Calculation of the inductance of conductive nonmagnetic conductors by means of FEM simulations

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Abstract

This article analyzes the inductance of different conductive nonmagnetic conductors' configurations under alternating current (AC) supply. The inductance is a key design parameter in tracks of electronic devices, power transmission and distribution systems, and lightning, grounding, and bonding systems. Inductance highly relies on the problem geometry, and under AC supply, it is also influenced by skin and proximity effects. The inductance significantly determines voltage drop in conductors, thus increasing reactive power consumption and limiting conductors' ampacity. Although this is an important topic, it is seldom studied in detail in undergraduate and even in graduate physics and engineering studies. To this end, this paper compares the results provided by existing closed formulas for simple conductors' configurations with those attained through two-dimensional finite element method (FEM) simulations. FEM-based simulations are increasingly being incorporated in the syllabuses of graduate and undergraduate courses due to their accurate solutions and flexibility, since FEM models can be applied in a wide range of electrical frequencies and configurations, some of which do not have an analytical solution. The FEM-based approach presented in this paper has been found a valuable complement to the lectures and assignments in electricity courses for engineering students.

1. INTRODUCTION

Technological universities are working hard to improve students' practical knowledge and skills for real industry situations ¹. However, the university often has to face limitations related to the lack of experimental equipment ¹ that can be partially overcome by means of computer simulations. Such simulations can help students to improve learning performance because this approach enables a better understanding of electrical concepts in a more flexible and interactive manner. Therefore, computer-aided learning tools can contribute in improving the education quality, especially in scientific and engineering disciplines and can be a valuable complement in both practical and lecture sessions. Computer-aided tools also allow reducing some difficulties and costs related to experimental tests ². During the last years the finite element method (FEM) has been introduced in regular electrical engineering courses due to its flexibility and accuracy ³. This paper aims at fostering students' motivation and interest in the topic of determining the inductance of a given configuration by confirming the calculations proposed during the lecture sessions by means of the application of existing formulas and a further validation by means of FEM simulations.

Many manufacturers are currently applying simulation tools like FEM to reduce costs, and development time, while increasing productivity and product quality. Due to the accurate results, the incorporation of FEM into the design process, allows designer engineers to verify that the product will fulfill the clients' specifications during the design and optimization stages. This strategy allows accelerating the product development stage, and in case that the new design do not meet expectations, it can be optimized before the prototyping phase, thus saving time and R&D associated costs.

The society is demanding new learning practices to overcome several teaching application difficulties. These learning methods are focused to boost students' interest and improve knowledge retention, by encouraging students to participate more actively in the learning activities. One possible approach is to instruct students to use advanced methods applied in industry, such as FEM, since it is widely applied to analyze engineering problems and to design different devices in different industrial sectors ⁴.

Current engineering education is always evolving and adapting to new social and market needs, and thus, active learning methods are appealing. However, introducing new competences into the already concentrated engineering courses is a challenging problem for many instructors ⁴.

This paper describes a proposal to improve student learning. It is done by applying active learning sessions to determine the inductance of different conductors' geometries by means of FEM simulations and analytical expressions. This experience has been carried out to meet the requirements of many industries, since they require graduate engineers with different skills such as simulation, analysis or optimal design capability.

