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Teaching Engineers in the Seventeenth Century: European Influences in Portugal

Antónia Fialho Conde[a] and M. Rosa Massa-Esteve[b]

[a]CIDEHUS, Universidade de Évora, Évora, Portugal; [b]Universitat Politècnica de Catalunya, Barcelona, Spain

ABSTRACT

The practice of mathematics underwent a major transformation in the seventeenth century due to new procedures and concepts that also showed their utility for military architecture. The circulation of this knowledge can be found in several works. In this paper, we focus on an early work on Portuguese fortifications, *Methodo Lusitanico de Desenhar as Fortificaçoens das Praças Regulares & Irregulares* (*Lusitanic Method of Drawing Fortifications of Regular and Irregular Military Posts*), published posthumously in 1680, the author of which was the leading Portuguese Chief Cosmographer and Chief Engineer, Luis Serrão Pimentel (1613–1679). *Methodo Lusitanico* was a novel work containing the author's own theoretical explorations of the art and science of fortification in Portugal arising from the theoretical investigations and military education sponsored by the Portuguese Crown. The aim of our contribution is to show the European influences on Portuguese science in the seventeenth century through the analysis of an early work on modern fortifications written in the Portuguese language and by a Portuguese scholar. As far as the contents of the book are concerned, we show that Serrão Pimentel analyzes and reviews the published methods of fortification, and modifies them by introducing new procedures to improve the use of mathematics in the teaching of engineers. Our analysis also shows that Serrão Pimentel was a leading mathematician and a skillful teacher who had read the main mathematical works published at his time, such as Stevin's decimal arithmetic, and used them in practice.

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Introduction

Innovations in military engineering in the seventeenth century took place alongside the emergence of new languages and new fields of knowledge in the early modern period. For the explanation of structures and theoretical concepts, images, and illustrations became more important and more sophisticated.[1] However, given the fallibility of the senses, the use and practice of mathematical concepts and scientific instruments as complements to written discourse were also significant and were strongly recommended by the mathematicians of the Society of Jesus.[2] One of the clearest examples of the application of these practical means is found in military engineering, whose practitioners were excellent

CONTACT M. Rosa Massa-Esteve m.rosa.massa@upc.edu

mathematicians, some of them with a Jesuit education. Military engineering was a field *par excellence* for the application of mathematical knowledge.

Fortresses and strongholds of this period reflected concurrent scientific and technological advances. They also incorporated the growing specific knowledge of military engineering. This knowledge contributed to the appearance of schools and to an immense corpus of treatises which circulated all over Europe.[3] Indeed, the scientific character of these fortifications and works on military engineering allow us to demonstrate the strength of the relationship between science and technology from the sixteenth to the nineteenth century.[4] Beyond their military function, fortifications of this period testify to the advance of sciences such as mathematics, as well as offering an example of how military techniques developed in consonance with their historical moment.

Furthermore, the circulation of masters and ideas in the most diverse domains characterizes the modern period at a European level. Architecture and engineering featured prominently in this panorama. In the case of Portugal, the presence of mathematicians from the Jesuit College in Rome, and architects and engineers from Italy, the Netherlands, France, and other countries, would eventually influence the creativity brought to bear on this endeavor. Military architecture has been overlooked by the historiography, perhaps because many contemporary treatise writers regarded it as a minor and strictly functional branch in the classical division of architecture into civilian, religious, and military branches. Nevertheless, the advent of pyro-ballistics and the greater technical demands on military architecture in the modern period led to it being increasingly considered as a science that should be grounded in practical knowledge gained in the fieldwork of military engineers.[5]

The main objective of this paper is therefore to emphasize how science was brought up to date in the field of military art at the European periphery. We do so by demonstrating the level of mathematical knowledge exhibited in the *Methodo Lusitanico*, a work published by the Portuguese mathematician and engineer Luis Serrão Pimentel. Two factors in Serrão Pimentel's life particularly enabled him to demonstrate the knowledge he had acquired at *Aula da Esfera* in Lisbon.[6] Firstly, Portugal was involved in the Restoration War (*Guerra de Restauração*) (1640–1668), a conflict that arose from the restoration of Portugal's independence from the kingdom of Castile, whose crowns had been united for 60 years. This conflict was especially acute near the frontier that divided Portugal from Spain for 28 years, and required the permanent presence of architects and engineers. Their task was to ensure the reinforcement of the border with new constructions and to repair or adapt those already in existence to the bastioned model. The abilities of the engineers were verified on the field (drawing or adapting the plans),[7] work in which Serrão Pimentel always stood out. Secondly, Portugal had the benefit at that time of some of the best teachers of mathematics thanks to the presence of the Society of Jesus, which could count on a number of followers in the country.[8]

Luís Serrão Pimentel was one of the disciples who benefited from a Jesuit education, especially in Mathematics and Fortification. He lived at a time when theoretical, technical, and scientific requirements were making themselves felt in several military domains: the recruitment and training of army personnel; the strategic position of the armies in the field (*Castrametação*); territorial studies focused on the development of fortified buildings that could efficiently meet the defensive needs of the population; and the design and construction of new machines, based on the available technology, which enabled the monetary costs of construction to be minimized.[9]

Throughout the modern period, especially in the mid-sixteenth century, a number of foreign experts in fortification and military architecture circulated throughout Portugal, spreading new knowledge based on recent texts published in Europe, which appeared during the Italian Renaissance. The emergence of new conditions conducive to fresh research stemmed not only from practice but also from theoretical, rational, and humanist knowledge, of which Serrão Pimentel would be one of the principal proponents in Portugal. There is a growing body of literature on Serrão Pimentel, consisting of master's theses, articles, and books, some of which are focused on Serrão Pimentel's work in Military Architecture[10] and others on the man himself,[11] while other research is more concerned with the influence of Jesuit training in Mathematical practice.[12] The aim of this article, however, is to highlight the exceptional education provided by Serrão Pimentel for engineers in Portugal by analyzing the contents of his *Methodo Lusitanico*. The title of this book stresses Serrão Pimentel's conviction that the Portuguese were also capable of producing scholarly works written in their mother tongue, based both on tradition and the theories circulating internationally at the time, as was already the custom in other European countries. In his *Methodo*, the author sets out an easier method of fortification by combining geometry and the rigorously drawn figures in his mathematical deliberations.

