A Novel Approach to MISO Interference Networks under Maximum Receive-Power Regulation

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Abstract: An aggressive frequency reuse is expected within the next years in order to increase the spectral efficiency. Multiuser interference by all in-band transmitters can create a communication bottleneck and, therefore, it is compulsory to control it by means of radiated power regulations. In this work we consider received power as the main way to properly measure radiated power, serving at the same time as a spectrum sharing mechanism. Taking into account the constraints on the maximum total receive-power and maximum transmit-power, we first obtain the transmit powers that attain the Pareto-efficient rates in an uncoordinated network. Among these rates, we identify the maximum sum-rate point for noise-limited scenarios. Next, in order to reach this working point using as less power as possible, we design a novel beamformer under some practical considerations. This beamformer can be calculated in a non-iterative and distributed fashion (i.e. transmitters do not need to exchange information). We evaluate our design by means of Monte Carlo simulations, compare it with other non-iterative transmit beamformers and show its superior performance when the spectrum sharing receive-power constraints are imposed.

Key–Words: Beamforming, Spectrum Sharing, Cognitive beamforming, Interference Channel, Open Spectrum, Time Area Spectrum, Interference Management.

1 Introduction

Currently, a high percentage of the data traffic is delivered to the final user via short or medium range systems that work in unlicensed or open sharing spectrum bands (i.e.the Industrial Scientific and Medical or ISM band). One example is the MuLTEfire alliance[1], which combines the best of LTE and WiFi. Indeed, the potential of these communication systems relies on its 'free' conception, as any transmitter can send information within a maximum radiated power. Although limiting the transmit power may avoid long range communications, it provides a good coexistence framework between different systems. At the same time, Fifth Generation communication systems (i.e. 5G) are foreseen to provide a huge capacity increase

with respect to Fourth Generation ones. This forces to identify both: new spectrum bands and new spectrum management techniques, such as LSA (Licensed Spectrum Access) in Europe, SAS (Spectrum Access System) in USA [27] or the recent model of micro-licenses [24]. Either for unlicensed or for licensed systems, this paper studies how the spatial spectrum constraints can be approximated not only by the transmit, but also, by the receive-power constraints (e.g. see [20]). The present work focuses on the unlicensed band. In order to control interference to unintended users and help in the coexistence of simultaneous communications by different transmitters, this work deals with transmit beamforming or MISO (Multiple Input Single Output) systems. Next we revisit those references that have been useful to frame the present work. We classify them into those that deal with cog-

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[1] https://www.multefire.org/

nitive scenarios and those that deal with the general MISO IC.

1.1 Previous work

In the second half of the past decade there were plenty of works on beamforming and power control techniques for spectrum sharing scenarios, most of these scenarios were cognitive and underlay. In these scenarios secondary communications must maintain their Quality of Service (QoS) together with that of the primary or incumbent users, which may be unwilling or unable to cooperate, as it is the case of legacy systems. Thereby, the cognitive scenario entails important difficulties and it has triggered interesting works and technical advances in order to overcome them. Beamforming or *MISO designs for cognitive scenarios* incorporate different kinds of quadratic constraints with the goal of controlling specific QoS in the primary and secondary users. Semidefinite programming or second order cone programming, with convex relaxations, are useful tools to carry out these beamforming designs. The book by Eldar and Palomar [29] provides interesting tutorials on these tools. Convex optimization has been the key tool to obtain many of the optimal downlink beamforming designs, offering numerically stable solutions that in most cases are obtained through iterative procedures. In [11, 39, 23, 34] the authors apply them with further mathematical developments in order to obtain beamformers that can be useful for cognitive scenarios and that work with partial channel state information. These papers contain new and interesting findings about the hidden convexity of apparently non-convex problems. In addition, if the beamformer is designed with normalized weights, as it is the case in the present article, power control is needed. For instance, we refer to [17], where the authors design a power control scheme for the cognitive coexistence not only between terrestrial systems, but also between terrestrial and satellite communications. In [16] the authors carry out a thorough and clear review of the different constraints that can be accounted for power control in spectrum sharing scenarios, either for licensed or for unlicensed (i.e. the so-called open sharing) models. Finally, we note that, although not limited to cognitive networks, a good general framework on power control in wireless communications can be found in [9].

A workhorse in cognitive networks is the knowledge at transmission of the produced interference. The most common solution for that is to consider a centralized coordination of the secondary transmitters. However, if this is not the case, in order to get this information, channel reciprocity or system duality are common strategies as can be learned from [28, 40].

These strategies are precisely the rationale behind the so-called virtual SNIR beamformer (signal-to-noise-and-interference-ratio). The virtual SNIR metric considers the interference that the transmitter creates in all the unintended receivers, or also known as leakage, instead of the interference that is received by the terminal of interest. Its good performance is shown in [36] and the beauty of this "general-purpose" beamformer is its closed-form design, which contrasts with others that, although taylored to more specific cognitive scenarios, require a recursive implementation. As such, the virtual SNIR beamformer was devised to solve the problem of a general MISO interference channel (MISO IC) [37], which is the basic system model that lies in cognitive scenarios and, in general spectrum sharing ones.

Concerning beamforming not only for cognitive channels, but for the general *MISO interference channel* (i.e. without coordination among transmitters), there are significant amount of references. Good news for this model is that a global optimization approach exists to find the sum-rate optimal operating point in [4]. Under some realistic assumptions, there are also solutions that are less computational costly as in [33]. In [26] the authors present the optimal beamformers that attain the boundary of the Pareto Gain region in IC, which is useful for resource management. In [6] the authors study the Pareto Rate Region for multi-antenna IC. In these works, for the sake of optimality, full channel state information (CSI) is available. In order to obtain realistic designs in interference channels, incomplete CSI must be considered. In this regard, an interesting work is [25], which studies decentralized beamforming designs for the IC. These references and more information on MISO IC systems can be found in [33]. Finally, it is worth mentioning that there are also interesting works on decentralized beamforming for the interfering broadcast channel that are close to the MISO IC solutions, such as [14] and [15]. They apply successive convex approximations and the alternating direction method of multipliers in order to obtain the decentralized solution.

1.2 Contribution and motivation

This paper focuses on the design of a decentralized MISO system for a general IC under receive-power constraints. The proposed work is different from the already mentioned papers in that it obtains a closed-form and non-iterative beamformer, which is designed for unlicensed spectrum sharing that is regulated with maximum receive-power constraint. We note that although receive-power constraints are partially addressed in cognitive radio scenarios, as the maximum total interference power received by the primary user,

the aim of the present work is open to any spectrum sharing situation; thus, having architectural implications that are different from a cognitive scenario, as Section 2 explains. Indeed, this paper studies the 'time-area-space' (TAS) licenses, first presented in [10], and provides a complete open spectrum system that is properly managed by the total received power so that the regulator can control the contamination of the radio spectrum. In the present work, we use the studies in [10] on how the capacity of the system is modified when only receive-power constraints are taken into account. The diverse and heterogeneous cell deployments that are foreseen in 5G will create some important technical challenges, such as communicating in a rich, uncoordinated and spectrum sharing ecosystem. The present article aims at providing a new physical layer scheme in order to address these uncoordinated scenarios.

The rationale behind the proposed design is a formal one and it results in a non-convex and NP-hard problem that requires full CSI. This fact motivates the relaxation of the problem and the adoption of simplifying assumptions, such as homogeneous channel properties among users, and simple power constraints. Surprisingly, the obtained beamformer, which is sub-optimal from the sum-rate point of view, can be interpreted as optimal from an array directivity perspective; thus, explaining its better performance when compared with the existing non-iterative and in closed-form transmit beamformers. The interest of this novel interpretation or focus is that it offers a natural and direct transmit beamformer design that does not come from any receiver duality principle (i.e. from translating any receiver beamforming criterion).

To sum up, the contributions of the paper are:

- A transmit power and beamformer design that is based not only on the constraint on available power, but also on the received one, in order to implement spectrum sharing in terms of a feasible regulation;

- a decentralized design, as there is no coordination among transmitters, and presents a closed-form and non-iterative solution;

- a new strategy to design transmit beamformers, based on the maximization of the directivity for IC;

- in unlicensed scenarios, under total transmit and receive-power constraints, the proposed technique proves with extensive simulations that: i) it has a performance that converges to the maximum sum-rate in the high SNR regime; ii)

in terms of sum-rate it outperforms the existing closed-form solutions and other related iterative designs; iii) in contrast to other closed-form transmit beamformers it controls the created interference level.

- Although sub-optimal, the proposed design is framed and justified in front of the state-of-the-art solutions.

