A punching shear mechanical model for reinforced concrete flat slabs with and without shear reinforcement

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ABSTRACT

A punching shear strength mechanical model for RC flat slabs with and without shear reinforcement, based on a beam shear model previously developed by the authors, is presented. The differences in resisting actions between beam shear and punching shear have been identified and incorporated into the governing equations and failure modes, resulting in simple but accurate punching design equations. The model consistently explains and quantifies some experimentally observed phenomena, such as the higher contribution of the concrete to the punching shear strength of slabs than in the case of beams subjected to shear, mainly due to the multi-axial state of stresses that takes place near the support. Furthermore, the model provides physical meaning to some parameters used in the design, such as the position of the critical perimeter or the effective stress of the punching reinforcement, among others. Very good agreement has been obtained between the model predictions and the results of 560 punching tests of concentrically loaded slabs, with and without shear reinforcement, included in two available large databases. The mechanical character of the model allows its extension to post-tensioned flat slabs, border or corner columns, steel fiber and FRP reinforced concrete slabs or different strengthening systems in a consistent way.

KEYWORDS: Punching shear, slabs, reinforced concrete, shear reinforcement.

1. INTRODUCTION

Punching capacity of slabs has been extensively studied in the past, both from the experimental and theoretical viewpoints [1–29]. As a result of these research works, several approaches have been developed for predicting the punching strength of reinforced concrete slabs with and without shear reinforcement. Even though some developed models reproduce quite well the experimental results, there is not yet a generally

accepted design model which combines accuracy with the necessary simplicity for daily design, adaptable to the variety of situations that can take place in practice. This is evidenced by the differences in the treatment of the punching strength in relevant codes provisions, such as EC2 [30], ACI [31] and Model Code 2010 [32], or by the changes produced along the time in some essential design parameters. Some examples of aspects still in discussion are the position of the critical perimeter, the effective stress in the shear reinforcement at ULS or the influence of the presence of shear reinforcement on the punching concrete contribution, among others. In fact, many of the punching strength code provisions are based on empirical models, adjusted to tests results, but without a consistent theory behind.

Certainly, advanced numerical models are more and more capable to simulate the local and global observed punching behavior [33–37]. However, there is still the need to improve the objectivity of the models, which are excessively dependent on the materials parameters used (i.e., post-cracking and softening behavior, bond...) in order to obtain reliable predictions of the experimental results without requiring too much effort and time. Nevertheless, numerical methods have become very useful tools to provide support to the development of conceptual models, by allowing the verification of certain assumptions and quantifying the influence of certain variables by performing parametric studies.

Since punching shear is a brittle -and therefore undesirable- failure, in order to reach the required safety level without an unaffordable cost, simplified, but safe and accurate design models are needed. In order to achieve these characteristics, such models should be based on the principles of concrete mechanics and should be verified with available experimental results.

In this paper, a new mechanical model for the estimation of the punching shear strength of reinforced concrete flat slabs with and without shear reinforcement is presented. The punching shear model presented in this paper is an adaption of a previously existing model for beam shear strength, developed by the authors in [38–40], which incorporates the contribution of the main shear resisting mechanisms. For this purpose, the relevant differences between the shear in beams and punching shear resisting actions have been identified and accounted for into the governing equations and into the failure criteria used in the mechanical model. Numerical simulations using a non-linear finite element model have been used to verify some of the assumptions made. Different authors have developed very complete and comprehensive databases on punching tests performed on concentrically loaded slabs without and with shear reinforcement [41–44]. The

predictions of the proposed model have been compared with the tests results included in the databases developed by Siburg [44] and Walkner [42], and with the predictions of EC2 [30] and MC2010 [32].

2. SUMMARY OF THE MULTI-ACTION SHEAR STRENGTH MODEL FOR BEAMS (MASM).

According to experimental observations, as the load increases in a RC beam failing in shear, damage concentrates around a critical shear crack, originally a flexural crack, which reaches the flexural neutral axis (Figure 1.a). Under incremental loading, a second branch of the crack develops inside the un-cracked longitudinally compressed concrete chord, which will connect the first branch of the crack and the load application point, producing failure. One of the main assumption of the MASM, supported by empirical observations of various authors [45,46], is that when the second branch of the critical crack develops, the load does not significantly increase, as softening of the concrete in the compression zone initiates, see Figure 1.b.

Figure 1. a) Shear critical crack evolution [41]. b) Load-displacement curve of a beam failing in shear [45].

According to what is generally accepted [31], the Multi-Action Shear Model considers that the shear strength, V_u in Eq. (1), is the sum of the shear resisted by the transverse reinforcement, if it exists, V_s , by the shear resisted in the un-cracked compression chord, V_c , the shear transferred across the critical shear crack, V_w , and the dowel action in the longitudinal reinforcement, V_l . Figure 2 shows these components in the portion of a beam placed over the first branch of the critical crack, placed in the tension zone. These contributions interact and their relative value depend on the state of strains and stresses and on the critical crack opening (i.e. the wider the crack, the less shear is transferred across it). The shear strength must be lower than the shear force that produces failure in the concrete struts, $V_{u,max}$, given by Eq. (2).

75

76 Figure 2. Shear strength components according to the MASM. a) Elements without stirrups. b) Elements with
77 stirrups.

78
$$V_u = (V_c + V_w + V_l) + V_s = f_{ctm} b d (v_c + v_w + v_l) + f_{ctm} b d v_s \quad (1)$$

79
$$V_{u,max} = \alpha_{cw} b_w z v_1 f_{cm} \frac{\cot \theta}{1 + \cot^2 \theta} \quad (2)$$

80 Table 1 shows the dimensionless equations governing each contributing component, for the particular case of
81 beams with rectangular cross-section. See reference [38] for further information related to these equations,
82 factors, and parameters. The principles used to derive each shear resistant component are briefly explained in
83 the following.

Shear resisting action	Dimensionless equations
Compression chord	$v_c = \zeta \left[(0.88 + 0.70 v_s) \frac{x}{d} + 0.02 \right] \quad (3)$
Cracked concrete web	$v_w = 167 \frac{f_{ctm} b_w}{E_{cm} b} \left(1 + \frac{2 G_f E_{cm}}{f_{ctm}^2 d} \right) \quad (4)$
Longitudinal reinforcement	$\text{if } v_s > 0 \rightarrow v_l = 0.23 \frac{\alpha_e \rho_l}{1 - x/d} \quad (5a)$
	$\text{if } v_s = 0 \rightarrow v_l = 0 \quad (5b)$
Shear reinforcement	$v_s = (d_s - x) \cot \theta \frac{A_{sw} f_{yw}}{s f_{ctm} b d} \approx \frac{0.85 d_s A_{sw} f_{yw}}{s f_{ctm} b d} \quad (6)$

84 Table 1. Summary of dimensionless shear contributing components, according to the MASM.

85 The component V_c is the shear resisted by the compressed concrete chord when the principal stresses (σ_1 , σ_2)
 86 at any point of the compression chord, produced by the shear force V and the bending moment M , reach the
 87 Kupfer's biaxial stresses failure [47], in the tension-compression branch, see Figure 3.

Figure 3. Biaxial failure envelope for plain concrete adopted [47].

