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Article publicat / *Published paper*:

Pastrana, D., [et al.]. Large-eddy simulations of the vortex-induced vibration of a low mass ratio two-degree-of-freedom circular cylinder at subcritical Reynolds numbers. *Computers and Fluids*, Setembre 2018, vol. 173, p. 118-132. DOI: <<https://doi.org/10.1016/j.compfluid.2018.03.016>>.

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PII: S0045-7930(18)30114-2
DOI: [10.1016/j.compfluid.2018.03.016](https://doi.org/10.1016/j.compfluid.2018.03.016)
Reference: CAF 3777

To appear in: *Computers and Fluids*

Received date: 9 February 2017
Revised date: 8 February 2018
Accepted date: 1 March 2018

Please cite this article as: D. Pastrana, J.C. Cajas, O. Lehmkuhl, I. Rodríguez, G. Houzeaux, Large-eddy simulations of the vortex-induced vibration of a low mass ratio two-degree-of-freedom circular cylinder at subcritical Reynolds numbers, *Computers and Fluids* (2018), doi: [10.1016/j.compfluid.2018.03.016](https://doi.org/10.1016/j.compfluid.2018.03.016)

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Highlights

- Detailed letter for the reviewer
- High-fidelity (LES) study of a 2dof VIV of a low mass ratio circular cylinder is presented
- A low-dissipative spatial and temporal discretization has been used
- Numerical results are extensively compared with available experimental and numerical data
- Significant improvements respect to previous high-fidelity numerical simulations are done
- Transition between SU branch and L branch is explored

Large-eddy simulations of the vortex-induced vibration of a low mass ratio two-degree-of-freedom circular cylinder at subcritical Reynolds numbers

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Abstract

The vortex induced vibration phenomenon of a low mass ratio ($m^* = 2.6$) two-degree-of-freedom circular cylinder at subcritical Reynolds numbers ($Re = 3900, 5300, 11000$) has been investigated by means of large-eddy simulations. A low-dissipative spatial and time discretisation finite element schemes have been implemented and combined with the Wall-Adapting Local-Eddy viscosity (WALE) subgrid-scale model to solve the filtered incompressible flow equations. Several values of the reduced velocity in the range $3 \leq U^* \leq 12$ have been considered. The numerical results are ~~extensively~~ **extensively** compared with available experimental and numerical data. Particular interest has been placed in the region of maximum cross-flow amplitudes, the super-upper branch, where previous high-fidelity numerical simulations have underestimated the peak amplitudes compared with experimental results. The transition between the super-upper and lower branches is also shown and described. The numerical simulations successfully reproduce the three-branch response maximum oscillation amplitudes and associated vortex formation modes. The 2T vortex formation mode, i.e. two triplets of vortices per oscillation period, has been observed to occur in the super-upper branch, for the three different values of the Reynolds number investigated. These results contradict the claim made in previous works ("Two-degree-of-freedom vortex-induced vibrations of a circular cylinder at $Re=3900$ ", Journal of Fluids and Structures (67), 2016) that the vortex formation mode in the super-upper branch is Reynolds number dependent. Beats are observed to appear prior the transition from the super-upper to the lower branch. It is argued that they may be related with the the coherence and strength of the third vortex shed at the shoulder of the cylinder each half-cycle, which is finally suppressed in the transition to the lower branch.

Keywords: Vortex induced vibration, turbulent flow, large-eddy simulation, vortex shedding, low mass ratio, two-degree-of-freedom system, fluid-structure interactions

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1. Introduction

It has been known for more than a century that the vortex shedding from a bluff body leads to transient flow-induced periodic forces. The fluctuating forces may induce the vibration of the body if it is deformable or elastically mounted. This complex fluid-structure interaction (FSI) is better known as vortex induced vibration (VIV). Historically, VIV phenomenon has been object of great interest and extensive research because of its importance on the design of great variety of offshore structures such as raisers, marine cables, oil platforms, mooring lines and hawsers, where predicting accurately the maximum loads acting on the structure is crucial [1]. Recently, VIV has become a renewable energy harvesting paradigm. This is reflected in the plethora of publications about VIV related with different prototypes exploring various transduction mechanisms for low power devices [2, 3, 4, 5, 6, 7, 8] and the work in progress to develop a commercial device capable of harvesting high amounts of power [9, 10, 11]. The previous highlights the importance of the phenomenon and the challenges ahead, regarding large scales applications and the underlying need to better understand the fundamental physics.

Numerous papers devoted to VIV have been written and some remarkable concise reviews can be found in the literature (see for instance [12, 13, 14, 15, 16, 17]). The most important advances have been achieved through the study of the classical problem of a rigid circular cylinder forced or free to oscillate in one-degree-of-freedom (1dof, cross-flow only motion) or two-degrees-of-freedom (2dof, cross-flow and in-line motion). It has been established that the largest amplitude response of the system takes place in the so-called lock-in, synchronisation or resonance range. This range is an interval of the reduced velocity $U^* = U_\infty / (f_n D)$ (defined as the inverse of the natural frequency f_n non-dimensionalised with the free stream velocity of the flow U_∞ and the diameter of the cylinder D) where the natural frequency of the cylinder is close to the vortex shedding frequency.

Different **different** branches of response have been described depending whether the system is a 1dof or a 2dof (see figure 1). In the figure, a diagram with the different branches for 1dof and 2df and corresponding maximum amplitude depending **on** the value of the reduced velocity are represented. For the free to oscillate 1dof system[18], two branches of response have been found, namely the initial branch (I), with the highest amplitudes, and lower branch (L) with peak amplitudes of about $0.5D$ for high values of the mass ratio m^* ($m^* = 4m / (\rho \pi D^2 L)$). The mass ratio m^* is a parameter that measures the relation between the mass of the cylinder and the mass of an equivalent volume of fluid. Different modes of vortex shedding have been directly associated with each of these branches: the 2S mode indicating 2 single vortices shed per cycle and the 2P mode meaning 2 pairs of vortices per cycle. The 2S mode is related with the initial branch; whereas the 2P mode with the lower branch. These modes are also represented in figure 1. At low values of the mass ratio, a new higher-amplitude branch of response appears between the other branches. This is the upper branch (U), where maximum amplitudes of oscillation of about $1D$ occur. The upper branch is also accompanied by the 2P vortex formation mode (see figure 1). A remarkable feature of this system is the hysteric transition between the I and L branches and the intermittent switching in the transition between the U and L branches [18].

