of a change of coordinates. The controller is designed for a nominal case, without disturbances, and then the presence of unknown terms is analysed. Following the idea in [14], we introduce a new state variable, $\mathbf{z}_e \in \mathbb{R}^3$, which is used to enforce the equilibrium point of the closed-loop system to the desired one, and a change of variables $\mathbf{z} = \mathbf{f}(\mathbf{q}, \mathbf{p}, \mathbf{z}_e)$. We define the target system, with the form of (6), as

$$\begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{z}} \\ \dot{\mathbf{z}}_e \end{bmatrix} = \begin{bmatrix} O_3 & J(q_3) & J(q_3) \\ -J^T(q_3) & -R_z & O_3 \\ -J^T(q_3) & O_3 & O_3 \end{bmatrix} \begin{bmatrix} \partial_{\mathbf{q}} H_{de} \\ \partial_{\mathbf{z}} H_{de} \\ \partial_{\mathbf{z}_e} H_{de} \end{bmatrix} \quad (31)$$

where $R_z = R_z^T \geq 0$, is a 3×3 matrix to be defined. The desired Hamiltonian function is defined as

$$H_{de}(\mathbf{q}, \mathbf{z}, \mathbf{z}_e) = \Psi(\mathbf{q}) + \frac{1}{2} \mathbf{z}^T M^{-1} \mathbf{z} + \frac{1}{2} \mathbf{z}_e^T K_e \mathbf{z}_e \quad (32)$$

where $K_e = \text{diag}\{k_{e1}, k_{e2}, k_{e3}\} > 0$. As in the static case, $\Psi(\mathbf{q})$ must be designed with a minimum at the desired regulation point. Then, the whole Hamiltonian function (32) has a minimum at $(\mathbf{q}^*, \mathbf{z}^*, \mathbf{z}_e^*) = (0, 0, 0)$.

Matching the $\mathbf{q}$ dynamics, from (10) and (31), the change of variables $\mathbf{z}$ is defined as

$$\mathbf{z} = \mathbf{p} - MK_e \mathbf{z}_e \quad (33)$$

The state feedback control law is obtained from the second row of (31) and the time derivative of (33)

$$\boldsymbol{\tau} = -K_p J^T(q_3) \partial_{\mathbf{q}} \Psi - K_D M^{-1} \mathbf{p} + K_I \mathbf{z}_e - J^T(q_3) \mathbf{b} \quad (34)$$

$$\dot{\mathbf{z}}_e = -J^T(q_3) \partial_{\mathbf{q}} \Psi \quad (35)$$

where we define

$$K_p := MK_e + I_3 \quad (36)$$

$$K_D := R_z - D \quad (37)$$

$$K_I := K_e \quad (38)$$

Similarly to the static controller in the previous section, using $\mathbf{p} = MJ^T(q_3) \dot{\mathbf{q}}$, the control law (34)–(35) takes the following form

$$\begin{aligned} \boldsymbol{\tau} = & -K_p J^T(q_3) \partial_{\mathbf{q}} \Psi - K_I \int J^T(q_3) \partial_{\mathbf{q}} \Psi dt \\ & - K_D J^T(q_3) \dot{\mathbf{q}} - J^T(q_3) \mathbf{b} \end{aligned} \quad (39)$$

which has the same structure as a non-linear PID controller with a feedforward term, $J^T(q_3) \mathbf{b}$.

Proposition 2: Assume that $\mathbf{q}$ is measurable, and that the disturbances vector $\mathbf{b}$ and the matrices M and D are known. If $K_e = \text{diag}\{k_{e1}, k_{e2}, k_{e3}\} > 0$, and $\Psi(\mathbf{q})$ has a (local) minimum at the origin, $\mathbf{q} = 0$, then the system (10) in a closed-loop with (39), is (locally) asymptotically stable at the point $(\mathbf{q}, \mathbf{z}, \mathbf{z}_e) = (0, 0, 0)$.

Furthermore, if $\mathbf{q} = 0$ is a global minimum of $\Psi(\mathbf{q})$, then the origin is globally asymptotically stable.

