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TITLE : PAPR reduction in OFDM systems

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AUTHOR: Natalia Revuelto

DIRECTOR: Prof. Dr. R. Mathar

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Autor: Natalia Revuelto

Director: Prof. Dr. R. Mathar

Data: September 18, 2008

Resum

En los últimos años las comunicaciones inalámbricas han experimentado un rápido crecimiento debido a la gran movilidad que permiten. Por otro lado, las comunicaciones inalámbricas tienen una serie de desventajas, como la selectividad en frecuencia, que dificultan su utilización. OFDM es una modulación que trabaja eficientemente en estos canales. Los avances en procesamiento digital de señal y en 'very large scale integrated circuits' (VLSI) permiten eficientemente y sin costes elevados la implementación de las operaciones de FFT, convirtiendo a OFDM en una solución atractiva para los canales inalámbricos. Una desventaja importante de OFDM es su elevado 'peak-to-average power ratio' (PAPR) que degrada las señales OFDM forzando al amplificador a trabajar en la región de saturación, distorsionando así las señales y provocando un aumento del consumo de potencia por parte del amplificador.

Las diferentes maneras de reducir el PAPR de las señales OFDM se pueden dividir en dos grandes grupos: técnicas que distorsionan la señal no linealmente y técnicas que reducen el PAPR sin distorsionar la señal. Una técnica muy simple que pertenece al primer grupo es clipping, que consiste en recortar la señal cuando ésta sobrepasa un máximo permitido con el coste de degradar el BER y de emisiones fuera de banda. Clipping no añade información extra a la señal y los picos elevados ocurren con poca probabilidad así que la señal se distorsiona con poca frecuencia. Las radiaciones fuera de banda se pueden reducir mediante un filtro en el transmisor, el filtro usado en este proyecto consiste en un par de funciones FFT-IFFT, que resulta más fácil de implementar que el filtro FIR normalmente usado y que además permite la repetición de las funciones de clip&filter iterativamente para reducir el crecimiento que sufren los picos cuando se filtran. La degradación del BER se puede mitigar mediante la reconstrucción de la señal una vez ha llegado al receptor. En este proyecto se han analizado diversas técnicas que intentan estimar los símbolos originales iterativamente y se ha propuesto una mejora para uno de éstos métodos. Los métodos se han analizado mediante Matlab y de los resultados de las simulaciones se han sacado conclusiones.

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Overview

In the last years wireless communications have experienced a fast growth due to the high mobility that they allow. However, wireless channels have some disadvantages, like multi-path fading, that make them difficult to deal with. A modulation that efficiently deals with selective fading channels is OFDM. The advancements in digital signal processing and very large scale integrated circuits allow efficient and cost-effective implementation of the FFT operations making OFDM an attractive solution for wireless channels. An important disadvantage of OFDM systems is their high peak-to-average power ratio (PAPR). High PAPR degrades OFDM signals by forcing the analog amplifier to work in the nonlinear region, distorting this way the signal and making the amplifier to consume more power. There are several ways to reduce the PAPR of OFDM signals which can be divided in two groups: the techniques that non-linearly distort the signal and the ones that reduce the PAPR without distorting the signal. A simple technique, belonging to the first group, is to clip the signal to a maximum allowed value, at the cost of BER degradation and out-of-band radiation. Clipping does not add extra information to the signal and high peaks occur with low probability so the signal is seldom distorted. Out-of-band radiation can be reduced by filtering at the transmitter, the filter used in this project consists on a FFT-IFFT pair which is easier to implement than traditional FIR filters and allows the implementation of the clip&filter set several times in order to reduce the peak regrowth that filtering introduces. The BER degradation can be mitigated by reconstructing the signal at the receiver. We analyzed the performance of some commonly used techniques that iteratively try to guess the original symbols and proposed an improvement for one of those techniques. The methods have been tested with Matlab and from the simulation results some conclusions are extracted.

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INTRODUCTION

In the last years wireless communications have experienced a fast growth due to the high mobility that they allow. However, wireless channels have some disadvantages, like multi-path fading, that make them difficult to deal with. A modulation that efficiently deals with selective fading channels is orthogonal frequency division multiplexing (OFDM), which consists on N orthogonal subcarriers generated and modulated in frequency domain. The fact that the subcarriers are orthogonal allows the usage of the Fourier transform without introducing inter-carrier interference (ICI). The advancements in digital signal processing and very large scale integrated circuits allow efficient and cost-effective implementation of the fast Fourier transform (FFT) operations making OFDM an attractive solution for wireless channels.

An important disadvantage of OFDM systems is their high peak-to-average power ratio (PAPR). When the OFDM signal is transformed to time domain, the resulting signal is the sum of all the subcarriers, and when all the subcarriers add up in phase the result is a peak N times higher than the average power. High PAPR degrades performance of OFDM signals by forcing the analog amplifier to work in the nonlinear region, distorting this way the signal and making the amplifier to consume more power.

A simple technique used to reduce the PAPR of OFDM signals is to clip the signal to a maximum allowed value, at the cost of BER degradation and out-of-band radiation. Clipping does not add extra information to the signal and high peaks occur with low probability so the signal is seldom distorted. Out-of-band radiation can be reduced by filtering at the transmitter, the filter used in this project consists on a FFT-IFFT pair which is easier to implement than traditional FIR filters and allows the implementation of the clip&filter set several times in order to reduce the peak regrowth that filtering introduces. The BER degradation can be mitigated by reconstructing the signal at the receiver. We analyzed the performance of the decision-aided reconstruction (DAR) and improved DAR (IDAR) techniques that iteratively try to guess the original symbols and proposed an improvement for one of those techniques. The methods have been tested with Matlab and the simulation results show that the proposed technique performs better than the existing ones.