The effect of the 2dof on VIV was studied by Jauvtis and Williamson [19] (hereafter referred to as JW2004). For values of the mass ratio down to $m^* \approx 6.0$, including the in-line direction motion (direction aligned with the fluid flow) hardly affects the system response, i.e. the same 1dof characteristic branches, peak amplitudes and vortex wake modes appear. For lower values of m^* , they showed that adding a degree of freedom in the flow direction has serious implications in the behaviour of the system. The extra degree of freedom increases the number of important

Figure 1: Three branches response and their vortex formation modes for an elastically mounted circular cylinder. At the higher branch a distinction between the U branch for 1dof systems and SU branch 2dof has been made.

parameters and allows a more complex two-dimensional motion. JW2004 experimentally identified a high-amplitude mode of response for lower values of the mass ratio $m^* < 6$ and identical natural frequency and mass ratio in each direction ($Re \approx 1000 - 15000$). This super-upper branch (SU), with maximum amplitudes of about $1.5D$, is not present in the 1dof system (see figure 1). The authors also showed that the emergence of the high-amplitude response mode is related with to the arising of the 2T vortex formation mode (two triplets of vortices per oscillation period), not previously reported, which maximizes the fluid-structure energy transfer. The 2T and multi-vortex shedding modes were also related with the appearance of a third and higher harmonics in the lift force [20]. JW2004 also reported that the transition between the SU and L branches is hysteric with a stable mode at the SU branch despite the very high amplitudes of oscillation.

Most of the numerical efforts in order to simulate the VIV phenomenon are limited to low Reynolds numbers, mainly because the massive computational resources and extra modelling efforts required to properly describe the fluid flow beyond the laminar region (see for instance [21, 22, 23]). In the last years valuable contributions have been done in the turbulent regime, but few studies have reproduced the three-branches response reported in the experiments and captured the large amplitude of the U and SU branches for low mass-damping systems. Lucor et al. [24] performed direct numerical simulations (DNS) of 1dof rigid circular cylinder with low mass ratio and zero damping. The study was focused on numerically capturing the large amplitude of response and its sensitivity to the Reynolds number for $Re = 1000 - 3000$. They successfully reproduced the three-branch response but pointed out that the width and magnitude of the U branch is Reynolds number dependent. Navrose and Mittal [25] investigated free vibrations of a 2dof circular cylinder via DNS for $Re = 1000$ and $m^* = 10.0$. The three-branch response of the system was observed, but a value of maximum oscillation amplitude smaller than that register for larger Reynolds numbers

was reported. Also, they reported the 2S vortex formation mode at the U brach, different to the 2P mode observed in experiments for higher Reynolds numbers. Hence, they proposed that the vortex shedding mode on the U branch is affected by the Reynolds number. The variation of the vortex shedding flow and its transition from two-dimensional to three-dimensional flow was conducted by Zhao et al. [26]. They found that the three-dimensionality of the flow is stronger in the U branch and weaker in the I branch, and that in the U branch 2S and 2P modes coexist along the cylinder span. Recently, Gsell et al. [27] (hereafter referred to as G2016) studied a 2dof elastically-mounted circular cylinder for $Re = 3900$. Despite reproducing the three-branch response, the high amplitudes in the SU branch were underestimated and the 2T vortex formation mode was not reported. Instead, the 2S mode in the SU branch was observed.

The challenge of VIV numerical simulations on capturing the maximum oscillation amplitude has been recently addressed in [28, 29], where improvements to Unsteady Reynolds-Average Navier-Stokes (URANS) traditional models are presented in order to properly capture the maximum transverse amplitudes for low mass ratio systems. Although these models succeeded at capturing the main characteristics, such as the maximum amplitude of the oscillation and vortex-shedding frequency, it seems to be difficult to these models to well capture the maximum amplitude in the SU branch. In this sense, large-eddy simulations (LES) of the flow, which have been proven accurate at capturing the massive separated flow behind captive cylinders at sub-critical, critical and super-critical Reynolds numbers (see for instance [30, 31, 32, 33, 34]), might be a good alternative. However, to the best of the authors' knowledge there are no relevant publications that properly describe the VIV phenomenon from a large-eddy simulations (LES) approach.

In this sense, the present work addresses the VIV numerical simulation of a 2dof circular cylinder by means of LES in order to show the capabilities of this kind of modelling to well capture not only the free vibrations of the cylinder due to its interaction with the fluid but also to reproduce the flow features observed in experiments regarding the super-upper branch, the maximum amplitude of the system and its hysteric behaviour, as well as, the later transition between the super-upper and the lower branches. To do so, 2dof free vibrations of a low mass ratio circular cylinder at subcritical Reynolds numbers ($Re = 3900, 5300$ and 11000) for different values of the reduced velocity are investigated. For this purpose, low-dissipative spatial and time discretisation schemes have been implemented and combined with the wall-adapting local-eddy viscosity (WALE) subgrid-scale (SGS) model. The three predicted branches of response are mapped and its characteristics are discussed and compared with available experimental and numerical data. The sensitivity of the maximum amplitudes and vortex formation modes to the Reynolds numbers are also investigated. Finally, the transition between the SU and L branches is described.

2. Problem statement and numerical formulation

In this work, the VIV of a rigid circular cylinder of diameter D elastically-mounted and free to oscillate in cross-flow (y-axis) and in-line (x-axis) combined motion immersed in a fluid-flow is numerically studied. Fig. 2 shows a not-to-scale schematic representation of the system and computational domain with its most important characteristics. Under such conditions the equations of motion that describe the behaviour of the flow and its interaction with the cylinder are stated in the following subsections.

Figure 2: Not-to-scale schematic representation of the modeled physical system.

