

Analysis of the torsional mode T(0,1) propagating in a bending pipe

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Abstract

In operation, some structural members are subjects to bending conditions. Velocity variations of guided waves propagating in a stressed specimen are expected due to the acoustoelasticity effect. In this work, a numerical study of the fundamental torsional mode is performed in a hollow cylinder subjects to bending stress. Bending produces different stresses along the horizontal axis of the pipe and through the specimen thickness. Therefore, stress magnitudes are dependent on the longitudinal axis distance and the radius. Acoustoelasticity predicts change in bulk velocities when the propagation is in a stressed waveguide. Thus, the estimation of the phase velocity of a guided wave in a bowing specimen is a complex task.

The objective of this study is to assess changes in the wavepacket dynamics due to stress variations in a bending environment. In the numerical analysis by using Finite Element Modeling (FEM), the stress gradient is imposed on the wave propagation analysis by importing the results of the quasi-static simulation (bending) to the explicit solution FEM scheme (Wave propagation).

Several bending's stresses behavior are simulated (variable and constant along the waveguide) and implemented to establish a relationship between the bending stress and the change in the phase magnitude velocity. Variations in velocity are determined in the time domain by comparing the current signals with the wavepacket without bending.

The importance of this study lies on the verification of the effects in the wave field (e.g. velocity variations) caused by the bending and determine if they are enough to be used to track stresses variations in the waveguide in an SHM system by using a pitch-catch configuration.

1. Introduction

Stresses in structures have great influence in the performance during the operation, affecting its strength, expected operational life and dimensional stability. Mechanical stresses are present in many real installations which can be subject to monitoring using guided waves. Guided waves in plates and cylindrical specimens are used to detect and locate material discontinuities in the investigated specimen. This technique has the potential to volumetrically explore the material covering long distance having low attenuation. The interaction of ultrasonic guided waves with discontinuities in structures is a widely studied topic.

Although the propagation of guided waves is described as a uniform motion with no acceleration, some factors may yield changes in velocities such as the variations in the thickness or the presence of stress in the waveguide, as predicted by the Acoustoelasticity effect. The study of ultrasonic guided wave propagation in stressed mediums is based mainly on the application of the acoustoelasticity principle. Acoustoelasticity is the stress influence in the acoustic bulk wave velocities i.e. shear and longitudinal velocities in elastic media.

On the other hand, the fundamental torsional mode has special characteristics which require attention. This mode is not dispersive, i.e. it preserves its shape as it propagates since phase and group velocities of the shear bulk wave do not change. Besides, as it will show ahead, the propagation velocity of this mode is the shear bulk velocity of the material. This condition unable this particular mode to be used to detect changes in the thickness but it offers the possibility to be utilized to track stress.

Most of the research on this topic (analysis of the effect of stress in the propagation of guided waves) has been focused mainly on the determination of the dispersion curves, or the velocity changes as a frequency function attributed to the acoustoelasticity effect. Some works are devoted to determine the load condition or the residual stress of the specimen based on the velocity of propagating of the waves (1-5). On the other hand, some researchers have studied the variations in the trace of the dispersion curves or the behavior of the different types of guided waves e.g. Rayleigh, Stone, Lamb (6) under different load settings (7). Some research has been centered on specific engineering applications such as bolted structural connections, grouted tendons, and steel stands, which have been aimed to monitoring using the acoustoelasticity effect (8-11).

The numerical analysis presented in this paper is aimed to study the influence of the bending stress in the propagation of the fundamental torsional mode for a cylindrical specimen. The simulation involves two different configurations to generate the bending stress: In the first one, the moment is variable along the waveguide (the bending produced for a force perpendicular to the beam). In the second one, the moment is constant and it is emulated by two bending moments perpendicular to the beam cross section.

2. Stress formulation

Bending stress can be produced in different ways, e.g. (i) by applying a perpendicular load to the beam and parallel to its cross-section. (ii) by applying bending moment perpendicular to the beam cross section as shown in Figures 1a and 1b respectively.