The first chapter of the project offers an overview of wireless communications and introduces OFDM. In the second chapter, OFDM is explained in depth as well as the PAPR problem and third chapter explains the methods used to reduce PAPR and its effects on the OFDM signal. The simulation results are shown and discussed in the fourth chapter and finally, the last chapter contains the conclusions, future work and environmental impact of the project.

CHAPTER 1. INTRODUCTION

In the last years wireless communications have experienced a fast growth, mainly due to the high mobility that they allow. This chapter analyzes these channels and their advantages and disadvantages and introduces the orthogonal frequency division multiplexing (OFDM) modulation, which is an efficient solution for wireless environments.

1.1. Digital communications

Digital communications experienced a fast growth with the success of Internet in the 90's, mostly using wired infrastructures; in this same period mobile communications became very popular and they together evolved in the necessity of wireless Internet. The application of wireless telegraphy to mobile communication systems started in the 1920's. This pre-cellular era began with the first land-based mobile wireless telephone system installed in 1921 by the Detroit Police Department to dispatch patrol cars and was followed in 1932 by the New York City Police Department[4].

The first analog cellular mobile communication systems were developed in the 80's. In Chicago, a service called advanced mobile phone service (AMPS) was started in 1983, in 1984-85 Motorola developed in the United Kingdom a system called total access communication system (TACS) based on AMPS. In Norway, Nordic Mobile Telephone (NMT) developed an analog cellular mobile communications system, the NMT-450, and other systems like C-450, RTMS and Radiocom-2000 were introduced in Germany, France and Italy respectively. In Japan, Nippon Telephone and Telegraph (NTT) developed a cellular mobile communication system in the 800-MHz frequency band and started the service in Tokio in 1979. Some years later, in 1991, started service narrowband TACS (NTACS), which reduced the required frequency band in half.

The incompatibility between the different analog cellular mobile communication systems in equipment and operation meant that the users had to change the mobile terminal whenever they moved to another country. Besides, these analog systems could not ensure enough capacity for the increasing number of users and the quality of the speech was not good. In 1982, 26 European companies began developing the global system for mobile communications (GSM), which in 1992 started service in Finland, operating in the 900-MHz frequency band. GSM is nowadays used in more than 200 countries and territories and its users represent approximately the 80 % of the world's cellular market. In 1990 the North American cellular network incorporated the IS-54B standard which used time division multiple access (TDMA) and increased this way the call capacity of the system.

The growth of Internet also led to the development of several wireless network standards to provide Internet users with mobile connectivity [7]. The main wireless technologies are the wireless global area network (WGANs), wireless wide area network (WWAN), wireless personal area networks (WPAN) and wireless broadband-personal area network (WB-PAN); Figure 1.1 shows the distances and data rates that each technology allows.

The demand for high-rate packet-oriented services such as mixed data, voice and video

Figure 1.1: Wireless technologies

services, which exceed the bandwidth of conventional systems, increases every day. An important objective of the next generation of wireless systems is the integration of the existing technologies in a common platform. Research and development are taking place all over the world to define the next generation of wireless broadband multimedia communications systems (WBMCS), 4G, also known as Beyond 3G. These systems should enable[3]:

- integration of existing technologies
- higher data rates in a given spectrum (i.e. maximizing the spectral efficiency)
- support for different cell configurations and automatic adaptation to the channel conditions
- simple protocol and air interface layers
- seamless adaptation of new standards and technologies in the future

Data rates exceeding 2 Mbps are commonly assumed for these new systems. The modulation that seems to better support those high data rates, with robustness to radio channel impairments, is orthogonal frequency division multiplexing (OFDM) [11], which is the target of this project.

The demand for high speed wireless applications and the limited RF bandwidth has impelled the development of power and bandwidth efficient air interface schemes. Wireless communications have several advantages which have made them very popular in the last years: mobility, cheap to maintain, easy to add/remove devices, but they have also some disadvantages that make them not so easy to implement, actually they are considered the most difficult channels to deal with [11]. They strongly depend on the environment and are usually affected by the following phenomenon: path loss attenuation with distance, shadowing, multipath-fading, Doppler shift and interference. Some of the advantages that make OFDM so popular are related to the robustness against frequency selective, or multipath-fading, channels. Also the advancements in the 80's in digital signal processing (DSP) and

Figure 1.2: White Noise power spectral density

very large scale integrated circuits (VLSI) technologies allow efficient and cost-effective implementation of the fast Fourier transform (FFT) and inverse FFT (IFFT) operations.

1.2. Noise and channel interference

The main sources of wireless signals' performance degradation are additive white Gaussian noise (AWGN) and multipath fading, also Doppler effect should be taken into account for mobile systems.

AWGN . Unwanted signals are always present in electrical systems. Those unwanted signals mix with the information signals masking them and making the receiver to interpret symbols incorrectly and limiting therefore the data transmission rate. Noise is generated by man-made and natural sources. The main sources of man-made noise are electrical machines, switching devices, pulsed electronic devices, receiver oscillators and AC power lines; natural noise sources include the atmosphere, the sun and other cosmic sources. Much of this noise can be eliminated but the thermal noise can not be eliminated. Thermal noise is caused by the motion of electrons in a resistor, wire or any other conductor element and can be characterized as follows:

The noise $w(t)$ is an additive random disturbance of the information signal $s(t)$ so the received signal is given by

$$r(t) = s(t) + w(t) \quad (1.1)$$

The noise is white which means that it has a constant power spectral density (psd) over the signal band. A thermal noise source generates an equal amount of noise power per unit bandwidth at all frequencies as shown in Figure 1.2.