2.1. Fluid turbulence model

The spatially filtered Navier–Stokes equations for a fluid moving in the domain Ω bounded by $\Gamma = \partial\Omega$ during the time interval (t_0, t_f) consist in finding a filtered velocity $\bar{\mathbf{u}}$ and a kinematic pressure p such that

$$\partial_t \bar{\mathbf{u}} + (\bar{\mathbf{u}} \cdot \nabla) \bar{\mathbf{u}} - \frac{2}{Re} \nabla \cdot \mathbf{S}(\bar{\mathbf{u}}) + \nabla p - \mathbf{f} = -\nabla \cdot \tau_{ij}(\bar{\mathbf{u}}) \quad \text{in } \Omega \times (t_0, t_f), \quad (1)$$

$$\nabla \cdot \bar{\mathbf{u}} = 0 \quad \text{in } \Omega \times (t_0, t_f), \quad (2)$$

where $Re = U_\infty D / \nu$ is the Reynolds number based on the cylinder diameter D and the free-stream velocity U_∞ , $\mathbf{f}$ the vector of external body forces and $\mathbf{S}(\bar{\mathbf{u}})$ is the large-scale rate-of-strain tensor. In equation (1) $\tau_{ij}(\bar{\mathbf{u}})$ is the subgrid scale (SGS) stress tensor, which must be modelled. Its deviatoric part is given by

$$\tau_{ij}(\bar{\mathbf{u}}) - \frac{1}{3} \tau_{kk}(\bar{\mathbf{u}}) \delta_{ij} = -2\nu_{sgs} \nabla \cdot \mathbf{S}(\bar{\mathbf{u}}) \quad (3)$$

where δ_{ij} is the Kronecker delta. The formulation is closed by an appropriate expression for the subgrid-scale viscosity, ν_{sgs} . In this paper the wall-adapting local-eddy viscosity model (WALE) [35] is used. This model, based on the invariants of the velocity gradient tensor, accounts for the effects of both the strain and the rotation rates of the smallest resolved turbulent fluctuations, and has an appropriate near wall scaling for the eddy viscosity (i.e. $\nu_{sgs} \propto y^3$). In WALE, the SGS viscosity is approximated as:

$$\nu_{sgs} = (c_{wale}\Delta)^2 \frac{(\bar{\mathbf{V}} : \bar{\mathbf{V}})^{3/2}}{(\bar{\mathbf{S}} : \bar{\mathbf{S}})^{5/2} + (\bar{\mathbf{V}} : \bar{\mathbf{V}})^{5/4}} \quad (4)$$

$$\bar{\mathbf{V}} = \frac{1}{2} (\nabla(\bar{\mathbf{u}})^2 + \nabla^T(\bar{\mathbf{u}})^2) - \frac{1}{3} (\nabla(\bar{\mathbf{u}})^2 : \mathbf{I}) \mathbf{I}$$

where c_{wale} is the model constant, here $c_{wale} = 0.325$, and $\mathbf{I}$ is the identity matrix. This model yielded good results in previous simulations of the static cylinder from sub-critical to super-critical Reynolds numbers [33, 34, 36].

In order to obtain the weak or variational formulation of the Navier–Stokes equations (eqns. (1) and (2)), we introduce the spaces of vector functions $\mathbf{V}_D = \mathbf{H}_D^1(\Omega)$, $\mathbf{V}_0 = \mathbf{H}_0^1(\Omega)$ and $Q = L^2(\Omega)/\mathbb{R}$. $L^2(\Omega)$ is the space of square-integrable functions, $H^1(\Omega)$ is a subspace of $L^2(\Omega)$ formed by functions whose derivatives also belong to $L^2(\Omega)$, $H_D^1(\Omega)$ is a subspace of $H^1(\Omega)$ that satisfies the Dirichlet boundary condition on Γ , $H_0^1(\Omega)$ is a subspace of $H^1(\Omega)$ whose functions are zero on Γ , and $\mathbf{H}_D^1(\Omega)$ and $\mathbf{H}_0^1(\Omega)$ are their vector counterparts in a two- or three-dimensional space. $(\cdot, \cdot)$ indicates the standard L^2 inner product.

For the unsteady case, $\mathbf{V}_t \equiv L^2(t_0, t_f; \mathbf{V}_D)$ and $Q_t \equiv \mathcal{D}'(t_0, t_f; Q)$ are introduced, where $L^p(t_0, t_f; X)$ is the space of time dependent functions in a normed space X such that $\int_{t_0}^{t_f} \|f\|_X^p dt < \infty$, $1 \leq p < \infty$ and Q_t consists of mappings whose Q -norm is a distribution in time. The weak form of problem (1, 2) with the boundary conditions is then: Find $\mathbf{u} \in \mathbf{V}_t$, $p \in Q_t$ such that

$$(\partial_t \bar{\mathbf{u}}, \mathbf{v}) + (\bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}}, \mathbf{v}) + \frac{2}{Re} (\mathbf{S}(\bar{\mathbf{u}}), \nabla \mathbf{v}) - (p, \nabla \cdot \mathbf{v}) + (q, \nabla \cdot \bar{\mathbf{u}}) - (\mathbf{f}, \mathbf{v}) = (\tau_{ij}(\bar{\mathbf{u}}), \nabla \mathbf{v}), \quad (5)$$

for all $(\mathbf{v}, q) \in \mathbf{V}_0 \times Q$.

In addition, in the previous equations the convective form of the nonlinear term has been discretized using a conservative scheme recently proposed by Charnyi et al. [37], in which the energy, momentum and angular momentum are preserved. The discretized convective term reads,

$$NL_{emac}(\bar{\mathbf{u}}) = 2\mathbf{S}(\bar{\mathbf{u}})\bar{\mathbf{u}} + (\nabla \cdot \bar{\mathbf{u}})\bar{\mathbf{u}} \quad (6)$$

A non-incremental fractional-step method is used to stabilise pressure. This allows for the use of finite element pairs that do not satisfy the inf-sup conditions, such as equal order interpolation for the velocity and pressure used in this work. The set of equations is time integrated using an energy conserving Runge-Kutta explicit method lately proposed by Cappuano et al. [38] combined with an eigenvalue based time-step estimator [39].

2.2. Solid model

The motion of the circular cylinder is governed by the two-dimensional forced damped second-order oscillator equation:

$$\ddot{\mathbf{x}} + \frac{4\pi\zeta}{U^*} \dot{\mathbf{x}} + \left(\frac{2\pi}{U^*}\right)^2 \mathbf{x} = \frac{2\mathbf{C}}{\pi m^*} \quad (7)$$

where $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$ and $\mathbf{x}$ stand for the non-dimensional in-line and cross-flow acceleration, velocity and displacement of the cylinder center of mass. In this equation, $U^* = U_\infty/(f_n D)$ is the reduced velocity, defined as the inverse of the natural frequency in vacuum $f_n = (2\pi)^{-1}\sqrt{k/m}$ non-

dimensionalised with U_∞ and D , and $\zeta = c/(2\sqrt{km})$ is the ratio between the structural damping and the critical damping. Here, it has been considered that the stiffness and damping are equal in both directions, $k_x = k_y = k$ and $c_x = c_y = c$. Finally, $m^* = 4m/(\rho\pi D^2 L)$ is the mass ratio (defined as the ratio between the mass of the cylinder and the mass of an equivalent volume of fluid) and $\mathbf{C} = 2\mathbf{F}/(\rho D U_\infty^2)$ represents the force coefficient vector computed by integrating the stresses acting on the surface of the cylinder. This set of equations are integrated in time using a fourth-order Runge-Kutta algorithm.