In [33] the authors propose a decentralized beamformer for the MISO IC that maximizes the sum-rate and which results in the same generalized eigenvalue-based beamformer that we obtain in this paper. However, in the present work the proposed beamformer incorporates a receive-power constraint that introduces a new power control parameter in the design. Although the formulation of both beamformers are similar in this paper we show twofold: i) that it is not straighforward to obtain the new proposed design and ii) the new power control parameter is key to obtain a good performance, as the simulations show.

1.3 Organization of the paper

The rest of the paper is organized as follows. Section 2 presents the system model and provides insights into the received power restrictions. We also further study the work presented in [30] and generalize that in [7] in order to obtain a working point of interest, which is the one that fulfils the received power mask with equality. This is a reference point for the design of the proposed beamformer, which is presented in Section 3 together with a review of the existing transmit beamformers. Section 4 presents the power control and Section 5 shows the numerical simulations of the proposed technique. Finally, Section 6 concludes.

Notation: We adopt the notation of using lower case boldface for vectors, $\mathbf{v}$, and upper case boldface for matrices, $\mathbf{A}$. The component-wise product is denoted by $\odot$. The transpose operator and the conjugate transpose operator are denoted by the symbols $(\cdot)^T$, $(\cdot)^H$ respectively. λ_{max} denotes the eigenvector associated to the maximum eigenvalue. $\mathbf{I}$ denotes the identity matrix. $\mathbb{C}$ denotes the complex numbers. $\preceq$ denotes vector component-wise inequality, $|\cdot|$ is used for the absolute value and $\|\cdot\|$ for the square of the Frobenius norm of a vector. $CN(.)$ denotes a complex normal statistical distribution.

2 System Model and Problem Statement

2.1 System Model

We consider a spectrum sharing scenario, where K Base Stations (BS) or Access Points (AP) transmit information to their intended receivers, sharing frequency and time resources without any hierarchy or labeling on primary or secondary communications. The receivers have one antenna and the transmitters have M antennas each. Coordination between transmitters is not supported and we assume that each transmitter (K in total) sends independent symbols to one intended receiver. This scenario is known as the MISO IC. We note that each transmitter uses its M antennas, or spatial degrees of freedom, to combat the interference that is created to other uncoordinated communications. We consider that this is a prioritary aspect in systems that use the unlicensed band if they are allowed to share frequencies. Therefore, we do not consider that the BS uses its M transmitting antenna for simultaneously communicating with more than one user. Another aspect is that in order to allow for transmit beamforming, every receiver has to broadcast in a common channel the respective channels experienced from any transmitted station. This can be done in non-colliding time division channels. In other words, transmitters cannot cooperate, but they have full information of the channel to the desired location and the channels to the interference locations. We denote $\mathbf{b}_k \in \mathbb{C}^{M \times 1}$ the transmit beamformer used by the k-th station. For notational convenience, we stack all the transmit beamformers in matrix $\mathbf{B} = [\mathbf{b}_1, \ldots, \mathbf{b}_K]$. The available power or needed power by the k-th station is p_k and we define the vector of transmit powers by $\mathbf{p} = [p_1, \ldots, p_K]^T$. Moreover, the available power for the k-th station is bounded by P_{max}^k, accordingly, and we define $\mathbf{P}_{max} = [P_{max}^1, \ldots, P_{max}^K]^T$. The complex baseband transmitted signal at the i-th BS is expressed as

$$\mathbf{x}_i = \sqrt{p_i}\mathbf{b}_i s_i \quad (1)$$

where s_i is $CN(0,1)$ and denotes the transmitted data symbols.

The link gain from the transmit beamformer i to the receiver j is

$$a_{ji} = \mathbf{b}_i^H \mathbf{R}_{ji} \mathbf{b}_i \quad (2)$$

where

$$\mathbf{R}_{ji} = \mathbf{h}_{ji}\mathbf{h}_{ji}^H \quad (3)$$

and $\mathbf{h}_{ji} \in \mathbb{C}^{M \times 1}$ is the spatial signature from the i-th base station to the j-th receiver. We assume flat block fading channels between the base stations and all users. Matrix $\mathbf{A} \in \mathbb{R}^{K \times K}$ contains all the link gains of the considered network $[\mathbf{A}]_{ji} = a_{ji}$.

The complex baseband received signal at the j-th terminal is expressed as

$$y_j = \sum_{i=1}^{K} \mathbf{h}_{ji}^H \mathbf{x}_i + n_j \quad (4)$$

where n_j is $CN(0, \sigma^2)$ and denotes the antenna noise at user j. Without loss of generality this work considers $\sigma^2 = 1$.

Finally, the data rate of user j in [bits/sec/Hz] is

$$r_j = \log_2 \left(1 + \frac{a_{jj}p_j}{\sum_{i \neq j}^{K} a_{ji}p_i + \sigma^2}\right), \quad (5)$$

which can be written as

$$r_j = \log_2 \left(1 + \frac{a_{jj}p_j}{I_j + \sigma^2}\right) \quad (6)$$

where $I_j = \sum_{i \neq j}^{K} a_{ji}p_i$ is the total amount of interference that is experienced by user j.

2.2 Problem Statement

For regulation purposes, in order to allow coexistence of different wireless services, the system designer must take into account the amount of total signal power that can be received by one terminal. This is restricted to ρ, which, without loss of generality, is assumed to be the same for all the standard receivers. Note that, in practice, this constraint is specified by each technology provider, who produces, for the network operators, receivers that comply with standard qualifications (e.g. satellite receivers have a higher sensitivity and, therefore, lower ρ than terrestrial wireless ones). For notational convenience we define $\boldsymbol{\rho}^T = [\rho \quad \ldots \quad \rho] = \rho\mathbf{1}$. In the proposed scenario this regulation can be formulated as

$$\sum_{i=1}^{K} a_{ji}p_i \preceq \rho \quad j = 1 \ldots K \Rightarrow \mathbf{A}\mathbf{p} \preceq \boldsymbol{\rho} \quad (7)$$

In general, power of overlapping signals at each location should not exceed the maximum power flux density (Watts/Hz) allowed by radio regulations [1], which translates into a received power when it is evaluated in the working bandwidth of the receiver.

As we have commented in Section I, oriented to a best use of the radio-spectrum, together with a continuously increasing demand of wide area communications, regulators might start to adopt this TAS licensing system [10, 16, 32]. Whenever the transmitter

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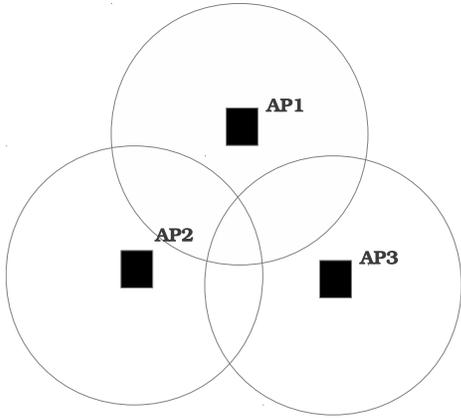Miguel Angel Vazquez, Miguel Angel Lagunas

Figure 1: Difficulties arise when several base stations are located within a TAS area. The amount of created interference impacts on the total received power and limits the transmit power and, therefore, the range of the communication system.

supports any power demand, the rate for a single user in a scenario that is free from interference will be

$$r_i = \log_2\left(1 + \frac{\rho}{\sigma^2}\right) \quad \text{[bits/sec/Hz].} \tag{8}$$

Clearly the power control will adapt the transmit power such that the global received signal is set to the regulation level:

$$p_i = \frac{\rho}{a_{ii}}. \tag{9}$$

A more complicated scenario is when more than one micro-cell is using the TAS. Furthermore, it is clear that the range of several APs overlap in a given area. This scenario is depicted in Fig.1. The possibility that three receivers, each corresponding to a different AP, stay on the overlapped area of the three ranges poses a more difficult problem to obey the mask, mainly because power control from each receiver works only for its corresponding access point, i.e. receivers are not coordinated. In addition, and for logistic reasons in services deployment, the APs are also not coordinated since they may be associated to a different vendor of communication services. Assuming that mask ρ is fulfilled with equality in (7), the rate delivered to each user in this new scenario with interference is given by

$$r_i = \log_2\left(\frac{\sigma^2 + \rho}{\sigma^2 + I_i}\right) \quad \text{[bits/sec/Hz].} \tag{10}$$

Note that the channel gains in (2), and in consequence the rates in (10), depend on the transmit beamformers. If they null out the interference in the denominator of (10) (e.g. $a_{ji} = 0$ for $i \neq j$), r_i is maximized

and the maximum sum-rate of the system becomes

$$R_{sum} = \sum_{j=1}^{K} r_j = K \log_2\left(1 + \frac{\rho}{\sigma^2}\right) \quad \text{[bits/sec/Hz].} \tag{11}$$

In other words, the Zero Forcer (ZF) beamformer is the optimal design for the maximum sum-rate when only the total received power is constrained (see [10]). Nevertheless, this design may imply that when the channel of desired and the channel of interference are co-linear, the transmit power requirements would be enourmous in order to fulfil the constraint implicit in (10). As a consequence, the power needed by user i would be far above the available power. Thereby, we have to take into account also an additional constraint on the available power for each transmitter, P_{max}.