The noise is a stationary and zero mean Gaussian random process. The output of every linear noise measurement is a zero mean Gaussian random variable that does not depend on the time instant when the measurement is done [9].

Multipath propagation . The electromagnetic wave often follows paths with different lengths, see Figure 1.3, it propagates with different propagation times and reaches the receiver as

Figure 1.3: Multipath dispersion in wireless systems

an incoherent superposition of many signals with different phases. The different paths can be classified as follows:

- direct path, the wave follows the direct line between the transmission and the reception points.
- reflected path, waves are reflected off obstacles like building façades, the ground and mountain sides.
- diffracted paths, corresponding to the possibility for waves of travelling round obstacles by diffraction (clumps of vegetation, buildings, edges of roofs, hills, etc.)
- scattered paths, waves are scattered either when they strike rough surfaces like the vegetation, the ground or the sea, or flat surfaces but with small dimensions compared to the wavelength.

The interferences can be either constructive, when the different paths arrive in phase, leading to a signal reinforcement, or destructive, causing a fading of the signal. Multipath-fading results in a doubly dispersive channel that causes dispersion in both time and frequency domains. Time dispersion causes intersymbol interference (ISI) that in OFDM systems can be mitigated by frequency domain equalization. The presence of the multiple paths implies that the attenuation and selectivity in frequency of the propagation channel should be examined. For short wavelengths, small changes in the location of the transmitter, receiver and/or scattering objects in the environment will cause large changes in the incident wave components [9, 10, 7, 3].

1.3. OFDM

Wireless networks should be designed to minimize the degradation effects of wireless channels. In wireless systems, the high data rates and QoS that wired networks provide can not be preserved. OFDM overcomes the adverse effects of wireless channels with low complexity, it reduces the influence of multipath with no need of complex equalizers. OFDM transmits a data stream over N lower-rate subcarriers. Parallel-data transmission and frequency division multiplexing (FDM) were developed in the mid-1960s [8]. Multicarrier transmission splits the high-rate data stream into N substreams of lower data rate. Classical parallel-data systems divide the available bandwidth into N nonoverlapping subchannels. Each subchannel is modulated with a separate symbol and then the N subchannels are frequency multiplexed. This way interchannel interference is eliminated but it leads to poor spectral efficiency. In order to use the spectrum more efficiently it was proposed to use overlapping subchannels, but in overlapping subchannels it is necessary to eliminate crosstalk between subcarriers, which means that the different modulated subcarriers have to be orthogonal to each other. Orthogonal means that there is a precise mathematical relationship between the frequencies of the carriers of the system.

Frequency selective channels are characterized either by their delay spread or by the channel coherence bandwidth. In a single carrier system, a single fade or interference can cause the whole link to fail, but in multicarrier systems only a few of the total subcarriers will be affected. An important design goal in OFDM systems is that the channel can be considered as time-invariant and that fading per subchannel can be considered as flat. Therefore, the duration of one OFDM symbol should be smaller than the coherence time and the subcarrier spacing should be smaller than the coherence bandwidth of the channel [3, 11].

In 1971 Weinstein and Ebert [11] applied the discrete Fourier transform (DFT) to parallel data transmission system as part of the modulation and demodulation process. Recent progress in digital signal processing (DSP) and in very large scale integrated circuits (VLSI) and the introduction of the fast Fourier transform (FFT) highly simplified the implementation of OFDM systems.

In the last years some wireless standards have been developed and some of them use OFDM. The first ones to use OFDM were asymmetric digital subscriber loop (ADSL) and digital audio broadcasting (DAB) in the 90s. OFDM was chosen for the European and Japanese standards for digital terrestrial video broadcasting (DTVB) in 1995 and later on, WLAN standards HIPERLAN/2 and IEEE802.11a, that provides wireless connectivity up to 54 Mb/s, chose it as well [11]. Other more recent standards like IEEE802.16 wireless metropolitan area network (WMAN) for broadband fixed wireless access (BFWA), IEEE802.20 mobile broadband wireless access (MBWA) and the interactive terrestrial return channel (DVB-RCT) also use OFDM [7, 11].

1.4. Thesis objective

Although OFDM has some advantages that make it suitable for fading channels, it presents a high peak-to-average power ratio (PAPR) , which is one of the main drawbacks of OFDM systems. A simple technique used to reduce the PAPR of OFDM signals is to clip the signal to a maximum allowed value, at the cost of BER degradation and out-of-band radiation, other techniques reduce PAPR without degrading the BER but at the cost of adding extra information.

The goal of this thesis is to analyze the techniques used to reduce the noises introduced by clipping and propose an improvement for one of the techniques.

1.5. Thesis outline

The second chapter analyzes the basic principles of OFDM as well as the main advantages and drawbacks and analyzes the PAPR problem. The third chapter analyzes some of the techniques used to reduce high PAPR in OFDM systems, an improvement for one of those methods is proposed at the end of the chapter. All the techniques are simulated using Matlab and the results of the simulations are shown and analyzed in Chapter 4. The last chapter contains the conclusions, future work and environmental impact of the project.