It is well known that conductors excited with a high-frequency source are affected by the skin effect ⁵⁻⁷, and therefore the electric current tends to flow within the external layer of its cross section, since the eddy currents induced and the accompanying magnetic fields modify the electrical current distribution within the conductor. This nonuniform current distribution across the conductor's cross section is further boosted due to the influence of nearby conductors ⁸ due to the proximity effect. Consequently, conductor's resistance and inductance ⁹ are greatly influenced. The effective resistance of the conductor increases with frequency, whereas the internal inductance falls ¹⁰, thus altering the overall conductor's impedance, the voltage drop and the operating temperature. The behavior of several systems, including power cables and lines, lightning, grounding and bonding protection systems is greatly influenced by their own inductance, especially at high-frequency operation. For example protective grounding, bonding and lightning systems must offer a low inductance path, since high frequency impulse currents generate a fast voltage drop along the conductor, which can cause electrical and safety problems ¹¹, thus compromising the efficacy of such systems. Although the inductance and the resistance of a coil can be determined experimentally by measuring the phase shift between the voltage and the current through the coil ¹², the calculation of the inductance in conductors' systems, which depends upon the analyzed geometry ¹³, is a very interesting subject. However, analytical methods to calculate the inductance can only deal with the simplest geometries ¹⁴ and most of the available analytical formulas for more complex geometries neglect eddy currents effects ¹⁵. As a consequence, it is required to apply numerical methods specifically intended for solving this type of problems, such as FEM-based approaches, which offer flexibility and accuracy ^{6,7} while allowing to deal with a wide range of frequencies and geometries. This paper calculates the inductance of different conductors' configurations in a wide frequency range under sinusoidal steady-state AC supply by applying FEM simulations. The results provided by such simulations are compared with those attained by applying analytical formulas found in the technical literature. It is worth noting that the FEM approach presented in this work can be a useful educational tool

for both graduate and undergraduate engineering and physics students to analyze the influence of frequency and geometry on the inductance.

2. INDUCTANCE OF AN ISOLATED INFINITELY LONG RECTILINEAR CONDUCTOR

At high frequency operation, because of the skin effect, the current within a conductor tends to flow on the outer shell of the conductor, therefore increasing the effective resistance and decreasing the effective internal inductance¹⁶. As shown in Fig. 1, at low frequency operation, that is, when the skin effect is negligible, the current is uniformly distributed across the conductor's cross section, thus generates a circular magnetic flux density field B within the whole cross section. However, at very high frequency operation, the current and the magnetic flux density field are confined in the periphery of the conductor because of the skin effect⁸, so both the magnetic energy stored within the conductor and the internal inductance are very low. The proximity effect can also affect the current distribution across the cross section of adjacent conductors, as shown in Figs. 1.d and 1.e. According to Ampere's law, the surface current does not generate magnetic field inside the conductor, thus decreasing the internal conductor's inductance¹⁷.

Fig. 1. a) Cross-section of an isolated round solid conductor carrying a current I flowing into the drawing. b) Isolated aluminum conductor (radius = 3 mm, 50 Hz). c) Isolated aluminum conductor (radius = 3 mm, 10 kHz). d) Two conductors carrying currents of the same polarity (radius = 3

mm, distance between centers = 10 mm, 10 kHz). e) Two conductors carrying currents of opposite polarity (radius = 3 mm, distance between centers = 10 mm, 10 kHz). f) Total domain applied in the FEM simulations ($R = 5000$ m) and triangular mesh. g) Detail of the triangular mesh around the conductors.

Although inductance is often associated with closed loops, an isolated straight or rectilinear conductor's segment also generates an inductance, assuming that the rectilinear segment generates a loop with the return segment placed at infinity. This inductance is known as partial inductance, and it is calculated as the magnetic flux that penetrates the surface between the segment and infinity, divided by the electric current flowing through this segment¹⁸. By applying this concept, the partial inductance of a very long and isolated conductor can be calculated¹⁹, although strictly speaking there is no loop current. The concept of partial inductance allows explaining, for example, the voltage induced in a grounding conductor, since it is mainly due to the voltage induced in the partial inductance due to a sudden transient current.

Therefore, in this paper the concept of inductance is associated to configurations forming a closed loop, whereas partial inductance is associated to configurations which do not form a closed loop, such as an isolated conductor or open return circuits.

The total partial inductance of the isolated conductor is therefore calculated from the addition of the internal and external partial inductances¹⁴. The external inductance, which is the main component²⁰, is due to the magnetic field distribution outside the conductor, whereas the internal inductance is owing to the distribution of the magnetic field within the conductor²¹. The external inductance is almost independent of the supply frequency, as it will be shown in next section, and it dominates the total inductance, especially at high frequency operation²².

Inductance in a loop can be calculated from (1),

$$L = \phi / I \quad (1)$$

ϕ being the total magnetic flux that penetrates the loop surface area and I the electric current in the loop¹⁴. From (1) it is deduced that the inductance in nonmagnetic regions is mainly influenced by the area bounded by the loop current, so it is only influenced by the system's geometry, thus being independent of the amount of electrical current.