In general terms, we would like to obtain a decentralized design to solve the following Multi-Objective Optimization problem (MOP)(see [5]) P_1

$$
P_1 \quad
\begin{aligned}
\underset{\mathbf{p},\mathbf{B}}{\text{maximize}} \quad & \mathbf{r} \\
\text{subject to} \quad & \mathbf{Ap} \preceq \boldsymbol{\rho} \\
& \mathbf{0} \preceq \mathbf{p} \odot \tilde{\mathbf{b}} \preceq \mathbf{P}_{max}
\end{aligned}
\tag{12}
$$

where $\mathbf{r}$ is a vector containing all the achievable rates, $\mathbf{r} = [r_1, \ldots, r_K]^T$. We note that in the constraints the link matrix $\mathbf{A}$ depends on the beamformers $\mathbf{b}_k$, also $\tilde{\mathbf{b}} = [\| \mathbf{b}_1 \|, \ldots, \| \mathbf{b}_K \|]^T$. Note that P_1 is a MOP, where the set of transmit beamformers, $\mathbf{B}$, and powers, $\mathbf{p}$, need to be calculated in a decentralized way since no cooperation between them is allowed. This fact contrasts with the broadcast channel, where the computations are carried out in a centralized way. We note that P_1 is not convex because the rates r_i are not convex on $\mathbf{p}$. Also, both, the cost function and the constraints are coupling the design of all the transmitters; thereby difficulting to obtain a decentralized solution. In spite of the potential relevance of P_1 for radio contaminated scenarios, to the best of our knowledge in the literature there is not yet a decentralized solution to this problem. The focus of the existing works is: i) to obtain beamformers that maximize the sum-rate under specific QoS w/o constraints on the interference temperature and ii) to offer centralized solutions or iterative decentralized ones. Unlike them, in this paper we want to explore an alternative focus: the challange is to find a standalone closed-form (i.e. non-iterative) and decentralized transmit beamformer and power control. Let us first comment on the QoS aspects next.

2.3 QoS and interference temperature

We consider that for the regulator the priority is to control the contamination of the radio spectrum and not the QoS of the communications operating in the corresponding frequency band. In any case, as it is clearly stated in [16], maximum receive-power constraints cannot be translated to QoS constraints, which are expressed as a minimum rate or SNIR per user location. This latter are formulated for each user as

$$\text{SNIR}_i = \frac{a_{ii}p_i}{\sum_{j \neq i}^{K} a_{ij}p_j + \sigma^2} \geq \gamma_i \ i = 1..K \quad (13)$$

where γ_i is the minimum SNIR. In fact, in [30] the authors show the equation that relates the SNIR thresholds in (13) and the regulation mask in (7), so that both can be fulfiled. The paper also shows that whenever maximum sum rate is the goal and the optimal solution fulfils (7) with equality, then this solution presents a $SNIR_i$ that is greater than γ_i. However, the paper points out the problem of obtaining a decentralized power control that fulfils both type of constraints.

Regarding the constraints on the maximum interference temperature, they can be mathematically equivalent to the maximum receive-power. This is the case in cognitive radios that consider the total interference power caused by the secondary network on a primary receiver (see e.g., [12, 21, 17]). However, the practical settings in these cognitive networks are different from the settings in our open licensed model. In the case of cognitive networks, due to the different communication hierarchies (i.e. primary and secondary users) additional mechanisms of message passing among transmitters or reverse network operation are devised; while in completely open sharing networks, which is our case of study, all transmitters have the same hierarchy and are assumed completely non-cooperative.

In next sub-section we frame the power control into the formal context of P_1 and its corresponding Pareto rate region, when the beamformers are fixed. After that, next Section 3 relaxes the problem in order to obtain the decentralized and non-iterative beamforming design that can be interpreted as optimizing the array directivity for interference channels. Although heuristic, the final beamformer presents the best behavior, in terms of simulation results, when compared with other competing solutions in the unlicensed spectrum scenario that is under consideration.

2.4 Rate and power Pareto regions in P_1

Let us consider that the beamformers have unitary norm and are known, i.e. the link matrix $\mathbf{A}$ is known.

Under this assumption, in [30] the authors found the powers that lead to all optimal rate pairs in a communication system with the constraints formulated in P_1 (12). These optimal points are the solution of the following problem:

$$\begin{aligned} \underset{\mathbf{p}}{\text{maximize}} \quad & \mathbf{r} \\ \text{subject to} \quad & \mathbf{A}\mathbf{p} \preceq \boldsymbol{\rho} \\ & \mathbf{0} \preceq \mathbf{p} \odot \tilde{\mathbf{b}} \preceq \mathbf{P}_{max} \end{aligned} \quad (14)$$

From the proof in [30] (i.e. section 3) we formulate the following two corollaries:

Corollary 1: The boundary points of the Pareto-efficient rate region in (14) are achieved with the power values that are at the border of the power feasible set.

Corollary 2: As a consequence of Corollary 1, if the system fulfils (7) with equality, which results in power values that are on the boundary of the Pareto-efficient power region, the system is rate efficient.

We note that the power-tuple $\mathbf{p}^*$, which fulfils (7), that is

$$\mathbf{A}\mathbf{p}^* = \boldsymbol{\rho}, \quad (15)$$

is the Most Upper Right Corner (MURC) of the Pareto power region whenever $\mathbf{p}^* \geq 0$. For completeness, Appendix A comments on the sum-rate optimality of the MURC for noise-limited scenarios. It is precisely the MURC that is the starting or motivating point to take a new focus in the design of the transmit beamformer in next section. Let us remark that in the stated problem there is no cooperation among transmitters and, therefore, there is not full channel state information (CSI) at transmission. This fact motivates in the following sections the need for a decentralized beamformer design and also for a sub-optimal power control, which is not able to implement the design in (15).

3 Decentralized transmit beamforming

This section presents a new decentralized, closed-form and non-iterative beamformer for the decentralized scenario, under both, receive and transmit power constraints. The proposed design departs from P_1 and although it results in a sub-optimal design, due to the different simplifying assumptions, the simulation section shows that it offers very good results in terms of sum-rate compared to other closed-form decentralized designs. Next we present the design of the new beamformer. After that, existing transmit beamformers are reviewed in order to frame the novelty of the proposed design.

3.1 Proposed Transmit Beamformer

In this section we depart from P_1 to design the new beamformers. However, as a closed-form and decentralized design is not possible to optimally solve P_1, we select the MURC as the desirable working point, instead of working in all the Parto-efficient points that P_1 represents. The beamformer is designed in this section so that it uses as less power as possible from the available one, P_{max}, in order to work in the MURC. In order to properly follow the derivations towards the final design, we note that: i) there is not complete CSI at the transmitters and, therefore, we have to assume certain channel symmetries to cope with this lack of knowledge; ii) in the interest of obtaining a close-form and decentralized design we adopt some convex relaxations that consider some bounds and also the harmonic mean of the constraints, as explained below.

Assuming that all receiver locations fulfil the mask with equality as in (15), if we apply the Cramer's rule to obtain the optimal powers, p_k^* that solve this system of equations, we can express this powers as

$$p_k^* = \frac{\rho \sum_{q=1}^{K} A_{qk}}{\Delta} = \frac{\rho \left(1 - \sum_{q\neq 1}^{K} \frac{|A_{qk}|}{A_{kk}}\right)}{a_{kk} - \sum_{q\neq k} a_{qk} \frac{|A_{qk}|}{A_{kk}}}, \quad (16)$$

where Δ is the determinant of matrix $\mathbf{A}$ and A_{qk} is the cofactor of element a_{qk}. In (16) we have considered that

$$A_{qk} \leq 0 \quad k \neq q. \quad (17)$$

This is a realistic assumption since it is expected that after beamforming the channel gain from transmitter k towards the desired receiver is bigger than the gains towards the unintended receivers. An important observation in what follows is that the smaller the powers p_k^* at the MURC are, the easier is for the transmitter k to attain this desirable point with the available power P_{max}. Therefore, our goal is to design each beamformer $\mathbf{b}_k$ so that p_k^* in (16) is minimized. However, this would entail the full CSI knowledge at all the transmitters. Indeed, the co-factors A_{qk} with $q, k = 1, ..., K$ are computed based on all transmit beamformers. In here, we do not assume full CSI and we consider a decentralized scheme where the transmitters do not exchange information among themselves. For this reason next we derive an upper bound for p_k^* that allows to obtain the decentralized design.