CHAPTER 2. OFDM

This chapter analyzes the basic principles of OFDM as well as the main advantages and drawbacks, where the high peak-to-average power ratio (PAPR) plays an important role.

2.1. General principles

OFDM is a particular type of FDM modulation. The first step in the OFDM system is to convert a serial data stream into a parallel stream and then modulate the symbols, using the quadrature amplitude modulation (QAM) or the phase shift keying (PSK) modulation. The transmitter and receiver can be implemented by means of the IDFT and DFT respectively. After the symbols are modulated the data stream is converted to frequency domain by means of an IFFT. The length N_{FFT} of the FFT function should be chosen significantly larger than the number of useful subchannels N to ensure that the edge effects are negligible at half the sampling frequency and to ensure that the shape of the reconstruction filter of the digital-to-analog converter (DAC) does not affect the significant part of the spectrum. Figure 2.1 shows the shape of an OFDM spectrum for $N = 48$ and $N = 1536$ subcarriers and how the side lobes decay steeper as the number of subcarriers increases. At the receiver, an FFT of the same length is performed and the N useful coefficients will be extracted from the N_{FFT} spectral coefficients [9]. Figure 2.2 shows a block diagram of an OFDM system. The output signal of the IFFT block is a discrete signal which is processed by the DAC. This discrete signal has a periodic spectrum so the DAC has to filter it by a low-pass filter in order to eliminate the unnecessary replicas. As said before, these filters attenuate the subcarriers close to the Nyquist frequency, so these subcarriers should be avoided for data transmission [8].

The signal bandwidth is divided into N small subchannels and the frequency of each subcarrier is chosen to form an orthogonal set of frequencies, which are known at the receiver. This structure with multiple subchannels makes OFDM suitable for multiuser communications, allocating several subcarriers to each user. Allowing the subcarriers to overlap reduces the required bandwidth and increases the spectral efficiency. The OFDM modulated signal can be expressed as

$$x(t) = \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, \quad (0 < t < T_s) \quad (2.1)$$

Where T_s is the duration of an OFDM symbol, $\frac{1}{T_s}$ is the distance between subcarriers (or subchannel space) in frequency domain (Δf) and S_n a block of N data complex symbols chosen from a signal constellation like QAM or PSK. Each subcarrier is located at

$$f_n = \frac{n}{T_s} \quad (0 < n < N - 1) \quad (2.2)$$

In order to maintain the orthogonality between the OFDM symbols, the symbol duration and subchannel space must meet the condition $T_s \Delta f = 1$. When $x(t)$ is sampled every $\frac{T_s}{N}$,

Figure 2.1: OFDM spectrum for N subcarriers

Figure 2.2: block diagram of an OFDM system

Figure 2.3: OFDM signal in frequency domain

the mathematical expression of the sampled signal is

$$x_k = x(kT_s) = \sum_{n=0}^{N-1} X_n e^{j2\pi f_n \frac{kT_s}{N}} \quad (2.3)$$

Setting $f_0 = 0$ then $f_n T_s = k$ and x_k becomes

$$x_k = \sum_{n=0}^{N-1} S_n e^{j2\pi \frac{kn}{N}} = IDFT S_n \quad (2.4)$$

The FFT and IFFT operations reduce the number of complex multiplications from N^2 to $\frac{N}{2} \log_2 N$ highly simplifying the implementation of the OFDM modulation.

The total bandwidth of the OFDM signal is $B = N \cdot \Delta f$. Figure 2.3 shows the OFDM basis functions and how they orthogonally overlap. At the center frequency of each subcarrier all the rest are zero, so they do not interfere with each other since the receiver calculates the spectrum values at the peak point of each subcarrier; this way intercarrier interference (ICI) is avoided which is an important advantage of OFDM systems.

If more subcarriers are added into the bandwidth used by the OFDM signal, then the distance between subcarriers Δf will decrease which in time domain means that the duration of a symbol T_s increases. In frequency domain, Δf is usually much smaller than the total available bandwidth, making frequency synchronization more difficult. Having more time for each symbol will increase the probability of receiving it correctly since there is some transient before the symbol is correctly set; if the symbol duration is too short, it may happen that there is no time after the transient to read the symbol and so there is a high probability of interpreting it wrong. The OFDM symbol should be large compared to the impulse response of the channel τ_{max} . However, the OFDM symbol must be short enough so that the temporal variations due to movements of the transmitter and/or the receiver do not result in a time-varying channel within the OFDM symbol. Since the length of the OFDM symbol increases linearly with the number of subcarriers, the number of subcarriers (and the duration of the OFDM symbol) are limited by the coherence time of the channel.

Figure 2.4: OFDM signal in time domain

Because of the frequency selective fading properties of wireless channels, some of the subcarriers of the OFDM signal are enhanced and others suffer fading. If N is large enough, each subcarrier is narrow compared to the coherence bandwidth of the channel and even under severe fading conditions they suffer flat fading.

Multipath delay causes ISI between OFDM symbols, which occurs when an OFDM symbol is distorted by the previous one. A simple solution to this problem is transmitting a guard interval before each OFDM symbol. A cyclically extended guard interval is added to the OFDM symbol so that it is preceded by the last N_g IFFT coefficients of itself. The total symbol duration now is $T_{total} = T_s + T_g$, where T_g is the guard interval duration and T_s is the useful symbol duration. The duration of the guard interval depends on the application, usually, since the insertion of the guard interval reduces the data throughput, T_g is chosen so that $T_g < T_s/4$. The guard interval absorbs time synchronization errors as well as the echoes from the delayed signals.