For the first case (Figure 1a), variations in the bending moment are obtained by changing the magnitude of the load located in the middle part of the cylindrical waveguide. Under this scenario, the cylinder can be treated as a beam with constant cross-sectional area which loads and reactions are applied perpendicular to its axis. It is assumed that loads and reactions are located in a simple plane (x,y plane). Due to the applied loadings, beams develop stress, internal shear force V and bending moment M that, in general, all of them vary from point to point along the axis of the beam and through the cross-section.

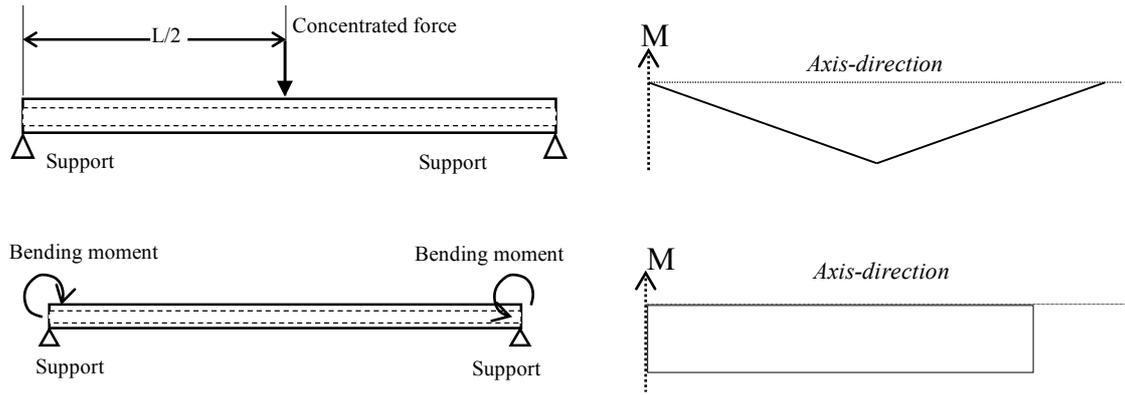


Figure 1. Moment diagrams in a cylindrical beam produced by a) concentrated force, b) bending moments

The maximum bending stress for a specific x coordinate is located in the outer y distance (exterior radius for the case of a homogeneous cylinder). In this way, the moment varies linearly with the x -coordinate until reaches its maximum value where the load is applied, in this case, in the middle of the beam.

For the second case (Figure 1b), the bending stress is obtained by applying bending moments perpendicular to the beam cross section at the ends of the cylindrical waveguide. In this case, the stress, internal shear force V and bending moment M developed in the beam, are constant along the cylinder.

3. Acoustoelasticity Effect

The acoustoelasticity theory establishes the mathematical relationship between ultrasonic bulk velocities and mechanical stresses in the studied material. In (12), it is considered uniaxial stress and derived expressions for changes in shear and longitudinal wave velocities as a function of applied stress for known material properties. Thus, for isotropic materials subject to uniaxial stress, in addition to the two Lamé constants, λ and μ , three additional constants, the Third Order Elastic Constants (TOEC), l, m, n are required to describe the relation between stress and velocity. The TOEC highly depends on the material processing, such as casting, rolling, or drawing.

The Acoustoelastic effect is small, typically of the order 0.001% per MPa of applied stress, for metals and it is influenced by the material structure (13). Although acoustoelasticity effect establishes the change of ultrasonic bulk velocities, this effect also has influence in the guided waves. Therefore, the wave field of guided waves propagating under stress not only is affected by the dispersion but also by the stress. In order to determine the change of velocity of the wave, the Time of Flight (TOF) have to be determined. Experimentally measures of TOF consider also the TOF induced by the elongation effect for the applied load. Therefore, in order to isolate acoustoelasticity effect, it is necessary to deduce this elongation from the TOF to characterize only the acoustoelastic effect.

4. Symmetric torsional waves propagating in cylindrical waveguides

The torsional modes are characterized by the fact that their displacement is primarily in $\hat{\theta}$ direction. Furthermore, the fundamental torsional mode belongs to a uniform twisting of the entire cylindrical waveguide. The highest order torsional modes exhibit more complicated behavior. However, the angular displacement is not constant through the radius of the cylinder. Different locations through the radius of the cylinder can twist in different directions and nulls of displacement can exist. In the studied case, we consider that the particle vibrations (displacements and velocities) for the torsional modes are located in a plane that is quasi-parallel to the surfaces of the layer. This is depicted in Figure 2, where the wave propagates in the z direction and the particle displacements are prescribed in $\hat{\theta}$ direction.