Note that in (17) $\sum_{q\neq k}^{K} \frac{|A_{qk}|}{A_{kk}} \leq 1$ in order to obtain $p_k^* \geq 0$. In addition, due to the lack of full channel information at the transmitters we consider assume a scenario with symmetrical links. In consequence

$$\frac{|A_{qk}|}{A_{kk}} \leq \frac{1}{K-1} \quad k \neq q \quad (18)$$

can be assumed. Next, from (18) it follows that

$$p_k^* \leq \frac{\rho \left(1 - \sum_{q\neq 1}^{K} \frac{|A_{qk}|}{A_{kk}}\right)}{a_{kk} - \frac{1}{K-1} \sum_{q\neq k} a_{qk}}. \quad (19)$$

If we define a_{kI} as the aggregated interference power that is created by transmitter k

$$a_{kI} = \sum_{q\neq k} a_{qk} = \mathbf{b}_k^H \left(\sum_{j\neq k} \mathbf{R}_{jk}\right) \mathbf{b}_k = \mathbf{b}_k^H \mathbf{R}_{kI} \mathbf{b}_k \quad (20)$$

then (19) reads as

$$p_k^* \leq \frac{\rho \left(1 - \sum_{q\neq 1}^{K} \frac{|A_{qk}|}{A_{kk}}\right)}{a_{kk} - \frac{1}{K-1} a_{kI}}. \quad (21)$$

Our goal is to design each beamformer $\mathbf{b}_k$ so that the upper bound of p_k^* in (21) is minimized. For this purpose, we focus the proposed design on the maximization of $a_{qq} - \frac{1}{K-1} a_{qI}$. Note that the numerator in (21) entails a centralized coupled maximization and; thus, we do not consider it in our approach. Later, we evaluate by means of simulations which is the final power that results from the "only-denominator" optimization.

Also, by inspecting the maximum sum-rate under regulatory constrints, which is given in (11), having low values in the terms that are out of the diagonal of the matrix $\mathbf{A}$ helps in attaining this optimum rate. As a matter of fact, despite the maximization of $a_{qq} - \frac{1}{K-1} a_{qI}$ would reduce the generated interference by the transmitter q, in cases where the intended user channel has a small gain, the generated interference to unintended terminals might increase severely. In order to control it, we propose to incorporate to our beamforming design the following constraint:

$$p_q^* a_{qI} \leq \alpha \rho \quad \forall q. \quad (22)$$

The parameter α is a percentage of the regulation mask that should not be violated by the interference that is created by transmitter q. For instance, in spectrum sharing scenarios it is common practice to consider that the total interference level at the receiver must be at least 10 dB below the noise level. However, the question is how much should be the interference that each secondary transmitter is allowed to create. This is the meaning of $\alpha \rho$. Note that the answer is not straight forward as we are considering secondary networks with uncoordinated transmitters. Later, section 3 is devoted to the setting of α. Next, in order to complete the beamforming design we include one constraint on the transmit power:

$$p_q^* ||\mathbf{b}_q|| \leq P_{max} \quad (23)$$

Summing up, the resulting optimization problem is P_2 and is formulated as follows:

$$P_2 \quad \underset{\mathbf{b}_q}{\text{maximize}} \quad a_{qq} - \frac{a_{qI}}{K-1}$$
$$\text{subject to} \quad p_q^* ||\mathbf{b}_q||^2 \leq P_{max} \quad (P_{2-a}) \quad (24)$$
$$\qquad p_q^* a_{qI} \leq \alpha\rho \quad (P_{2-b}),$$

This problem is an indefinite quadratic optimization problem with quadratic constraints, which is, in general, non-convex. Only for the case of no more than three constraints, for the complex case, the solution can be found iteratively by resorting to semi-definite programming (SDP)[23]; this means that the SDP relaxation is tight in this case. However, we are interested in a closed-form solution and for this purpose, next, we propose an innovative convex relaxation. This relaxation consists in substituting all the constraints by just one constraint on its harmonic mean. First, note that the two inequalities in (22) and (23) are constraining the same variable p_q^* and only one of the two constraints can be active (i.e. with equality) at the optimum. Therefore, we can say that the only constraint to be verified is

$$p_q^* \leq min\left(\frac{\alpha\rho}{a_{qI}}, \frac{P_{max}}{||\mathbf{b}_q||}\right) \quad (25)$$

As

$$\left(\left(\frac{\alpha\rho}{a_{qI}}\right)^{-1} + \left(\frac{P_{max}}{||\mathbf{b}_q||}\right)^{-1}\right)^{-1} \leq min\left(\frac{\alpha\rho}{a_{qI}}, \frac{P_{max}}{||\mathbf{b}_q||}\right) \quad (26)$$

then

$$p_q^* \leq \left(\frac{a_{qI}}{\alpha\rho} + \frac{||\mathbf{b}_q||}{P_{max}}\right)^{-1}. \quad (27)$$

Therefore, we can relax the initial beamforming design with two constraints in P_2 per transmitter q and substitute it by

$$\underset{\mathbf{b}_q}{\text{maximize}} \quad a_{qq} - \frac{a_{qI}}{K-1}$$
$$\text{subject to} \quad a_{qI} + \alpha\frac{\rho}{P_{max}}||\mathbf{b}_q|| \leq \frac{\alpha\rho}{p_q^*}. \quad (28)$$

We note that the single constraint in (28) is tighter than the two constraints in P_2 and, therefore, (28) is a lower bound of P_2. By Lagrangian theory (see also chapter 4 in [29]), (28) is a generalized eigenvalue problem. Therefore, the optimal solution of (28) takes the form of the optimal solution of the following Rayleigh quotient

$$P_3 \quad \underset{\mathbf{b}_q}{\text{maximize}} \quad \frac{a_{qq} - \frac{a_{qI}}{K-1}}{a_{qI} + \alpha\frac{\rho}{P_{max}}||\mathbf{b}_q||}, \quad (29)$$

and its solution (i.e. the norm of the eigenvector associated to the maximum eigenvalue) has to be scaled in order to fulfil the constraint in (28). Therefore, by resorting to P_3 we have decoupled the beamforming optimization and the power control design, which is a desirable feature.

We now focus on the beamforming design of norm equal to one. Later on in Section 4 we focus on the power control. An equivalent expression to (29), which helps later on to gain further insight into the proposed beamformer (i.e. as we comment in next Section 3.B), is

$$\frac{a_{qq} - \frac{a_{qI}}{K-1}}{a_{qI} + \alpha\frac{\rho}{P_{max}}||\mathbf{b}_q||} \quad (30)$$

which can be written as

$$\frac{a_{qq} + \alpha\frac{\rho}{(K-1)P_{max}}||\mathbf{b}_q||}{a_{qI} + \alpha\frac{\rho}{P_{max}}||\mathbf{b}_q||} - \frac{1}{K-1} \quad (31)$$

then P_3 results equivalent to

$$\underset{\mathbf{b}_q}{\text{maximize}} \quad \frac{a_{qq} + \alpha\frac{\rho}{(K-1)P_{max}}||\mathbf{b}_q||}{a_{qI} + \alpha\frac{\rho}{P_{max}}||\mathbf{b}_q||}, \quad (32)$$

where we can re-write the objective function such as

$$\frac{a_{qq} + \frac{1}{K-1}\gamma||\mathbf{b}_q||}{a_{qI} + \gamma||\mathbf{b}_q||} = \frac{\mathbf{b_q}^H\left(\mathbf{R}_{qq} + \frac{\gamma}{K-1}\mathbf{I}\right)\mathbf{b}_q}{\mathbf{b_q}^H\left(\mathbf{R}_{qI} + \gamma\mathbf{I}\right)\mathbf{b}_q} \quad (33)$$

whose solution, $\mathbf{b}_q^*$, is the eigenvector associated to the maximum eigenvalue, $\lambda_{\max}$, in

$$\left(\mathbf{R}_{qq} + \gamma\frac{1}{(K-1)}\mathbf{I}\right)\mathbf{b}_q^* = \lambda_{\max}\left(\mathbf{R}_{qI} + \gamma\mathbf{I}\right)\mathbf{b}_q^*, \quad (34)$$

where

$$\gamma = \alpha\frac{\rho}{P_{max}}, \quad (35)$$

and it can be interpreted as a sort of Signal-to-Interference Ratio inverse. We call the proposed optimal beamformer to P_3, $\mathbf{b}_q^*$, in (34) MIO beamformer (MIOB). Note that, ultimately, this beamformer is an approximation to the optimal solution to P_2 and produces a value for the denominator of (16) that is lower than the optimal one. This is the prize to pay in order to obtain a practical solution. In any case, the simulation section proves that the proposed beamformer has a performance that is above that of other low complexity schemes.

