DOI: https://doi.org/10.14311/TPFM.2017.033

ENVIRONMENTAL PATTERNS AND INTERMITTENT CASCADES

J. M. Redondo[1], A. Carrillo [1*], M. Diez[12], J. Jorge [1], E. Sekula[1]**

[1] Department of Physics, UPC Barcelona Tech. B5 Campus Nord *(Terrasa) **(Manresa) Barcelona 08034, Spain.

[2] Ports de la Generalitat, UPC-ERCOFTAC, Harbour; Vilanova i la Geltru 08800, Barcelona, Spain

Abstract

Real environmental flows are non-homogeneous, of fundamental interest is to determine and quantify turbulent diffusion from the available conditions of the flow, because the role of buoyancy and rotation modify the flow topology with often the dominant scale occurring when these two forces are in equilibrium. In geophysical flows both in the Atmosphere and the Ocean, the main forcing occurs at the Rossby deformation Radius with both direct and inverse energy cascades [1,2]. The role of the spectra of steady and decaying turbulence is important as well as its scale to scale conditions, so that a large range of scales has to be taken into account. When mixing and dispersion processes are studied, the behaviour of reactants or pollutants is seen to depend of both the intermittency of the vorticity and energy spectra. If irreversible molecular mixing has to be accounted, the range of scales spans from hundreds of Kilometres to the Bachelor or Kolmogorov sub millimeter scales. It is important to evaluate mixing and compare with oscillating grid experiments, Redondo [3], across a density interface measuring entrainment and grid decaying non steady mixing. These experiments are evaluated and compared with results of a Kinematic simulation model, Castilla [4]. The local vorticity is evaluated confirming the trapping of tracers in the strong vertical regions in 2D flows, but showing also that hyperdiffusion may also occur. Intermittency was evaluated using numerical evaluation of higher order moments in different types of 2D and 3D turbulence.

Keywords: turbulence cascade, intermittency, vorticity, kinematic simulation

1. Introduction

Both Oceanic and atmospheric flows may be considered as turbulent motions under the constraints of geometry, stratification and rotation. At large scales these flows tend to be along isopycnal surfaces due to the combined effects of the very low aspect ratio of the flows (the motion is confined to thin layers of fluid) and the existence of stable density stratification. The effect of the Earth's rotation is to reduce the vertical shear in these almost planar flows. The combined effects of these constraints are to produce approximately two-dimensional turbulent flows called geophysical turbulence.

In a strictly two-dimensional flow with weak dissipation, energy input at a given scale is transferred to larger scales, because these constraints stop vortex lines being stretched or twisted. Physically this upscale energy transfer occurs by merging of vortices and leads to the production of coherent structures in the flow that contain the energy the appearance of order from chaos [5-7].

This scenario is an attractive model for geophysical flows which are known to contain very energetic vortices, mesoscale oceanic eddies as those depicted in figure 1 that show the dominant length scale at the Rossby deformation Radius near the gulf of Lions. The upscale transfer of energy is inhibited at the Rossby deformation scale by baroclynic instability at larger scales, which accounts for the dominant observed size of geophysical vortices. Large-scale atmospheric models rely on small scale parameterisation of vertical mixing, and the ability to identify the local processes, which determine mixing, is very important in order to increase the accuracy of

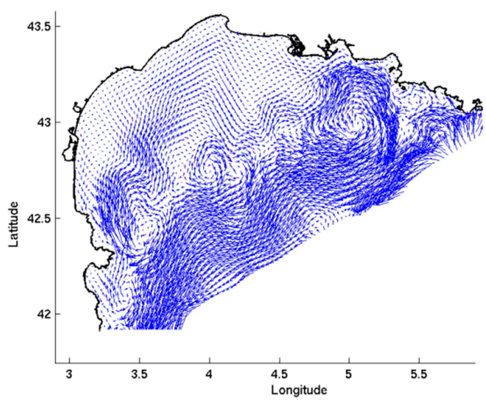

Figure 1: Eddies in the NW Mediterraneas sea.

forecasting. Inversions often reduce drastically the dilution of contaminants and in order to reduce local atmospheric pollution, industrial emissions need to take into account the scale to scale transfers.