This section analyzes rectilinear conductors with isotropic nonmagnetic material properties and uniform cross sections.

2.1 Infinitely long isolated rectilinear round conductor

From Maxwell's equations, a plane wave in a good conducting media can be described by the diffusion equation ²³,

$$\nabla^2 E - \sigma\mu \frac{\partial E}{\partial t} = 0 \quad (2)$$

Since the conduction current satisfies $\vec{J} = \sigma\vec{E}$ and assuming $J(t) = Je^{j\omega t}$, Eq. (2) can be written as ²⁴,

$$\nabla^2 J - j\omega\sigma\mu J = 0 \quad (3)$$

By supposing that the conductor has cylindrical symmetry and the current is directed towards the z axis and varies only through the radial axis due to the skin effect, so $\vec{J} = J(\rho)\hat{z}$, Eq. (3) can be transformed into cylindrical coordinates as,

$$\frac{d^2 J}{d\rho^2} + \frac{1}{\rho} \frac{dJ}{d\rho} - j\omega\sigma\mu J = 0 \quad (4)$$

Which is a Bessel differential equation, whose solution is given by ²⁴,

$$J(\rho) = J_o \cdot \frac{ber(\sqrt{2}\rho/\delta) + bei(\sqrt{2}\rho/\delta)}{ber(q) + bei(q)} \quad 0 \leq \rho \leq a \quad (5)$$

J_o being the current density at the surface ($\rho = a$, a being the radius of the conductor), $q = \sqrt{2}a/\delta$, and $\delta = 1/\sqrt{\pi \cdot f \cdot \mu_r \cdot \mu_0 \cdot \sigma}$ the skin depth, σ and μ_r being, respectively, the electrical conductivity and the relative permeability of the conductor's material, μ_0 the permeability of free space and f the electrical frequency. Note that ber and bei are the real and imaginary components of the first kind zero-order Kelvin functions ²⁵, whereas bei' and ber' are their derivatives. The series expansion of the Kelvin functions can generate an arithmetic overflow of the computer's capacity for large values of q , i.e. when operating at very high frequency or when dealing with large conductor's cross sections.

The current is obtained by integrating the current density over the conductor's cross section,

$$I = \int_0^{2\pi} \int_0^a J(\rho) d\phi d\rho = \int_0^{2\pi} \int_0^a J_o \cdot \frac{ber(\sqrt{2}\rho/\delta) + bei(\sqrt{2}\rho/\delta)}{ber(q) + bei(q)} d\phi d\rho \quad (6)$$

Thus resulting in,

$$I = \sqrt{2}\pi a \delta J_o \cdot \frac{ber'(q) + jbei'(q)}{ber(q) + bei(q)} \quad (7)$$

As shown in Fig. 2, the phasor voltage V between the ends of the conductor of length l can be calculated from the line integral of the electric field along the path $a-b$ on the conductor's surface,

$$V = -\int_a^b \vec{E}_0 \cdot d\vec{z} = -\int_l^0 E_o \cdot dz = E_o \cdot l = J_o \cdot l / \sigma \quad (8)$$

Fig. 2. Isolated round solid conductor of radius a .

Finally, the impedance of the conductor is determined as,

$$Z = \frac{V}{I} = \frac{J_o \cdot l / \sigma}{\sqrt{2\pi a \delta J_o} \cdot \frac{ber'(q) + jbei'(q)}{ber(q) + bei(q)}} = \frac{j}{\sqrt{2\pi a \delta \sigma}} \cdot \frac{ber(q) + bei(q)}{ber'(q) + jbei'(q)} \cdot l \quad (9)$$

Therefore, the internal partial inductance of the conductor per unit length is calculated as ^{26,27},

$$L_i / l = \text{imag} \left(\frac{Z}{2\pi f} \right) / l = \frac{\mu_o}{2\pi q} \cdot \frac{ber(q) \cdot ber'(q) + bei(q) \cdot bei'(q)}{[ber'(q)]^2 + [bei'(q)]^2} \text{ [H/m]} \quad (10)$$

The external partial inductance is due to the magnetic flux surrounding the conductor. To calculate this inductance, an imaginary cylinder of radius R is considered, as shown in Fig. 3.