Serrão Pimentel's work was innovative for its pedagogical and didactic character in which image and text complement each other with the aim of combining mathematical theory, geometric representation and military practice in an accessible way. It also attests to the author's expertise as a teacher of Fortification and Military Architecture in the classroom (1647), inaugurating a trend that Manuel de Azevedo Fortes would continue in the Eighteenth century with the publication of handbooks for the teaching of military engineering containing a marked emphasis on Arithmetic, Geometry, and Plane Trigonometry.[13] Throughout his work, Serrão Pimentel laid stress on the importance of exact demonstration in his endeavor to show that, in the training of engineers, theory and practice constituted an essential alliance for the exercise of military engineering, which could not be dissociated from the birth of modern science in the context of Europe.[14] The Iberian 'Century of Lights' in the eighteenth century would demonstrate the academic influence exerted by Serrão Pimentel on subsequent generations charged with the training of military engineers in Portugal. These included Manuel de Azevedo Fortes and Manuel da Maia, among others, many of whom remain anonymous. These engineers proved themselves capable of continuing and consolidating his Cartesian spirit.[15]

In this article, we first we focus our attention on Serrão Pimentel's biography. This is followed by a proposal for the analysis of his book, for which priority is given to the structure of the work. We emphasize Serrão Pimentel's authorial choices in the different chapters of the book, in order to demonstrate his mastery in various matters. In this analysis, two central aspects are then considered. First, we contextualize his proposal for innovation in bastion fortification, which he deals with in his book in order to provide 'exercise in practice' for the formation of military engineers. Their training was based on this discourse by Serrão Pimentel in accordance with his own model rather than by any foreign method or discourse. Second, we demonstrate Serrão Pimentel's extensive knowledge of mathematics, which enabled him to teach practical geometry, decimal calculation in accordance with Simon Stevin's work and Euclid's *Elements* from the Euclidian work by the Jesuit Christophorus Clavius (1538–1612), teacher at the *Collegium Romano*. We show how Serrão Pimentel addressed the original training of seventeenth-century engineers, and how he

provided practical engineers with some examples of geometrical constructions to deepen their mathematical knowledge in that period.

Luis Serrão Pimentel (1613–1679)

Luis Serrão Pimentel was the son of Jorge Serrão Pimentel and Ana de Tovar e Miranda. Born in Lisbon, he was baptized in the parish of Santa Justa on 4 February 1613. He was of Jewish descent, which caused the family some problems concerning the purity of blood,[16] although they lived in the manner of a noble family with links to trade in the East. At the age of 18, he embarked for India with an uncle. This project in India (according to Machado, they really arrived in Brazil)[17] was not carried out, since Pimentel returned to Lisbon shortly after. There he married Isabel Godines, who was also of Jewish descent (*conversos*). They had three children: Jorge Pimentel, Manuel Pimentel, and Francisco Pimentel.[18]

His training began in 1631 at the Jesuit College of Santo Antão, where he was a student of the Irish priests Ignacio Stafford (1599–1642) and Simon Fallonio, who taught there from 1604 to 1642.[19] He engaged in the study of Mathematics for 10 years (1631–1641), also studying under the Chief Cosmographer[20] Valentim de Sá. During this time, Serrão Pimentel also mastered Latin, French, and Italian. The recognition of his knowledge in the field of cosmography led to his appointment in 1641[21] as Chief Cosmographer of the kingdom (temporarily, because of the impediment of the owner of the position, António de Mariz Carneiro). He regained the position in 1647 when, with the approval of the King João IV, he oversaw the creation of a Class of Fortification and Military Architecture[22] (or Class of Mathematics, in which he taught Mathematics, Navigation, and Military Architecture). This class was merged with the existing Class of Nautical Studies before being transferred from Ribeira das Naus to the Terreiro do Paço, and by the eighteenth century it was designated a Military Academy.[23]

Serrão Pimentel would later be appointed Chief Engineer of Alentejo and lieutenant general of artillery, exercising his duties in all the provinces of Portugal in 1663.[24] By order of the monarch, he visited all the fortifications of the kingdom, making suggestions for reform in several of them as well as participating directly in several episodes of the Restoration War.

In 1671, the king recognized his contributions to the field of cosmography, in particular because Serrão Pimentel established routes for Portuguese travels to India and Italy, and made official that the property of the position of Chief Cosmographer belonged to Serrão Pimentel, indicating at the same time the end of his activities as Chief Engineer in Alentejo. However, the king also asked Serrão Pimentel to *read* (i.e. teach) the art of Mathematics and Navigation in Ribeira das Naus.[25] The number of his students progressively increased, and first the manuscript of *Methodo Lusitanico*, and later the book, began to be used in various contexts other than that of the dry border, e.g. for the training of engineers in Portugal who would later travel to India and Brazil, Africa, and the Orient.[26] For over 30 years, he taught at the *Aula de Fortificação*, where almost all the engineers of the time were his students (for example, Jena Dontel, Nicolau de Langres' son, was examined by Serrão Pimentel). Among his requirements for engineering education, he regarded it as essential to 'exercise in practice'; i.e. not only was it necessary to master the theory of construction, but also to have the opportunity of providing on-site assistance.

He was a well-known bibliophile, and his personal library (with works in the fields of Mathematics, History, Philosophy, Astronomy, Navigation, Geography, Military

Architecture, or about instruments, such as *Theatro Estromentorium*) reveals his erudition. He was also known for maintaining relations with the scholars of his time, such as D. Francisco Manuel de Melo, especially at the Academy of Generous, where Serrão Pimentel imparted several lessons in Mathematics.

Besides the *Methodo Lusitanico*, Serrão Pimentel wrote *Roteiro do mar Mediterrânico (Road of the Mediterranean Sea)*, published in 1675, and *Arte prática de navegar e regimento de Pilotos repartida em duas partes* (. . .) (*Practical art of sailing and Pilots' regiment divided into two parts*), also published posthumously (1681). He also left several works in manuscript, especially in the domain of bastion fortifications, most of them written by students who attended his classes.