90 The position of the critical point depends on M/V and, for common slender beams, is placed at a distance of
 91 the flexural neutral axis around $0.425 x$, being x the neutral axis depth. The shear stress at the critical point
 92 can be related to the normal and principal stresses using the Mohr's Circle of stresses. Thus, assuming a
 93 parabolic distribution of shear stresses with zero values at both ends of the parabola, the shear force V_c is
 94 obtained through direct integration of shear stresses along the concrete chord depth, by Eq. (7).

$$v_c = \frac{V_c}{f_{ctm}bd} = 0.682\zeta\frac{x}{d}\sqrt{1 - \frac{\sigma_x + \sigma_y}{\sigma_1} + \frac{\sigma_x\sigma_y}{\sigma_1^2}} \quad (7)$$

96 where σ_1 is the principal tensile stress at failure, σ_x and σ_y are the normal stresses produced by the bending
 97 moment, σ_y are the normal stresses produced by the bending moment and the vertical confinement stresses
 98 produced by the stirrups, and ζ is a size effect factor (see Table 2, Eq. 12) associated to the softening of the
 99 concrete in the compression chord.

100 The normal longitudinal stresses at the critical point, σ_x and σ_y can be expressed, respectively, as function of
 101 the bending moment, and of the force in the stirrups crossing the crack, assumed yielded, see Eq. (6). Then,
 102 by setting the equilibrium of forces and moments in the portion of beam shown in Fig. 2, Eq. (7) can be
 103 expressed in a dimensionless form as a function of the bending moment, the concrete tensile and
 104 compressive strengths, the shear resisted by the stirrups, and the relative neutral axis depth x/d . Once solved

iteratively, Eq. (7) provides results that are almost exactly a linear function of x/d and v_s , resulting in Eq. (3) of Table 1. It is relevant to observe that v_c depends on x/d , and therefore on the longitudinal reinforcement ratio, and that the presence of stirrups enhances the shear strength of the concrete chord due to confinement. The shear resisted by the web, V_w , is obtained by computing the residual stresses that can be transferred across the crack, according to the cohesive crack model, which depend on the crack opening, on the concrete tensile strength and fracture energy, and on the crack inclination. Note that the shear resisted by the web, V_w , is due to the residual tensile stresses in this mechanical model and not to aggregate interlock. In fact, the share of shear resistance actions is variable in the different regions of a beam for a given load state. Therefore, there may exist simultaneous governing actions. Bairán et al. [48] showed that aggregate interlock can be an effective action to transfer the stresses through cracks in the zone of the beams where cracks are rather vertical. However, in the critical shear region, cracks are generally more inclined (around or less than 45°) and the stresses components in the crack plane mostly produce direct tension, with fairly small shear in the crack plane [48]. For this reason, and due to the geometry of the critical shear crack considered in this mechanical model, only residual tensile stresses are considered in the web. Finally, dowel action in the longitudinal reinforcement, V_l , is computed assuming the longitudinal bars rigidly connected to the stirrups and subjected to an imposed vertical displacement due to the inclined crack opening and the shear deformation. As the load and the critical crack width increase, components V_w and V_l decrease, while V_c increases, as normal flexural compressive stresses enhance the shear capacity of the concrete chord. Then, for practical reasons, the multi-action model was simplified by merging these three components into a single one (V_c) and for this reason the simplified model was called the Compression Chord Capacity Model (CCCM) [40]. The resultant equations for rectangular sections are shown in Table 2. Factor 1.4 in Eq. (11) accounts for the vertical confinement stresses introduced by stirrups in the uncracked concrete chord, thus enhancing its shear capacity. Bazant's size effect law [5] is adopted, including also the effect of the shear span a/d . The derivation of this size and slenderness effect expression is presented in [40]. The factor depending on a/d was taken from previous empirical work performed with genetic programming [49,50], where it was seen that the term $(a/d)^{0.21}$ correctly predicted the influence of this variable [40].

Equations	Expressions	
Shear strength	$V_{Rd} = V_{cu} + V_{su} \leq V_{Rd,max}$	(8)
Strut crushing	$V_{Rd,max} = \alpha_{cw} b z v_1 f_{cd} \frac{\cot \theta + \cot \alpha}{1 + \cot^2 \theta}$	(9)
Concrete contribution	$V_{cu} = 0.3 \zeta \frac{x}{d} f_{cd}^{2/3} b d \nless V_{cu,min} = 0.25 \left(\zeta K_c + \frac{20}{d_0} \right) f_{cd}^{2/3} b d$	(10)
Shear reinforcement	$V_{su} = 1.4 \frac{A_{sw}}{s} f_{ywd} (d_s - x) \sin \alpha (\cot \theta + \cot \alpha)$	(11)
Factors	Expression	
Size and slenderness effect	$\zeta = \frac{2 \left(\frac{d}{a} \right)^{0.2}}{\sqrt{1 + \frac{d_0}{200}}} \nless 0.45$	(12)
Relative neutral axis depth	$\frac{x_0}{d} = \alpha_e \rho_l \left(-1 + \sqrt{1 + \frac{2}{\alpha_e \rho_l}} \right) \approx 0.75 (\alpha_e \rho_l)^{1/3}$	(13)
Crack inclination	$\cot \theta = \frac{0.85 d_s}{d_s - x} \leq 2.5$	(14)

Table 2. Summary of the simplified equations for shear design according to the CCCM.

3. PUNCHING STRENGTH OF REINFORCED CONCRETE FLAT SLABS WITHOUT SHEAR REINFORCEMENT

3.1. Phenomenology

Consider a reinforced concrete flat slab without shear reinforcement, supported on isolated columns, and an interior column subjected to a centered axial load, see Figure 4. Due to the two-directional flexural work of the slab, two families of orthogonal bending moments, M_r producing radial stresses, and M_ϕ producing tangential stresses, take place around the column. The normal stresses, σ_r and σ_ϕ , produced respectively by both families of moments, compress the bottom and stretches the top of the slab. When these stresses exceed the concrete tensile strength, flexural cracks, noticeably normal and parallel to the column perimeter, appear.

144

145 Figure 4. Punching shear cracks and internal forces in a concrete slab around an internal column.

146 Under increasing load, damage tends to concentrate in a tangential crack that develops inclined trough the
 147 slab depth, crossing the compressed chord and reaching the slab bottom at its intersection with the column
 148 face, thus producing a punching shear failure. Along the loading process, redistributions between radial and
 149 tangential bending moments take place due to changes in the stiffness produced by cracking and eventually
 150 yielding of the flexural reinforcement.

151 A relevant aspect is that the concrete placed at the bottom of the slab is compressed both by the radial and
 152 tangential bending moments. In addition, in the vicinity of the column, the slab is also subjected to vertical
 153 compressive stresses, since the forces transmitted from the slab to the column concentrate on the column
 154 periphery, as indicated by Figure 5. Such vertical compressive stresses in the uncracked concrete zone of the
 155 slab enhance the capacity of this region to resist shear stresses.

156

157 Figure 5. Schematic profile of vertical stresses near the bottom of the slab and state of stresses at a point.

158 Figure 6 shows the distribution of vertical stresses computed by means of a non-linear finite element analysis
159 performed using program FEA-Midas [51] to simulate the tests performed by Adetifa and Polak [52],
160 similarly to the analysis performed by Genikomsou and Polak [35].

161 Figure 6. Distribution of vertical stresses along the area enclosed by the critical perimeter of the slab tested
162 by Adetifa and Polak [52].