Fig. 3. Imaginary area ($a \leq \rho \leq R$) applied for calculating the external partial inductance.

The magnetic field generated by the conductor at any external point situated at a radius ρ is given by,

$$B(\rho) = \frac{\mu_o \cdot i}{2\pi \rho} \quad (11)$$

The external partial inductance of an infinitely long round conductor of radius a can be calculated as ²⁸,

$$L_e / l = \phi_e / i = \int_a^R \frac{\mu_0}{2\pi\rho} d\rho = \frac{\mu_0}{2\pi} \ln(R/a) \text{ [H/m]} \quad (12)$$

R being the radius of the imaginary cylinder, which is shown in Fig. 3. However, Eq. (12) depends on the particular value of the radius R selected. This is because Eq. (12) ignores the influence of the return current. In most practical configurations the complete circuit including the return path must be considered²⁹. The total conductor's partial inductance is obtained from the addition of the external and internal inductances.

2.2 Infinitely long isolated rectilinear tubular conductor

Following a similar procedure that the one applied to obtain Eq. (10), the internal partial inductance per unit length of a tubular conductor with external remote return, as shown in Fig. 4, results in^{26,30},

$$L_{i,ac} / l_u = \text{imag} \left[j \cdot \frac{m}{2\pi\sigma a_2} \cdot \frac{\text{ber}(m \cdot a_2) + j \cdot \text{bei}(m \cdot a_2) - C \cdot \text{ker}(m \cdot a_2) - jC \cdot \text{kei}(m \cdot a_2)}{\text{ber}'(m \cdot a_2) + j \cdot \text{bei}'(m \cdot a_2) - C \cdot \text{ker}'(m \cdot a_2) - jC \cdot \text{kei}'(m \cdot a_2)} \right] / (2\pi f) \text{ [H/m]} \quad (13)$$

where a_1 and a_2 are the inner and outer radius of the conductor, $C = \frac{\text{ber}'(m \cdot a_1) + j \cdot \text{bei}'(m \cdot a_1)}{\text{ker}'(m \cdot a_1) + j \cdot \text{kei}'(m \cdot a_1)}$ and

$$m = (2 \cdot \pi \cdot f \cdot \sigma)^{1/2}.$$

Fig. 4. Tubular conductor of circular cross-section

The external partial inductance per unit length of this configuration is obtained from Eq. (12) as,

$$L_e / l = \frac{\mu_0}{2\pi} \cdot \ln(R/a_2) \text{ [H/m]} \quad (14)$$

3. INDUCTANCE OF A RETURN CIRCUIT OF INFINITELY LONG RECTILINEAR CONDUCTORS

When considering a return circuit, the proximity effect cannot be neglected, that is, the influence of the magnetic field generated by one conductor on the current density distribution of the other.

Since this is a more complex problem, most of the “exact” formulas found in the literature cannot deal with this effect, and therefore a numerical tool such as FEM becomes essential to obtain accurate results.

3.1 Infinitely long return circuit of rectilinear round conductors

The total partial internal inductance of two parallel infinitely long conductors carrying opposite currents (see Fig. 5) is two times that shown in Eq. (10),

$$L_i / l = \frac{\mu_o}{\pi q} \frac{ber(q) \cdot ber'(q) + bei(q) \cdot bei'(q)}{[ber'(q)]^2 + [bei'(q)]^2} \text{ [H/m]} \quad (15)$$

Eq. (15) takes into account the skin effect. However, it does not consider the proximity effect, thus being an approximate formula.

Fig. 5. Return circuit composed of two round conductors of infinite length.