Teaching fortifications in Portugal

The work of fortification was first considered as a science and an art form in the mid-sixteenth century, because it relied upon mathematical principles in the search for forms and proportions. The activities of engineers and architects had been very closely associated since Antiquity; the essential difference resided in the fact that engineers were familiar with the practice of war, as their professional origins were in the military. For Sebastiano Sérlio, engineers were 'architects of war' who designed other types of buildings in peacetime. Brunelleschi and Leonardo, among others, adapted fortified constructions for the use of artillery, giving rise to a new polygonal bastion that would revolutionize the conception of fortresses during the second quarter of the sixteenth century. This geometric organization of space with angular bulwarks and regular layouts had several foreign interpreters in Portugal from the 1500s on. They were instrumental both in the education of national master builders and in Portuguese military architecture itself. These interpreters (approximately 100 French, Flemish, Italian, Dutch, Swedish, and English engineers and architects), remained in Portugal after the reign of the 'Philip Kings' (Philip II, III, and IV),[27] and later participated in the reinforcement of the frontier during the Restoration War.

Indeed, the fortresses dating from this period were mainly conceived by European engineers, some of whom were already in the Peninsula at the service of the Philip Kings and came from regions that had been war zones for many years: the Eighty Years War (1568–1648), the Thirty Years War (1618–1648)), or religious uprisings such as those in Holland and France. Their work reflected a departure from the Italian model in favor of Nordic models. A new attitude towards the importance of a defensive reinforcement had taken root with the ascension to the throne of King John IV in December 1640.[28] Parallel to the appearance of a permanent army and auxiliary bodies, the Council of War and the Border 'Junta' (Assembly) were created in Portugal, each with very well defined functions for the purpose of inspecting and dealing with all matters relating to fortifications.

According to Mateus, seventeenth-century books provided national variations of fortification systems,

> (. . .) All of these books were reciprocally influenced and influential. Due to their frequent reprints, citations, and translations into a variety of foreign languages, special mention should be given to *Les fortifications* (issued between 1628 and 1672) and *De la charge des gouverneurs de places* (reprinted from1639 till 1708), both by Antoine de Ville; *Les fortifications* by Blaise François de Pagan, (printed from 1645 till 1689); *Les travaux de Mars* by Manessont Mallet (reprinted 1671–84); *La nouvelle manière de fortifier les places* by François Blondel (reprinted

> 1683–1711); the same title by Arnold de Ville (reprinted 1689–1748); *Véritable manière de bien fortifier,* by Vauban, first published in 1692 and then translated into French, Italian, English, German and even into Turkish in 1794; the *Traité de fortification* (reprinted 1694–1711) by Ozanam and finally the *L'ingénieur français*, by Naudin, published between 1695 and 1757. (. . .)[29]

As one may see, prior to Serrão Pimentel's death, only the works by Antoine de Ville, Pagan, Manessont Mallet, and Blondel appear on this list. His approach should be understood in the context of a period in which the translation of works by G. Fournier (*Architecture Militaire* ou *Fortification Moderne*, 1645) and Henrique Villegas (*Fortification de Plazas* y *Nuevo Metodo e Fortificar*, 1651), as well as Simon Stevin's book, renewed and disciplined the teaching of drawing and building fortifications in the *Aula de Fortificação* through *Methodo Lusitanico*'s work, as explained in the following sections.

Methodo Lusitanico (1680): fortifications and practical geometry

The *Methodo Lusitanico* consists of 666 pages and is divided into two parts. The first, entitled 'Operative', explains Serrão Pimentel's method, which states that from the drawing of outer polygons to the inside, and from proportions, it is possible to prepare all kinds of terrain, regular or otherwise, in a regular frame.[30] The second, entitled 'Qualitative', seeks to qualify with events and demonstrations the operations made in the first part.[31] These two long parts are followed by two appendices of comments on the works of the Count of Pagan, in which Serrão Pimentel defends the 90° angle against Pagan's obtuse angle. The book finishes with two mathematical texts: the first entitled *Trigonometria Practica Rectilinea* (*Rectilinear Practice Trigonometry*), and the other *Compendio de alguns problemas da Geometria practica, & Theoremas da especulativa* (*Compendium of some problems of Practical Geometry, and Theorems of Speculative*). These were essential materials to those who wished to realize the *Methodo* and to intensify the practice of mathematics for military engineers according to Serrão Pimentel.

The publication of the text underwent some delay, mainly due to literary censorship (the work was completed in 1666, but 11 years later Serrão Pimentel was still asking the king to allow its publication). The license for publication was granted in 1678 and the book appeared two years later, one year after Pimentel's death in 1679. The opinions expressed by the censors enables us to estimate the critical reception of this work, its acceptance and the recognition of the author at that time. In the opinion of General Diogo Gomes de Figueiredo, the *Methodo* is more accurate than the European theories in use at that time, especially with regard to the angles of construction of the walls, 'generally in all kinds of regular and irregular squares'. João Duarte, connoisseur of mathematical disciplines, considered the main advantage of the *Methodo* to be that of fanciful design, hitherto done more for 'art' than for science. João Mendes de Vasconcelos had a similar opinion: he considers it to be an important work for national military engineering, the result of 'great work, study and science' by the author.

However, Francisco Correia de Lacerda commented that there was a danger that engineers might neglect various aspects of the art of fortification when using the new *Methodo*, since they could not be broken down into detail in just one manual. Furthermore, the third Count of Ericeira, D. Luís de Meneses, who had met with Serrão Pimentel, was of the opinion that the training of an engineer based solely on this treatise was insufficient because fortification required a great deal of experience. As Serrão Pimentel himself explained at the

beginning of the published work, 'Experience is very necessary to practice . . . Science and experience are not enough by themselves; both are needed to train a good Engineer.'[32]

Fortifications in the *Methodo Lusitanico*

Within the context of the seventeenth century, Serrão Pimentel's work signified a turning point in the application of notions of trigonometry and logarithmic calculus and in the progressive replacement of the Italian model with treatises written in the Low Countries. As theoretical models, the works by Simon Stevin (*Oeuvres Mathématiques*, 1585), Daniel Speckle (*Arquitectura das Fortalezas*, 1589), and Jean Errard Bar-Le-Duc (*Demonstrée et Reduict en Art*, 1594), which was the foundation of the French fortification school) were of crucial importance for Pimentel's work.