163

3.2. Differences between beam shear and punching shear behavior to incorporate them in a mechanical model

3.2.1. Position and inclination of the critical crack

In a two-way slab supported by isolated columns, the radial bending moments law $m_r(r)$ is different than that of the bending moments in a beam or in a one-way slab. As a result, the distance from the column face to where the radial moment reaches the cracking moment, where the critical crack initiates, s_{crack} , is generally less than $1.5d$ and the compressed concrete zone of the slab lies close enough to the column face to be considered to be located inside a D (discontinuity) region (Figure 7). In fact, the previously mentioned vertical compressive stresses makes the crack inclination steeper than in beams so that, unlike what happens in slender beams or one-way slabs subjected to shear, the punching critical crack around a column follows an almost straight path from its initiation to the intersection of the slab bottom with the column face. Then, the perimeter where the critical crack reaches the compression zone of depth x , identified in this model as the “critical perimeter”, will be placed at a distance s_{crit} from the column face (Figure 7), given by Eq. (15):

$$s_{crit} = x \cot \theta = s_{crack} \frac{x}{d} \quad (15)$$

By equaling the radial bending moment per unit width $m_r(r)$ to the slab cracking moment per unit width, the value of r_{crack} can be obtained. According to the elastic theory of plates, for a uniformly distributed load, $m_r(r)$ is given by Eq. (16):

$$m_r(r) = \frac{V}{4\pi} (1 + \nu) \ln \frac{r_0}{r} = m_{crack} \quad (16)$$

where V is the total shear transferred by the slab to the column, ν is the Poisson coefficient, r_0 and r are the distances to the column axis from the zero bending moment point and from the point where the moment is calculated, respectively. Then, solving Eq. (16) and taking into account Eq. (15), the distance from the critical perimeter to the column face and the inclination of the critical crack are given by Eq. (17):

$$\frac{s_{crit}}{d} = \frac{s_{crack} x}{d} = \frac{r_{col}}{d} \left(\frac{r_0}{r_{col}} e^{-10.5 \frac{m_{crack}}{V_{Ed}}} - 1 \right) \frac{x}{d}; \quad \frac{s_{crack}}{d} = \cot \theta \leq 2.5 \quad (17)$$

where r_{col} is the radius of a column with equal perimeter than the actual column. Eq. (17) shows that the position of the critical perimeter depends on the column size, slab depth, distance r_0 , concrete tensile

190 strength, neutral axis depth (or longitudinal reinforcement ratio), and on the shear force transferred to the
 191 column. Since for design purposes it is desirable a simpler way to define the critical perimeter, the value of
 192 s_{crit}/d was evaluated through Eq. (17), using for V_{Ed} the experimental punching strength for those punching
 193 tests included in the database compiled by Siburg [44], with ratios slab to effective depth $d/u_o > 0.15$ and
 194 reinforcement ratios $\rho < 2\%$ obtaining values of s_{crit}/d between $0.4d$ and $0.7d$. The slabs with higher value
 195 of s_{crit}/d are those with lower ratio d/u_o and higher reinforcement ratios, since the cracking moment is lower
 196 and the neutral axis depth x/d is higher.

197

198

199 Figure 7. Critical crack, critical perimeter and forces in a portion of slab above the critical crack.

200

However, considering that, in order to avoid flexural failure, the longitudinal reinforcement amount in the tests was higher than usual in actual slabs, a conservative value of the distance from the critical perimeter to the column face $s_{crit} = 0.5d$ will be adopted in this work, as shown in Fig. 7. This decision is aligned with the punching provisions by ACI Code [31], Model Code [32], and the revised EC2 model proposed by Kueres et al. [26].

3.2.2. *Effect of the multi-axial stress state in the slab compressed chord*

Vertical stresses in the slab in the vicinity of the column, σ_v , enhance the concrete capacity to resist shear stress in the radial vertical plane, as indicated by Eq. (7) where σ_y represents the slab vertical stress. Such stresses are not provided in the punching experiments available in the databases consulted. Thus, in order to estimate σ_v , a numerical study has been made of an axisymmetric case in which the reinforcement ratio and the ratio between the slab effective depth and the column perimeter (d/u_o) have been varied. The results obtained indicate that there is a non-uniform radial distribution of stresses, see Fig. 5, and that the maximum vertical stress at the slab between the column face and the critical perimeter ranges between 1.25 and 1.50 times the average vertical stress produced in the area enclosed by the critical perimeter. Then, a value $\alpha = 1.25$ is adopted resulting the following approximate value of the vertical stress in the concrete:

$$\frac{\sigma_v}{f_{ct}} = \frac{\alpha V}{f_{ct} \pi r_{crit}^2} = \frac{2\alpha v}{\frac{r_{crit}}{d}} \simeq \frac{2.5v}{\frac{r_{crit}}{d}} \quad (18)$$

where $v = V/(u_{crit} \cdot d)$ is the non-dimensional shear resisted by concrete, which includes the contributions of the uncracked chord and the cracked web.

Normal stresses due to tangential moments, σ_ϕ , jointly with vertical stresses σ_v , confine the concrete in the vertical tangential plane, thus incrementing its compressive strength in the radial direction. However, the effect of such confining stresses is moderate, since a tri-axial compressive stresses state is never reached, because the normal stresses are accompanied by shear stresses, generating always a tensile principal stress in the bottom region of the slab. Consequently, failure will take place in a compression-compression-tension state of principal stresses (Fig. 8.a). Thus, in order to account for the effect of those confining stresses, a modified tension-compression branch of Kupfer's biaxial failure envelope is adopted (Fig. 8.b), in which the

226 confined concrete strength f_{cc} is used in spite of the unconfined one, f_c , so that a higher shear stress is needed
 227 to reach failure.

228
 229 Figure 8. a) Principal stresses at the uncracked slab region; b) modified Kupfer biaxial failure envelope

230 The Eurocode 2 [30] proposal for the confined concrete strength in the vertical radial plane is adopted, see
 231 Eq. (19):

$$232 \quad f_{cc} = f_c \left(1.125 + 2.5 \frac{\sigma_{cc}}{f_c} \right) \quad \text{if } \frac{\sigma_{cc}}{f_c} \geq 0.05 \quad (19)$$

233 The confinement stress σ_{cc} adopted is the mean value between the transverse and vertical stresses σ_ϕ and σ_v .
 234 The vertical stress is given by Eq. (18) while the transverse stress can be estimated by assuming that it is
 235 proportional to the radial stress, as the slab is cracked in both radial and tangential directions, and the
 236 reinforcement ratios in both orthogonal directions are similar ($\rho_{lx} \approx \rho_{ly}$), for interior columns with similar
 237 span lengths. The tangent normal stress is, then, obtained by multiplying the radial stress by the ratio
 238 between the tangential and the radial moments, m_ϕ/m_r :

$$239 \quad \frac{\sigma_\phi}{\sigma_r} \simeq \frac{m_\phi(r)}{m_r(r)} = \frac{\frac{V}{4\pi} \left[(1+\nu) \ln \frac{r_0}{r} + 1 - \nu \right]}{\frac{V}{4\pi} \left[(1+\nu) \ln \frac{r_0}{r} \right]} = 1 + \frac{1-\nu}{(1+\nu) \ln \frac{r_0}{r}} \quad (20)$$

240 where r_0 is the distance from the column axis to the point of zero radial bending moment (contraflexure
 241 point) which may be approached by $0.15L$, being L the span length; and r is the distance from the support
 242 axis to the considered perimeter; ν is the concrete Poisson's ratio, that considering the existence of radial
 243 cracks may be considered higher than for uncracked concrete. The value $r_0 = 0.15L$ has been adopted
 244 assuming that redistribution of bending moments takes place due to cracking and yielding of the

reinforcement around the column, what is desirable in order to avoid a sudden shear punching failure. For $\nu = 0.3$ and usual dimensions of the column, slab depth and slab slenderness, a conservative value of $m_\phi/m_r=1.5$ can be adopted. Then, the confinement stress becomes:

$$\sigma_{cc} = 0.5(\sigma_\phi + \sigma_v) \simeq 0.5(1.5\sigma_r + \sigma_v) \quad (21)$$

The radial stress is obtained by assuming linear behavior in the uncracked concrete zone, after setting the equilibrium of forces in the portion of slab around the column indicated in Figure 9.