Sharp density interfaces are commonly found in the ocean and transport through them is very important in determining the residence times of deep water. The dynamics of the thermocline is controlled by the mixing due to many different processes. Also, boundary layer results from mixing near the bottom of the sea. These processes merge near coastal regions, where tides often produce fronts. Sometimes exchanges between two different masses of water are locally controlled by interfacial mixing. The same is true in estuaries and river basins where fresh river water is mixed with all seawater. Turbulence theory is often non-homogeneous and based on these conditions. Grid-generated turbulence in the well-controlled environment of a laboratory is useful to parametrize the flows with the Rossby and the Richardson numbers [5,6].

1.1Basic Equations in Environmental Flows

The basic equations are the continuity and momentum equations, for an incompressible fluid the continuity equation reduces to

$$\Delta \cdot u = 0 \tag{1}$$

The equation of motion of an incompressible, homogeneous geophysical fluid with constant viscosity is:

$$\frac{\partial u}{\partial t} + u_i \frac{\partial u}{\partial x_i} = -\frac{1}{\rho} \nabla p + \nabla \varphi - 2\Omega * u + \upsilon \nabla^2 u \tag{2}$$

where p is the pressure, ϕ the gravitational potential, Ω the angular velocity of Earth and ν kinematic viscosity. If the motion is primarily horizontal, then the flow is approximately two-dimensional i.e. u = (u, υ, 0) where u, υ are independent of depth.

In a strictly 2D flow of this form there is only one non-zero component of vorticity (the vertical) and ω = (0, 0, ω), so in the absence of dissipation vorticity is conserved

$$\frac{D\omega}{\mathrm{Dt}} = 0 \tag{3}$$

Defining a stream function ψ as

$$u = \left(\frac{\partial \psi}{\partial y}, -\frac{\partial \psi}{\partial x}, 0\right) \tag{4}$$

$$\omega = -\nabla^2 \psi \tag{5}$$

The equation of vorticity conservation becomes

$$\frac{\partial \omega}{\partial t} + J(\varpi, \psi) = 0 \tag{6}$$

In strictly 2D flows with no dissipation we have conservation of

Energy $$\overline{u^2} = \int_0^{\infty} E(k,t)\, \mathrm{dk} \tag{7}$$

Enstrophy $$\overline{\omega^2} = \int_0^{\infty} k^2\, E(k,t)\, \mathrm{dk} \tag{8}$$

Conservation of enstrophy results from conservation of vorticity ω (no stretching or twisting). In a flow (2D) but with weak dissipation the above results hold approximately. The basic nondimensional parameter that describes the effects of stratification in stable situation is the Richardson number, *Ri*, described as a gradient *Ri* or as a flux *Ri* that relates buoyancy flux to turbulent production by shear or the other causes. The basic nondimensional parameter used to describe the effects of Rotation is the Rossby number considered as the ratio of the local fluid induced vorticity to the part of the absolute vorticity induced by the overall external rotation, Rotation produces also a 2D appearance of the flows looked from the rotation Axis but the structure is very different to that of a 2D strongly stratified situation. Also used to indicate the stability of the shear flows is the gradient Richardson number Turner et al. (1973).[8,9] It relates the variables that control the mixing at interface. The change in density of the fluid with depth is the density gradient $\partial\rho/\partial z$, that controls the natural buoyancy frequency often named after Brunt-Väisälä:

$$N = \left(-\frac{\rho}{g} \frac{\partial \rho}{\partial z}\right)^{1/2} \tag{9}$$

The linear internal waves propagate at an angle θ with a dispersion relationship given by $\omega^2 = N^2 \cos\theta$. That process is a powerful mechanism for the radiation of energy away from the mixing front or density produced by the local velocity gradient $(\partial u/\partial z)$. A similar role for rotation with frecuency f is produced by inertial wave radiation. A simpler parameterization of the effects of buoyancy as a local gradient Richardson number in terms of gradients [10-14]:

$$\mathrm{Ri} = \frac{g\,\partial\rho/\partial z}{\rho(\partial u/\partial z)^2} \tag{10}$$

where g is gravity, and ρ is the average fluid density.