The per unit length external partial inductance of the return circuit under the assumption $D \gg a$ is two times that of a single conductor, thus resulting in ³¹,

$$L_{e,2c} / l = \frac{\mu_0}{\pi} \ln\left(\frac{D}{a}\right) \text{ [H/m]} \quad (16)$$

3.2 Infinitely long tubular round conductor with internal return

According to Russell ³², the per unit length partial internal inductance of a tubular conductor with inner return (see Fig. 6), is given by the exact formula,

$$L_{i,2c} = \frac{m}{4\pi^2 \cdot \sigma \cdot r_1 \cdot Y(mr_1)} [ber(mr_1) \cdot ber'(mr_1) + bei(mr_1) \cdot bei'(mr_1)] + \frac{m \cdot ber'(mr_1)}{4\pi^2 \cdot \sigma \cdot r_1 \cdot Y(mr_1)} [M \cdot ber(mr_2) + N \cdot bei(mr_2) + O \cdot ker(mr_2) + P \cdot kei(mr_2)] + \frac{m \cdot bei'(mr_1)}{4\pi^2 \cdot \sigma \cdot r_1 \cdot Y(mr_1)} [M \cdot bei(mr_2) - N \cdot ber(mr_2) + O \cdot kei(mr_2) - P \cdot ker(mr_2)]$$

(17)

where,

$$m = (2\pi f \mu_0 \sigma)^{1/2}$$

$$S(mx) = ber'(mx) \cdot ker'(mx) + bei'(mx) \cdot kei'(mx)$$

$$T(mx) = \text{bei}'(mx) \cdot \text{ker}'(mx) - \text{ber}'(mx) \cdot \text{kei}'(mx)$$

$$Y(mx) = \text{ber}'^2(mx) + \text{bei}'^2(mx)$$

$$M = \frac{-O \cdot S(mr_3) + P \cdot T(mr_3)}{Y(mr_3)}$$

$$N = \frac{-O \cdot T(mr_3) - P \cdot S(mr_3)}{Y(mr_3)}$$

$$O = -\frac{(r_1 / r_2) \cdot Y(mr_3) \cdot [Y(mr_3) \cdot S(mr_2) - Y(mr_2) \cdot S(mr_3)] \cdot [\text{bei}'(mr_1) \cdot \text{bei}'(mr_2) + \text{ber}'(mr_1) \cdot \text{ber}'(mr_2)]}{[Y(mr_3) \cdot S(mr_2) - Y(mr_2) \cdot S(mr_3)]^2 + [Y(mr_2) \cdot T(mr_3) - Y(mr_3) \cdot T(mr_2)]^2} +$$

$$+ \frac{(r_1 / r_2) \cdot Y(mr_3) \cdot [Y(mr_2) \cdot T(mr_3) - Y(mr_3) \cdot T(mr_2)] \cdot [\text{ber}'(mr_1) \cdot \text{bei}'(mr_2) - \text{bei}'(mr_1) \cdot \text{ber}'(mr_2)]}{[Y(mr_3) \cdot S(mr_2) - Y(mr_2) \cdot S(mr_3)]^2 + [Y(mr_2) \cdot T(mr_3) - Y(mr_3) \cdot T(mr_2)]^2}$$

$$P = -\frac{(r_1 / r_2) \cdot Y(mr_3) \cdot [Y(mr_2) \cdot T(mr_3) - Y(mr_3) \cdot T(mr_2)] \cdot [\text{ber}'(mr_1) \cdot \text{ber}'(mr_2) + \text{bei}'(mr_1) \cdot \text{bei}'(mr_2)]}{[Y(mr_3) \cdot S(mr_2) - Y(mr_2) \cdot S(mr_3)]^2 + [Y(mr_2) \cdot T(mr_3) - Y(mr_3) \cdot T(mr_2)]^2} -$$

$$- \frac{(r_1 / r_2) \cdot Y(mr_3) \cdot [Y(mr_3) \cdot S(mr_2) - Y(mr_2) \cdot S(mr_3)] \cdot [\text{ber}'(mr_1) \cdot \text{bei}'(mr_2) - \text{bei}'(mr_1) \cdot \text{ber}'(mr_2)]}{[Y(mr_3) \cdot S(mr_2) - Y(mr_2) \cdot S(mr_3)]^2 + [Y(mr_2) \cdot T(mr_3) - Y(mr_3) \cdot T(mr_2)]^2}$$

Fig. 6. Inner return tubular conductor with radiuses a_1 , a_2 and a_3 .