2.3. Mesh deformation

In order to track the cylinder movement, the Arbitrary Lagrangian Eulerian (ALE) formulation is employed. Some examples of the ALE formulation applied to FSI problems can be found in [40, 41, 42, 43]. In this formulation, a moving/deforming Eulerian reference frame is used, in which the governing equations (1) and (2) of the fluid take the form

$$\partial_t \bar{\mathbf{u}} + [(\bar{\mathbf{u}} - \mathbf{u}_m) \cdot \nabla] \bar{\mathbf{u}} - \frac{2}{Re} \nabla \cdot \mathbf{S}(\bar{\mathbf{u}}) + \nabla p - \mathbf{f} = -\nabla \cdot \tau_{ij}(\bar{\mathbf{u}}) \quad (8)$$

$$\nabla \cdot \bar{\mathbf{u}} = 0 \quad (9)$$

where $\mathbf{u}_m$ represents the velocity of the mesh points, and is calculated from the mesh displacement $\mathbf{d}_m$. In our case, $\mathbf{d}_m$ is obtained as the solution of the diffusion equation:

$$\nabla \cdot (c_m \nabla \mathbf{d}_m) = 0 \quad (10)$$

with $c_m = AR/V$ is an isotropic diffusion coefficient (defined as the ratio between the aspect ratio AR and the volume V of the elements) which controls the element stiffness. This form of c_m is quite helpful in order to preserve the form of the boundary layer elements.

2.4. Fluid-Structure interaction (FSI)

The FSI problem is solved by means of a massively parallel partitioned approach [44]. At each time-step the fluid mechanics problem is solved (eqns.(8), (9)). Forces ~~in~~ on the wet surfaces are computed and used to solve the rigid body equations (eqn. (7)). The resulting configuration of the solid body is used as boundary condition to solve the diffusion equation for the nodes displacement (eqn.(10)) and deform the mesh. This strategy may result in an unstable numerical scheme when the densities of the solid and the fluid are similar [45, 46]. Therefore, a special treatment of the coupling algorithm would be needed in such cases. However, in the present study a high enough densities ratio is considered and no special stabilization techniques are required.

2.5. Details of the computations

Computations are carried out for three values of the Reynolds number in the subcritical regime, $Re = 3.9 \times 10^3, 5.3 \times 10^3$ and $Re = 11 \times 10^3$, with zero structural damping $\zeta = 0$, a fixed value of the reduced mass ratio $m^* = 2.6$, and different values of the reduced velocity U^* . The chosen parameters, summarised in Table 1, have been selected in order to cover the three solution branches reported experimentally for 2dof systems [19]. All parameters are kept constant for the entire simulation, except for the $U^* = 6.7$ case. In this case, the Reynolds number and the reduced velocity are gradually increased from 0 to 5.3×10^3 and from 0 to 6.7, respectively. Once the

Reynolds number (Re) $\times 10^{-3}$	Reduced velocity parameter (U^*)
3.9	6.0
5.3	3.0, 5.5, 6.0, 6.5, 6.7, 7.0, 8.0, 9.0, 11.0, 12.0
11	3.0, 5.5, 9.0

Table 1: List of parameters used in the present study. All computations were carried out without structural damping $\zeta = 0$ and a fixed value of the mass ratio $m^* = 2.6$.

targeted values are reached, they are kept constant for the rest of the simulation (see section 3.4 for details).

All numerical simulations are performed with Alya, a high performance computing multi-physics code developed at Barcelona Supercomputing Center [44]. Alya is based on a finite element formulation and parallelised using an hybrid MPI/OpenMP strategy. Alya has been extensively tested and proved to give accurate solutions in different complex fluid mechanics problems [47, 48, 49]. The mesh used in this study of about 10^7 linear elements (see section 2.6) is partitioned in sub-domains and distributed among 576 CPU on the MareNostrum IV supercomputer. The wall-clock time per iteration depends on the reduced velocity; for $U^* = 6.5$ is about 2s.

2.6. Domain characteristics and mesh resolution

The characteristics of the computational domain can be seen in Fig. 2, where lengths have been scaled with the diameter of the cylinder. The inlet and outlet boundaries are located at $10D$ and $20D$ from the center of the cylinder, while the top and bottom boundaries are at $10D$. The total span-wise length (z -axis) is of $6D$. A discretisation domain with these characteristics has already proved to properly capture the wake three dimensionality of the turbulent flow around a static circular cylinder at $Re=5000$ [50]. At the boundaries of the domain, a no-slip condition is imposed at the cylinder surface while at the inlet and outlet boundaries a Dirichlet condition and a stress vector equals to zero are specified, respectively. At the top and bottom boundaries, the tangential component of the stress vector and the normal component of the velocity are set to zero. Periodic boundary conditions are considered in the span-wise direction. The computational mesh consists of 128 two-dimensional unstructured planes of 7.27×10^4 linear elements each, uniformly distributed along the span-wise direction resulting in a three-dimensional mesh of 9.3×10^6 linear elements.

3. Results

Numerical solutions are advanced in time up until the initial transient is washed out and stationary statistical time is achieved. Then, numerical results are collected during about 50 cycles in order to study the unsteady behaviour of the fluid and its interaction with the cylinder surface. Additionally, a special treatment for an unstable case in the SU branch is used in order to get statistically independent results. In this case, the reduced velocity and Reynolds number are gradually incremented in an initial stage of the simulation, until a target Re number and reduced velocity are achieved. Then the system oscillates in the SU branch for twenty cycles and undergoes a transition to the L branch.