1.2. Evaluations of Mixing, Mixing Efficiency and Entrainment

There exist several mechanisms that produce mixing across density interfaces; it depends on the rate of transfer from kinetic to potential energy, elevating the center of gravity of the initially stratified flow. The Prandtl number $\left(\mathrm{Pr} = \frac{\nu}{\kappa}\right)$ is also important relating the momentum diffusivity ν and the mass diffusivity κ. The mixing across a density interface may be evaluated by a general entrainment law [15,16] as.

$$E = \frac{\mathrm{Ue}}{u'} = c(\mathrm{Pr}) \cdot \mathrm{Ri}^{-n(\mathrm{Ri},\mathrm{Pr})} \tag{11}$$

For oscillating grid experiments Turner et al. (1968, 1973) [17,18] proposed that the entrainment velocity Ue defined as $U\mathrm{e} = \mathrm{d}D/\mathrm{d}t$, where D is the depth of the turbulent layer, is given by a simple law of the form

$$E \propto \mathrm{Ri}^{-n} \tag{12}$$

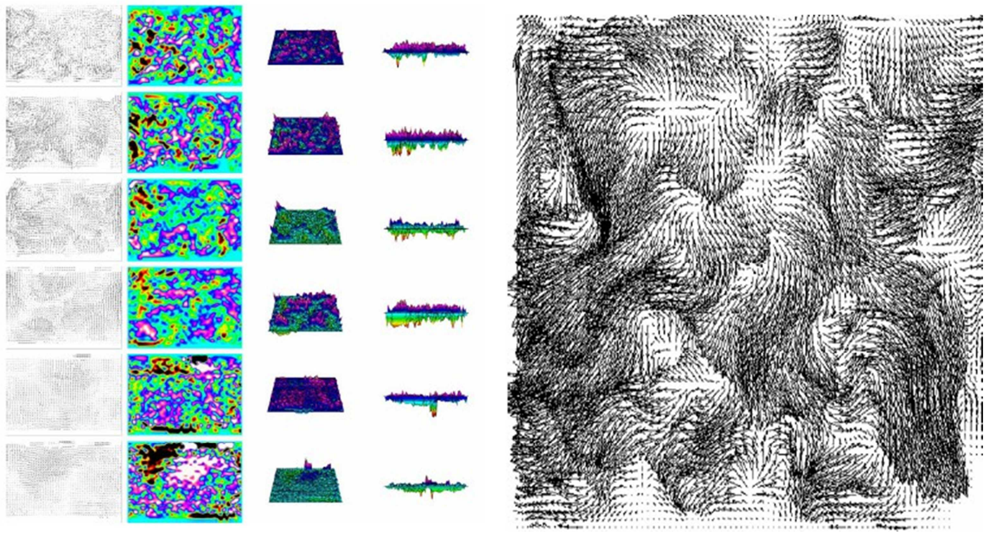

Figure 2: Velocity plot of a 2D rotating stratified flow (left) and Evolution of the velocity and vorticity of turbulence generated by a grid in time, notice the increase in size of the vorticity (right).[11]

2. Vorticity and Kinetic Energy Evolution

We may write the kinetic energy transport equation for vertical mixing in a shear flow, as:

$$\frac{1}{2}\frac{\partial q^2}{\partial t} + \frac{\partial}{\partial z}\overline{\left(w'\left(\frac{p'}{\rho} + \frac{q^2}{2}\right)\right)} = -g\frac{\overline{p\prime w\prime}}{\rho} - \varepsilon + \overline{u'w'}\frac{\partial \overline{U}}{\partial z} \tag{13}$$

in equilibrium and steady flow the first term is zero, the second term is transport and the lastis turbulent energy producción P *due to shear*, E is dissipation., and then if we separate mixing M, waves W and viscosity:

$$P = -g\frac{\overline{p\prime w\prime}}{\rho} - \varepsilon = \varepsilon_M + \varepsilon_W + \varepsilon_\nu \tag{14}$$

$\varepsilon_W = f(N)$ is a complex non linar function of N, the Brunt-Väisällä or buoyancy frecuency. This term seems to control the molecular irreversible mixing, acting like a catalizer for the scale to scale intermittency and transport. Due to dimensional analysis, the scaling is like that of an action i.e. $\varepsilon_{M,W,\nu} = \alpha \frac{U'^3}{l} \Rightarrow u_i l_i$ so it is the Monin-Obukhov length scale, used in the atmosphere as the main parameter, that determines whether the friction induced mechanicaly by shear dominates, or the thermal non-homogeneity forces the turbulence [19 - 22]:

$$L = \frac{\overline{\rho}u^3}{\mathrm{Kg}\overline{\rho' w'}} = -\frac{u^3}{\mathrm{KgH}/c_p\rho\overline{\theta}} = -\frac{u^3}{\mathrm{KB}} \tag{15}$$