Proof: The closed-loop system (10) with (39) takes the form of (31). Then, the Hamiltonian function (32) is a Lyapunov-candidate function and its time derivative is

$$\dot{H}_{de} = -\mathbf{z}^T (M^{-1})^T R_z M^{-1} \mathbf{z} \leq 0 \quad (40)$$

Then, the stability can be proved invoking LaSalle's invariance principle. $\square$

Equation (39) is a rather general formulation of the control law. What follows are two special cases depending on two different formulations of the energy shaping (32).

5.3 Quadratic energy shaping

Let us first take the same quadratic energy shaping as in Section 4, $\Psi(\mathbf{q}) = (1/2) \mathbf{q}^T K \mathbf{q}$ with $K = \text{diag}\{k_1, k_2, k_3\} > 0$, the Hamiltonian function (32) becomes

$$H_{de1}(\mathbf{q}, \mathbf{z}, \mathbf{z}_e) = \frac{1}{2} \mathbf{q}^T K \mathbf{q} + \frac{1}{2} \mathbf{z}^T M^{-1} \mathbf{z} + \frac{1}{2} \mathbf{z}_e^T K_e \mathbf{z}_e \quad (41)$$

with a global minimum at the origin, $(0, 0, 0)$. The control law (39) becomes

$$\boldsymbol{\tau} = -K_p J^T(q_3) K \mathbf{q} + K_I \mathbf{z}_e - K_D J^T(q_3) \dot{\mathbf{q}} - J^T(q_3) \mathbf{b} \quad (42)$$

$$\dot{\mathbf{z}}_e = -J^T(q_3) K \mathbf{q} \quad (43)$$

From Proposition 2 the controller (42)–(43) ensures the global asymptotic stability.

5.4 Trigonometric energy shaping

The trigonometric energy shaping proposed in the previous section can also be considered for the extended controller. Taking again

$$\Psi(\mathbf{q}) = \frac{1}{2} \mathbf{q}_{12}^T C_{12} \mathbf{q}_{12} + c_3 (1 - \cos q_3) \quad (44)$$

where $\mathbf{q}_{12} = [q_1 \ q_2]^T$ and $C_{12} = \text{diag}\{c_1, c_2\}$, and $c_1, c_2, c_3 > 0$ the desired Hamiltonian function (32) becomes

$$\begin{aligned} H_{de2}(\mathbf{q}, \mathbf{z}, \mathbf{z}_e) = & \frac{1}{2} \mathbf{q}_{12}^T C_{12} \mathbf{q}_{12} + c_3 (1 - \cos q_3) \\ & + \frac{1}{2} \mathbf{z}^T M^{-1} \mathbf{z} + \frac{1}{2} \mathbf{z}_e^T K_e \mathbf{z}_e \end{aligned} \quad (45)$$

As previously mentioned, this function has multiple local minimums at $\mathbf{q} = (0, 0, a2\pi)$, $a \in \mathbb{Z}$. From Proposition 2, this case ensures local asymptotic stability, and the control law (39) becomes

$$\boldsymbol{\tau} = -K_p J^T(q_3) C \begin{bmatrix} q_1 \\ q_2 \\ \sin q_3 \end{bmatrix} + K_I \mathbf{z}_e - K_D J^T(q_3) \dot{\mathbf{q}} - J^T(q_3) \mathbf{b} \quad (46)$$

$$\dot{\mathbf{z}}_e = -J^T(q_3) C \begin{bmatrix} q_1 \\ q_2 \\ \sin q_3 \end{bmatrix} \quad (47)$$

where $C = \text{diag}\{c_1, c_2, c_3\}$.

5.5 Analysis in presence of disturbances

The main contribution of the extended dynamics excels in presence of unknown disturbances. As before, let us to analyse the closed-loop system in presence of disturbances. First, we consider constant (or, at least, very slow) disturbances. Taking the perturbed system (25) with the designed control law (39), we obtain

$$\begin{bmatrix} \dot{q} \\ \dot{z} \\ \dot{z}_e \end{bmatrix} = \begin{bmatrix} O_3 & J(q_3) & J(q_3) \\ -J^T(q_3) & -R_z & O_3 \\ -J^T(q_3) & O_3 & O_3 \end{bmatrix} \begin{bmatrix} \partial_q \Psi \\ M^{-1} \tilde{z} \\ K_e \tilde{z}_e \end{bmatrix} + \begin{bmatrix} O_3 \\ I_3 \\ O_3 \end{bmatrix} \Delta \quad (48)$$