The partial external inductance of this geometry is given by ³³,

$$L_{e,2c} / l_u = \frac{\mu_0}{2\pi} \cdot \ln \left(\frac{a_2}{a_1} \right) \quad [\text{H/m}] \quad (18)$$

Finally, the total partial inductance can be calculated by adding Eq. (17) and Eq. (18).

4. INDUCTANCE OF A CIRCULAR LOOP OF ROUND CONDUCTOR

There is not an exact closed-form solution for the inductance of a round conductor forming a circular loop, which takes into account the effect of the eddy currents induced. However, an approximate formula for the total inductance which assumes an azimuthal current in a ring of major radius R with circular cross section of radius a , as shown in Fig. 7, is given by ^{34,35}

$$L = \mu_0 \cdot R \cdot \left\{ \left[1 + \frac{2y+1}{8} \cdot (a/R)^2 \right] \cdot \ln \frac{8R}{a} - \frac{7}{4} + \frac{(y-1) \cdot (y-2/3)}{16} \cdot (a/R)^2 \right\} \quad [\text{H/m}] \quad (19)$$

where $y = 0$ for uniform current distribution, that is, low frequency operation, whereas $y = -1$ corresponds to the natural current distribution.

Fig. 7. Round conductor forming a circular loop.

5 RESULTS

In this section FEM simulations are taken as the reference method, due to the flexibility and accuracy they provide⁶, and because in the technical literature it is difficult to find internationally recognized experimental inductance measurements of the simple geometries analyzed in this work. Therefore, the results provided by the analytical formulas detailed in Sections II and III are compared with the results obtained from FEM simulations by using the free-license two-dimensional FEMM package³⁶.

FEM calculates the magnetic energy stored in a region of a material by means of the integral³⁷,

$$W_{mag} = \frac{1}{2} \iiint_v B \cdot H dv \quad (20)$$

The partial inductance of an isolated nonmagnetic rectilinear conductor can be calculated from the magnetic energy as,

$$W_{mag} = \int_0^i L \cdot i \cdot di = \frac{1}{2} \cdot L i^2 \quad (21)$$

Finally, by equaling Eq. (20) and Eq. (21), the partial inductance L is obtained through FEM simulations. The internal or external partial inductances can be evaluated by selecting the appropriate region in which the stored magnetic energy is calculated, as shown in Fig. 3. In order to account for the effect of the partial external inductance, a sufficient large air region around the analyzed geometry must be selected, the magnetic vector potential A being set to zero in the boundaries of such region, as shown in Fig. 3.

5.1 Inductance of an infinitely long isolated round conductor

Solid round conductors are used in many applications, including grounding, bonding and lightning protection systems. Therefore, it is interesting to analyze the frequency dependency of the inductance.

Figs. 8 show the dependence of the partial inductance with the frequency in the 0 Hz – 1 MHz range for an AWG-0000 round conductor of copper with radius $a = 5.842$ mm.

Fig. 8. Infinitely long isolated round conductor. a) Total partial inductance per unit length obtained from exact analytical formulas given by Eq. (10) + Eq. (12) and FEM simulations of a copper round solid conductor type AWG-0000 with radius $a = 5.842$ mm (107.2 mm^2) calculated within a cylindrical region with an outer radius $R = 5000$ m. b) External and internal partial inductances (FEM results).

Results from Figs. 8 show that FEM provides very accurate results, since the mean error in Fig.8a is about 1.0% (0.2% for the internal partial inductance). The small error between the results of FEM and the “exact” analytical equations depends on the size of the mesh, the “infinite” domain simulated ($R = 5000$ m) or the boundary conditions, among others.

Fig. 8b also proves that the main component of the partial inductance is the external inductance, which is independent of the frequency, since it is not affected by the eddy currents.

5.2 Inductance of an infinitely long isolated tubular conductor

Figs. 9 show the dependence of the partial inductance with the frequency in the 0 Hz – 1 MHz interval for a tubular round conductor of copper with inner and outer radius, $a_1 = 5$ mm and $a_2 = 19.64$ mm, respectively.