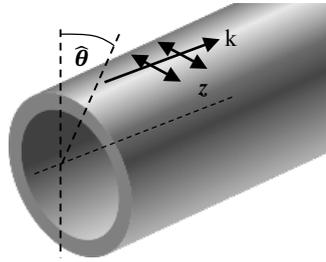


Figure 2. Schematic representation of the cylindrical waveguide.

The frequency equation for the torsional modes is derived from the motion equation (cylindrical coordinates) when no external stress is present in the boundaries of the waveguide. The deduction of this equation is omitted for space concerns but it can be consulted in some references such as (14-15). Focusing on the axisymmetric torsional guided waves, the frequency equation may be obtained by using the boundary condition $\tau_{r\theta} = 0$, resulting

$$J_2(\beta b)Y_2(\beta a) - J_2(\beta a)Y_2(\beta b) = 0, \quad (1)$$

$$\beta^2 = \frac{\omega^2}{C_2^2} - k^2, \quad (2)$$

where a and b are the inner and outer radius of the cylinder, J_2 and Y_2 are Bessel functions of first and second order respectively, C_2 is the shear bulk velocity and k is the wavenumber. The lowest axisymmetric torsional mode, $n = 0$, in which involves the rotation of each transverse section of the cylinder as a whole about its center is not adequately described by the Bessel equations (16). This mode belongs to $\beta = 0$ and, therefore using Equation 2 states:

$$\omega = kC_2. \quad (3)$$

As guided wave phase velocity is the relation between ω and k , Equation 3 clearly shows that the fundamental torsional mode propagates at a constant phase velocity, with no dispersion, that is equivalent to the bulk shear velocity of the material.

5. FEM approach

In order to investigate the influence of the mechanical bending stress in the propagation of T(0,1), a steel pipe of 1 inch schedule 40 (outer diameter of 33.4 mm and wall thickness of 3.38 mm) is modelled as a hollow cylinder with an axial length of 0.42 m. Changes in the bending stress are configured varying the magnitude of a concentrated force located in the middle part of the waveguide. To generate a constant moment, a couple of bending moments are applied at the left and right ends of the cylinder. Simulations are performed launching a 50 KHz modulated pulse through of the cylindrical specimen by axisymmetric surface loading. The material properties are: Density = 7830 kg/m³, Young's modulus (E) = 210 GPa and Poisson's ratio = 0.33. To ensure an adequate mesh refinement level, the minimum allowed inter-nodal length L_{min} is calculated, using as a reference the shortest wavelength, which in this case is shear wave speed (C_2). The meshing criterion proposed by (17) is implemented. Considering the frequency and the steel shear wave velocity, L_{min} is calculated as follows:

$$\frac{2\pi C_2}{\omega L_{min}} > 10 \quad (4)$$

where ω is the studied circular frequency. Considering a frequency of 50 kHz and a shear velocity of around 3200 m/s, the minimum element length results, approximately 6.4 mm. Therefore, seeds size of 2 mm can be considered as a sufficient mesh refinement.

In addition, the simulation is executed in two sequentially stages one static and the other dynamic. In the static step, two load settings are used, first, a concentrated force produces a variable moment along the cylinder, and in the second setting two bending moments are applied to generate a constant moment along the waveguide. The stress and strain fields resulting are used as predefined fields in the next stage. The first stage is executed using a standard step and the next stage by an explicit scheme. Explicit schemes are preferred as time marching process to simulate guided waves. In general, simulation accuracy can be increased with increasingly smaller integration time steps (ΔT) but punished by a higher computational cost. It is recommended to have at least five to ten time steps in one wavelength (14). A ΔT of 5 ns satisfy this criterion and it is used to solve the model. Meshing is performed by linear eight node brick element (C3D8). The torsional wave is produced by a shear load at the left end face of the cylinder by a 5 cycles Hanning-window tone burst of 50 kHz. The model is configured in such way the torsional wave freely propagates along the cylinder. The propagated pulse is captured 0.12 m ahead of the excitation surface. This distance assures unwanted reflections during the 200 milliseconds of the time period of simulation.