Figure 9. Forces in a portion of slab above the critical crack.

$$C = T_l + V_w \tan\theta \quad (22)$$

$$V_{Ed} = V_c + V_w \quad (23)$$

$$Cz + m_\phi d\phi \beta d = m_{crack} \frac{r_{crack}}{r_{crit}} + V_c \beta d + V_w \beta_w d (\tan\theta + \cot\theta) \quad (24)$$

where the differential term $m_\phi d\phi \beta d$ due to the tangential moment can be neglected. The distance βd is the horizontal projection of the critical crack in the tensile zone of the slab depth which, according to the assumptions made, is equal to $0.5d(d-x)/x$, being x the neutral axis depth, see Fig. 7. V_w is the resultant of the residual stresses crossing the crack that take place where the crack width is smaller. Therefore, its distance $\beta_w d$ to the crack initiation is lower but very similar to βd , and can be conservatively adopted as $2/3\beta d$. Thus, adopting for the flexural lever arm $z = d-x/3$, the radial stresses at the critical point inside the compressed chord, placed at a distance $\lambda = 0.425x$ [38] of the flexural neutral axis, will be:

$$\sigma_r(\lambda) = \frac{2 \lambda C}{x z} = \frac{2 \lambda \left(m_{crack} \frac{r_{crack}}{r_{crit}} + V_c \beta d + V_w \beta_w d (\tan \theta + \cot \theta) \right)}{x \left(d - \frac{x}{3} \right)} \quad (25)$$

It can be observed that the effect of radial geometry (the perimeter length increases as the distance to the column increases) is accounted for in the formulation by multiplying the cracking moment per unit length by the ratio r_{crack}/r_{crit} .

3.3. Simplified equations for design

3.3.1 Punching strength when flexural reinforcement is not yielded

Eq. (7), derived for beams subjected to shear, provides the shear punching force resisted by the concrete in a slab if the width b is substituted by the critical perimeter u_{crit} , see Figure 7. Eqs. (18), (21) and (25) show that all terms of Eq. (7), in which x and y must be considered as the radial and vertical axes respectively, can be expressed as a function of the shear force V , except x/d . The solution of Eq. (7) needs to be solved iteratively, since initially the principal stress σ_l is not known as it depends of the shear stress, τ , that need to be calculated from the shear force. Once solved, V_c results to be an almost linear function of the relative neutral axis depth, x/d , the tensile concrete strength, the slab depth and the column dimensions (through the critical perimeter).

In addition, the shear transferred across the crack, V_w , given by Eq. (4), must be added to V_c . Since V_w is much smaller than V_c , average values are adopted for f_{ct} , E_{cm} and G_f , assuming a compressive concrete strength of $f_c = 35$ MPa and an effective depth of $d = 350$ mm, resulting a minimum value of $v_w = V_w/(f_{ctm} \cdot u_{crit} \cdot d) = 0.05$. Therefore, the total punching strength of a slab without shear reinforcement is given by:

$$V_{cu} = V_c + V_w = \zeta \left(1.125 \frac{x}{d} + 0.425 \right) f_{ctm} u_{crit} d \quad (26)$$

where ζ is the size effect associated to the softening of the uncracked concrete zone, see Eq. (6), f_{ctm} is the mean tensile strength of the concrete, limited to 4.6 MPa, i.e. the value at corresponding to $f_{ck} = 60$ MPa, and u_{crit} is the critical perimeter (Fig. 7) placed at a distance $s_{crit} = 0.5d$ from the column face.

Eq. (26) can be expressed as a function of the concrete compressive strength, f_{ck} as follows:

$$V_{cu} = 0.3\zeta \left(1.125 \frac{x}{d} + 0.425 \right) f_{ck}^{2/3} u_{crit} d \quad (27)$$

Alternatively, V_{cu} could be expressed as a function of the flexural reinforcement ratio,

$$V_{cu} = 0.125\zeta \left(1 + 2 (\alpha_e \rho)^{1/3} \right) f_{ck}^{2/3} u_{crit} d \quad (28)$$

For slab floors in buildings subjected to distributed loads, the shear span, a , to be used in the size effect parameter, ζ , defined in Eq. (12), can be estimated as the average distance from the position of the line of zero radial bending moment to the edge of the column, $l_0 = \sqrt{l_{0x} \cdot l_{0y}}$, where $l_{0x} \cong 0.15 l_x$ and $l_{0y} \cong 0.15 l_y$, l_x and l_y are the span lengths in the x and y directions. The neutral axis depth x/d should be obtained using the average of the longitudinal reinforcement ratios, ρ_{lx} , ρ_{ly} , in the two orthogonal directions, adopting an effective slab width $b_{s,eff}$ approximately equal to the column side or diameter plus 3 times the slab effective depth at each side of the column.

3.3.2 Punching strength when flexural reinforcement is yielded

Flexural reinforcement may yield before punching failure occurs, leading to a flexural punching failure, what may happen in usual slabs with flexural reinforcement ratios less than 0.75%, approximately. Since the tangential moment per unit length is higher than the radial moment, first yielding will take place when the tangential moment reaches the lower yielding moment of the two orthogonal directions. Due to the internal redundancy of the system, since the equilibrium between radial moments, tangential moments, and shear forces must be satisfied in any portion of the slab, when yielding in the tangential direction occurs, a redistribution of moments may take place, so that under increasing loads, the radial moments must increase while the tangential moment remains constant. Assuming perfect plasticity, the radial moment will also reach the plastic moment in the radial direction. Then, the associated shear force can be obtained as follows:

$$m_r = \frac{V_{Ed}}{4\pi} \left[(1 + \nu) \ln \frac{r_0}{r_{crit}} \right] \simeq \frac{V_{Ed}}{2\pi} = m_y = \rho f_{yk} d^2 \left(1 - \frac{\rho f_{yk}}{2 f_{ck}} \right) \quad (29)$$

In Eq. (29) it has been assumed, for the sake of simplicity, a mean value of $r_0/r_{crit} = 5$ and a Poisson's ratio at Ultimate Limit State for cracked concrete of $\nu = 0.3$. Then the shear force associated to reinforcement yielding is:

$$V_{Rd} \leq V_y \simeq 2\pi m_y \simeq 2\pi \rho f_{yk} d^2 \left(1 - \frac{\rho f_{yk}}{2 f_{ck}} \right) \quad (30)$$

312 Similar equations for V_y were developed, based on the yield line theory, by different authors [1,12,18,53].

313 3.3.3 Minimum punching strength

314 For slabs, where both the depth and reinforcement ratio are generally small, the portion of shear transferred
315 along the crack is higher than in the case of beams, as x/d is small and v_w (Eq. 4) is inversely proportional to
316 the effective depth d . Therefore, similarly to what was done in beam shear, see references [40] and [54] for
317 further information, a minimum punching strength is defined, that incorporates a simplified form of the
318 component v_w , to explicitly account for the slab depth, and in which the neutral axis depth is limited to $x/d =$
319 0.20 to be consistent with the initial assumption.