Time synchronization can easily be achieved by introducing a time stamp, like repeating periodically a certain known OFDM reference symbol of known content. Usually time synchronization uses training sequences which are introduced in the OFDM stream. Sometimes time synchronization uses the cyclic prefix of the OFDM symbols since it is a periodic sequence and can be used to estimate the phase changes suffered by each OFDM symbol. Frequency synchronization is important in order to keep the orthogonality between subcarriers and to deal with phase rotations at the received symbols, this is specially important in systems that use differential modulations.

Although the signal is built in frequency domain, the data transmission takes place in time domain. After shaping the signal, the IFFT block converts it to time domain and produces N output samples, corresponding to the N subcarrier of the OFDM system and which make a single OFDM symbol, however, as said before, not all the N subcarrier carry data. The resulting signal is the sum of all the modulated subcarriers which form the baseband signal. Figure 2.4 shows how an OFDM signal looks like in time domain.

The different subcarriers (blue and green waves) in Figure 2.4 are added up to form the OFDM signal (red wave). The shape of the red wave has several peaks much greater than the average power. When all the subcarriers are added up constructively, the resulting power is much higher than the average power of the OFDM signal. This causes a very large peak which increases as the number of subcarriers increases, leading to a very high PAPR. High PAPR is an important drawback of OFDM systems.

Hereafter are summarized the main advantages and drawbacks of OFDM systems [3, 11].

First the main advantages:

- high spectral efficiency. The signal is built in frequency domain so it can be shaped in order to use the available bandwidth as efficiently as possible.
- efficient in multipath environments. OFDM increases the symbol duration by N , being N the number of subcarriers and this together with the cyclic prefix can completely eliminate ISI. Because the bandwidth of each subcarrier is narrow compared to the coherence bandwidth of the channel, fading can be considered as flat which reduces the complexity of the receiver.
- simple digital realization by using the FFT operation. Thanks to the advances in digital signal processing and VLSI, the realization of the FFT operations is now simple to implement.
- low complex receivers due to avoidance of ICI and ISI. Each sub-channel can be considered separately since ICI does not affect the signal and each channel suffers flat fading so complex equalizers can be avoided.
- different modulation schemes can be used on individual sub-carriers. This way more robust modulations can be used at the subcarriers that suffer more fading, if channel estimation is used.

And the main drawbacks are:

- accurate frequency and time synchronization is required
- more sensitive to Doppler spreads than single-carrier schemes
- sensitive to frequency offset and phase noise caused by imperfections in the transmitter and the receiver oscillators
- guard interval causes loss in spectral efficiency
- high PAPR

2.2. PAPR

When in time domain all the N subcarriers are added up constructively, they produce a peak power that is N times greater than the average power of the signal. The *PAPR* is calculated by the following equation

$$PAPR = \frac{\max(x^2(t))}{\text{mean}(x^2(t))} \quad (2.5)$$

Where $x(t)$ is the amplitude of the signal.

The peak power of the OFDM signal, regarding the worst case when all the subcarriers are added-up constructively, is the sum of all the N subcarriers: $1 \cdot N = N$. The mean power

Figure 2.5: CCDF for different number of subcarriers

of the OFDM signal is the sum of all the values of the signal, which is actually N , divided by the total number of subcarriers, which is also N . Therefore the maximum $PAPR$ is:

$$PAPR_m = \frac{N}{N} = 1 \quad (2.6)$$

This maximum $PAPR$ increases whenever the number of subcarriers increases. Thus, if $N \rightarrow \infty \Rightarrow x_k$ becomes Gaussian distributed, for $k = 1, \dots, N$, which means that

$$P(x_k < PAPR_m) < 1 \quad (2.7)$$

When the number of subcarriers tends to ∞ this probability gives

$$\lim_{N \rightarrow \infty} \prod_{k=1}^N P(x_k < PAPR_m) = 0 \quad (2.8)$$

If the above statement represents the probability of a signal x_k to have a smaller $PAPR$ than the given one $PAPR_m$, the probability of the signal to have a $PAPR$ greater than $PAPR_m$ is

$$\lim_{N \rightarrow \infty} (1 - \prod_{k=1}^N P(x_k < PAPR_m)) = 1 \quad (2.9)$$

The above statement can be better understood graphically in Figure 2.5, which shows the complementary cumulative distribution function (CCDF) of an OFDM signal. The CCDF denotes the probability of a signal to have a higher $PAPR$ than a threshold $PAPR_m$, so in the figure, horizontal and vertical axes represent the threshold values of $PAPR$ and the CCDF respectively. The CCDF simulation compares the CCDF of signals with different number of subcarriers (from 128 to 1024) and it corroborates the theory. For all the different signals of the simulation, the probability of having higher $PAPR$ than a fixed $PAPR_m$ decreases as this fixed value increases. However, this probability increases as the number of subcarriers increases, and if N keeps increasing the probability tends to be 1 for any $PAPR_m$ so the

Figure 2.6: OFDM transmitter basic scheme

$PAPR$ of the signal when $N \rightarrow \infty$ will always be greater than $PAPR_m$. The probability of having a peak higher than $PAPR_m$, though, decreases as N increases because the more subcarriers there are, the less probable it is that all them add-up constructively at the same time, but still it may happen.