5. FEM approach

This section gives the results of a series of simulations conducted under the conditions described above. Slightly variations in amplitude are the only consistent and quantifiable observation in all torsional waves sensed when they are compared each other. Contrary, to the expressed by the acoustoelasticity effect, variations in the bending stress are not

enough to produce any significant change in the TOF of the torsional guided wave under the simulated conditions. On the other hand, as it is expected, no dispersion is observed and the velocity of propagation of the captured wavepacket was around of 1% of the material shear velocity. No other modes were detected in the simulations. Different moment scenarios are emulated, first, the normal condition is determined considering the absence of concentrated force in the middle. Later on, the magnitude of the concentrated force is increased for the rest of stressed scenarios in the gravity direction. The concentrated force and the corresponding maximum bending stress are presented as follows (Stress in parentheses): 100 N (4.21 MPa), 500 N (21 MPa), 1000 N (42.1 MPa), 2000 N (84.2 MPa) and finally, 10000 N (421 MPa). In the case of constant moments, the stressed scenarios are the following: 5 Nm (2.22 MPa), 10 Nm (4.44 MPa), 20 Nm (8.88 MPa) and 100 Nm (44.4 MPa). The test is configured in such way that the stresses cover the whole elastic region.

In Figure 3, it is presented a zoom view of the highest peak of the captured wavepacket by the simulated sensor. As it is noted in the Figure, amplitude changes are observed and small lags among the signals are just noticeable. Time trace changes are found by a cross-correlation analysis between nominal pulse and the simulated signals belonging to the stressed scenarios, as shown in Figure 4. In this figure it is indicated the variation of TOF of the signals obtained for the different loads with the one obtained from unstressed pipe. The velocity resolution obtained for the simulated cases is around 8.5 m/s, which belongs to 0.26% change of velocity. This variation is presented for bending stresses above of the 42.1 MPa (around 20% of yield strength).

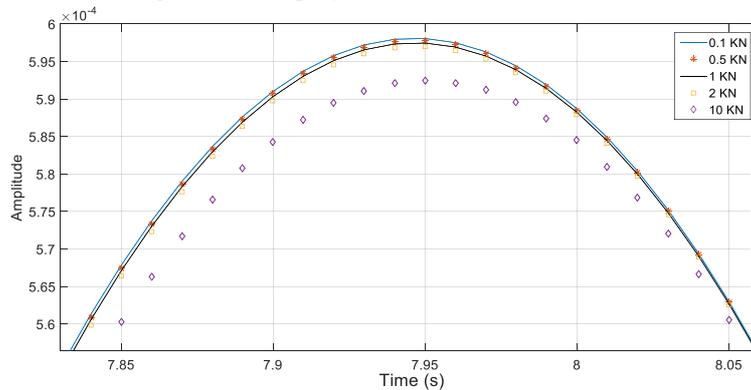


Figure 3. Zoom view of the highest peak of the propagated T(0,1) for variable bending moment using concentrated loads from 100 N to 10 KN.

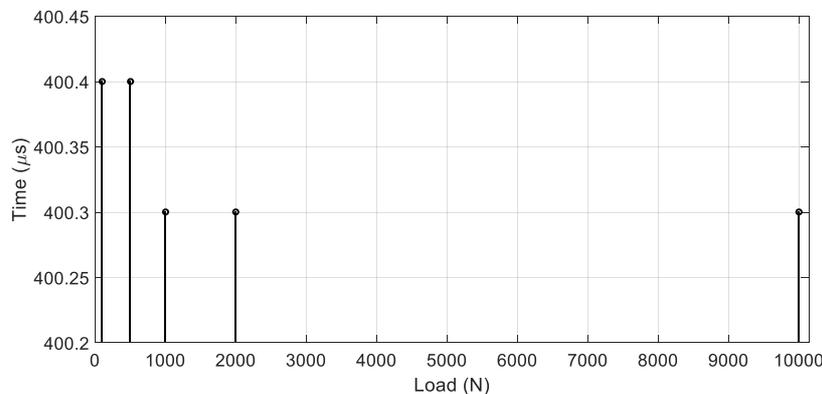


Figure 4. Delay among the different stressed scenarios for variable bending moment

In the case of the uniform bending moment along the cylinder, a nonlinear amplitude change is observed in the wave field as shown in Figure 5. In the first peak, the higher moment, the higher amplitude, but in the second peak, the effect is opposite. In addition, the propagated pulse is deformed compared with the as shown in Figure 6.