$$320 \quad V_{cu,min} = \left[\zeta(1.125 k_c + 0.375) + \frac{10}{d_0} \right] f_{ctm} u_{crit} d \quad (31)$$

321 where $k_c = x/d \leq 0.2$ and $d_0 = \max(d, 100\text{mm})$

322

323 4. PUNCHING SHEAR STRENGTH OF SLABS WITH SHEAR REINFORCEMENT

324 When $V_{Ed} > V_{cu}$, the necessary shear reinforcement to provide the punching strength is:

$$325 \quad V_{su} = V_{Ed} - V_{cu} = A_{sw} f_{yw,eff} \sin \alpha \quad (32)$$

326 A_{sw} is the total area of the shear reinforcement placed around the column that crosses the inclined crack, that
327 can be approximated by considering the reinforcement placed between $0.5 d$ and $1.5 d$ from the support face
328 (Figure 7), α is the angle between the shear reinforcement and the plane of the slab and $f_{yw,eff}$ is the effective
329 design strength of the punching shear reinforcement, given by:

$$330 \quad f_{yw,eff} = K_s \frac{\tau_b l_b u}{A_s} = K_s \frac{4.5 f_{ctm} d}{\phi_w} \leq f_{yw} \quad (33)$$

331 where τ_b is the bonding stress ($\tau_b = 2.25 f_{ctm}$, according to EC2), A_s and u the area and perimeter of the bar used
332 as shear reinforcement, l_b is the anchorage length of the shear reinforcement, ϕ_w is the diameter of the shear
333 reinforcement, f_{ctm} is the mean concrete tensile strength, and K_s is a factor that accounts for the effectiveness
334 of the anchorage of the shear reinforcement.

335 Eq. (33) has been derived assuming that anchorage is produced only by bond along the straight part of the
336 reinforcement, and considering an anchorage length of the transverse reinforcement, l_b , equal to $d/2$. The
337 effective stress given by Eq. (33) may be increased by improving the anchorage effectiveness (for example

338 using shear studs or other adequate devices). Then, the value of K_s may be obtained by testing, but in absence
 339 of other data, a value $K_s = 1.0$ can be adopted. Even assuming perfectly anchored studs, the effective design
 340 strength needs further research, since it is possible that not all the studs can develop its maximum capacity,
 341 as was shown by Ferreira et al. [25] being its stress depending on the position of the stud in the conical
 342 critical crack.

343
 344 Figure 10. Front view and plan view of the shear reinforcement arrangement.

345 There is not a full agreement about the value of V_{cu} to use in Eq (32). According to Fernandez and Muttoni
 346 [23] the increment of capacity provided by the stirrups increases the cracks opening and reduces the concrete
 347 contribution. Furthermore, the presence of stirrups increases the angle of inclination of the crack, which
 348 becomes more vertical and, therefore, the critical perimeter is smaller. However, the increment of loading
 349 capacity produces more compressive stresses in the uncracked zone, thus increasing its shear capacity.
 350 Therefore in this work, the same value has been adopted for the concrete contribution in slabs with and
 351 without shear reinforcement. The confinement effects of the transverse reinforcement introduced through
 352 factor 1.4 in Eq. (11) for beam shear, may not be ensured in slabs with the usual transverse reinforcement
 353 arrangements, so they will not be considered in Eq (32).

354 If shear reinforcement is required, the perimeter u_{out} where shear reinforcement is no longer required (see
 355 Figure 10) can be estimated equaling the value of V_{cu} to V_{Ed} , but taking into account that since the u_{out}
 356 perimeter is far from the column, the confinement effects produced by the vertical stresses and the tangential
 357 bending moment can be neglected. Therefore, the punching shear problem at perimeter u_{out} is assimilated to a

beam shear problem, and Eq. (10) may be used. This equation can be rearranged to specifically solve this problem, see Eq. (34):

$$V_{cu,u_{out}} = 0.3\zeta\frac{x}{d}f_{ck}^{2/3}u_{out}d \leq V_{cu,min} = 0.25\left(\zeta K_c + \frac{20}{d_0}\right)f_{ck}^{2/3}u_{out}d \quad (34)$$

Perimeter u_{out} is usually placed at a location with relatively low bending moments, where the shear-bending moment interaction is not critical. For this reason, for calculation the size effect factor, ζ (Eq.12), for obtaining the punching capacity at this perimeter, a/d may be taken equal to 3.

The ultimate shear strength is limited by the maximum shear that can be resisted in the perimeter of the column u_0 , i.e. the shear that produce failure at the concrete struts. According to current EC2 [30], taking into account the Amendment 1 from 2014, the maximum punching shear capacity, $V_{Rd,max}$, is defined as a multiple of the punching shear strength of an identical slab without shear reinforcement:

$$V_{Ed} \leq V_{Rd,max} = \alpha_{max}V_{Rd,c} \quad (35)$$

with α_{max} equal to 1.5 for stirrups and bent-up bars, and 1.8 for double-headed studs. A similar approach was first adopted by Model Code 2010, considering a general value of α_{max} equal to 2, or higher values in some specific conditions. Note that in Model Code 2010, the factor referred here as α_{max} (K_{sys} in Model Code) multiplies the design shear resistance attributed to the concrete in a slab with punching reinforcement and, this concrete contribution is lower than the punching resistance of an identical slab without shear reinforcement. Consequently, the values of α_{max} in Eurocode and Model Code 2010 are not totally equivalent. In this research, the same approach used by EC2 [30] will be followed. The exact definition of α_{max} is out of the scope of this paper. In any case, $\alpha_{max} = 1.75$ will be considered in general, and $\alpha_{max} = 2$ for double-headed studs. Recently, interesting works regarding the performance of different punching shear reinforcement systems have been published [27,55].

According to the position of the critical section, the inner perimeter of shear reinforcement should be placed at a distance between 0.2 and 0.5d from the column face. The outermost perimeter of shear reinforcement should be placed at a distance not greater than 0.5d within u_{out} .

5. COMPARISON WITH EXPERIMENTAL RESULTS

The proposed model has been used to predict the punching shear strength of 328 tests on concentrically loaded circular and square flat slabs without shear reinforcement, obtained from Siburg [44], and of 232 tests on slabs with shear reinforcement from the database developed by Walkner [42]. The exact databases previously published have been used, although the authors had access to new test results. This decision was taken to ensure that the full databases used, and the criteria adopted for their development, are fully accessible for other researchers. The same database with 328 slabs without shear reinforcement has also been recently used in Kueres [26]. Table 3 summarizes the range of the parameters in both databases.

Number of slabs	Slabs without shear reinforcement		Slabs with shear reinforcement	
	328		232	
	min	max	min	max
Effective depth, d (mm)	30	500	83	476
Column diameter (mm) – circular	51	901	102	400
Column side (mm) - square	51	520	120	520
Compressive strength, $f_{cm,cyl}$ (MPa)	9.4	125.0	13.0	92.0
Flexural reinforcement ratio, ρ_l (%)	0.2	3.7	0.3	3.1
Yielding strength of longitudinal reinforcement (MPa)	270	811	270	917
Shear span to depth ratio, a/d	1.6	14	2.0	11.9
Number of rows of punching shear reinforcement	-	-	1	20
Distance between the rows (mm)	-	-	40	315
Diameter of shear reinforcement (mm)	-	-	3.7	22
Yield strength of shear reinforcement (MPa)	-	-	278	1100

Table 3. Range of parameters in the used databases.