In the introduction (*Proemio*), Pimental mentions other authors, some of whom were his contemporaries,

> (. . .) Also for these Methods we avoid the multitude of tables that Marolois, Fritach, Dogen, Cellario, Goldman, & others bring, not even one, as does Antonio de Ville, in order to make proportions, because although I bring some that I have accommodated in the Qualitative part, it is not because they are necessary, but only so that in them as in a mirror it is possible to see the correspondence of the angles and the measures that are taken between the parts of the fortification drawn by any of the said Methods. I only made the Table 14.B (see Figure 1) in the form of the said Authors to make proportions, but founded on different and improved proportion (. . .) for if anyone still wants to use their way of making proportions by tables for greater or less regular figure than the side that was taken by supposition. (. . .) And because the Fortification of the Count of Pagan is famous in Europe, I made a Summary of it by Appendices, with censure about the faults that I recognized in it. (. . .)[33]

Thus, the *Methodo Lusitanico* provides us with an insight into the European influences on Portuguese science in the seventeenth century, given that this is the first book written in

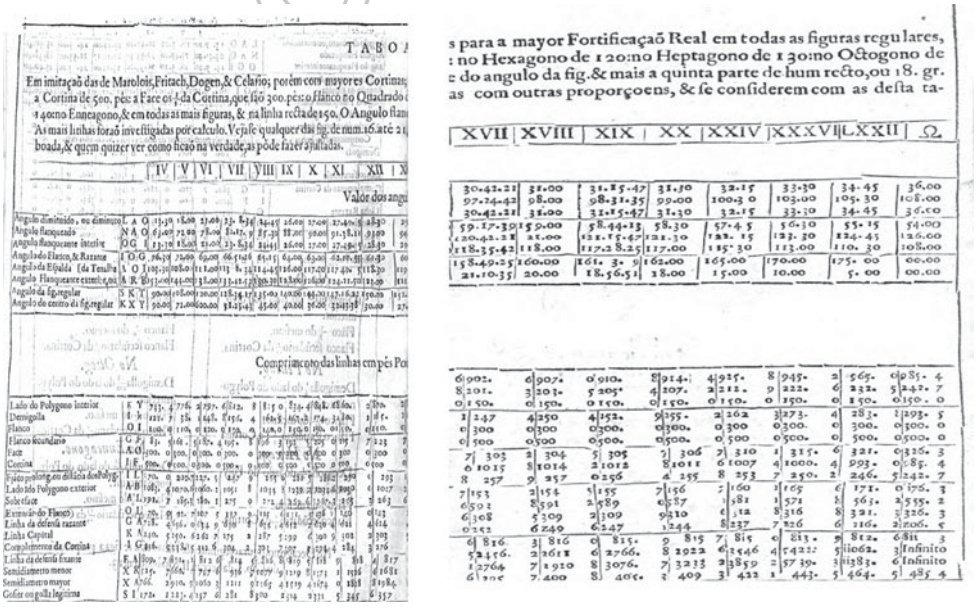


Figure 1. Table 14B. Pimentel, *Methodo*. BNP-Cota: sa-1138-a.

Portuguese about modern fortifications by a Portuguese author. From a theoretical point of view, Serrão Pimentel was basically inspired by the work of Adam Freitach Mathias Dogen, Nicolau Goldman, Samuel Marolois, Coheorn, Simon Stevin, and Antoine de Ville, in different editions, while from a practical point of view he was contemporaneous with some foreigners active in Portugal at the time. He criticizes some European theorizers, such as Pagan, who belonged to the so-called second French school of fortification, before the appearance of Sébastien Le Prestre de Vauban. Serrão Pimentel was himself inspired by the examples in Holland.[34]

As far as construction is concerned, the practical application of these treatises can be found in several fortresses in the south of Portugal, particularly in Elvas. In this case, the application to the terrain of the most important geometric features of the so-called *first Dutch method* of fortification (starting in Leiden in 1575) resulted from the designs of the Dutch Jesuit João Paschasio Cosmander, in 1643, who received from the king John IV the rank of colonel superintendent of engineers and engineer in the province of Alentejo. He praised the capability of Dutch fortifications, which had proved their worth by their resistance to the Spaniards.[35] He achieved his ambition of educating national master builders in the area of military engineering, which would be continued in the following century by Azevedo Fortes.

As mentioned above, while there was an increasing number of treatises on ground warfare, Serrão Pimentel distinguished his *Methodo Lusitanico* from these by elevating rationality as the essence of fortification as represented by the seventeenth-century Portuguese scientific practice. He did so by quoting and criticizing Euclid[36] and at the same time developing a structured and scientific mathematical praxis. This praxis is also present in the use of mathematical instruments (the protractor and the graduated ribbon), and in the dictated or copied notes of the *Aula da Esfera*, which were quoted by his son Manuel, who succeeded him as Chief Cosmographer. For example, he introduces the Portuguese foot (the sixth), although the Rijnland foot, standardized in the Netherlands, was the unit he used most frequently during the early years of his career.[37]

The method proposed by Serrão Pimentel considered the military art of fortification an exact science, stressing the importance of reason, mathematics, and geometry in the construction of fortresses, and thereby lending perfection to the imperfections of the land on which the buildings would stand. This principle of placing the terrain to be fortified in a geometric representation of a regular or irregular polygon was regarded by Serrão Pimentel as his own invention and the novel contribution of his work. He applied it to the angles of defense walls as well as to the duplication of defenses, 'in general, in all kinds of regular and irregular military forts'.[38] Everything was conceived as being part of a machine, and according to Serrão Pimentel that accuracy of perfection was an integral part of his own method, which improved upon the European theories of his time. The justification for this method was the result not only of his experimentation in the field but also of the Portuguese experience, as well as his theoretical learning and self-education at an international level (reading different authors in different languages, some of them his contemporaries already mentioned in this text – S. Marolois, M. Dogen, Andreas Cellarius, A. Fritach, N. Goldman, Antoine de Ville, Blaise François Pagan, Francesco Tensini). We emphasize the educational role that Serrão Pimentel assumed with this work, which foresaw the creation of a talented body of military engineers in Portugal.

In addition, theoretical and practical mathematics are well represented in the work on fortifications by Serrão Pimentel as a textbook for training military engineers, as explained in the following section.