3.1. Cylinder response and synchronization

In Fig. 3 the maximum transverse and longitudinal non-dimensional amplitudes of vibration for the three different values of Reynolds number are presented as a function of the reduced velocity. The maximum amplitude has been estimated as the average of the highest 10% amplitudes [51, 27]. As the experiments of JW2004 were performed in water, for comparison purposes, and in order to match the current definition of U^* , their experimental data have been scaled according to the empirical relation $f_w/f_n = \sqrt{m^*/(m^* + C_a)}$, f_w being the natural frequency in still water, f_n the natural frequency in vacuum and $C_a \approx 1$ the added mass coefficient whose value was determined experimentally by Khalak & Williamson[18] (for more details the reader is referred to [18, 19]). It can be observed that the cylinder response is characterised by three branches: the initial (I), super upper (SU) and lower (L) in very good agreement with the experimental data. For the longitudinal vibration there is some scattering in the maximum amplitudes $x/D \approx 0.35$ due to the Reynolds number. This scattering is not present in the transverse maximum amplitudes where present calculations follows the experimental trends regardless the Reynolds number. The maximum transverse amplitude before the hysteric region (red arrows) is about $y/D \approx 1.355$ at $U^* = 6.5$. After this point, computational data show an early transition from the SU to the L branch (black arrow) at $U^* \approx 6.7$. At this reduced velocity the cylinder vibrates in the SU branch with a maximum transverse non-dimensional amplitude of $y/D \approx 1.412$ for about twenty cycles (135TU) and undergoes its transition to the L branch. In the L branch, there is a fair agreement with the experimental data except at $U^* = 11.0$ where present LES over-estimate the transverse maximum amplitude. This discrepancy between experimental data and present results may be related to differences in the structural damping as is discussed by G2016. Notably, the maximum amplitudes in the most challenging cases of the SU branch addressed are well reproduced, as well as an acceptable amplitude decay in the L branch, which is a significant improvement from the previous high-fidelity simulations of the system (G2016).

Figure 4 shows the ratio between the frequency of the cylinder transverse oscillations f_y to the natural frequency of the system f_n . Also the Strouhal frequency for the static cylinder case $St_{sc} \approx 0.2$ and the natural frequency (horizontal dashed line) are plotted for comparison purposes. Here, present LES results match the experimental and computational data and there are no important differences regardless the Reynolds number, as can be seen from the figure. In the I branch, the frequency ratio slightly follows the St_{sc} trend. In the SU branch, the frequency ratio highly departs from the St_{sc} value and at $U^* \approx 5.5$ starts to linearly grow approaching to the natural frequency. Then, an important jump takes place due to the SU-L transition. Finally, the frequency ratio is approximately constant in the L branch. As is characteristic for systems with low mass ratio the frequency ratio is different from unity. This kind of lock-in has been referred to as “soft lock-in” and it has been shown to be one of the mechanisms that limits the VIV maximum oscillation amplitude [22].

The trajectory of the cylinder in the (x, y) plane for each branch for Reynolds number $Re = 5300$ are depicted in Fig. 5. As can be inferred for the shape of the trajectories and corroborated by the power spectrum density (PSD) in Figure 6, for the three branches the frequency of the in-line response is twice the main cross-flow response frequency, $f_y \approx 2f_x$. The difference in their characteristic pattern is given by the phase difference θ between in-line and cross-flow displacement. For these cases, the phase difference is $\theta \approx 180^\circ$, 270° and 315° respectively, as reported by JW2004. Moreover, the trajectories can also be compared with the equations $x = A_x/D \sin(4\pi f_y + \theta)$, $y = A_y/D \sin(2\pi f_y)$ represented as grey curves in Fig. 5, which are a good representation of the

Figure 3: Non-dimensional Cross-flow and in-line maximum amplitudes of oscillation as a function of the reduced velocity. Present LES results are compared with the experimental data of JW2004 and the computational data of G2010.

Figure 4: Frequency ratio as a function of the reduced velocity. Present LES results are compared with the experimental data of JW2004 and the computational data of G2010.

Figure 5: (x, y) trajectories for three selected values of the reduced velocity at $Re = 5300$. Each reduced velocity value represents the response of the system at each branch.

$Re \times 10^3$	U^*	C_d	$C_{d_{rms}}$	$C_{l_{rms}}$	St	$f_y D/U_\infty$	$f_x D/U_\infty$	y_{rms}/D	x_{rms}/D
3.9	6.0	3.105	1.185	1.357	0.136	0.137	0.274	0.886	0.153
	3.0	1.709	0.053	1.658	0.165	0.166	0.331	0.127	0.058
	5.5	3.235	1.352	1.833	0.139	0.139	0.278	0.840	0.192
	6.0	3.206	1.201	1.364	0.135	0.135	0.270	0.884	0.188
	6.5	3.342	1.264	1.040	0.133	0.132	0.265	0.893	0.150
	6.7	3.270	1.048	0.899	0.131	0.130	0.261	0.907	0.156
	5.3	6.7	1.718	0.312	0.317	0.160	0.161	0.321	0.458
	7.0	1.560	0.244	0.258	0.153	0.155	0.305	0.442	0.043
	8.0	1.388	0.194	0.222	0.134	0.133	0.267	0.463	0.044
	9.0	1.211	0.131	0.156	0.120	0.120	0.239	0.452	0.022
5.3	11.0	0.975	0.041	0.092	0.100	0.100	0.200	0.264	0.018
	12.0	0.919	0.020	0.043	0.215	0.094	0.064	0.025	0.018
	3.0	1.684	0.045	1.679	0.167	0.168	0.337	0.129	0.054
	5.5	3.168	1.466	1.946	0.139	0.138	0.275	0.858	0.215
	9.0	1.175	0.111	0.182	0.120	0.120	0.241	0.416	0.036
	3.9 [†]	–	1.037	0.046	0.155	0.215	–	–	–
	5.0 [†]	–	1.050	0.042	0.188	0.210	–	–	–
	3.0	1.684	0.045	1.679	0.167	0.168	0.337	0.129	0.054
	5.5	3.168	1.466	1.946	0.139	0.138	0.275	0.858	0.215
	9.0	1.175	0.111	0.182	0.120	0.120	0.241	0.416	0.036

Table 2: Statistical flow parameters for the studied cases. Comparison with static cylinder values from the literature. Drag coefficient (C_d), drag coefficient rms($C_{d_{rms}}$), lift coefficient rms($C_{l_{rms}}$), Strouhal number (St), non-dimensional frequency of the cross-flow displacement ($f_y D/U_\infty$), non-dimensional frequency of the in-line displacement ($f_x D/U_\infty$), cross-flow displacement rms(y_{rms}) and in-line displacement rms(x_{rms}). [†]Values for the static cylinder from data reported in [52, 50], respectively.

displacement of the cylinder for these cases. Note also the change of scale in the axis, the largest displacement in both cross-flow and in-line being for the case with reduced velocity $U^* = 6.5$ (see also values reported in Table 2).