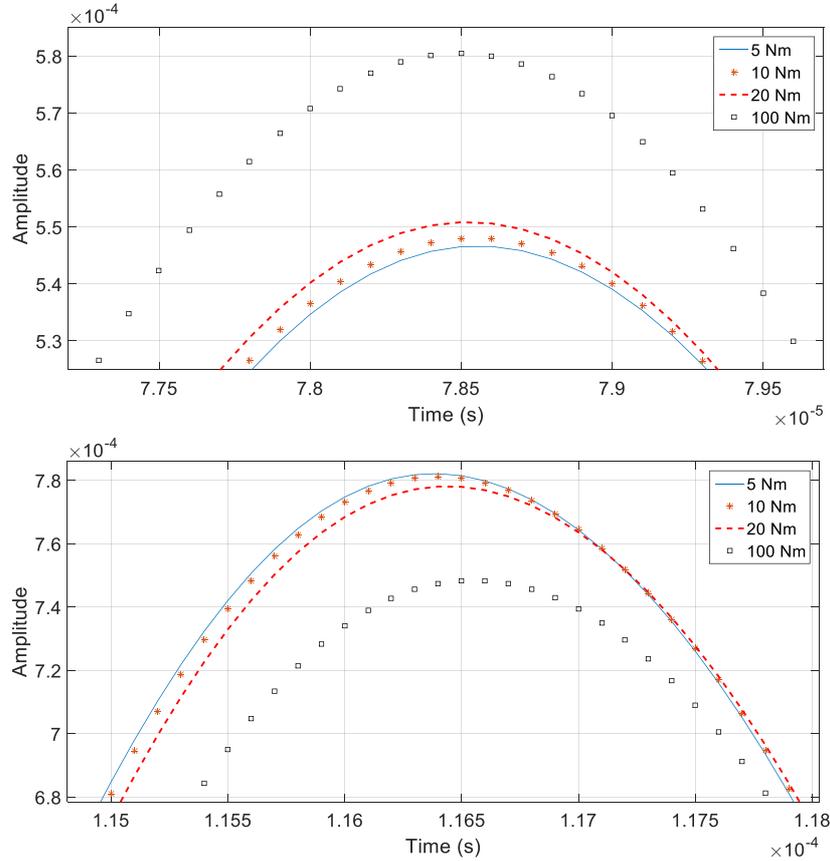


Figure 5. Zoom view of the highest peaks of the wavepacket of the simulated scenarios for a constant bending moment

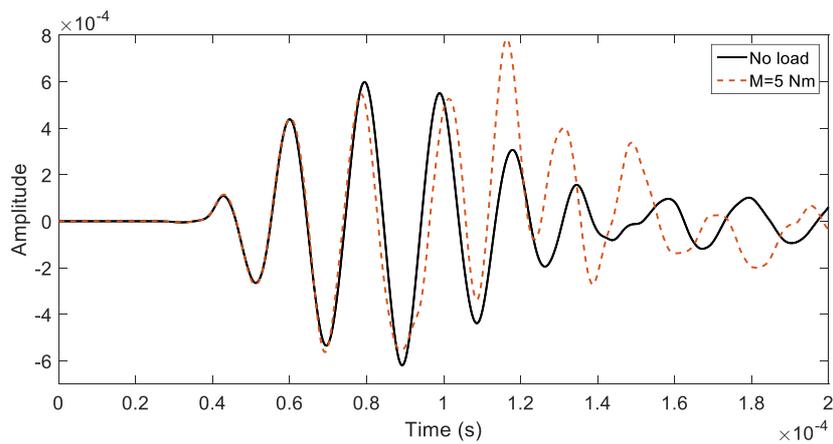


Figure 6. Distortion of the propagated pulse in presence of a constant bending moment.