Table 4 shows the results of the proposed method in terms V_{test}/V_{pred} , being V_{test} the shear punching strength obtained in the real test and V_{pred} the prediction of the shear punching strength by different formulations. In addition to the proposal model, the results are shown for Eurocode 2 [30] and the level of approximation I and III from Model Code 2010 [32]. For the 328 slabs without shear reinforcement, very good results have been obtained by the proposed method in terms of mean value (1.188) and coefficient of variation (15.1%). The safety of the proposal is also adequate, with a 5% percentile of V_{test}/V_{pred} equal to 0.905. For the 232 slabs with shear reinforcement, the results are very similar for the proposed model, with a mean value, coefficient of variation and 5% percentile equal to 1.173, 14.9% and 0.929 respectively. The calculations have been performed without including partial safety factors and considering mean values of the mechanical properties. For slabs without shear reinforcement, the key equation proposed in this paper to calculate the punching shear strength is Eq. (27), with a minimum punching strength given by Eq. (31).

Moreover, the shear force associated to reinforcement yielding, Eq. (30), was also checked. For slabs with shear reinforcement, the contribution of the shear reinforcement has been taken into account using Eq. (32). In this case, it has also been needed to check the punching shear capacity at a perimeter located at $0.5d$ beyond the shear reinforcement, by means of Eq. (34), and the maximum shear that produce failure at the concrete struts, by Eq. (35). As previously commented, efficiency factor $K_s=1$ (Eq. 33) and $\alpha_{max} = 1.75$ (Eq. 35) will be considered for performing the calculations, except for slabs reinforced with double-headed studs ($\alpha_{max} = 2$).

	Slabs without shear reinforcement				Slabs with shear reinforcement			
	Proposal	EC2	MC-2010 LoA I	MC-2010 LoA III	Proposal	EC2	MC-2010 LoA I	MC-2010 LoA III
Number	328	328	328	328	232	232	232	232
Mean	1.188	1.171	1.895	1.141	1.173	1.067	2.129	1.251
Stand. dev	0.179	0.206	0.633	0.176	0.175	0.191	0.707	0.199
CoV	0.151	0.176	0.334	0.154	0.149	0.179	0.332	0.159
Max	1.746	2.038	4.896	1.696	1.880	1.757	4.263	1.755
Min	0.676	0.697	0.722	0.613	0.774	0.642	0.831	0.733
5%	0.905	0.843	1.084	0.810	0.929	0.774	1.136	0.909

Table 4. Comparison of models predictions with tests results.

The correlation between the predictions and the tests results are shown in Figures 11 and 12 in terms of the slab effective depth, d , and the amount of flexural reinforcement, ρ_l (%). It may be seen that Eurocode 2 underestimates the size effect and the influence of flexural reinforcement. In summary, the results for the four studied formulations are shown in Table 4 and Figures 11 and 12 show that: 1) the statistical results of the proposed method in front of the two slab databases, without and with shear reinforcement, are very homogeneous; 2) Eurocode 2 is the most unsafe method, with 5% percentiles of V_{test}/V_{pred} equal to 0.843 and 0.774; 3) the results of the Level of Approximation (LoA) I of Model Code 2010 are too disperse even for a simplified method; 4) the results of LoA III of Model Code 2010 are reasonable, but the model is less safer for slabs without shear reinforcement than slabs with shear reinforcement. Moreover, LoA III of MC2010 is an iterative method for evaluating the punching strength of existing slabs, meanwhile the proposed model is direct, without the need of performing iterations, both for designing of new structures and assessment of existing structures.

In 2012, Lips et al. [56] reported a series of slabs without and with shear reinforcement. The experimental results of slabs PV1 (without shear reinforcement), PL11 (with a low amount of shear reinforcement), PL12 (medium amount of shear reinforcement) and PL7 (high amount of shear reinforcement) are very interesting,

428 as the failure mode changed from a punching shear failure without shear reinforcement (PV1), or a failure
 429 within the shear reinforced area (PL11 and PL12; Failure mode PF in Table 5), to excessive stresses in the
 430 strut (failure in the perimeter of the column due to maximum punching capacity; Failure mode Pmax in
 431 Table 5). The proposed method is able to fully reproduce the failure mode of the four slabs, as can be seen in
 432 Table 5, and the predicted results correlate very well with the observed shear punching capacities. These four
 433 tests were also recently studied in [26].

436 Figure 11. Correlation between the predictions and the experimental results in terms of slab depth, d (mm).

	d (mm)	Column side (mm)	f_{cm} , reported (MPa)	ρ_l (%)	$A_{sw, 1st}$ row (mm ²)	V_{test} (kN)	Failure mode	V_c (kN)	V_s (kN)	V_{pred} (kN)	Failure mode	V_{test}/V_{pred}
PV1	210	260	34	1.50	0	974	PS	882	0	882	PS	1.10
PL11	201	260	34.2	1.57	628	1176	PS	765	310	1075	PS	1.09
PL12	201	260	34.6	1.57	1257	1683	PS	770	625	1395	PS	1.21
PL7	197	260	35.9	1.60	2463	1773	Pmax	-	-	1541	Pmax	1.15

437 Table 5. Experimental results and predictions by the proposed model for 4 slabs reported in [56].

Figure 12. Correlation between the predictions and the experimental results in terms of the amount of flexural reinforcement, ρ_l (%).

In the proposed model (Eq.27) the punching strength depends on the term $f_{ck}^{2/3}$. However, the punching strength also depends on x/d , which relies on the modulus of elasticity of the concrete and, indirectly, on the concrete compressive strength. For this reason, the real dependence does not follow the 2/3 power. Figure 13 shows that correlation between the predictions of the proposed model and the experimental results in terms of the concrete compressive strength, f_{cm} . It may be seen that there is not any significant scatter depending on f_{cm} .

Figure 13. Correlation between the predictions and the experimental results in terms of the concrete compressive strength, f_{cm} (MPa).

As discussed in Section 4, the proposed model considers that the punching concrete contribution of a slab with shear reinforcement equals the punching strength of an identical slab but without shear reinforcement. Moreover, the proposed model for slabs with shear reinforcement, uses an effective design strength of the punching shear reinforcement (Eq. 33). The average efficiency of the transverse reinforcement found with Eq. (33) using $K_s=1$ is around 50%, which is very low but similar to the values obtained on punching tests of slabs where the transverse reinforcement strains have been measured [52] and [57]. Figure 14 shows the correlation between the predictions by the proposed model and the experimental results in terms of the amount of shear reinforcement in the critical region, that is, the amount of shear reinforcement placed between $0.5d$ and $1.5d$ from the support face. The figure shows the results of the slabs that the proposed model predicts that will fail through a critical crack crossing the shear reinforcement. To avoid noise, the slabs predicted to fail due to the concrete maximum shear capacity around the perimeter of the column (Eq. 35), due to punching-flexure failures (Eq. 30), and due to a critical crack beyond the shear reinforcement (Eq. 34) have not been included in the graph. The figure shows that the results are very similar independently of the amount of shear reinforcement provided.

Figure 14. Correlation between the predictions and the experimental results in terms of the amount of shear reinforcement provided.