3.2. Force coefficients

Lift and drag coefficients response, and non-dimensional in-line x/D and cross-flow y/D displacements are shown in Fig. 6 together with their PSD for the three branches at $Re = 5300$. In the I branch, for $U^* = 3$ Fig. 6a and 6b, the drag coefficient exhibits a quite irregular behaviour resembling that of the static case (see for instance Figure 2 in [53]) and low fluctuations (see values reported in Table 2). But, even-though in-line displacement variations are small (see Fig. 8a), the C_d magnitude increases by 70%, from $C_d \sim 1$ in the static case to $C_d \sim 1.7$ (see Table 2). The PSD of the x/D displacement exhibits one important peak, while in the spectrum of the drag coefficient both the primary peak, corresponding with twice the vortex shedding, and its harmonic are dominant. In contrast, the lift coefficient and y/D displacement exhibit a more regular trend, and their power spectra is clearly dominated by the non-dimensional frequency $fD/U_\infty = 0.165$. This value is smaller than the frequency of the static cylinder $St_{sc} \sim 0.2$, which is a characteristic effect of the lock-in phenomena. A remarkably feature is the eight-fold increase in the lift coefficient fluctuations over the stationary case; from $Cl_{rms} \sim 0.18$ to $Cl_{rms} \sim 1.6$. This is more relevant as at these Reynolds numbers (i.e. $Re = 3900 - 5300$) the lift fluctuations of the stationary cylinder are at a minimum due to the changes in the transition to turbulence mechanism as discussed in [50]. At $Re = 11000$, the differences are reduced but still there is a three-fold increase in the lift

Figure 6: For caption see facing page

Figure 6: Time history of displacement and force coefficients (left column), and their power spectra (right column) in the three branches response. (a, b) I branch, (c, d) SU branch (e, f) L branch.

fluctuations (according to Norberg[54], in the range of $Re = 5.4 \times 10^3 - 2.2 \times 10^5$ the lift fluctuation follows $Cl_{rms} = 0.52 - 0.06X^{-2.6}$ being $X = \log(Re/1.6 \times 10^3)$ which for $Re=11000$ yields a $Cl_{rms} \sim 0.51$).

For $U^* = 6.5$, in the SU branch (figures 6c and 6d), a more regular behaviour and a significant increment in the C_d and x/D fluctuations can be appreciated (see also Table 2 and Fig. 8a). At this reduced velocity the drag coefficient reaches its highest value $C_d \sim 3.342$, almost three times the drag coefficient for the static case; its energy spectrum presents a dominant peak at twice frequency of the fluid and its harmonics. However, attention is drawn to the large increase in the drag fluctuations by more than 22 times due to the in-line motion of the cylinder. For this U^* the magnitude of the displacement of the cross-flow direction almost matches the amplitude of the lift. This is the reason why both cross-flow and lift fluctuations spectra almost perfectly match, as can be seen from Fig. 6d. In addition, it should be noticed that the main frequency $fD/U_\infty = 0.133$, is well below that of the static cylinder $St_{sc} \sim 0.2$. The most important frequencies for both lift and cross-flow fluctuations are the first and the third harmonic; the third harmonic can be interpreted as the footprint of the 2T vortex formation mode (two triads of vortices per oscillation cycle, see section 3.3), which also corresponds with the two shoulder peaks in the lift temporal evolution. This structure, i.e. the 2T mode, seems to appear only at low mass reduced velocity 2dof systems, as was pointed out by JW2004.

Within the L branch (figures 6e and 6f), the fluctuations of the drag coefficient and the in-line displacement are reduced (see figures 8a and 8c). The drag mean value and its fluctuations fall down to $C_d \sim 1.21$ and $C_{d,rms} \sim 0.13$, although these fluctuations surpass those of the static cylinder by three-fold. It is important to note that the mean value is closer to the static cylinder case than the case with $U^* = 3.0$ but fluctuations are higher. As expected, the dominant peak of

Figure 7: Time history of force coefficients and non-dimensional displacement for three different values of the reduced velocity at the SU branch. (a, d) $U^* = 5.5$, (b, e) $U^* = 6.5$ and (c, f) $U^* = 6.7$.

the spectrum of the fluctuations occurs at twice the frequency of the fluid main frequency. However, the energy content of the in-line cylinder fluctuations is lower than that of the drag fluctuations, which can also be observed in figure 6e. At this reduced velocity, drag and in-line displacement variations are no longer in phase and this leads to a considerable reduction of the magnitude of in-line displacement fluctuations ($x_{rms}/D \sim 0.022$). Moreover, lift fluctuations also experiences an important reduction, being close to the values reported for the static cylinder at $Re = 5300$, but far below that at $Re = 11000$. This suggests that after the cylinder enters the L branch fluid and cylinder are out-of-phase and thus, the forces acting on the cylinder are consequently reduced. This is in contrast with the $U^* = 3.0$ case where, even-though cross-flow fluctuations are about 50% smaller, lift coefficient fluctuations are eight-fold higher as both fluid and cylinder oscillations are in-phase, as can be seen from figure 6a. As a matter of fact, an important feature of the L brach is a change of phase of 180° between displacements forces, which is quite evident in the cross-flow and lift time histories. The peak in the energy spectrum of cross-flow fluctuations falls down up to a frequency of $fD/U_\infty = 0.12$, a value further away from the static case $St_{sc} \sim 0.2$ but nearer to the natural frequency as can be appreciated in Fig. 4.

The desynchronisation state is reached at $U^* = 12$, when the Strouhal number of the static cylinder case is recovered and the non-dimensional displacement fluctuations are at their lowest. However, the drag coefficient and force coefficients variations are lower than the static cylinder (see Table 2). This is probably due to the cylinder out-of-phase displacements, which although are close to zero, are still noticeable and thus decrease the magnitude of the force coefficient fluctuations.