Practical geometry in the *Methodo Lusitanico*

Many changes occurred in the practice of mathematics in the seventeenth century.[39] Trigonometry, logarithmic calculus, and practical geometry all underwent a major development throughout this century. All these subjects proved their utility for solving problems in natural philosophy or physics, in architecture and also in military fortifications.[40] Furthermore, technical developments in military and art, as well as in scientific instruments, increasingly made mathematical disciplines a universal tool.[41] Thus, in the textbooks entitled *Cursus Mathematicus*, the mathematical disciplines, classified into pure and 'mixed' mathematics, were used for training engineers.[42] Theoretical and practical education was necessary for military engineers, above all training in practical geometry for making all kind of fortifications possible.

Although Serrão Pimentel emphasized that knowledge of theoretical mathematical questions is not essential for practical work, since engineers need only to follow a set of rules, at the end of his book he adds two theoretical mathematical texts on practical geometry containing trigonometry with logarithmic calculus, the notation for the decimal numbers together with their arithmetical operations, as well as some propositions and theorems of practical geometry from Euclid's *Elements*. For example, in the *Proemio* of this work, he justifies the inclusion of these mathematical texts for people who wish to go deeper into their researches,

> I did not start by writing about the principles of practical Geometry or decimal Arithmetic and Trigonometry, despite the fact that they may appear at the beginning, since they serve in the practice of Fortification. (. . .) But I have placed them at the end, because those who wish to deepen their studies in this area and those who intend to draw on the paper regular shapes may use them without being limited to the existing standards.[43]

Pimentel explains that all these mathematical concepts and propositions are useful for the practice of building fortifications and for becoming a good engineer. For that reason, the *Trigonometria practica rectilinear* begins with a section entitled: 'The Practice of Decimal Arithmetic or *Dizima*' (*Practica da Arithmetica Decimal ou Dizima)*. He justifies the order by claiming that this part 'is extremely necessary . . . for fortification calculations'.[44]

This part of mathematics dealing with the operations of decimal numbers was in fact an invention of Simon Stevin (1548–1620), one of the greatest mathematicians of his time, mainly known in the history of mathematics for his decimal calculus in 'La Disme' section of *L'Arithmetique* (1585), and for his concept of number.[45] Stevin stressed the benefit of the practice for humanity and claimed that this calculus was easier than fractional calculus.[46]

In the prologue of this part of the *Methodo Lusitanico,* entitled *Dizima* in Portuguese, Serrão Pimentel acknowledges Stevin as the inventor of decimal arithmetic and praises it:

> The Dizima is an invention by Simon Stevin of Bruges, which many of our contemporaries will find excellent for an immediate and very approximate determination of broken numbers, using only four kinds of common Arithmetic as well as the common way to obtain more and increasingly approximate square and cubic roots (. . .) which by their nature were irrationals according to the 'dignidades' of numerous or specious Algebra.[47]

He explains the great utility of decimal arithmetic and claims that he was already using this calculus in his teaching practice, which shows that he had not seen Stevin's work beforehand. In fact, from 1647 to 1666, Serrão Pimentel taught at the Aula of Fortification using a manuscript, and the book based on this manuscript *Hercotectonica Militar*[48] was subsequently accepted for publication in 1678. Furthermore, Serrão Pimentel claims that he had not previously been acquainted with Stevin's work, and that after having read it he did not have anything more to add to his own text.[49]

Thus, Serrão Pimentel repeats Stevin's definition in Portuguese and goes on to emphasize the difference in his word 'proporçao' rather than Stevin's word 'progressao' as well as justifying the relevance of his invention for human accounting,

> Dizima is a kind of arithmetic invented by the tenth proportion (progression), consisting of characters of ciphers, whereby a certain number is described and by which also all accounts which happen in human affairs are solved by whole numbers, without fractions or broken numbers.[50]

Q5

Although Serrão Pimentel's definition of *Dizima* is very similar to the one set out in Stevin's text, he introduces a very different notation for decimal numbers and justifies this choice. He explains that placing the character above the number (see Figure 2(a)) may lead to confusion with the algebraic expressions, and thus he proposes a new notation of his own:

> Some say the whole, first, second, third, etc. with the following character etc, or several other numbers placed above the numbers to which they belong, or only the last character placed above the last numeric letter. These are used by some authors of Algebra; but they are cumbersome and it would be easier (to cast) a zero as an exponent of the integers numbers; a tiny line in the first, two in the second, etc.[51]

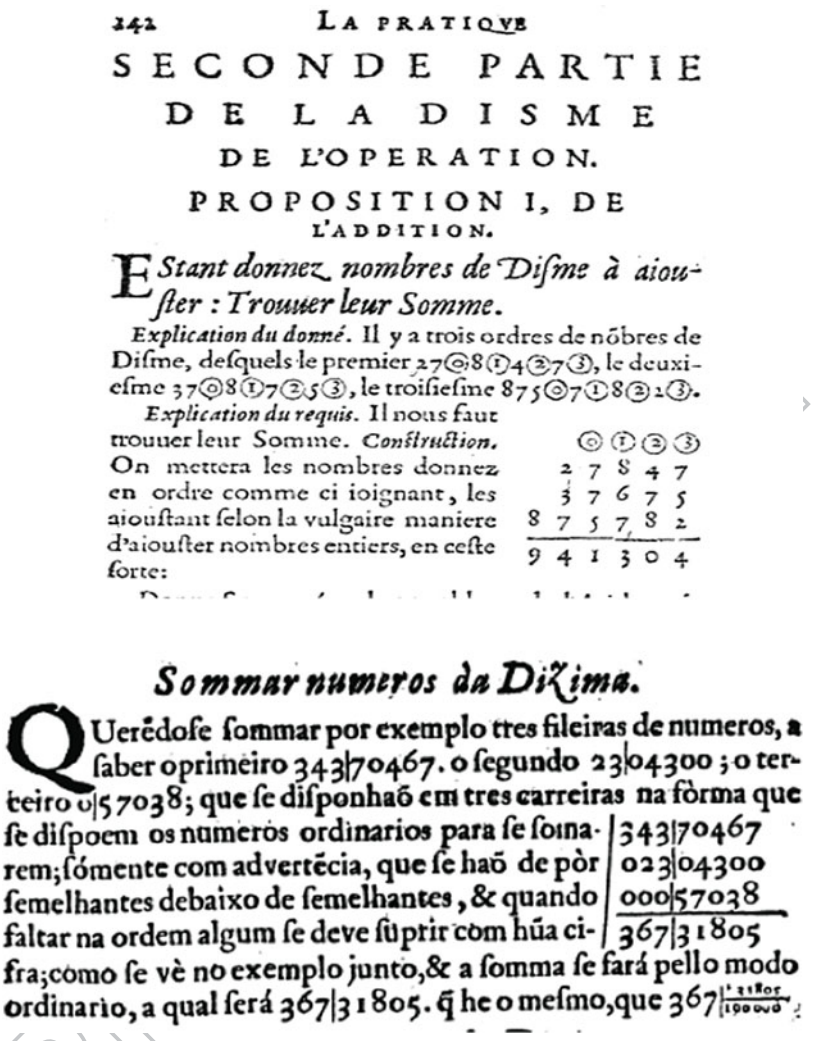
Serrão Pimentel also introduces a further novelty; he proposes a new way to represent the decimal numbers with a vertical line between the whole number and the decimal part. He states as follows (see Figure 2(b)):