From the z_e dynamics, and because of $\partial_q \Psi|_{q=0} = 0$, this system has the following equilibrium point.

$$q^* = 0 \quad (49)$$

$$z^* = MR_z^{-1} \Delta \quad (50)$$

$$z_e^* = -K_e R_z^{-1} \Delta \quad (51)$$

Note that, contrary to the static controller, in presence of unknown disturbances, the origin is still the equilibrium in the position coordinates. It can be easily checked that for the case of the trigonometric controller, the equilibria becomes $q^* = (0, 0, a2\pi)$.

The question that now arises is about the stability of (48). A simple stability analysis is possible using the error coordinates $\tilde{z} = z - z^*$ and $\tilde{z}_e = z_e - z_e^*$. Shifting the energy function (32) to the new equilibria, we obtain

$$\tilde{H}_{de}(q, \tilde{z}, \tilde{z}_e) = \Psi(q) + \frac{1}{2} \tilde{z}^T M^{-1} \tilde{z} + \frac{1}{2} \tilde{z}_e^T K_e \tilde{z}_e, \quad (52)$$

and (48) can be written as

$$\begin{bmatrix} \dot{q} \\ \dot{\tilde{z}} \\ \dot{\tilde{z}_e} \end{bmatrix} = \begin{bmatrix} O_3 & J(q_3) & J(q_3) \\ -J^T(q_3) & -R_z & O_3 \\ -J^T(q_3) & O_3 & O_3 \end{bmatrix} \begin{bmatrix} \partial_q \Psi \\ M^{-1} \tilde{z} \\ K_e \tilde{z}_e \end{bmatrix} \quad (53)$$

This closed-loop system still has the desired structure (6), with a energy function, that has a minimum at (49)–(51). The asymptotic stability is automatically derived.

In a more realistic case, the disturbance could be considered as a vector that depends on time, that is, $\Delta = \Delta(t)$. Then, we can no longer study the stability of an equilibrium point, and we only can expect that the solution of the system becomes bounded because of the special structure of the closed-loop system, that can be seen as a forced oscillator with damping in the $\tilde{z}$ coordinates.

6 Simulations

In order to test the performance of the designed controllers we performed some numerical simulations. For this validation, we used the data of a supply ship from [2]. The (Bis-scaled non-dimensional [5]) matrices M and D are given by

$$M = \begin{bmatrix} 1.1274 & 0 & 0 \\ 0 & 1.8902 & -0.0744 \\ 0 & -0.0744 & 0.1278 \end{bmatrix}$$

and

$$D = \begin{bmatrix} 0.0358 & 0 & 0 \\ 0 & 0.1183 & -0.0124 \\ 0 & -0.0124 & 0.0308 \end{bmatrix}$$

Fig. 5 Simulation results: ship position trajectories in the q_1, q_2 plane, for the static quadratic controller with extra damping (solid line) and the original damping coefficient (dashed line)

The bias vector has been set to $b = (0.05, 0.05, 0.01)$. For all simulations, we considered that the initial conditions of the ship are $(q_1, q_2) = (-10, -10)$, and the heading angle $q_3 = 4$ rad, and the desired stabilisation position is the origin. Precisely, the starting heading angle is set greater than π to show the ability of the so-called trigonometric controller to stabilise to the closer minimum, in this case 2π .

6.1 Simulation results for the static controller

In this subsection, we present the simulation results for the static (quadratic and trigonometric) control laws (21) and (24), respectively. The gain matrices we used are $K = C = \text{diag}\{0.05, 0.05, 0.01\}$ and $R_p = \text{diag}\{0.75, 0.75, 0.1\}$. Precisely, for this system, we enlarged the damping (about one order of magnitude in the first and third components), to improve the performance. In Fig. 5, we show the trajectories of the same static controller (the quadratic case) with extra dissipation, setting R_p to the values proposed before, and keeping the original damping, $R_p = D$. This comparison justifies the use of the extra damping to obtain more suitable paths.