3. Scaling and Fractal Results

Intermittency is related to the spaced filled by disipation, if *i* is the intensity of velocity, svalar or vorticity. *D (i)* also as a function of the scale *e* of the image. This dimension is usually calculated as:

$$D\,(i) = -\,Log\,(N(i))\,/\,Log\,(\,e,\,i\,), \tag{16}$$

where *N(i)* is the number of boxes of size e, needed to cover the image contour of intensity *i*. The algorithm used by ImaCalc [23] operates dividing the 2D surface into smaller and smaller square boxes and counting the number of them which have values of *i* within an interval.

The number of dominant vortices varies with time as shown in figure 2, the histogram of the energy and vorticity is clearly non Gausian and the higher order moments, Skewenes, Kurthosis, etc may be evaluated comparing the instantaneous vorticity-stream function diagrams [21-34]

The moment of third order - skewness of the vorticity - further confirms the existence of stronger positively skewed vorticity in flow far away from spires (Fig. 5c). The positive clock-wise vorticity, associated with frequent number of clock-wise vortices, is prevailing features in the fully developed turbulent boundary layer. On the other hand, the partially developed boundary layer apparently exhibits more Gaussian distribution with lower skewness, resulting in more equal occurrence of both the clock-wise and anticlock-wise vortices.

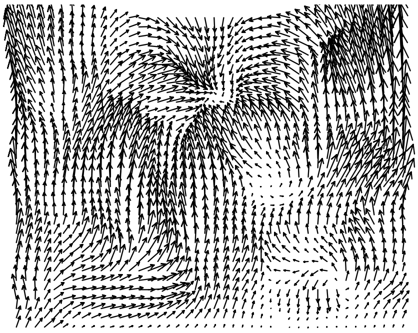

Figure 3: Velocity of a stratified layer showing non zero divergence, sinks or 3D lines because the real flow is not divergence free.

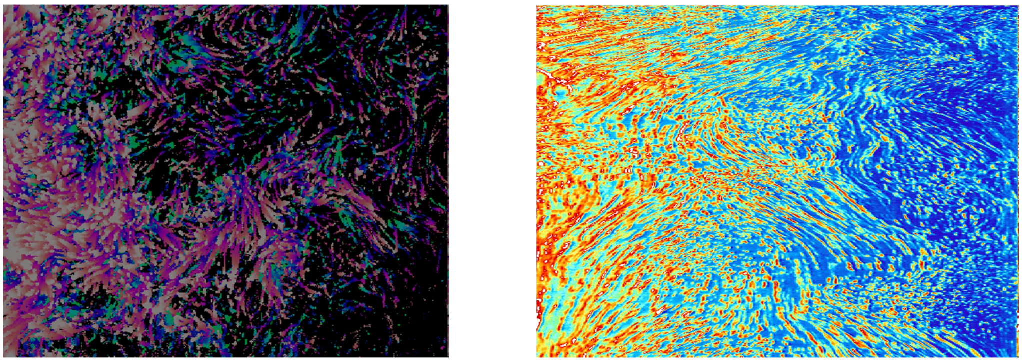

Figure 4 Instantaneous traces of PIV and particle traking PT techniques (left) Internal Waves superimposed to local horizontal flow [22].