Fig. 9 Infinitely long isolated tubular conductor. a) Total partial inductance per unit length obtained from exact analytical formulas given by Eq. (13) + Eq. (14) and FEM simulations of a round tubular conductor when the current returns outside the conductor remotely with $a_1 = 5$ mm and $a_2 = 19.64$ mm calculated within a cylindrical region with an outer radius $R = 5000$ m. b) External and internal partial inductances (FEM results).

Results from Figs. 9 show that FEM results provide very accurate results, since the mean error in Fig. 9(a) is about 0.3%. Fig. 9(b) proves that the main component of the partial inductance is, once more, the external partial inductance.

5.3 Inductance of a return circuit of infinitely long round conductors

In this subsection a return circuit composed of two conductors of circular cross section is analyzed. Figs. 10 show the evolution of the partial inductance as predicted by Eq. (15) + Eq. (16) and FEM simulations in the frequency interval 0 Hz – 1 MHz for a return circuit of circular cross section of 200 mm^2 ($a = 7.979 \text{ mm}$), in which the two conductors are axially spaced $D = 50 \text{ mm}$.

Fig. 10 Return circuit of infinitely long round conductors. a) Total partial inductance per unit length obtained from exact analytical formulas given by Eqs. (15) and (16) and FEM simulations of a return circuit composed of two round conductors ($a = 7.979 \text{ mm}$, $A = 200 \text{ mm}^2$) axially spaced $D = 50 \text{ mm}$. b) External and internal partial inductances (FEM results).

Results from Figs. 10 show a mean difference between FEM results and Eqs. (15) and (16) of 1.6%. It must be pointed out that Eqs. (15) and (16) do not take into account the proximity effect,

which is more pronounced when decreasing the spacing between conductors. In this case, the analytical formulas given by Eqs. (15) + (16) generate a numerical overflow when dealing with frequencies above 0.1 MHz.

5.4 Inductance of an infinitely long tubular conductor with internal return

Coaxial conductors are applied in numerous applications such as communication systems, radar or television. Figs. 11 show the change in the partial inductance predicted by Eq. (17) + Eq. (18) and FEM simulations in the interval 0 Hz – 1 MHz for a return circuit consisting of a tubular conductor with radiuses $a_1 = 5$ mm, $a_2 = 15$ mm and $a_3 = 17$ mm.

Fig. 11 Infinitely long tubular conductor with internal return. a) Total partial inductance per unit length obtained from the “exact” analytical formulas (17) + (18) and FEM simulations of a coaxial conductor with inner return and radiuses $a_1 = 5$ mm, $a_2 = 15$ mm, and $a_3 = 17$ mm (see Fig. 6). b) External and internal partial inductances (FEM results).

Figs. 11 show that Eq. (17) generates a numerical overflow when dealing with frequencies above 0.1 MHz, for the selected geometries. The average difference between FEM results and Eq. (17) + Eq. (18) is 0.5 % for the analyzed geometry.

5.5 Inductance of a circular loop of round conductor

In this subsection, the inductance of a circular loop of round conductor is analyzed for a particular geometry in which $a = 5$ mm and $R = 20$ mm, as detailed in Figs. 12.

Fig. 12 Circular loop of round conductor. a) Total inductance obtained from the analytical formula (19) and FEM simulations of a circular loop of round conductor with $a = 5$ mm and $R = 20$ mm. b) External and internal inductances (FEM results).

As shown in Figs. 12, the analytical formula given by Eq. (19) does not take into account eddy

currents effects and, therefore, the average difference between FEM results and Eq. (19) is 9.2% for the total inductance.

6 EDUCATIONAL APPLICATION AND ASSESSMENT

In this work, the inductance of different conductive nonmagnetic conductors' configurations under sinusoidal steady-state AC supply has been analyzed by means of analytical formulas found in the specialized literature, and FEM simulations. FEM simulations have been introduced to improve learning skills and capabilities of undergraduate engineering students during practical sessions at the Electrical Engineering Department of the Universitat Politècnica de Catalunya (UPC, Spain). The proposed simulation approach allows the students determining the inductance offered by different conductors' configurations in a wide frequency range, which would be very complex and would require expensive material if carried out in a conventional laboratory. The proposed approach allows students applying the theoretical concepts discussed in the lecture sessions in a practical and realistic way, so that they become familiarized with the topic of systems simulation and modeling, which is gaining importance in engineering courses.