As $\mathbf{b}_q^*$ is an eigenvector, it has norm equal to one. For this reason, Section 4 presents the corresponding power control. Before that, next sub-section comments on the design of α in order to attain the desired behaviour.

3.2 Parameter setting

When MIOB is implemented, the optimal value of the Rayleigh quotient is

$$\frac{a_{qq} - \frac{a_{qI}}{K-1}}{a_{qI} + \alpha \frac{\rho}{P_{max}}||\mathbf{b}_q||} = \lambda_{\max} - \frac{1}{K-1} \qquad (36)$$

Thus, α is parametrizing the maximum eigenvalue and it should be such that $\lambda_{\max}$ is maximized. However, the optimization with respect to α is difficult to solve. On the other hand this paper is aiming at a practical beamformer design that should fulfil important practical aspects. The key one is that, as the design is for uncoordinated BSs, the resulting beamfomer in (34) should not depend on the other link gains and should be designed without any a priori knowledge. This fact motivates the following practical approach that aims at attaining a well-behaved beamformer in the aysmptotic regim:

- when $P_{max} \rightarrow \infty$ the proposed beamformer in (34) should tend to the ZF beamformer (ZFB); thus, $\gamma \rightarrow 0$ in (35);

- when the mask level $\rho \rightarrow \infty$ (i.e. unregulated scenario) the interference constraint should not go to infinite (i.e. $\alpha\rho$ is bounded). This is because the transmission must be possible also in this case and the beamformer should tend to the EIG beamformer (EIB) of [33], which maximizes the sum-rate in the unregulated scenario and where $\gamma = \frac{\sigma^2}{P_{max}}$;

- when the number of transmit antennas is much higher than the number of receivers the parameter α should be designed so that the beamformers tend to the ZF beamformer (ZFB); thus, $\gamma \rightarrow 0$.

By designing

$$\alpha = \frac{\sigma^2}{D\sigma^2 + \rho} < 1 \Rightarrow \gamma = \frac{\sigma^2 \rho}{(D\sigma^2 + \rho) P_{max}} < 1 \qquad (37)$$

with $D = M - (K-1)$, the practical aspects that have been identified are fulfilled and γ presents the described asymptotic behaviour. Note that both, α and consequently γ are adimensional and that D represents the extra degrees of freedom. Also, as $\alpha < 1$ the imposed constraints in the beamformer design, namely, $P_{(2-a)}, P_{(2-b)}$, garantee that $p_q^* > 0$ (i.e. see appendix A). Finally, note that in the proposed setting if $\rho \rightarrow 0$ then $\gamma \rightarrow 0$ and the proposed beamformer tends to the ZFB, which makes sense in order to achieve the zero regulated receive power. The situation when $P_{max} \rightarrow 0$ is not considered as it ends up in no transmission.

Although heuristic, the proposed design for α is key for the good performance of the proposed beamformer. This is proved in the simulation section. We further stress that the lack of information of the channel at transmission justifies the homogeneous user assumptions that have been considered all along this section. As examples of the values that α may take and its implications if the proposed design is the one in (37), let us consider for instance $K = 3$ BSs with $M = 3$ antennas each one. If the noise power and the regulation mask are $\sigma^2 = \rho = 0dB$ then $\alpha = 1/2$ and by $P_{(2-b)}$ the allowed amount of total interference that the transmitter q is half the regulation mask.

3.3 Existing Transmit Beamformers

In the literature, none of the existing beamformers at transmission takes into account in their design the receive-power constraint, which is our case of interest. Let us first review those beamformers that present a closed-form.

In general, the optimal transmit beamformer in a MISO IC depends on the desired and interference signal power levels with respect to the noise level. Indeed, when the scenario is dominated by the noise (i.e. the SNR is low) the optimal design for transmitter k in the two user case ($k = 1, 2$) is the Matched Beamformer (MB)

$$\mathbf{b}_k^{\text{MF}} = \frac{\mathbf{h}_{kk}}{\sqrt{||\mathbf{h}_{kk}||}}, \qquad (38)$$

whereas when SNR is very high (or equivalently there is no constraint on P_{max}) and the number of transmit antennas is equal to or greater than the number of receivers, the Zero-Forcing Beamformer (ZFB) is the best option

$$\mathbf{b}_k^{\text{ZF}} = \frac{(\mathbf{I} - \mathbf{R}_{kI}) \mathbf{h}_{kk}}{\sqrt{|| (\mathbf{I} - \mathbf{R}_{kI}) \mathbf{h}_{kk}||}}. \qquad (39)$$

These results where obtained in [22].

There are also two additional designs, ones is the virtual-SNIR beamformer (VB) [37]

$$\mathbf{b}_k^{VS} = \frac{\left(\sum_{j=1,j\neq k} \gamma_k \mathbf{R}_{kj} + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{h}_k}{\sqrt{\left\| \left(\sum_{j=1,j\neq k} \gamma_k \mathbf{R}_{kj} + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{h}_k \right\|}}, \qquad (40)$$

which presents an intermediate behaviour between the MB and the ZFB. Note that $\gamma_k, \quad k = 1, \ldots, K$ are degrees of freedom that are not easy to design. The most used scheme is when $\gamma_k = P_{max}$ or the also so-called MMSE transmit beamformer [31]. However,

other values can be used as for instance it was done in [38] in the context of multicell communications.

The other option is the EIG Beamformer (EIB)

$$\left(\mathbf{R}_{kk} + \sigma^2 P_{max}^{-1}\mathbf{I}\right)\mathbf{b}_k^{EIG} = \lambda_{\max}\left(\mathbf{R}_{kI} + \sigma^2 P_{max}^{-1}\mathbf{I}\right)\mathbf{b}_k^{EIG},$$
(41)

which maximizes the sum-rate under specific assumptions on the interference level and that can outperform the VSB in some cases [33]. We note that even though EIB and the proposed MIOB present the same generalized eigenvalue structure, EIB was obtained under different rational than MIOB and without imposing the total receive-power constraint. In the previous sub-section IV.B we have commented when (41) collapses into the same design as MIOB in (34).

It is also important to remark that, amazingly, it is the design in (34) the one that maximizes the array factor directivity, D_{RD}, that is proposed in [19] for scenarios that are dominated by interference:

$$D_{RD} = \frac{||\mathbf{b}_k^H \mathbf{h}_{kk}|| - \frac{\mathbf{b}_k^H \mathbf{R}_{kI}\mathbf{b}_k}{K-1}}{||\mathbf{b}_k||\xi + \mathbf{b}_k^H \mathbf{R}_{kI}\mathbf{b}_k},$$
(42)

where the design of ξ is still to be solved. The aim of this paper is precisely to propose a design for ξ that is well-behaved in the aymptotic regim and that is validated by means of extensive simulations. Let us comment that for the case of $K = 2$ receivers, whose total received power is ρ, (42) is also the formulation of the secure rate for a MISO interference channel as adapted in [2]. However, while the secrecy rate for more than two receivers is still unknown, the directivity framework that is introduced in [18] is valid for any number of simultaneous transmissions.

So far, we have commented on closed-form designs, since this is the goal of this paper. As we have commented before, whenever complexity is not an issue iterative beamforming techniques can be used. For example, we cite [13], where the authors design downlink beamforming for wireless systems that co-exist in the region of interest and operate on the same frequency band. The goal is the minimization of the total transmission power under different constraints and the resulting semi-definite programming problem is studied and it is identified when the problem relaxation has always a rank-one optimal solution. This work generalizes the existing research and considers not only SNIR constraints (i.e. quality of service constraints) or the individual shaping constraints, but also the so-called soft-shaping constraints on the beamforming vector. Although formulated and treated differently, [13] also resorts to the receive-power constraint that is presented in this paper. Also, not completely zero interference constraint is the goal, but it

suffices to keep the interference level under the required threshold.

Because of its simplicity and application to spectrum sharing scenarios, another interesting work is the one in [11]. This work is the first solution that designs transmit beamforming from only rudimentary CSI to jointly maximize the signal to noise ratio of the secondary users and mitigate interference to the primary user. Of specific interest to this work is the formulation of one of the steps of the procedure for a scenario with one secondary user and one primary one:

$$\begin{aligned}\underset{\mathbf{b}}{\text{maximize}} \quad & \mathbf{b}^H \mathbf{R}\mathbf{b} \\ \text{subject to} \quad & \mathbf{b}^H \mathbf{R}_I\mathbf{b} \le \rho \\ & ||\mathbf{b}|| \le P_{max}\end{aligned}$$
(43)

which resembles P_2 and can be solved by resorting to the same QCQP procedure in [35]. Note, however, that the cost function in (43) is convex and the underlying problem that is modeled by P_2 involves more than two users; thus, involving important and different discussions about the parameter setting.

Finally, to complete the proposed design next section presents a decentralized and open-loop power control.