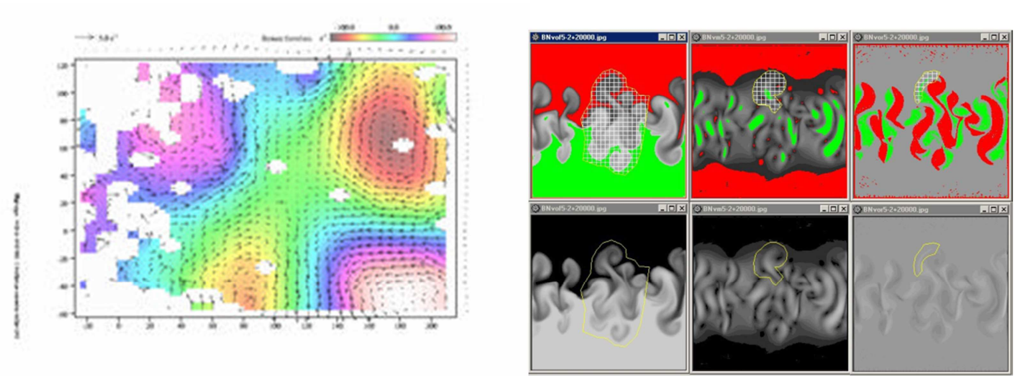

Figure 5: Horse-Saddle structure of the local vortices and shear layer with both senses of rotation (left) Different structures , with different fractal spectra for density, velocity modulus and vorticity [23,24] (right) in a buoyancy driven front [25,26] in the flow.

4. Discussion

On the one hand, from the stable stratification case; the aim is to explain and model the flat structure dynamics by the zig-zag instabilities as precursors of quasi 2D or under the horizontal and vertical shear conservation arguments. This field is open to an anisotropy and non-linear statistic theory of ‘wave-vortex’, supported by the direct numerical simulations. Recent results of Lagrangian dispersion by passive tracers (couples); showed that diffusion looks dominated by the linear modes of movement, essentially anisotropic dispersive waves and the quasi-geostrophic modes. While the organized eddies from non-linear interactions play a minor role in the tracer trapping. Those results are found into other domains in the environment. Paradoxically the measurement of eddy structures is easy to visualize than the more elusive wave manifestations. At high Reynolds, we cant quantify the portion of organized eddies and their role in dispersion. Those studies are going to give information on the force and time scales of the organized eddy structures and the specific promoter instabilities.[20-22]

Laboratory experiments will be described here comparing the mixing efficiency and the main flow descriptors of the interaction of several Jets and plumes in a confined space. Matulka et al (2014) [11, 23], used in a different configuration, where the plume interacted with a density interface a numerical model using the Boussinesq set of equations in a two dimensional grid. Their scaling assumed a constant turbulent viscosity defined in terms of a Smagorinski type of dimensional scaling with a simple turbulent parametrization based on local mixing length scales as $v = l^2 / \Delta t$ and taking the integral scale initially as a constant in terms of the mesh size as $l = 0.23 (\Delta x \, \Delta z)^{1/2}$ Different aspect ratios of the convective flow, generated by buoyancy were classified then. There the crucial condition was whether the plume/Jet was able or not to break the interface. The plumes are formed by injecting a dense fluid from a small source (from one to nine orifices) into a stationary body of lighter brime (saline solution) contained in a tank. The source fluid was dyed with fluorescein and we use the LIF technique. The plumes were fully turbulent and we have both momentum and buoyancy driven distinct regimes. The mixing process is generated also by the evolution of a bidimensional array of forced turbulent plumes [21]. The conclusions of the experiments where no Jet structure was formed, but a 2D array was used to measure mixing efficiency and the volume of the final mixed layer as functions of the Atwood number, (0.010 to 0.134) was discussed by [11] The mixing efficiency has an upper limit of 0.18 compared with the maximum mixing efficiency (0.5) in comparable experiments [26]. An explanation to understand the smaller mixing efficiencies uses the reduction in possible mixing volume induced by the interaction of the array of plumes, and its interaction with the side walls that clearly modify the overall mixing efficiency, so it depends strongly on initial conditions and the structure of the jet entrainment boundaries. The reduction of the overall mixing efficiency when the flow starts as an array of plumes may be explained because there is less volume where contact may exist and once the potential energy is lost by a falling plume no mixing may take place locally above the Ozmidov scale. Field data from wind shows fractal behaviour as may be seen in figure 6 from Eulerian measures, but also Lagrangian Fractal measures in the ocean detect instabilities such as vortex dipoles (figure 7).

One of the most important roles of Stratification and Rotation in environmental turbulence, and in general of all body forces, including magnetic fields; is to modify the slope of the spectral energy cascade.