6. CONCLUSIONS AND FUTURE RESEARCH

A punching strength mechanical model, based on a beam shear model previously developed by the authors, has been presented. The following main conclusions can be drawn from the present work:

1. The distribution of radial bending moments in a slab supported on isolated columns, different from those produced in cantilever beams, produces that the distance of the initiation of the critical crack to the column face is, in general, less than $1.5d$. Consequently, the critical crack is partially developed inside a D (discontinuity) region. In such a D region, the critical crack develops directly to the intersection between the column face and the compressed slab face. Therefore, the position and inclination of the critical crack can be approached in an objective way.
2. According to the adopted failure criterion, the critical perimeter is the perimeter where the critical crack reaches the un-cracked compressed zone. Therefore, its distance to the column face depends on the span length, on the load level, on the longitudinal reinforcement ratio, and on the cracking moment per unit length. Its value ranges from $0.4 d$ to $0.7 d$, so for simplicity reasons an average value of $0.5 d$ is adopted in this work.

3. Due to the proximity of the critical perimeter to the column face, the confining vertical stresses introduced by the column affect the state of stresses at the critical point where failure initiates, thus enhancing the shear capacity of the slab.
4. The tangential bending moments around the column produce compressive stresses that also confine the concrete compression chord of the slab. Such confinement in the tangential direction increases the concrete compression strength in the radial direction, affecting the Kupfer's biaxial failure envelope in the C-T branch and enhancing the concrete chord capacity to withstand shear stresses. This effect, due to the two-dimensional work of a slab is almost negligible in beams.
5. A punching shear failure may occur because of flexural yielding (flexural punching failure). A limit value has been proposed in the present model, corresponding to the load producing the fully yielding of reinforcement in both directions, after redistribution between radial and tangential moments takes place (Eq. 30). As pointed out, similar equations were previously developed, based on the yield line theory, by different authors.
6. The efficiency of shear reinforcement depends very much on its position and anchorage capacity. An expression for the maximum stress that the reinforcement may develop before losing its anchor capacity, based on bond, has been presented, showing good agreement with stresses experimentally measured in the reinforcement in some tests. According to such expression, the mean stress in the shear reinforcement found for the data base of punching tests in slabs with transverse reinforcement is around 50% of the steel yielding strength.
7. The outermost perimeter of shear reinforcement should be placed at a distance not greater than $0.5d$ within the perimeter u_{out} , the control perimeter at which shear reinforcement is not required. At this perimeter, the confinement effects produced by the vertical stresses and the tangential bending moment can be neglected. Therefore, the punching shear problem at the perimeter u_{out} is assimilated, in this model, to a beam shear problem.
8. The predictions of the proposed model have been compared with the results of 560 punching tests on slabs, 328 without and 232 with shear reinforcement. The results obtained by the model are very good (mean=1.188, CoV=0.151 of the ratio V_{test}/V_{pred}), for slabs w/o shear reinforcement and mean=1.173 and CoV=0.149 for slabs with shear reinforcement.

515 According to the above conclusions tests on slabs with different types and amounts of shear reinforcement,
516 adequately instrumented to capture the contribution of the concrete and the reinforcement are still necessary,
517 as well as tests to evaluate the reinforcement efficiency factor, K_s .

518 The model herein presented may be extended to slabs that eccentrically transfer the load to the column, by
519 considering the changes produced by the bending moments distribution on the cracking perimeter and on the
520 confinement and shear stresses in the critical perimeter. Furthermore, due to its mechanical character, the
521 model can be extended, in a natural way, to post-tensioned slabs, SFRC and FRP reinforced concrete slabs,
522 to account for forces redistributions, membrane effects or to evaluate the effects of strengthening
523 interventions

524

525 **ACKNOWLEDGMENTS**

526 The authors want to acknowledge the financial support provided by the Spanish Ministry of Economy and
527 Competitiveness (MINECO) and the European Funds for Regional Development (FEDER), through
528 Research projects: BIA2015-64672-C4-1-R and BIA2015-64672-C4-3-R.

529

530 **NOTATIONS**

531 a shear span. For slab floors in buildings subjected to distributed loads, the shear span, a , to be used in the
532 size effect parameter, ζ , can be estimated as the average distance from the position of the line of zero
533 radial bending moment to the edge of the column, $l_0 = \sqrt{l_{0y} \cdot l_{0z}}$, where $l_{0y} \cong 0.15 l_y$ and $l_{0z} \cong 0.15 l_z$, l_y
534 and l_z are the span lengths in the y and z directions. See reference [40] for complete definition
535 regarding shear in beams

536 b width of the cross-section of a beam. For T or I-shaped is equal to the flexural effective compression
537 flange width

538 b_w width of the web on T, I or L beams. For rectangular beams $b_w = b$

539 d effective depth of the cross-section

540 d_0 effective depth of the cross-section, d , but not less than 100 mm

- 541 d_s distance between the maximum compressed concrete fibre and the centroid of the mild steel tensile
 542 reinforcement. In the case of prestressed elements without mild reinforcement, d_s shall be taken equal
 543 to d_p
- 544 d_p distance between the maximum compressed concrete fibre and the mechanical centroid of the
 545 prestressing tendons placed at the tension zone
- 546 f_{cc} confined concrete compressive strength
- 547 f_{cd} design value of concrete compressive strength
- 548 f_{ck} characteristic compressive strength of concrete
- 549 f_{cm} mean compressive strength of concrete
- 550 f_{ctm} mean tensile strength of concrete, equal to $0.30 \cdot f_{ck}^{2/3}$ in MPa, not greater than 4.60 MPa
- 551 f_{yw} mean yield strength of the shear reinforcement
- 552 f_{ywd} design yield strength of the shear reinforcement
- 553 m_{crack} slab cracking bending moment per unit width
- 554 m_r bending moment per unit length producing radial stresses around the column
- 555 m_φ bending moment per unit length producing tangential stresses around the column
- 556 r radial distance from the column axis
- 557 r_0 radial distance from the column axis to the point of zero radial bending moment (contraflexure point)
- 558 r_{col} radius of a column with equal perimeter than the actual column
- 559 r_{crack} distance from the starting point of the critical crack (due to bending) to the column axis, see Fig. 7
- 560 r_{crit} distance from the critical perimeter to the column axis, see Fig. 7
- 561 s radial distance
- 562 s_{crack} distance from the starting point of the critical crack (due to bending) to the column face, see Fig. 7
- 563 s_{crit} distance from the critical perimeter to the column face, $s_{crit} = 0.5d$, see Fig. 7

564 u_{crit} critical perimeter (Fig. 7) placed at a distance s_{crit} from the column face

565 u_{out} perimeter where shear reinforcement is not longer required (Fig. 10)

566 x neutral axis depth of the cracked section, obtained assuming zero concrete tensile strength

567 x_0 neutral axis depth of a RC member or of a PC member considering $P = 0$ and the same amounts of

568 reinforcements

569 z inner lever arm. In the shear analysis of reinforced concrete beams without axial force, the approximate

570 value $z \approx 0.9d$ may normally be used. See Eq. (2)

571 A_{sw} cross-sectional area of the shear reinforcement. For punching, A_{sw} is the total area of the shear

572 reinforcement placed around the column that crosses the critical inclined crack that can be

573 approximated by considering the reinforcement placed between $0.5d$ and $1.5d$ from the support face