4 Decentralized power control and final algorithm

With the goal of fulfiling the mask with equality as in (15) in [30] the authors propose an iterative and distributed power control, where the receiver feedbacks the difference between the received power control and the mask ρ. However, under certain link gain circumstances (15) may lead to negative powers. For this reason and also in order to devise a simple non-iterative scheme, in the present paper we propose an open-loop control scheme that consists in guaranteeing that $\forall$ transmitter q

$$\begin{aligned}\text{find } & p_q \\ \text{subject to} \quad & p_q \le P_{max} \quad \text{(a)} \\ & p_q \le \frac{\alpha\rho}{a_{qI}} \quad \text{(b)} \\ & p_q \le \frac{\rho}{a_{qI} + a_{qq}} \quad \text{(c)}.\end{aligned}$$
(44)

Conditions (a) and (b) come from the problem statement P_2, under the assumption that $||\mathbf{b}_q|| = 1$. Condition (c) guarantees that the regulation mask is not violated under the assumption of facing a completely symmetric scenario (i.e. where the link matrix $\mathbf{A}$ is symmetric).

Depending on which is the active power control condition, the achievable rate per user q will have different bounds as we detail in the following:

- constraint (a) is active

$$
2^{r_a} = \frac{1 + P_{max}\,(a_{qq} + a_{qI})}{1 + a_{qI}P_{max}}
$$
$$
\leq 2^{r_a(a_{qImin})} = \frac{1 + \rho}{1 + a_{qI}P_{max}}
$$
$$
\geq 2^{r_a(a_{qImax})} = \frac{1 + P_{max}\,(a_{qq} + a_{qI})}{1 + \alpha\rho}
$$
$$(45)$$

- constraint (b) is active

$$
2^{r_b} = \frac{1 + \alpha\rho\frac{a_{qq}+a_{qI}}{a_{qI}}}{1 + \alpha\rho}
$$
$$
\leq 2^{r_b(a_{qImin})} = \frac{1 + \rho}{1 + \alpha\rho}
$$
$$
\geq 2^{r_b(a_{qImax})} = 1
$$
$$(46)$$

- constraint (c) is active

$$
2^{r_b} = \frac{1 + \rho}{1 + a_{qI}\frac{\rho}{a_{qq}+a_{qI}}}
$$
$$
\leq 2^{r_b(a_{qImin})} = 1 + \rho
$$
$$
\geq 2^{r_b(a_{qImax})} = \frac{1 + \rho}{1 + \alpha\rho}
$$
$$(47)$$

Note that constraint (c) is the one that may allow to achieve the maximum rate, which is the ultimate goal as it has been initially presented in P_1 (i.e.(12)). Also r_b attains the lower rate values (i.e. equal to 1) and the bounds that limit r_a are contained within the ranges of r_b and r_c. These bounds will help to interpret the results in the next section on simulations.

The purpose of the proposed power control is to allow to compare in the next section the different beamforming schemes in terms of sum-rate. We stress that the comparison of different power control schemes is out of the scope of this paper. Next, we formulate the final algorithm that each BS should implement.

> **for** $q = 1$ **to** K **do**
> Compute R_{qq}, R_{qI};
> Compute MIOB: b_q^* in (34) and γ in (35);
> Implement the open-loop power control of
> (44);
> **end**

Algorithm 1: Proposed algorithm for decentralized beamforming and power control in an interference network with K BSs

5 Simulations

In order to prove the validity of the proposed beamformer MIOB, formulated in (34), we consider a block fading channel, where the pathloss and shadowing have already been compensated by the existing Automatic Gain Controls. For this reason, the elements that form each spatial channel $\mathbf{h}_{ij}$ in (3) are complex Gaussian, such that their modulus is Rayleigh distributed with power equal to one. We consider a non-line-of-sight scenario. Note also that the worst-case situation in terms of interference is when the user terminals of the colliding communications are close together, as it is the case of the intersection zone of the three communications in Fig.1; thus, when all communications present very similar pathloss and shadowing. First we consider a scenario with two BSs. Unless stated otherwise, the noise power is set to $\sigma^2 = 1$ and 5000 Monte Carlo Runs are considered to attain each point in the different plots. For each run the procedure is the following: i) realization of the channels $\mathbf{h}_{ij}$; ii) computation of the transmit beamformer; iii) implementation of the power control; iv) evaluation of the figure of merit.

First, Fig.2 considers two BSs with 2 antennas each and regulation mask equal to $\rho = 0dB$, and plots the average value of the power at the MURC for one of the BSs, when either MIO or VS beamforming is applied. More specifically, for each beamformer the corresponding channel gains a_{ij} $i, j = 1...K$ are computed, and subsequently, the powers that solve (15) (i.e. the powers at the MURC) are obtained for each of the two BS. These are the optimal powers in case there were a node with the information from all the channel links and could implement a centralized power control. This figure shows that MIOB is the technique that allows higher power at the MURC for a given P_{max} and ρ, which is a direct consequence of the beamformer design of Section 3. This behavior explains the better sum-rate performance that next figures show.

In order not to have negative powers at the MURC, the condition formulated in (50) (see Appendix A) has to be fulfilled and Fig.2b shows that although MIOB is not designed with this goal explicitly, it achieves it indirectly. This is thanks to the constraints in P_2. Specifically, Fig.2b shows for each of the beamformers in Fig.2a and also $\rho = 0$ dB the percentage of realizations that the powers in the MURC are negative within a certain range of P_{max}. The outages result in a low sum-rate and degrade the corresponding results. We note that VSB is the one that presents the worst behaviour.

Within a range of available power, the rate per user is computed following (8) and next figures show the mean sum-rate that it is attained for different
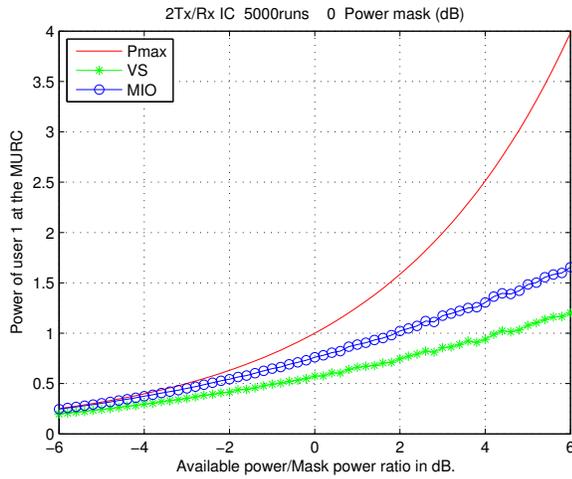
(a) Power user 1
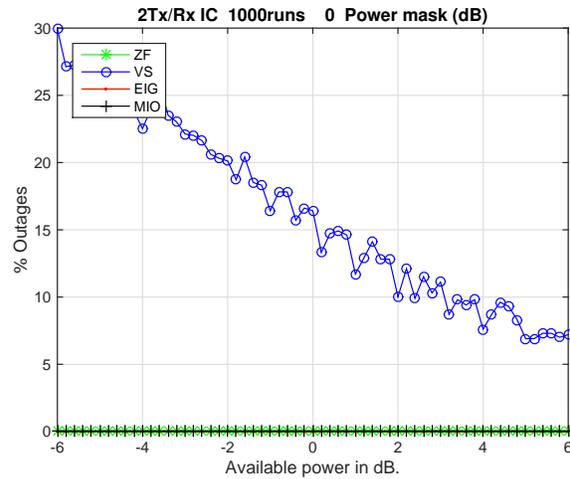
(b) Outage user 1

Figure 2: K= 2 BS with M= 2 antennas and the regulation mask is $\rho = 0$ dB. (a) Power at the MURC for one of the 2 BSs. The maximum avaliable power P_{max} is compared with the power that is allowed by each of the two closed-form beamformers VSB and MIOB; (b) Percentage of realizations that the powers in the MURC are negative.

beamformers. The evaluated ones are:

- closed-form transmit beamformers: ZFB, VSB, EIB, MIOB. In all cases the power control is the decentralized one that is proposed in Section V;

- iterative beamfomer and power control that solves P_2 (i.e. OptSDPEIG)

- iterative beamfomer that solves P_2 with the power control of Section V (i.e. OptSDP);

- iterative beamfomer that solves P_2, but with the additional power constraint (c) in (44) (i.e. OptSDP3C);

- closed-form beamformer, which follows the same design as MIOB, but with the additional constraint (c) in (44) (i.e. MIO3c).