Another experimental and numerical observation is that while the anisotropy of the Reynolds stresses is obviously linked with the non-homogeneity taking the vertical axis (in stratified flows) and the rotation axis (in rotating flows) ; Scalar behaviour in such flows has non-linear mixing properties Redondo et al. (2002) and Babiano et al. [5, 25]. There are similar effects that depart from Kolmogorov's K41 and also for K62 theories, not just in second order structure functions (and related spectra) for spatial non-homogeneity, for anisotropy and for spatial and temporal intermittency.[27,28]. The intermittency character in non-homogeneous flows is more complicated than the K62 log-normal model.

After the refined similarity hypothesis, different types of intermittency models were proposed to describe the turbulence cascade and particularly the behaviour of scaling exponents [28]. The success of these models can be evaluated especially on the basis of experiments and this is next reason why is so important to do experiments in this field. However, there are no models that agree with all experiments although each model works quite well within a limited group of available data. Some of the most popular models for fully developed turbulence are the She-Leveque model, the *p*-model, the *β*-model or the random *β*-model, which involve relating the structure functions of turbulent velocity to fractal measures [27, 28].

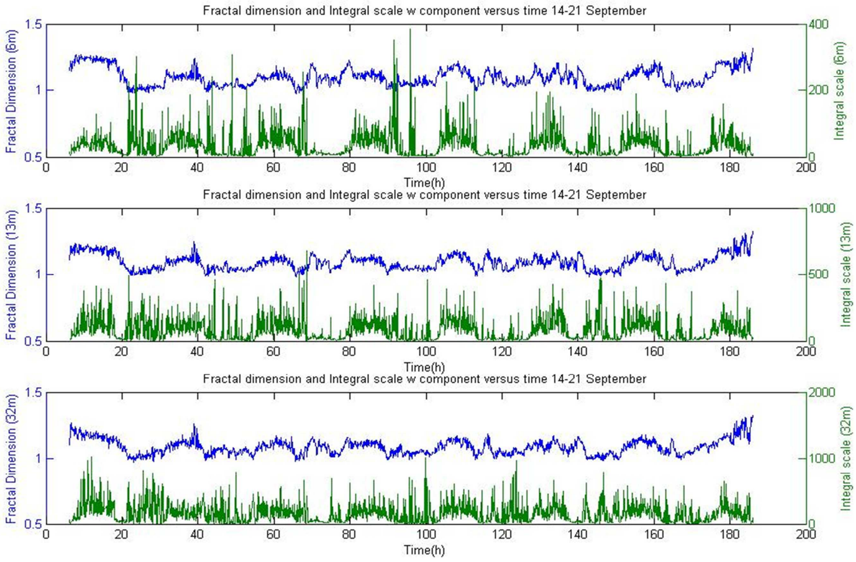

Figure 6: Fractality and intermittency of wind measurements [7, 21]

5. Conclusions

Structure function analysis allows the investigation of intermittent and scale to scale energy transfer even in local non equilibrium flows. The relative diffusion of tracers is strongly dependent on the slope of the energy spectra which tends to Richardson's law also for very steep spectra. Local turbulence is used to establish the geometry of the turbulence mixing, changes in the equilibrium (or not) cascade may lead to more physically realistic (and understandable) models to parametrize sub-grid scaling but care has to be taken when interpreting the direct 3D Kolmogorov cascade and the Inverse 2D Kraichnan Cascade [9].

It is very interesting to use ESS and the third order structure functions to investigate the scale to scale transfer of energy (and enstrophy) There is a range of spectral slopes (between 5/3 and 3) where hyper diffusion takes place, due to non-linear explosive interactions between eddies. A parameter space based on Richardson numbers, Rossby numbers and Reynolds Numbers can be used to determine the dominant instability in a stratified-rotating flow.