> And yet I feel better and easier for the operations to separate the integer from the broken integer with a line (vertical) up and down like in this number 3428l 76054, which means 3428 whole and 76054/100000 that are fifth.[52]

He goes on to explain the operations between the decimal numbers and uses examples for the addition, multiplication, and division. While they are very similar to those used in Stevin's text, Serrão Pimentel employs his own original notation with some differences; for example, he also adds zeros in the corresponding empty places. One may see these details in Figure 2(a,b), which represent the addition in Stevin's *Disme*'s and Serrão Pimentel's *Dizima*'s: The reference, definition, and the theoretical explanation for using this invention with Serrão Pimentel's own notation, written didactically and in Portuguese, provide an example that reveals the contemporary nature of his work within the scientific framework of his time. Moreover, Stevin's *La Disme* is used and quoted constantly throughout the book, so we may conclude that although Serrão Pimentel may have been familiar with *La Disme*, he decided not to use it and invented new notation and new procedures.[53]

Compendium of some problems of practical geometry and speculative theorems

This mathematical section consists of 21 pages and is divided into four chapters. Nine problems appear in the first chapter, entitled: 'The direction of the lines in the problems' (*Da direção das linhas em problemas*); two problems appear in the second, entitled: 'The design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SECONDE PARTIE DE LA DISME DE L'OPERATION.

PROPOSITION I, DE L'ADDITION.

Estant donnez nombres de Disme à aiouster : Trouuer leur Somme.

Explication du donné. Il y a trois ordres de nõbres de Disme, desquels le premier 27⓪8①4②7③, le deuxiesme 37⓪8①7②5③, le troisiesme 875⓪7①8②2③.

Explication du requis. Il nous faut trouuer leur Somme. *Construction.* On mettera les nombres donnez en ordre comme ci ioignant, les aioustant selon la vulgaire maniere d'aiouster nombres entiers, en ceste sorte:

⓪	①	②	③
2	7	8	4	7
3	7	6	7	5
8	7	5	7	8	2
9	4	1	3	0	4

Sommar numeros da Dizima.

QUerẽdose sommar por exemplo tres fileiras de numeros, a saber o primeiro 343|70467. o segundo 23|04300; o terceiro 0|57038; que se disponhaõ em tres carreiras na fórma que se dispoem os numeros ordinarios para se somarem; sómente com advertẽcia, que se haõ de pòr semelhantes debaixo de semelhantes, & quando faltar na ordem algum se deve suprir com hũa cifra; como se vè no exemplo junto, & a somma se fará pello modo ordinario, a qual será 367|31805. q̃ he o mesmo, que 367 1/100000.

343	70467
023	04300
000	57038
367	31805

Figure 2. (a) Stevin, *L'Arithmetique*, 142. (b) Pimentel, *Methodo Lusitanico*, 551. BNP-Cota: sa-1138-a.

of the regular figures' (*Da delineaçao das figuras regulares*); two further problems in the third chapter, entitled: 'The augmentation and diminution of the plane figures and solids' (*Do augmento, & dimunuçao das figuras planas, & corporeas*), and finally 33 theorems in the last part entitled: 'Theorems about the lines' (*Dos Theoremas a cerca das linhas*). The theorems in this last part correspond to some propositions from Euclid's *Elements* in Books I (13 propositions), II (1), III (5), V (1), VI (10), VII (2) and to the theorem from Ptolemy's *Almagest*. Serrão Pimentel enunciated the proposition, briefly explained the demonstrations rhetorically and added the numerated figures at the end of the book, leaving the detailed demonstrations aside.

If we compare it with Clavius' *Elements* (1574), his possible source, we are able to check the similarity in the enunciation, translated from Latin to Portuguese; the figures are the same and the corollaries also coincide both in order and in number. However, Serrão

Pimentel claims that these theorems are not all necessary for the Fortification Calculus, but since they are widely used and serve for several cases, he decided to refer to them in order to make them accessible for practical engineers and soldiers. He further added that he left their proofs for Speculative Geometry in another manner and another order, stating nevertheless that in the meantime anyone could read Euclid's proofs in any of his quoted expositions.[54]

As regards the constructions of figures in practical geometry, in the first four problems of the first chapter Serrão Pimentel shows how to design parallel and perpendicular lines to other given lines. The fifth problem explains how to divide a line into equal parts. The most relevant problems in this first chapter are the sixth and the seventh, in which Serrão Pimentel solved the problems of finding one and two means proportional between two given lines, respectively, as well as their use for obtaining a square root of a number or a cubic root of a number. At the end of the problem of finding two means proportional, Serrão Pimentel offers the sources of this geometrical construction.[55] In the example for squaring cubic roots, he also quotes Nunes, Tartaglia, Clavius, and Stevin.[56]

All these examples provide evidence of Serrão Pimentel's extensive mathematical knowledge, while at the same time showing his didactical intention for proper training of engineers, as expressed in the Portuguese language.

Final comments

We believe that Serrão Pimentel's training and his activities as an engineer, a teacher, and a military man on the ground should be considered in the context of the Jesuit influence in Portugal. It is also necessary to bear in mind the Jesuit tradition in education and in mathematical knowledge, which can be found in the Jesuits' own study plan. This gave rise to the presence of several specialists in Portugal, especially foreigners, sent there by the Superior General of the Jesuits in the seventeenth century. These specialists exerted a strong influence, directly or indirectly, on Serrão Pimentel's education in the *Aula da Esfera* at the College of Santo Antão, as mentioned above.