Fig.3a compares all eigth beamformers for two BSs, two antennas and a power mask equal to zero dBs (i.e. $\rho = 1$). If this received power mask is not surpassed, the maximum sum-rate, which is given by (11), is equal to 2 (i.e. $2log_2 (1 + 1) = 2bps/Hz$). Fig.3b zooms in a part of Fig.3a in order to better appreciate the mentioned comparative results. Certainly, the proposed MIOB is the closed-form beamformer that gets closer to this bound. The beamformer OptSDPEIG is the one that gives higher rates because its design in P_2 does not constraint the total received power. When the power control is the one that is introduced in Section V (i.e. OptSDP) then its performance

lags behind the one of MIOB. Therefore, imposing the more restrictive constraint (27) in MIOB results in a better sum-rate than not doing it in P_2. Only OptSDP3c, which is optimum for P_2 with the additional power constraint (c) of (44), presents a slight better behavior, but with a high computational load. MIOB and MIO3c have the same performance. We can conclude that in this figure there is not a competitive design for MIOB, either iterative or non-iterative.

With OptSDPEIG, in order to visualize the local sensitivity that the optimum value in P_2 has to the constraints, Fig.4 plots the dual variables that correspond to each of the two constraints in P_2 (i.e. OptSDPC1 for constraint P_{2-a} and OptSDPC2 for constraint P_{2-b}). Each dual variable gives us a quantitative measure of how active is each constraint at the optimum. If the dual variable is zero, that means that the constraint is not active (i.e. it is fulfilled only with inequality). The higher the dual variable is, the more sensitive is the optimum value with respect to perturbation of the constraint. Specifically, in this plot we see that the higher is the available power, the more important is the second constraint about the interference value, and this makes sense.

In the following plots, MIOB is only compared with the non-iterative beamformers: ZFB, VSB and EIB. The reason is that the comparison with the iterative techniques OptSPEIG, OptSDP and OptSDP3C follow the same conclusions as the ones that have been drawn from Fig.5. Next, Fig.5a repeats the same simulations as in Fig.3a, but plotting only the closed-form
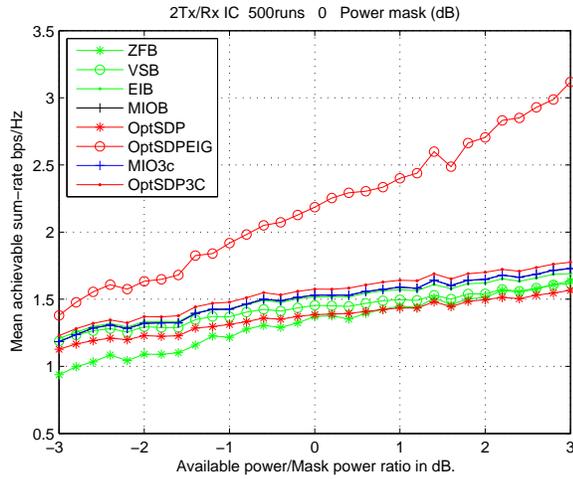
(a) Mean Sum-rate
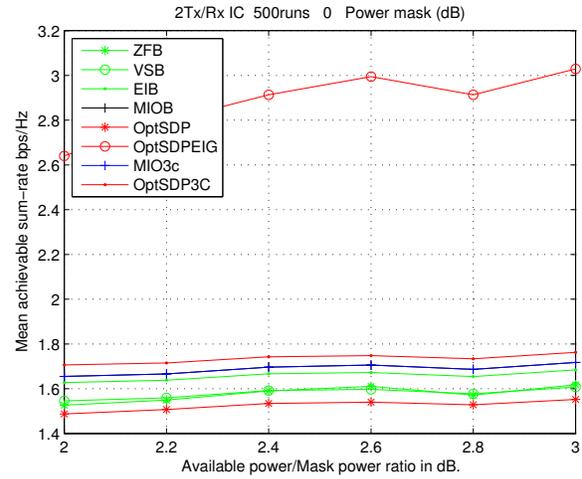
(b) Mean Sum-rate

Figure 3: K= 2 BS with M= 2 antennas and the regulation mask is $\rho = 0$ dB. (a) Average sum-rate;(b) zoom in.
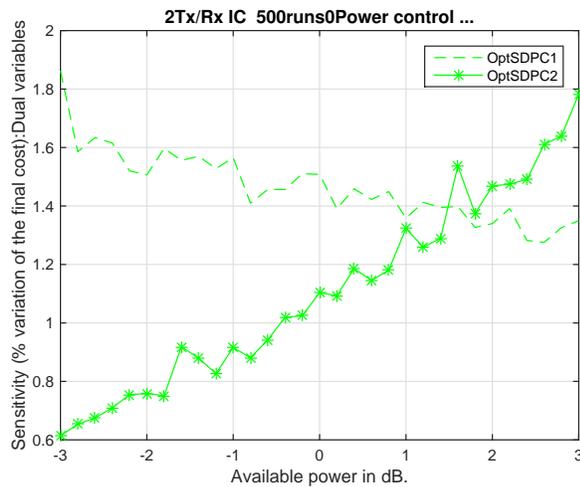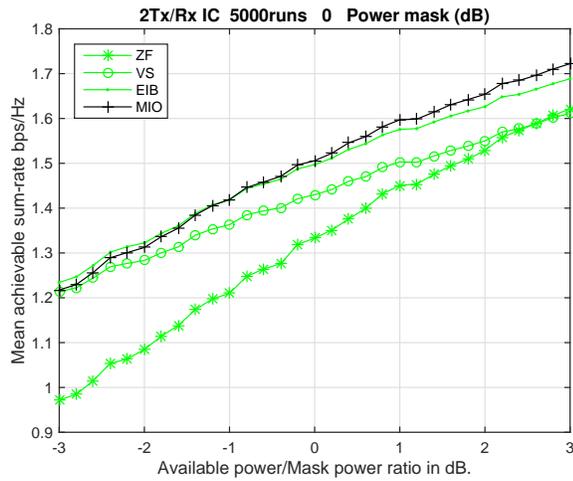

Figure 4: K= 2 BS with M= 2 antennas and the regulation mask is $\rho = 0$ dB. Sensitivity analysis of the constraints in P_2 (i.e. OptSDPC1 for constraint $P_{2-a} : p_q^* ||\mathbf{b}_q|| \leq P_{max}$ and OptSDPC2 for constraint $P_{2-b} : p_q^* a_{qI} \leq \alpha \rho$).

beamformers. Note again how our design can achieve the maximum sum-rate $K log_2 (1 + \rho)$ for high P_{max}. This behavior was predicted by (47). In Fig.5b a different power control has been considered with respect to the one in Fig.5b; that is, the proposed power control in Section V is applied only to the proposed beamformer MIOB, the rest of beamformers are working with the powers that results from (15). In this latter case, whenever the power results negative it is set to zero. In this way, we stress the benefit of the proposed novel beamforming and power control in front of the state-of-the-art techniques.

To complete the study of K=2 BS, M=2 antennas and $\rho = 0 dB$, next Fig.6a presents the sum-rate for the same setting as in Fig.5b for a wider range of P_{max}. Fig.6b shows the sum-rate for BS=2, M=2 and a regulation mask of $\rho = 3$ dB. The power control policy is the same as the one in Fig.5b. The superior performance of MIOB is observed, which aims at attaining the maximum sum-rate of $2 log_2 (1 + 10^{0.3}) = 3.16 bps/Hz$. We note that simulations with different values for the regulation mask have been carried out. We do not include them as similar conclusions can be drawn from them.