In some applications, the drawbacks of high $PAPR$ overcome the benefits of OFDM. To understand how and where the $PAPR$ of the signal becomes a problem, it is necessary to take a look at the OFDM transmitter, Figure 2.6 shows a basic scheme. The information data is generated and shaped in the Data block, this takes place in frequency domain, then comes the IFFT block and the signal is afterwards in digital time domain. To send the signal through the antenna, it must be first converted to analog time domain by means of an A/D converter and then amplified with an RF power amplifier. After the RF amplifier comes the antenna which sends the signal to the OFDM receiver over the wireless channel.

Taking a deeper look into the RF amplifier, actually into its characteristic curve (Figure 2.7), it clearly shows two operating regions, the linear and the saturation ones. The curve in Figure 2.7 is given by the following equation

$$F[x] = \frac{x}{(1 + (x)^{2p})^{\frac{1}{2p}}} \quad (2.10)$$

When the amplifier operates in the linear region, as its name indicates, signals are linearly amplified by the same factor A . However, when it operates in the saturation region, signals are not amplified anymore, they are flattened at the maximum output power of the amplifier. In this application (the OFDM transmitter), the system needs the amplifier to work in the linear region in order not to distort the data, the better performance will be achieved for an input backoff (IBO) = 0 dB. The IBO is defined as

$$IBO = 10 \cdot \log_{10}\left(\frac{P_{max}}{P_{av,in}}\right) \quad (2.11)$$

and gives the ratio between the input power and the maximum input power of the linear region so an $IBO = 0$ dB means that the amplifier is working at the upper limit of the linear region. The problem appears when the peaks of the OFDM signal are too large and do not fit in the linear region. In this case they are treated by the saturation region and so they are non-linearly modified, causing intermodulation among subcarriers and out-of-band radiation. To be able to hold these peaks, the operational amplifier (OA) of the transmitter needs a very large dynamic range. Increasing the linear region of an OA (increasing p in equation 2.10) is very expensive and it is inefficient in this case. If N increases, the peak power will also increase and so will do the $PAPR$. If N is large enough, the actual OA will not be able to hold the new peak power and it would be necessary to replace it. Besides, the larger the OA's dynamic range is, the more battery it consumes and since most OFDM systems are

Figure 2.7: Non-linear amplifier characteristic curve

power limited, an OA with a large linear region would consume most of the battery of the system, which is another reason why increasing the dynamic range of the OA is not a good solution. Lots of research is being made in this area in order to improve the performance of OFDM systems. The different methods designed to avoid high PAPR can be divided in two big groups. The techniques in the first group distort nonlinearly the transmitted signal by clipping the peaks and causing the bit error rate (BER) to increase. Methods in this group include clipping, peak windowing and peak cancellation. The techniques in the second group reduce the PAPR of the signal but the signal remains undistorted, they do not affect the BER but some of them need to transmit additional information so the spectral efficiency is reduced and most of them increase the computational complexity which affects also the time. This group includes coding techniques that, together with special forward error correction codes (FEC), exclude those symbols that produce high PAPR, and scrambling sequences, which choose the sequence that produces the smallest PAPR. The following section explains how clipping works, which is the PAPR reduction method chosen for our application, and several reconstruction methods that help mitigating the degradation the signal suffers at clipping, proposing an improvement for one of them.

CHAPTER 3. PAPR REDUCTION AND NOISE MITIGATION TECHNIQUES

This chapter analyzes some of the techniques used to reduce high PAPR in OFDM systems, and because the technique chosen for our application is clipping, some methods used to reduce the noise caused by clipping are also explained and an improvement for one of them is proposed.

3.1. PAPR reduction: Clipping

Clipping too large peaks is a simple solution to the PAPR problem. Clipping belongs to the group of techniques that reduce large peaks by nonlinearly distorting the signal [8]. It does not add extra information to the signal and too large peaks occur with low probability so the signal is seldom distorted. The maximum peak power allowed is determined by the system specifications, usually by the linear region of the power amplifier. A maximum peak amplitude A is chosen so that the OFDM signal does not exceed the limits of this region, symbols that exceed this maximum amplitude, will be clipped. The clipping function is performed in digital time domain, before the D/A conversion as shown in Figure 3.2, and the process is described by the following expression

$$x_k^c = \begin{cases} x_k & |x_k| \leq A \\ Ae^{j\phi(x_k)} & |x_k| > A \end{cases}, \quad 0 \leq k \leq N-1 \quad (3.1)$$

Where x_k^c is the clipped signal, x_k is the transmitted signal, A is the clipping amplitude and $\phi(x_k)$ is the phase of the transmitted signal x_k . The graphical expression of this function is shown in Figure 3.1.

The clipping ratio (CR) is defined as

$$CR = \frac{A}{\sigma} \quad (3.2)$$

Figure 3.1: Clipping function

Figure 3.2: Clipping in the transmitter

Figure 3.3: SCNR measured for different number of subcarriers

Figure 3.4: Out-of-band radiation

And the clipping ratio in dB is given by

$$CR_{[dB]} = 20 \log \frac{A}{\sigma} \quad (3.3)$$

Where σ is the root mean squared value of the unclipped OFDM signal and its mathematical expression is

$$\sigma^2 = \frac{1}{N} \sum_{k=1}^N x_k^2 \quad (3.4)$$

Clipping is a non-linear process so it introduces in-band distortion (Figure 3.3), also called clipping noise, and out-of-band radiation and inter-carrier interference (Figure 3.4), which degrade the system performance and the spectral efficiency.