Helicity local balance leads to a 7/3 Energy spectra, we believe that μ is not universal, as it varies from approximately 0.2 to 0.7, according to different experiments. The new energy spectra, $E(k)$, has a correction term in its power of k: -5/3 becomes $-5/3-\mu/9$, thus, the global form of the spectra is $E(k) \sim k\text{-}\beta$ The Fractal dimension is related to the intermittency and the spectral exponent, but the relationships that may be used to parameterise the sub-grid turbulence in terms of generalized diffusivities are not universal. They take into account the topology and the self-similarity of the Mixing. As an example, a relationship between the diffusivity, the exponent β, the intermittency μ, and $D(i)$, may be found for the volume fraction or the concentration, at the same time other locally measured parameters such as the enstrophy or the gradient alignment as well as their multi-fractal structures may turn out to be physically relevant indicators of the local turbulence and the mixing. Several methods of deriving local eddy diffusivity should give more realistic estimates of the spatial/temporal non-homogeneities (and intermittencies in the Kolmogorov 62 sense obtained as spatial correlations of the turbulent dissipation, or from structure functions) and these values may be used to parameterise turbulence at a variety of scales. The method involving the multi-fractal dimension measurements is much more elaborated and seems to have a better theoretical justification in the sense that it is possible that different concentrations showing different fractal dimensions may be due to different levels of intermittency (and thus different spectra, which are not necessarily inertial nor in equilibrium.

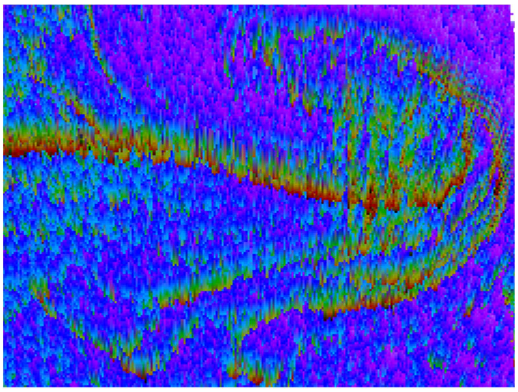

Figure 7: An ubiquitous structure in environmental turbulence is the vortex dipole.

Acknowledgements

The authors would like to thank Filip Ciesinski from Warsaw University of Technology and Dr. Alejandro Carrillo for help with the experiments reported in this study. Thanks also to Professor Jan Wojciechowski, to Dr. Ania Matulka and to Dr. Manuel Tijera. Further support was provided by grants MULTIFLOW and ESA . for its financial support. Emil Sekula acknowledges the grant “Beca predoctoral UPC para investigación” from Universitat Politecnica de Catalunya. The Xarxa Tematica de Dinamica de Fluids i Turbulencia Geofisica (XT96-00016) provided some support for the research as well as ERCOFTAC- PELNHT and the research group of Turbulencia Fluctuacions i Diffusio. (SGR99-00145).

References

[1]. Redondo J.M. (1996) Vertical microstructure and mixing in stratified flows. Advances in Turbulence VI. Eds. S. Gavrilakis et al., pp. 605-608.

[2]. Redondo J.M.(2001) Mixing efficiencies of different kinds of turbulent processes and instabilities, Applications to the environment” in Turbulent mixing in geophysical flows. Eds. Linden P.F. and Redondo J.M., pp. 131-157.

[3]. Tellez J., Gómez M., Russo B. & Redondo, J.M. (2015) A simple measuring technique of surface flow velocity to analyze the behavior of velocity fields in hydraulic engineering applications. Geophysical Research Abstracts

[4]. Carrillo, J.A., Redondo, J.M., Sánchez, M.A., and Platonov, A. (2001), Coastal and interfacial mixing laboratory experiments and satellite observation, *Physics and chemistry of the Earth* **26** (4), 305-311.

[5]. Babiano, A. (2002), On Particle dispersion processes in two-dimensional turbulence. In Turbulent mixing in geophysical flows. Eds. Linden P.F. and Redondo J.M., p. 2-33.

[6]. Dalziel, S.B., Linden, P.F. & Boubnov, B.M. (1995), Experiments on turbulence in stratified rotating flows; in Mixing in Geophysical Flows, Ed. Redondo & Metais; CIMNE, Barcelona, 195-208.

[7]. Tijera, M., Cano J.L., Cano, D., Bolster, B. and Redondo J.M.: Filtered deterministic waves and analysis of the fractal dimension of the components of the wind velocity. *Nuovo Cimento C.* Geophysics and Space Physics 31: (2008) pp. 653-667

[8]. Cuxart J., C. Yagüe, G. Morales, E. Terradellas., J. Orbe, J. Calvo, A. Fernández, M.R. Soler, C. Infante, P. Buenestado, A. Espinalt, H.E. Joergensen, J.M. Rees, J. Vilà, J.M. Redondo, I.R.Cantalapiedra, and Conangla, L.:(2000) Stable atmospheric boundary layer experiment in Spain (SABLES 98): A report. *Bound.-Layer Meteorol.*, 96: pp 337-370.