Time evolution of force coefficients and non-dimensional displacements at the SU branch for three values of the reduced velocity are plotted in Fig 7. As the reduced velocity increases in the SU branch a clear difference from its most stable part at $U^* \sim 5.5$ to the unstable region $U^* \sim 6.7$ can be appreciated. A substantial decrease in the lift fluctuations (about 50%), and the appearance of beats in the force coefficients and non-dimensional displacements time evolution precedes the transition to the L branch, accompanied with an important jump in the frequency ratio (see Fig. 4) and phase change. The increasing beating with the reduced velocity is due to the coherence of the third vortex shedding at the shoulders of the cylinder. As the reduced velocity increases the system loses its ability to completely form and shed the third vortex at the shoulders of the cylinder at each cycle. This is clearly observed at the last cycle prior the transition to the L branch in Fig. 13 (see section 3.4). Two clock-wise strong vortices are formed and shed at the beginning of the cycle, figures 13a, 13b and 13c, while at the beginning of the next cycle, just one clock-wise vortex is shed 13g, 13h and 13i, which has a strong impact in the lift amplitudes Fig. 13j.

Figure 8 shows the rms value for force coefficients and non-dimensional displacement as a function of the reduced velocity. The three-branch response and the hysteric region can be easily recognised. Force coefficients rms (figures 8a and 8b) are also compared with experimental and computational results. As can be seen from the figure, there is a good agreement with the experimental results. Moreover, the non-dimensional displacement fluctuations show a similar trend as the maximum amplitudes displacement. The peaks of the coefficients fluctuations take place at two different values of the reduce velocity, $C_{l_{rms}} \sim 2.2$ at $U^* = 5.0$ and $C_{d_{rms}} \sim 1.3$ at $U^* = 5.5$. Then, both force coefficients fluctuations drops quickly at the SU branch with and appreciable jump in the SU-L transition. It is also important to notice that the maximum in the $C_{l_{rms}}$ does not coincide with the maximum cross-flow displacement amplitude in the cylinder ($U^* = 6.7$) as could be intuitively expected. This is due to the phase difference between the lift force and the cylinder motion in the synchronized state. This phase difference results in regions where the lift favours the cylinder movement and regions where the lift opposes the movement (see Fig. 7a,b,c). Thus, it is possible to obtain large amplitudes with small forces provided that the regions where the lift favours the motions are long enough. This phenomenon has been reported and studied in [55].

3.3. Vortex shedding formation modes

Instantaneous non-dimensional pressure iso-contours for the fully developed flow in the three branches at $Re = 5300$ are shown in Figure 9. The vortex shedding configurations at each branch are consistent with the experimental results reported by JW2004. At the I branch (Fig. 9a), the 2S configuration is observed. In this mode two single vortices are shed per oscillation cycle. At the SU branch (Fig. 9b), the vortex formation mode is quite different to that observed in the upper branch for cross-flow-only systems where the 2P mode is reported. This is also in agreement ~~to~~ **with** the observations of JW2004. Actually, at the SU branch, the 2T vortex formation mode, i.e. two vortex triads per oscillation cycle can be observed. In the figure, each vortex has been identified following the same criteria as JW2004. These triple vortices are only observed in the vortex formation region and they seem to merge into a single one as the flow moves downstream. Moreover, in the SU branch due to the larger oscillation of the cylinder, the wake is also wider than in the initial branch, and contrary to this one, vortices move downstream off the wake centreline. The third additional vortex shed near the cylinder each half cycle seems to provide enough energy to the system to induce significative amplitude vibrations. The 2T vortex formation mode is also directly associated with the appearance of the third harmonic on the lift force [20], see Figure 7b,c.

Figure 8: Dependence with the reduced velocity. (a) rms of the drag coefficient; (b) rms of the lift coefficient; (c) rms of the non-dimensional in-line displacement; (d) rms of the cross-flow displacement.

Figure 9: Instantaneous non-dimensional pressure $p/(\rho U_\infty^2)$ iso-contours at $Re = 5300$ and (a) $U^* = 3.0$ (I branch), (b) $U^* = 6.5$ (SU branch), (c) $U^* = 9.0$ (L branch)

Figure 10: Instantaneous non-dimensional pressure $p/(\rho U_\infty^2)$ iso-contours at the SU branch. (a) $Re = 3900$ and $U^* = 6.0$, (b) $Re = 11000$ and $U^* = 5.5$.

The L branch (Fig. 9c) is characterised by the 2P vortex formation mode. In this mode, two pairs of vortices are shed during one cycle.

In addition, in the present simulations, the 2T vortex shedding structure is observed in the SU branch for the values of the Reynolds numbers considered. In Fig. 10, the 2T vortex formation mode is observed for $Re = 3900$ and 11000 . To the authors' knowledge this structure has not been reproduced before in high-fidelity 3D simulations, although experimental works mention its existence. In the DNS performed by G2016 for $Re = 3900$ the 2T vortex mode is not reported. The authors argued that it might have been due to the difference in the Reynolds number with the experiments of JW2004, and that it might be a Reynolds number dependence phenomenon. However, as can be seen from the figure, in both Reynolds numbers the 2T mode is easily observed, contradicting the hypothesis of G2016 of the Reynolds number dependence. This might also be the reason why they underestimated the peak amplitudes in the SU branch compared with the experimental results.

The 2T vortex dynamics is depicted in Fig. 11 for one cycle, where the instantaneous non-dimensional span-wise vorticity is plotted. At the beginning of the cycle (see Fig 11a), two strong clock-wise vortices are being formed. The first vortex is shed near the point of maximum amplitude in the cylinder trajectory, point (a) in Fig. 11 (g), while, the second one is shed together with a weaker counter-clock wise rotating vortex during the descent of the cylinder between points (a) and (b) of Fig. 11 (g). This behaviour is repeated in the second half of the cycle. Formation and existence of the 2T pattern is closely related to the amplitude of the oscillation. This has already been suggested by JW2004 and is supported by the present results. The 2T pattern is only present in the SU branch, and is the responsible of providing extra lift force to the cylinder. The latter can be observed clearly in Fig. 7a, where the peak in the lift related to the third harmonic provoked by the 2T pattern favours the acceleration of the cylinder in all the cycles shown. The previous makes the oscillation amplitude large enough to allow the existence of both, the vortices formation and shedding mechanisms of the three vortices. This is a complex self-sustained mechanism that finally exhibits an unstable behaviour for large enough values of the reduced velocity parameter U^* . In this work, this unstable behaviour was observed at $U^* = 6.7$, which is lower than the experimental value $U^* \sim 7$. The earlier transition may be directly related ~~with~~ to the numerical scheme used.