The *Aula de Fortificação e Arquitectura Militar* was created in 1647, in the context of the Restoration War, by the order of the king, and Serrão Pimentel was the first person to teach there. This Class had the aim both of providing a theoretical education (which led to the translation and production of manuals for use by students), and of monitoring the progress of trainees in their practice, since the Chief Engineer was responsible for examining instruments and fortification projects. It is these demanding circumstances surrounding the production of autonomous and critical treatises at a European level that are reflected in the *Methodo Lusitânico*. This text was written expressly to provide didactical texts and examples to teachers and students of Engineering and Military Architecture. It is no coincidence, then, that it was written by the first Portuguese professor and specialist in this field – Luís Serrão Pimentel – who was both a Chief Cosmographer and the kingdom's Chief Engineer.

Serrão Pimentel is also remarkable for being an outstanding and recognized mathematician who was familiar with the most recently published works in this field at the time. He knew how to use these works in practice and was able to justify his choices. He explained his reasoning as well as analyzed, criticized, and changed various methods, of which Stevin's *La Disme* is just one example. He also invented new procedures to improve the utility of

mathematics. At the end of the *Methodo,* he added a mathematical section in Portuguese for improving the training of Portuguese engineers in practical geometry.

The way in which Serrão Pimentel defended the principles of the emerging model of bastion fortification set out in the *Methodo* is the result of his own on-site experience with his students and of the Portuguese experience in that area. Furthermore, it is also due to the fact that he brought his theoretical knowledge, based on the Jesuit tradition, up to date within the European framework of his time.

Notes

1. Flores, "Saber, Tecnologia."
2. From 1540 until its expulsion in 1759, the Society of Jesus founded 26 Colleges in Portugal, in addition to the University of Évora (recognized as a University only in 1559), first a College, and two schools. See Rodrigues, *Historia da Companhia*.
3. Castillo, *Ejércitos e militares* and Massa-Esteve, Roca-Rosell, and Puig-Pla, "'Mixed' Mathematics."
4. Moreira, "Do Rigor Téorico."
5. Conde, "Alentejo (Portugal)."
6. Inaugurated in 1590, in the College of St.° Antão was the most important Jesuit scientific institution in Portugal. It operated regularly between 1590 and 1759, the date of the expulsion of the Society of Jesus from Portugal. On this subject, see Albuquerque, *A "Aula da Esfera"*; Leitão, *A Ciencia na*; Mota, *O estatuto da*.
7. Conde, Henriques, and Guiomar, "Análise dos espaços"; Moreau, *Arquitetura Militar*; and Moreira, "Escola de Arquitectura."
8. Specifically for the seventeenth century, alongside the Portuguese master builders, we find in the Colleges of the Jesuits Christopher Borri, Ignatius Stafford, Simon Fallonio, Thomas Berton, John Riston, Thomas Audueno, Hugo Colano, João Pascacio Cosmander, Valentine Estancel, among others. In the following century, despite the presence of some foreigners such as Domingos Capassi and João Carbone, there were fewer, although this did not correspond to a lower number of teachers of mathematics, since there were at that point more Portuguese.
9. Bebiano, "A arte da Guerra."
10. Macedo, "Luiz Serrão Pimentel"; Conde, "Alentejo (Portugal)"; Flores, "Saber, Tecnologia"; and Sousa, *O conjunto*.
11. Carvalho, *Luís Serrão Pimentel*; Pereira, "Luís Serrão Pimentel"; Ferreira, *Luís Serrão Pimentel*; and Sousa, *Tenente-General de Artilharia*.
12. De Lucca, *Jesuits and Fortification*; Ferrão, "Tratadística, Ensino"; and Leitão, "Jesuit Mathematical Practice."
13. The relevance of these works for the history of mathematics and engineering in Portugal is evident in publications such as Guimaraes' "Sciencias Physico-Mathematicas" and Guerra's *Historia da Engenharia*.
14. Rossi, *O nascimento da ciência*.
15. Correia, "Pragmatismo e utopismo."
16. Olival, "O acesso de uma família," 67–82.
17. Machado, *Biblioteca*, t. III, 133–5.
18. Machado, *Biblioteca*.
19. Moreau, *Arquitectura Militar*.
20. A position belonging to the nautical field, the Chief Cosmographer taught and examined pilots, cartographers, and constructors of instruments linked to maritime navigation; he also revised and updated existing Nautical Regulations, and compiled the information received from the voyages by elaborating or updating Road Maps. See Albuquerque, "Instrumentos de Alturas."
21. Lisboa, Arquivo Nacional Torre do Tombo, Chancelaria de D. João IV, L.° 18, fl. 298v [microfilme 1238]. The document is from 13 July 1647; officially, Serrão Pimentel had the charge from 1644, but he started functions before, as shown by this document.