Finally, no delays or velocity change are found by the correlation.

6. Conclusions

The effects of bending moment in the propagation of the fundamental torsional mode are studied by means of numerical simulations. FEM simulations were conducted for different magnitudes of concentrated forces and bending moments. It was observed that the influence of bending stress levels is few noticeable in the torsional wave velocity of the fundamental mode. On the other hand, notorious changes in the amplitude of the propagated wave were detected in all studied scenarios. This observation has more relevance in practical applications.

The above variations in amplitude of the wave could be attributed to the fact that boundary conditions influence the magnitude of the simulated torsional guided wave. This effect is severe in the case of the uniform bending moment because the presence of stress in the boundary condition is present all pathway long of the propagation.

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References

1. Chen F, Wilcox PD. The effect of load on guided wave propagation. *Ultrasonics* 2007;47:111–22. doi:10.1016/j.ultras.2007.08.003.
2. Shi F, Michaels JE, Lee SJ. An ultrasonic guided wave method to estimate applied biaxial loads. *Rev Prog Quant NDE* 2012;1430:1567–74. doi:10.1063/1.4716401.
3. Nikitina NY, Ostrovsky L a. An ultrasonic method for measuring stresses in engineering materials. *Ultrasonics* 1998;35:605–10. doi:10.1016/S0041-624X(97)00154-6.
4. Chaki S, Bourse G. Stress level measurement in prestressed steel strands using acoustoelastic effect. *Exp Mech* 2009;49:673–81. doi:10.1007/s11340-008-9174-9.
5. Zhu ZH, Post MA, Meguid SA. The Potential of Ultrasonic Non-Destructive Measurement of Residual Stresses by Modal Frequency Spacing using Leaky Lamb Waves. *Exp Mech* 2012;52:1329–39. doi:10.1007/s11340-011-9585-x.
6. Guz AN. Elastic waves in bodies with initial (residual) stresses. *Int Appl Mech* 2002;38:23–59. doi:10.1023/A:1015379824503.
7. Gandhi N. Determination of dispersion curves for acoustoelastic lamb wave propagation. Georgia Institute of Technology, 2010.
8. Wang T, Song G, Liu S, Li Y, Xiao H. Review of bolted connection monitoring. *Int J Distrib Sens Networks* 2013;2013. doi:10.1155/2013/871213.
9. Beard MD, Lowe MJS, Cawley P. Ultrasonic Guided Waves for Inspection of Grouted Tendons and Bolts. *J Mater Civ Eng* 2003;15:212–8. doi:10.1061/(ASCE)0899-1561(2003)15:3(212).
10. Chaki S, Bourse G. Guided ultrasonic waves for non-destructive monitoring of the stress levels in prestressed steel strands. *Ultrasonics* 2009;49:162–71. doi:10.1016/j.ultras.2008.07.009.

11. di Scalea FL, Rizzo P. Stress measurement and defect detection in steel strands by guided stress waves. *J Mater Civ Eng* 2003;15:219. doi:10.1061/(ASCE)0899-1561(2003)15:3(219).
12. Hughes D., Kelly JL. Second-Order Elastic Deformations of Solids. *Phys Rev* 1953;92:1145–50.
13. Stobbe D. Acoustoelasticity in 7075-T651 Aluminum and Dependence of Third Order Elastic Constants on Fatigue Damage. Georgia Institute of Technology, 2005. doi:http://hdl.handle.net/1853/7184.
14. Rose JL. *Ultrasonic Waves in Solid Media*. Cambridge University Press; 2014. doi:10.1017/CBO9781107273610.
15. [Viola E, Marzani A. *Exact Analysis of Wave Motions in Rods and Hollow Cylinders* n.d.
16. Armenakas A, Gazis D, Herrmann G. *Free Vibrations of Circular Cylindrical Shells*. 1st ed. London: Pergamon Press; 1969.
17. Galán JM, Abascal R. Numerical simulation of Lamb wave scattering in semi-infinite plates. *Int J Numer Methods Eng* 2002;53:1145–73. doi:10.1002/nme.331
 TH Tan, “Reciprocity relations for scattering of plane, elastic waves”, *J. Acoust. Soc. Am* 61(4), pp 928-931, 1977.