[9]. Kraichnan, R. (1975), Statistical dynamics of two-dimensional flow. *J. Fluid Mech*. 67, 155-175.

[10]. Linden, P.F. (1980), Mixing across a density interface produced by grid turbulence. Journal of Fluid Mechanics 100, 3–29.

[11]. Matulka, A. (2003), Environmental Turbulence. Effects of rotation and stratification on diffusion and mixing in geophysical flows, Master Thesis, UPC and T.U.Warsava.

[12]. Onsager, L. (1949), Statistical hydrodynamics. Suppl. Nuovo Cim. 6, 279-287.

[13]. Nicolleau, F.C.G.A.; Cambon, C.; Redondo, J.M.; Vassilicos, J.C.; Reeks, M.; Nowakowski, A.F. (Eds.)(2011)New approaches in modeling multiphase flows and dispersion in turbulence, fractal methods and synthetic turbulence, ERCOFTAC Series, Springer.

[14]. Redondo J.M. and Cantalapiedra I.R. (1993) "Mixing in horizontally heterogeneous flows", Applied Scientific Research, 51, 217-222.

[15]. Redondo, J.M., Sánchez, M.A. & Cantalpiedra, I.R. (1996), Turbulent mechanisms in stratified fluids, *Dyn. of Atmospheres and Oceans* **24**, 107-715.

[16]. Redondo, J.M. (2004), The topology of Stratified Rotating Flows in *Topics in Fluid Mechanics. Prihoda & K.Kozel, CAS, Praga* 129-135.

[17]. Turner, J.S. (1968), The influence of molecular diffusivity on turbulent entrainment across a density interface. *J. Fluid Mech.* **33**, 639-656.

[18]. Turner, S. (1973) Buoyancy Effects in Fluids. Cambridge University Press, Cambridge.

[19]. Castilla R., Redondo J.M., Gamez P.J. and Babiano A. (2007), *Non Linear Processes in Geophysics*, 14, pp. 139.

[20]. Redondo, J. M. Fernando J.H. and S. Pares (1995) Cloud entrainment by internal or external turbulence, Mixing in geophysical flows, J. M. Redondo and O. Metais (Eds), CIMNE, Barcelona (1995) pp. 379-392.

[21]. Sekula E., Redondo J. M. (2017) The structure of turbulent jets, vortices and boundary layers: Environmental Complex Flows and Dispersion in Turbulence, *ERCOFTAC BULLETIN* .

[22]. Fraunie P., Berreba S. Chashechkin Yu.D., Velasco D. and Redondo J.M. (2008) Large eddy simulation and laboratory experiments on the decay of grid wakes in strongly stratified flows. Il Nuovo Cimento C 31, 909-930.

[23]. Matulka, A., López, P., Redondo, J. M., and Tarquis, A.(2014) On the entrainment coefficient in a forced plume: quantitative effects of source parameters, Nonlin. Processes Geophys., 21, 269-278.

[24]. Castilla R., Oñate E. and Redondo J.M. (2007) Models, Experiments and Computations in Turbulence. CIMNE, Barcelona, 255.

[25]. Redondo, J. M. (1993) Fractal models of density interfaces. In: IMA Conf. Ser. 13, Farge, M., Hunt, J.C.R., Vassilicos J. C. (eds.) Oxford: Clarendon Press, 353–370.

[26]. Linden, P. F., Redondo, J. M. and Youngs, D. L.: 1994, “Molecular mixing in Rayleigh-Taylor instability”, J. Fluid Mech. 265, 97-124.

[27]. Mahjoub O.B., Redondo J.M. and Babiano A. “Hyerarchy flux in nonhomogeneous flows” in Turbulent diffusion in the environment Eds. Redondo J.M. and Babiano A. 249-260. 2000.

[28]. Valente, P.C. and Vassilicos, J.C.: The decay of turbulence generated by a class of multiscale grids. *Journal of Fluid Mechanics*, 687, (2011) pp. 300–340.