3.4. Unstable SU branch case and SU-L branch transition

In order to study in more detail the dynamical response ~~of~~ for the system in the SU branch and gain insight in the transition from the SU branch to the L branch, a special simulation was carried out. Fixing the value $U^* = 6.7$, the Reynolds number was gradually advanced in time following the relation

$$Re(t) = Re_f \tanh(t/\tau) \quad (11)$$

where Re_f is the target Reynolds number and τ is a parameter used to control the rate of growth of Re . In this case the selected values are $Re_f = 5300$ and $\tau = 70$, which allow to reach the target Re roughly after 200 TU. This numerical strategy proved to be useful to keep the system in the SU branch enough cycles to have statistically independent measurements. Once in the SU branch, the system oscillates steadily for 135TU until it reaches a counter phase synchronized state transitioning to the L branch.

To understand the dephasing process, we should go back to Fig. 7, where C_l and the cylinder displacement for the cases of $U^* = 6.5$ and $U^* = 6.7$ are plotted. Both cases follow similar trends but there are variations in the evolution of C_l for $U^* = 6.7$ observed. On the one hand, there are

Figure 11: Span-wise non-dimensional vorticity $w_z L / U_\infty$ at $z/D = 3$ at various instants of one cross-flow oscillation cycle for $U^* = 6.7$. The time instants for each image are specified in (g), where also the lift coefficient behaviour is shown.

Figure 12: Time history of force coefficients and non-dimensional displacements at the SU-L transition for $U^* = 6.7$.

large peaks in the lift coefficient related to cycles where the three vortices of the 2T pattern are fully formed, shed and transported downstream (see for instance at $tU_\infty/D = 230$ and 285). On the other, smaller peaks of C_l (e.g. at $tU_\infty/D = 240$ or 270) correspond to cycles where the third vortex of the 2T pattern is not completely formed or breaks down as soon as it is shed.

Fig. 12 shows the integral forces and displacements evolution during the dephasing of the fluid and the cylinder and the following transition to the L branch. It is noticeable that during the last in-phase cycle, around $tU_\infty/D = 340$, the lift suffers a considerable magnitude decrement and so does y/D displacement. As a consequence, due to the eventual reduction of the wake width and the distance between separated shear-layers, an increase in the frequency takes place.

Flow visualizations plotted in figure 13 show a multi-vortex shedding pattern in the first half of the last synchronized cycle (Figs. 13a,b,c) followed by immediate dephasing of the lift force and cylinder displacement in the second half of the cycle (Figs. 13d,e,f). The extra vortex shed in the first half of this cycle, clearly shown in Fig. 13b, affects the lift force and diminishes the amplitude of oscillation, which turns to be not large enough to allow the in-phase formation and shedding of the three vortices of the 2T pattern in the second half of the cycle. Note that lift opposes the movement of the cylinder in a large section of the second half of this cycle, almost entirely between points c, d, and e of Fig. 13j. This creates a further decrease in the lift force and oscillation amplitude, the 2T pattern is no longer formed and dephasing finally occurs, transitioning to the L branch. Figs. 13g,h,i show the lack of the 2T pattern in the first dephased half cycle.

4. Conclusions

In the present paper, a high-fidelity (LES) study of a 2dof VIV of a low mass ratio ($m^* = 2.6$) circular cylinder at subcritical Reynolds numbers ($Re = 3900, 5300$ and 11000) has been presented. A low-dissipative spatial and temporal discretization in a finite elements formulation has been used together with the WALE sub-grid scale model. Different values of the reduced velocity parameter U^* , which is related to the inverse of the natural frequency of the system, covering the three branches of solution reported experimentally by JW2004 were investigated with a zero structural

Figure 13: Span-wise non-dimensional vorticity $w_z L / U_\infty$ at $z/D = 3$ at various instants of one and a half cross-flow oscillation cycle for $U^* = 6.7$ in the SU-L transition. The time instants for each image are specified in (j), where also the lift coefficient behaviour is shown.

damping parameter ζ . The results of the maximum amplitude and frequency of motion in both directions show very good agreement with the experiments in all the studied range. There are some slight discrepancies in the in-line direction amplitude depending on the Reynolds number and an early transition from the SU branch to the L branch that may be related ~~with~~ to the numerical scheme used. Notably, the SU branch response of the cylinder was very well captured by the simulation, the amplitudes and frequencies are very well reproduced for the most challenging cases in this large amplitude response branch and an acceptable decay in the oscillation amplitude is obtained for large values of U^* in the L branch.

The force coefficients C_d and C_l , as well as their mean and rms values are reported and compared with available static cylinder case results. Very interesting deviations from the behaviour of the fixed cylinder case are observed depending on the solution branch of each case. Also, depending on U^* , a change ~~on~~ in the phase between the cylinder displacement and the force coefficients, which eventually ends in the decay of the oscillation amplitude, is obtained.

Regarding the vortex shedding patterns of each solution branch, it is important to note that all the patterns obtained coincide with those reported experimentally by JW2004. The 2T vortex pattern (where two triads of vortices are shed per oscillation cycle) in the SU branch has been observed at all Reynolds numbers studied. These vortices manifest as a two shoulder peaks in the transient evolution of the lift coefficient. The formation of this structure is responsible for providing an extra lift force to the cylinder, where the peak in the lift related ~~with~~ to the third harmonic in the spectrum favours the cylinder acceleration. Considering the results presented at different subcritical Reynolds numbers, it can be concluded that the 2T vortex pattern is an intrinsic characteristic of the flow in the SU branch which is independent of the Reynolds number. To the best knowledge of the authors, this is the first time high-fidelity simulations reproduce the behavior observed experimentally of the fluid-structure interaction in a 2dof circular cylinder at these Reynolds numbers.

Finally, the transition from the SU branch to the L branch is explored in the case of $U^* = 6.7$. In the last cycle on the SU branch, a multi-vortex shedding pattern is observed in the first half of the cycle, which is followed by an out-of-phase 2T pattern. The combination of these two phenomena diminishes drastically the oscillation amplitude and a counter-phase oscillation characteristic of the L branch is obtained.

Acknowledgments. We acknowledge Red Española de Supercomputación (RES) for awarding us access to the MareNostrum IV machine based in Barcelona, Spain (Ref. FI-2017-2-0016). J.C. Cajas acknowledges the financial support of the ‘Consejo Nacional de Ciencia y Tecnología (CONACyT, México)’ grant number 231588 290790. D. Pastrana acknowledges support of the CONACyT-SENER graduate fellowship program to study abroad 278102/439162.

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