The study and computer simulation detailed in this paper was taught to 45 students during the first semester of the 2017-2018 academic year in the Electrical Systems course within the Degree in Engineering of the EEI School of the UPC. It is a 15-week required course of the third-semester with a workload of 6 ECTS (European Credit Transfer and Accumulation System). The lectures meet for three on-hour lecture sessions per week, and every 2 weeks students have a two-hour practical session. Two lectures and a practical session were dedicated to this topic. The effects and the procedure to calculate the inductance were explained during the lecture sessions. Due to time restrictions, the students were provided with the Matlab codes of the analytical formulas detailed in Sections 2, 3 and 4. During the practical sessions the students had to prepare and run FEM simulations, compare the results with those obtained with the Matlab codes, and draw conclusions.

One week after the sessions, the students were enquired to fill a questionnaire to evaluate the effect of the computer-aided practical session in their learning performance. The questionnaire is an important feedback source for the instructors, since it allows to identify possible weaknesses and to apply corrective actions. The students were required to grade the questions according to five levels, as shown in Table 1. The average grade was 4.13/5.00, so it can be concluded that the

methodology applied is well-suited to help students consolidate their knowledge of the topics analyzed, while stimulating their interest in electrical engineering topics.

Table 1. Questions and students' responses

Question	1	2	3	4	5	Average
The computer experiment helps to understand the course topics	0	2	8	22	13	4.02
The topics covered in the computer experiment are in-line with the topics studied in the lectures	0	0	7	24	14	4.16
I recommend to apply this methodology in advanced courses	0	0	6	23	16	4.22
This methodology allows connecting theoretical and practical knowledge	0	1	9	20	15	4.09
This approach provides me with skills I expect to apply in the future	0	0	8	21	16	4.18
Average						4.13

1:strongly disagree; 2: disagree; 3: neutral; 4: agree; 5: strongly agree;

The theoretical derivation of analytical formulas to calculate the inductance of different conductors' configurations offers technical difficulties even for simple geometries, especially when considering eddy currents effects. Although textbooks and lectures related to this topic usually include examples based on simple geometries, students complain that the process supposes a complex task, which is difficult to follow. In addition, these examples are often the same or very similar among textbooks and the theoretical analysis of variations of these examples is not trivial. The feedback from the students after the practical sessions was very positive. Students informed about the suitability of the proposed approach to solve complex problems using software tools similar to those applied in the industry. Students reported that by means of FEM simulations they can digest and understand more easily and faster the theoretical concepts involved and the behavior of the analyzed systems. Therefore, by this way, the students' comprehension task is facilitated, while increasing learning performance.

7 CONCLUSIONS

In this work, the behavior of different exact and approximate formulas to calculate the inductance of different conductors' configurations carrying sinusoidal steady-state AC currents

has been analyzed for a wide frequency range. The results attained by applying the analytical formulas have been compared with those obtained through FEM simulations. The limitations of such formulas has been shown, especially regarding the limited number of configurations allowed, thus highlighting that FEM simulations offer higher flexibility since they allow dealing with an extended variety of configurations and frequencies, otherwise they would be very difficult to solve analytically. However, FEM is not an “exact” method, since the accuracy of the results depends on the mesh size, the “infinite” domain simulated or the boundary conditions, among others. The simulation method analyzed in this paper can be applied in computer-assisted laboratory experiments or lecture demonstrations in different engineering and physics graduate and undergraduate courses. The FEM simulation approach analyzed in this paper has been found a helpful complement to the lectures and assignments in electricity courses in engineering studies, which can be extended to other disciplines such as heat transfer, acoustics or mechanics among others. Students’ feedback was very positive, since they found very interesting to solve complex problems using similar software tools to those used in the industry.

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