From the previous simulations, one concludes the importance of the power control that is proposed in Section V. The next two figures in Fig.7 stand out some features that the proposed beamformer MIOB presents, irrespectively of the power control. For this purpose the same power control of Section V is applied to all the beamformers. Fig.7a shows the sum-rate for BS=2 when the number of antennas at each transmitter increases. Specifically, the scenario is the same as the one in Fig.5a but with N=4 antenna. In
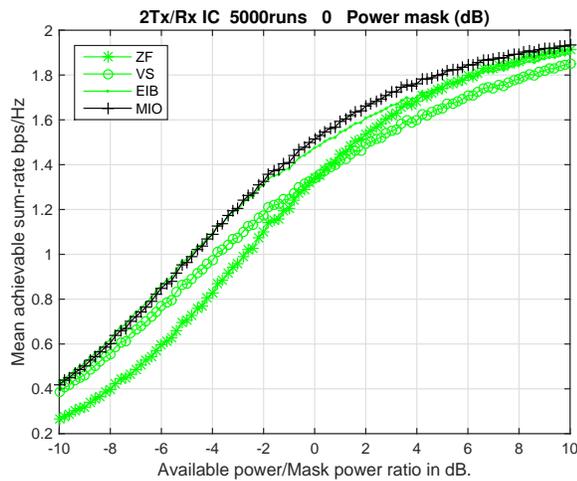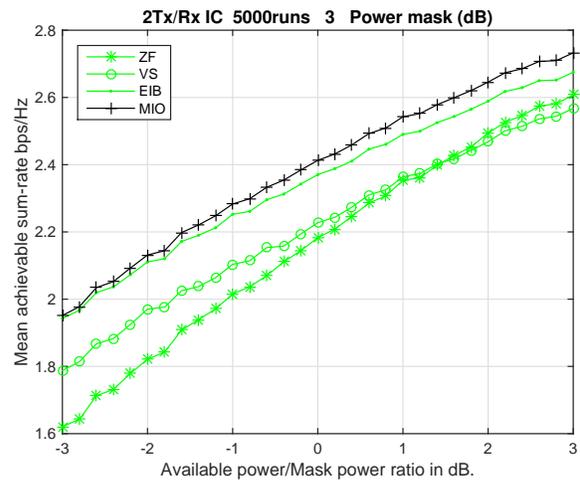
(a) Mean Sum-rate
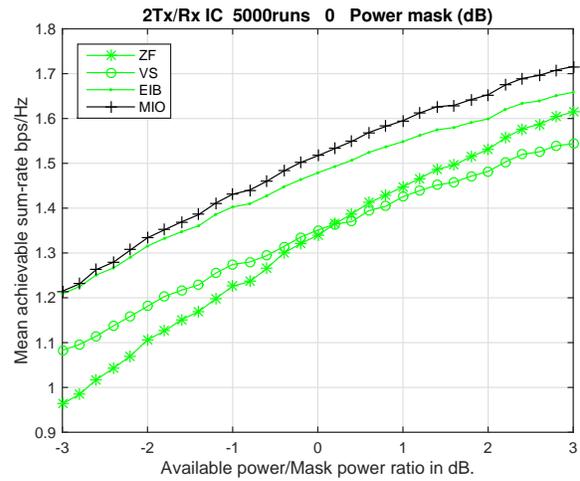
(b) Mean Sum-rate

Figure 5: K= 2 BS with M= 2 antennas and the regulation mask is $\rho = 0$ dB. (a) the power control used for all the beamformers is the one in Section V;(b) the power control of Section V is only applied to MIOB.

(a) Mean Sum-rate

(b) Mean Sum-rate

Figure 6: Sum-rate for K= 2 BS with M= 2 antennas and the power control of Section V is only applied to MIOB. (a) $\rho = 0 dB$;(b) $\rho = 3 dB$.

Fig.7b not only the number of antenna, but also the number of BSs increases. Specifically, BS=3 and N=3, the regulation mask is the same as in Fig.7a, that is $\rho = 0$ dB. In this case the maximum sum-rate is $3log_2 (1 + 1) = 3bps/Hz$. In these plots the superior performance of MIOB is verified, even in front of EIB, which is the beamformer that has shown a closer performance. This proves the effectiveness of the loading that is proposed in this paper.

6 Conclusions

In this paper we present a transmit beamformer and power control for the MISO IC channel. The design is non-iterative and has a closed-form. Unlike other transmit beamformers it ensures that a maximum total receive-power is not surpassed at each of the receivers in the network. Another unique feature is that the proposed beamformer optimizes a figure of merit that characterizes the transmitter, which is the directivity. Thereby, unlike other beamformer designs, we are working with an intrinsic feature of the transmitter. In order to maximize the directivity measure for the interference channel most of the paper is devoted to solve the design of the regularization parameter γ that it involves. This design is based on an heuristic and practical reasoning, whose validity is demonstrated by means of extensive simulations. Namely, in front of the existing closed-form beamformers the proposed MIOB controls the interference level that is creates and offers equal or the best sum-rate whenever the available power, P_{max} ranges $\pm 3dB$ from the regulation mask, ρ. The paper also frames the proposed beamformer within the extensive existing works on transmit beamfoming. Finally, the beamformer design is motivated by the future time-area-spectrum licensing systems.

7 Appendix A: Maximum sum rate

This appendix studies a condition under which the MURC of the Pareto power region in Section III attains maximum sum-rate. Let us consider first that the beamvectors are fixed and that only the trasnmit powers are to be optimized. Indeed, obtaining the maximum sum-rate power allocation for the IC is known to be complex [3]. However, the authors in [7] and [8] study the 2-user and the K-user scenario, respectively, and show that in noise limited scenarios (i.e. the amount of received interference is low w.r.t. the noise power level), the optimal power allocation strategy is that all transmitters work at the maximum available power. In other words, they work at the MURC of the Pareto power region. The derivation is done by

considering the convexity or concavity of the Pareto rate region as a function of the channel cross-gains. The maximum sum-rate point is the one that its tangent presents slope -1. However, in P_1 the feasible power set is not in general rectangular, but a polyhedron. Next it is shown at least one situation in P_1, where the MURC attains maximum sum rate.

Continuing with the case of K equal to 2, for the sake of clarity, the interest is to characterize the two contours of the rate region, i.e. from the upper left corner to the most upper right corner or MURC and from the MURC to the most right corner. Note that the MURC of the Pareto rate region is given by the MURC of the Pareto power region. Note also that once one of the contours is characterized, the other one can be inferred. Therefore, focusing on the curve between the most left corner and the MURC, the derivative of r_2 with respect r_1 is

$$\frac{dr_2}{dr_1} = -2^{r_1} \frac{\rho + \frac{a_{22}}{a_{12}}}{\rho + 1} \frac{\beta 2^{r_1}}{(\beta + 2^{r_1} - 1)^2}$$
$$\beta = \frac{a_{22}a_{11}}{a_{12}a_{21}} \qquad (48)$$

Note that the derivative is always negative and decreases as r_2 decreases. Concerning the second derivative of (48)

$$\frac{d^2 r_2}{dr_1^2} \leq 0$$
$$\text{if } r_1 \leq log_2 (\beta - 1). \qquad (49)$$

In consequence, this part of the rate region boundary is concave. With this condition, the optimum sum-rate point is achieved at the MURC of the feasible set; thus, when (15) is fulfilled. It is important to remark that $r_1 \leq log_2 (\beta - 1)$, and, therefore, the cross-gains must be lower than the gains of the direct links. Thus, again, as in [7], the scenario is a noise-limited one. As the link gains depend on the beamformer, this should introduce enough attenuation; thereby, showing the importance of the beamforming design. Finally, and for completeness, we note that a positive solution of (15) exists if

$$1 > \sum_{k \neq j}^{K} \frac{a_{jk}}{a_{jj}} \quad j = 1, \dots, K. \qquad (50)$$

As it is derived in [30].
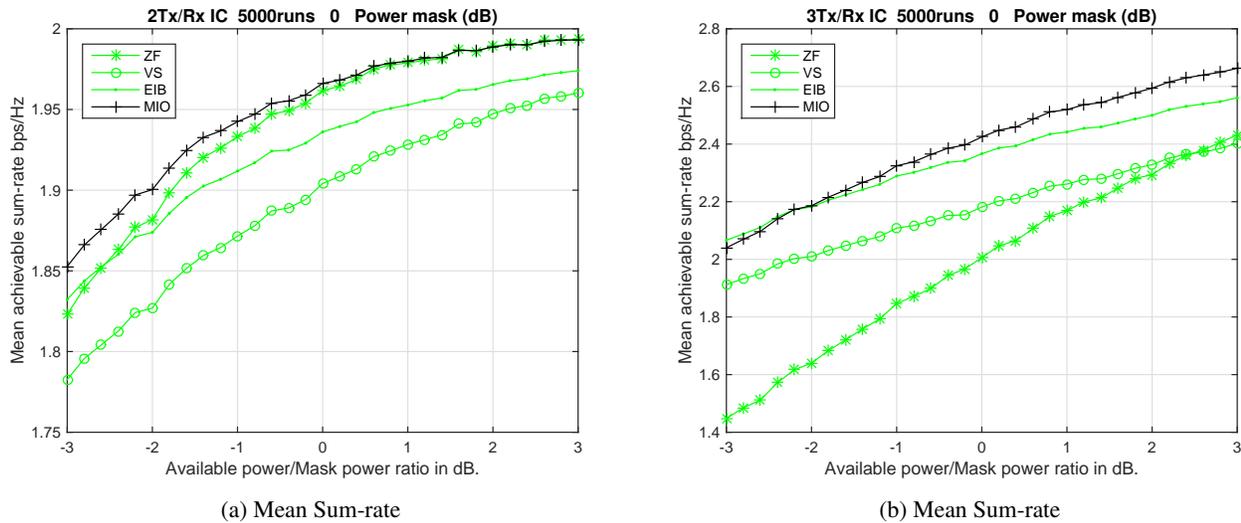
(a) Mean Sum-rate

(b) Mean Sum-rate

Figure 7: (a) Sum-rate for BS=2, M=4 antenna, $\rho = 0dB$ and the power control of Section V for all the beamformers;(b) Sum-rate for BS=3, M=3 antenna, $\rho = 0dB$ and the power control of Section V for all the beamformers.

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