22. About the teaching activity in the *Aula de Fortificação* (or *Aula Régia,* because of its creation by the Portuguese king), and already in the preparation of *Methodo Lusitanico*, there is a manuscript transcribed in 1659 from these classes, a copy signed by his disciple, D. Diogo Pardo Osório, entitled *Lição de Architectura Militar ou Fortificação*. Biblioteca da Ajuda, Ms. 49 III-4. Cf. also Patrocínio, "A sabedoria dos antigos."
23. Ferrão, "Tratadística, Ensino" and Vieira, "O ensino da Matemática."
24. In 3 September 1663, we have a consultation by the Council of War with the King to grant the post of Chief-Engineer of the Kingdom to Luis Serrão Pimentel. Lisboa, Torre do Tombo, Conselho de Guerra, Consultas, maço 17. Viterbo, *Dicionário Histórico*.
25. fl. 167v. Lisboa, Arquivo Nacional Torre do Tombo, Chancelaria de D. Afonso VI, L.° 29, fl. 167-167v.
26. Bueno, "Com as mãos sujas."
27. Carita, *Conhecimento e Definição*.
28. Espírito Santo, *Restauração*.
29. Mateus, "Science of Fortification," 2130.
30. In this part, and for measures, moats, walls, and buttresses, he compares the discourses of Pagan, Cellarius, Dogen, Fritach, Marolois, Goldman, George Fournier, Bonajuto Lorini, Jeronimo Cataneo, Errard de Bar-Le-Duc, Pietro Ruggiero, Congio Farnesiano, Vilhalpando, Wilhermo Dilichio, D. Diogo Henriques de Villegas, Medina Barba, Francisco de Marchi (1599), Daniel Barbaro (in his comments to Vitrúvio, 1556), Sardi, Rojas, Jerónimo Maggi, Jacome Castrioto, Pedro Sardi, Tensini, Firrufino, Pietro Ruggiero, Carlos Teti, Vicenzo Scamozzi, Sebastiano Serlio, Vegésio, Jerónimo Victor, Jacobo Witsio (engineer), Pedro Persevallo (engineer), João Bossio (engineer). For the polygon drawing and measurements, he quotes Villegas, Goldman, Antoine de Ville, Dogen, Marolois, Witsio, Persevallo, Bossio, Archimedes, Christopher of Grimberg, Apolonio Pergaeo, Kepler, and John the Baptist Benedict. All of these names are written like in the *Methodo*.
31. Here are mentioned Fritach, Antoine de Ville, Dogen, Goldman (the most cited author), Villegas, and Henrique Hondio. Luís Serrão Pimentel criticized the methods of diverse authors, like José Barca, Alonso de Cepeda, Pietro Ruggiero (1661, dedicated to D. João de Áustria), Manesson Mallet (1671, Paris, *Les travaux de Mars*), Sylvere de Bitainvieu, D. Vicente Mut, António Ramón de Cardona (1671, Naples), Sir Ionas Moore (1673, London), the king of Great Britain (it was the work of Pagan), Louis XIV, the Emperor Ferdinand II (published by Gaspar Schotto, 1662). All of these names are written like in the *Methodo.*
32. Pimentel, *Methodo Lusitanico*, "Proemio."
33. Ibid.
34. Paar, "Fortificações urbanas."
35. Cobos, "La fortificación española."
36. Pimentel, *Methodo Lusitanico*, 398.
37. Ibid., 99.
38. Ibid., 89.
39. An essential change was the establishment of a symbolic language as a formal language in mathematics, so that the new language of symbols and techniques could be used in operations to obtain new results and procedures in several parts of mathematics. See Massa-Esteve, "Role of Symbolic."
40. Dear, "Art, Nature."
41. Roberts, Schaffer, and Dear, *Mindful Hand*.
42. Massa-Esteve, Roca-Rosell, and Puig-Pla, "'Mixed' Mathematics."
43. Pimentel, *Methodo Lusitanico*, 'Proemio'.
44. Pimentel, *Methodo Lusitanico*, 'Proemio', without numeration.
45. Sarton, "First Explanation." Indeed, Struick stated as follows, 'It was also Stevin who first showed the advantage of a systematic decimal division of weights and measures.' Struik, *The Principal*, 373.
46. The Decimal Arithmetic used by Stevin went into many editions, such as the first to be written in Dutch, *De thiende* (1585); a translation into French entitled *La Disme,* made by Stevin in *L'Arithmétique* (1585); a new edition in *Oeuvres Mathématiques* by Girard (1625), and in English, *The Art of Tenths or Decimal Arithmetic* by Norton, 1608. For more explanations on Stevin's decimal

calculus, see also Cajori, *History of Mathematical Notations* and Sarton, "First Explanation." For Stevin's instruction for engineers, see Krüger, "First Mathematics."

47. Pimentel, *Methodo Lusitanico*, 548.
48. *Areotectonica ou parte oppugnatoria e reppugnatoria, por outro nome poliorçetica, da hercotectonica militar [manuscrito] / por luis serrão pimentel tenente general com exercissio em ql. quer das provinçias em que se acha eng.o mor dos exerçitos e provinçia do alentejo e reino, lente da fortificação e mathematicas*. National Library of Portugal Reserved manuscripts, COD. 1640.
49. Pimentel, *Methodo Lusitanico*, 548.
50. Stevin's definition,

> (. . .) Disme est une especе d'Arithmetique inventée par la Disiesme progresión, consistente es caracteres des ciffres, par lesquels se descript quelque nombre, & par laquelle l'on Depesche par nombres entiers sans rompuz, tous comptes se rencontrans aux affaires des hommes. (. . .). (*La Disme*, 1585, 139)

Serrão Pimentel's definition:

> (. . .) Dizima he húa especie de Arithmetica inventada pella decupla proporçao, consistente nos caracteres das cifras, pellos quaes se descreve qualquer numero, & pella qual se resolvem por numeros inteiros sem quebrados todas as contas, que intervem nos negocios dos homens. (. . .). (Pimentel, *Methodo Lusitanico*, 548)

51. Pimentel, *Methodo Lusitanico*, 549.
52. Ibid.
53. At the end, he quotes authors who treated Algebra, such as Diophanto Alexandrino, Cardano, Tartaglia, Pedro Nunes, Rafael Bombelles, Clavius, Stevin, Vieta, Descartes, Alberto Gerardo, and Renaldino. In the part devoted to the angles, he begins with Euclid and ancient authors who treated the proportions, such as Hipparcho, Mileo Romano, Ptolemy. After, he continues quoting Georgio Purbachio, John Regiomonte, Pedro Apiano and the Arabs. In the examples, he uses Regiomonte, Ullac, Clavius, Magino, and Nicholas Raymaro. In logarithms, he underlines the importance of John Neper (Napier), who is followed by Henique Biggio and Ullac. In the tables of logarithms, he quotes the existing tables by Kepler, Frobenio, Laurencio Eichtadio, Bonaventura Cavalerio, Gellibrando, and Ullac, noting that he uses the last one in his calculations. All of these names are written as in *Methodo*. Pimentel, *Methodo Lusitanico*, 644.
54. Pimentel, *Methodo Lusitanico*, 666.
55. We have checked the similarities in Tartaglia's work (Tartaglia, *General Trattato*, 44) and in Clavius' work (Clavius, *Geometria*, 297–8), in which Heron's demonstration of finding two means proportional is also reproduced.
56. All these proofs are analyzed and commented in Bos, *Redefining Geometrical*. Serrão Pimentel also quoted Viète for this construction and for squaring cubic roots. Pimentel, *Methodo Lusitanico*, 652.

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