574 E_{cm} secant modulus of elasticity of concrete, $E_{cm} = 22000(f_{cm}/10)^{0.3} \geq 39 \text{ GPa}$

575 E_s modulus of elasticity of reinforcing steel

576 G_f concrete fracture energy, $G_f = 0.028f_{cm}^{0.18}d_{max}^{0.32}$, in Eq. (4)

577 K_c factor equal to the relative neutral axis depth, x/d , but not greater than 0.20, in Eqs. (10) and (31)

578 K_s factor that accounts for the effectiveness of the anchorage of the shear reinforcement

579 M_{cr} cracking moment at the section where shear strength is checked calculated using the mechanical

580 properties of the gross concrete section and the flexural tensile strength

581 M_{Ed} concomitant design bending moment, considered positive

582 V_{Ed} design shear force in the section considered

583 V_{Rd} design shear resistance of the member

584 $V_{Rd,max}$ design value of the maximum shear force which can be sustained by the member, limited by

585 crushing of the struts

586 V_u shear resistance of the member calculated by the background mechanical model, Eq. (1)

587 $V_{u,max}$ maximum shear force which can be sustained by the member, limited by crushing of the struts in the
588 background mechanical model or multi-action model, Eq. (2)

589 α angle between shear reinforcement and the beam axis perpendicular to the shear force in Eq. (11) and Eq.
590 (32). In Eq. (18) α is a parameter taking into account the non-uniform distribution of the vertical
591 stresses

592 α_{cw} coefficient taking account the state of the stress in the struts. See EC2 [30] for further information.

593 α_e modular ratio, $\alpha_e = E_s/E_{cm}$

594 α_{max} parameter for the determination of the maximum punching shear capacity, Eq. (35)

595 ν Poisson coefficient

596 ν_1 strength reduction factor for concrete cracked in shear. See EC2 [30] for further information

597 θ angle between the concrete compression strut and the beam axis perpendicular to the shear force

598 ρ_l longitudinal tensile reinforcement ratio. The neutral axis depth x/d should be obtained using the average
599 of the longitudinal reinforcement ratios, ρ_{lx}, ρ_{ly} , in the two orthogonal directions, adopting an effective
600 slab width $b_{s,eff}$ approximately equal to the column side or diameter plus 3 times the slab effective
601 depth at each side of the column

602 σ_r normal radial stresses around the column produced by m_r

603 σ_ϕ normal tangential stresses around the column produced by m_ϕ

604 σ_v vertical stresses in the slab in the vicinity of the column, see Fig. 5

605 ζ size and slenderness effect factor, given by Eq. (12)

606

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tango
crack

Eurocode 2

Model Code 2010 - LoA 3

Proposal

Eurocode 2

Model Code 2010 - LoA 3

Proposal

× Slabs with shear reinforcement

Shear resisting action	Dimensionless equations	
Compression chord	$v_c = \zeta \left[(0.88 + 0.70 v_s) \frac{x}{d} + 0.02 \right]$	(3)
Cracked concrete web	$v_w = 167 \frac{f_{ctm} b_w}{E_{cm} b} \left(1 + \frac{2 G_f E_{cm}}{f_{ctm}^2 d} \right)$	(4)
Longitudinal reinforcement	$if\ v_s > 0 \rightarrow v_l = 0.23 \frac{\alpha_e \rho_l}{1 - x/d}$	(5a)
	$if\ v_s = 0 \rightarrow v_l = 0$	(5b)
Shear reinforcement	$v_s = (d_s - x) \cot \theta \frac{A_{sw} f_{yw}}{s f_{ctm} b d} \approx \frac{0.85 d_s A_{sw} f_{yw}}{s f_{ctm} b d}$	(6)

Table 1. Summary of dimensionless shear contributing components, according to the MASM.

Equations	Expressions	
Shear strength	$V_{Rd} = V_{cu} + V_{su} \leq V_{Rd,max}$	(8)
Strut crushing	$V_{Rd,max} = \alpha_{cw} b z v_1 f_{cd} \frac{\cot \theta + \cot \alpha}{1 + \cot^2 \theta}$	(9)
Concrete contribution	$V_{cu} = 0.3 \zeta \frac{x}{d} f_{cd}^{2/3} b d \leq V_{cu,min} = 0.25 \left(\zeta K_c + \frac{20}{d_0} \right) f_{cd}^{2/3} b d$	(10)
Shear reinforcement	$V_{su} = 1.4 \frac{A_{sw}}{s} f_{ywd} (d_s - x) \sin \alpha (\cot \theta + \cot \alpha)$	(11)
Factors	Expression	
Size and slenderness effect	$\zeta = \frac{2 \left(\frac{d}{a} \right)^{0.2}}{\sqrt{1 + \frac{d_0}{200}}} \leq 0.45$	(12)
Relative neutral axis depth	$\frac{x_0}{d} = \alpha_e \rho_l \left(-1 + \sqrt{1 + \frac{2}{\alpha_e \rho_l}} \right) \approx 0.75 (\alpha_e \rho_l)^{1/3}$	(13)
Crack inclination	$\cot \theta = \frac{0.85 d_s}{d_s - x} \leq 2.5$	(14)

Table 2. Summary of the simplified equations for shear design according to the CCCM.

Number of slabs	Slabs without shear reinforcement		Slabs with shear reinforcement	
	328		232	
	min	max	min	max
Effective depth, d (mm)	30	500	83	476
Column diameter (mm) – circular	51	901	102	400
Column side (mm) - square	51	520	120	520
Compressive strength, $f_{cm,cyl}$ (MPa)	9.4	125.0	13.0	92.0
Flexural reinforcement ratio, ρ_l (%)	0.2	3.7	0.3	3.1
Yielding strength of longitudinal reinforcement (MPa)	270	811	270	917
Shear span to depth ratio, a/d	1.6	14	2.0	11.9
Number of rows of punching shear reinforcement	-	-	1	20
Distance between the rows (mm)	-	-	40	315
Diameter of shear reinforcement (mm)	-	-	3.7	22
Yield strength of shear reinforcement (MPa)	-	-	278	1100

Table 3. Range of parameters in the used databases.

	Slabs without shear reinforcement				Slabs with shear reinforcement			
	Proposal	EC2	MC-2010 LoA I	MC-2010 LoA III	Proposal	EC2	MC-2010 LoA I	MC-2010 LoA III
Number	328	328	328	328	232	232	232	232
Mean	1.188	1.171	1.895	1.141	1.173	1.067	2.129	1.251
Stand. dev	0.179	0.206	0.633	0.176	0.175	0.191	0.707	0.199
CoV	0.151	0.176	0.334	0.154	0.149	0.179	0.332	0.159
Max	1.746	2.038	4.896	1.696	1.880	1.757	4.263	1.755
Min	0.676	0.697	0.722	0.613	0.774	0.642	0.831	0.733
5%	0.905	0.843	1.084	0.810	0.929	0.774	1.136	0.909

Table 4. Comparison of models predictions with tests results.

	d (mm)	Column side (mm)	f_{cm} , reported (MPa)	ρ_l (%)	$A_{sw, 1st}$ <i>row</i> (mm ²)	V_{test} (kN)	Failure mode	V_c (kN)	V_s (kN)	V_{pred} (kN)	Failure mode	V_{test}/V_{pred}
PV1	210	260	34	1.50	0	974	PS	882	0	882	PS	1.10
PL11	201	260	34.2	1.57	628	1176	PS	765	310	1075	PS	1.09
PL12	201	260	34.6	1.57	1257	1683	PS	770	625	1395	PS	1.21
PL7	197	260	35.9	1.60	2463	1773	Pmax	-	-	1541	Pmax	1.15

Table 5. Experimental results and predictions by the proposed model for 4 slabs reported in [46].