The clipping noise is related to the difference between the original signal x_k and the clipped signal x_k^c . The signal sent to the receiver is the clipped signal, which is different from the signal that we actually wanted to send. This difference is measured by the signal to clipping noise ratio (SCNR)

$$SCNR = \frac{|x_k^c|^2}{|x_k - x_k^c|^2} \quad (3.5)$$

The SCNR (Figure 3.3) is the ratio between the clipped signal x_k^c and the distortion introduced when clipping the signal, that is the difference between the unclipped signal and the clipped one. The more the signal is clipped, the greater the difference between clipped and unclipped signals will be and so the SCNR will decrease. On the other hand, the less

Figure 3.5: FFT filter block diagram

the signal is clipped, the smaller the difference is and so the SCNR will increase. Figure 3.3 shows the SCNR measured for signals with different number of subcarriers (from 128 to 512). For a low CR, the SCNR is low and approximately the same for all the different signals. This makes sense since it was already deduced in Figure 2.5 that for a very low $PAPR_m$, the probability of having a signal with higher PAPR is 1 independently of the number of subcarriers of the signal. When the CR is very high, the signal is not clipped any more and so from a certain CR the SCNR stops increasing, maintaining the same value for all the CR.

3.2. Noise mitigation: Clip and filter

Introducing a filter after the clipping operation helps mitigating the out-of-band radiation introduced by clipping. Filtering, however, distorts also the signal and causes peak regrowth [6]. A common filter used for this purpose is the FIR filter. In this application the filter used is the FFT-IFFT filter proposed by Armstrong in [1]. This filter achieves better performance than the FIR filter, it introduces less noise into the signal, causes less peak regrowth and needs less computation; since it operates symbol by symbol it causes no inter-symbol interference.

The filter itself consists on a set of FFT-IFFT operations where filtering takes place in frequency domain after the FFT function. The first step in order to implement this filter is introducing $N(I - 1)$ zeros (being I the oversampling rate) in the middle of the original signal, as shown in Figure 3.5, before transforming it to time domain.

After clipping the signal, the out-of-band radiation caused by clipping falls in the zeros and then the signal is passed to the FFT filter. The FFT function transforms the clipped signal x_k^c to frequency domain yielding X_n^c . The information components of X_n^c are passed unchanged to the IFFT block and the out-of-band radiation that fell in the zeros is set back to zero. The IFFT block of the filter transforms the signal to time domain and the obtained signal x_k^F is passed to the D/A converter. After filtering, the signal suffers peak regrowth so in order to minimize this effect the clip & filter process can be repeated several times.

Figure 3.6: CCDF for 1,2 and 3 iterations of clip&filter

Figure 3.6 shows the CCDF of an OFDM signal with 512 subcarriers and modulated with QPSK for one, two and three times clip & filter and it can be observed how the probability of having high peaks decreases as the number of iterations increases. It can be also observed that just one iteration of clip & filter gives less high peaks than just clipping.

The zeros added in the transmitter are removed in the receiver once the signal is in frequency domain. Figure 3.7 shows a simplified block diagram of the OFDM system including the FFT filter so that it can be seen where the zeros are added and removed.

After distorting the signal in the transmitter, the goal is to recover it in the receiver with the minimum possible number of errors. Figures 3.8 and 3.9 show the bit error rate (BER) of an OFDM signal with 256 subcarriers in flat and selective fading channels respectively. From the figures it can be observed that the better performance is achieved for just one iteration of clip & filter, this is logic since each iteration distorts the signal so the more iterations, the more distortion. In a flat fading channel, the signal-to-noise ratio (SNR) needed to get a BER of 10^{-3} is about 15.5 dB for one iteration, about 17.5 dB for two iterations and nearly 19 dB for three iterations; to get the same BER in a selective fading channel, the SNR needed is about 26 dB for one iteration, 27 dB for two iterations and 27.5 dB for three iterations. In both cases the results regarding BER are better when just clipping than when performing clip&filter more than once.

Figure 3.7: OFDM system with FFT filter block diagram

Figure 3.8: Flat fading channel

Figure 3.9: Selective fading channel

Figure 3.10: DAR algorithm block diagram

3.3. Noise mitigation: Decision-aided reconstruction (DAR) method

The goal of the clipping operation is to reduce the PAPR of the original signal x_k ; it introduces, however, some distortion that makes the recovering of x_k in the receiver more difficult. The DAR method was proposed by Kim and Stüber in [5] in order to mitigate the effects of the distortion introduced by clipping [5].

It is a nonlinear iterative reconstruction technique implemented in the receiver. To implement this technique, the receiver needs to know the clipping ratio A used at the transmitter. The signal sent by the transmitter in frequency domain can be expressed as

$$Y_n = \alpha_n X_n + C_n \quad (3.6)$$

Where α_n is the clipping distortion, which changes randomly from block to block, and C_n is the noise introduced by clipping. The received signal is therefore

$$Z_n = \eta_n Y_n + W_n = \eta_n \alpha_n X_n + Q_n \quad (3.7)$$

Where η_n is the complex channel gain, that can be accurately estimated, and Q_n is the sum of the additive white Gaussian noise (AWGN) and the clipping noise C_n . In all the simulations performed in this project ideal channel estimation is supposed. Figure 3.10 shows the block diagram of the DAR method, where r_k is the received signal Z_n in time domain. The steps to follow in order to implement the method are as follows:

1. Calculate and store in memory the time domain equalized signal:

$$\hat{Y}_n = \frac{Z_n}{\eta_n} \quad (3.8)$$

$$\hat{Y}_n \rightarrow IFFT \rightarrow \hat{y}_k \quad (3.9)$$

- This step is not repeated in the iterations.
2. Estimate the original signal by comparing each symbol of the received signal to the set of symbols of the modulation and choosing the one that less error gives. The symbols are multiplied by the complex channel gain so that they suffer approximately the same distortion as the received signal:

$$\hat{X}_n^{(I)} = \min_X |Z_n^{(I)} - \eta_n X| \quad (3.10)$$

Where X represents the set of symbols of the modulation and I is the iteration number, for $I = 0 \Rightarrow Z_n^{(I)} = Z_n$.