Fig. 6 shows the trajectories of the position coordinates q_1 and q_2 , and the heading angle, q_3 , of the quadratic and the trigonometric versions. Both controllers stabilise the ship at the desired position and angle. The notable point is the difference in the orientation profiles. While the quadratic controller stabilises the heading angle at $q_3 = 0$, the trigonometric controller does so at $q_3 = 2\pi$.

Fig. 7 compares the trajectories in the q_1, q_2 -plane for two versions of the static controller. In both cases, the performance is similar but, even the controllers for the q_1 and q_2 coordinates are the same (with the same gain values), the trajectories take different paths. This fact is associated with the different heading angle trajectories.

6.2 Simulation results in presence of disturbances

In this subsection, we present the simulation results in presence of disturbances. The key point is to show that the dynamic controllers proposed are able to reject unknown terms. For this scenario, we considered that the disturbance because of the bias term, b , is not available.

Fig. 6 Simulation results: ship position coordinates, q_1 and q_2 , and heading angle, q_3 , for the static quadratic (solid line) and trigonometric (dashed line) controllers

Fig. 8 Simulation results: ship position coordinates, q_1 and q_2 , and heading angle, q_3 , for the static and dynamic (quadratic and trigonometric) controllers

Fig. 7 Simulation results: ship position trajectories in the q_1q_2 -plane, for the static quadratic (solid line) and trigonometric (dashed line) controllers

Fig. 9 Simulation results: ship position trajectories in the q_1, q_2 plane, for the static and dynamic (quadratic and trigonometric) controllers

Consequently, the feedforward term, $J^T(q_3)\mathbf{b}$, is removed in all the tested controllers. The gain matrices for the static controllers, (21) and (24), are the same as in the previous subsection. The corresponding gain matrices used for the dynamic controllers, (43) and (47), are $R_z = \text{diag}\{0.75, 0.75, 0.4\}$, $K = C = \text{diag}\{0.05, 0.05, 0.025\}$ and $K_e = \text{diag}\{0.01, 0.01, 0.015\}$.

In Fig. 8, $\mathbf{q}$ trajectories for the four controllers are plotted. Clearly the dynamic controllers steer the ship to the desired equilibrium position, while the static controller fails to do so and have some steady-state error. This difference between the performance of the static and the dynamic controllers

can also be seen from the respective trajectory profiles; see Fig. 9.

7 Conclusions

A passivity-based approach called IDA-PBC is used to obtain a set of controllers for the dynamic positioning of a ship. This methodology is based on the port-Hamiltonian description, which gives a physical interpretation of the dynamical systems. Under this point of view, the controller design problem is addressed as to shape the energy function of the closed-loop system. After a general formulation we propose two different controllers: first with a quadratic

energy function and second, inspired by the physics of a pendulum, with a trigonometric energy function. Also, the presence of disturbances is studied and it turns out that the obtained static control laws do not stabilise the system at the desired position. This discrepancy is the starting point for a second set of controllers, which consists of a dynamic extension of the system that provides stability at the desired regulation point, also in presence of disturbances. Simulations are done to validate and compare the performance of the controllers designed.

It is worth to mention that the obtained control laws, with a general form of state feedback, can be easily converted to output feedback algorithms that only requires the position measurement. Furthermore, they exhibit a simple structure that can be interpreted as non-linear version of PID controllers.

Future work can be oriented in to determine other energy functions, $\Psi(\mathbf{q})$, to improve the performance, as well as to consider the optimisation of the resulting path. Further analysis depending on the nature of the disturbance vector (including the WF and wind models) are possible. Also, this work could be a starting point for a new design, using the port-Hamiltonian perspective, of the complete motion-control system (controller and observer) for a DP problem.

8 Acknowledgments

Shah Muhammad would like to express his gratitude to the Higher Education Commission (HEC) of Pakistan (Ref: 12-1/PM-OS/NETH/PHASE-IIB/2005), the Delft Institute of Applied Mathematics (DIAM-TU Delft), and Netherlands Universities Foundation for International Cooperation (NUFFIC). Arnau Dòria-Cerezo was partially supported by the Spanish government research project DPI2010-15110.

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