3. Convert the estimated signal to time domain

$$\hat{X}_n^{(I)} \rightarrow IFFT \rightarrow \hat{x}_n^{(I)} \quad (3.11)$$

4. Detection of the clipped samples

$$\hat{y}_k^{(I)} = \begin{cases} \hat{y}_k & |\hat{x}_k^{(I)}| \leq A \\ \hat{x}_k^{(I)} & |\hat{x}_k^{(I)}| > A \end{cases} \quad (3.12)$$

Where $\hat{y}_k$ is always the signal stored in memory in Step 1.

5. Convert to frequency domain the detected sequence $\hat{y}_n^{(I)}$

$$\hat{y}_n^{(I)} \rightarrow FFT \rightarrow \hat{Y}_n^{(I)} \quad (3.13)$$

6. Increment the iteration number $I = I + 1$ and calculate the new $Z_n^{(I)}$

$$\hat{Z}_n^{(I)} = \eta_n \hat{Y}_n^{(I-1)} \quad (3.14)$$

7. With the new Z_n , estimate again $\hat{X}_n^{(I)}$

- This is the end of the iteration, to continue iterations repeat steps from 3 to 7.

Although replacing the clipped samples in Step 4 with the decisions from Step 2 helps improving the received signal, the decisions in Step 2 are noisy because of decision errors so Step 4 may miss some of the clipped samples (miss detection) or replace some of the unclipped ones (false detection). If the CR is too low (this depends on the modulation), the performance of DAR is degraded because false detection increases.

3.4. Noise mitigation: Improved decision-aided reconstruction (IDAR) method

The DAR method was proposed to mitigate the clipping noise but it does not deal with the intermodulation noise. The inter-modulation products are reduced by setting the IBO of the

Figure 3.11: IDAR algorithm block diagram

power amplifier at the linear region; however, in order to maximize the power efficiency of the amplifier, the IBO is set nearly at the saturation region degrading this way the BER due to the high non-linearity. The improved DAR method mitigates both the clipping noise and the inter-modulation noise. The IDAR method [2], like the DAR one, is an iterative method applied at the receiver and was proposed by Boonsrimuang et al. in [2], Figure 3.11 shows the block diagram. It repeats at the receiver the clipping and the amplifying operations, so the receiver needs to know the clipping amplitude A and the amplifier characteristics. In Figure 3.11 can be seen that the method starts by estimating the original signal like in the DAR method, the steps to follow to apply the technique are the following ones:

1. Estimate the original signal by comparing each symbol of the received signal to the set of symbols of the modulation and choosing the one that less error gives:

$$\hat{X}_n = \min_X |\hat{Z}'_n - X| \quad (3.15)$$

- Where X represents the set of symbols of the modulation and in the first iteration $\hat{Z}'_n = Z_n$

2. Convert the estimated signal to time domain

$$\hat{X}_n \rightarrow IFFT \rightarrow \hat{x}_n \quad (3.16)$$

3. Clipping the estimated signal

$$\hat{x}_k^c = \begin{cases} \hat{x}_k & |\hat{x}_k| \leq A \\ Ae^{j\{arg(\hat{x}_k)\}} & |\hat{x}_k| > A \end{cases} \quad (3.17)$$

4. The output signal after the non-linear amplifier is given by

$$\hat{s}_k = F[|\hat{x}_k^c|] \cdot e^{j\{arg(\hat{x}_k^c)\}} \quad (3.18)$$

- Where $\hat{s}_k$ includes the transmitter noises.

5. The error signal including all noises is

$$\hat{e}_k = \hat{s}_k - \hat{x}_k \quad (3.19)$$

6. Convert to frequency domain the error signal

$$\hat{e}_k \rightarrow FFT \rightarrow \hat{E}_n \quad (3.20)$$

7. Calculate the new $\hat{Z}'_n$

$$\hat{Z}'_n = \hat{Z}_n - \hat{E}_n \quad (3.21)$$

8. Repeat Step 1 yielding a new estimated signal

$$\hat{X}_n = \min_X |\hat{Z}'_n - X| \quad (3.22)$$

- This is the end of the iteration, to continue iterations repeat steps from 2 to 8.

By using this method the intermodulation noise is mitigated, relatively even when the non-linear amplifier is operated at the saturation region, while with DAR the BER is severely affected by this noise. Although the method shows a good performance for different required signal to noise ratios, for CR lower than 3 dB the decisions made in Step 1 are too erroneous. The better performance of BER when using IDAR is obtained when the IBO is around 0 dB.

3.5. Proposed method: IDAR including the FFT-IFFT filter (IDARF)

The IDAR method shows a very good performance by mitigating the clipping and the intermodulation noises. However, in order to reduce the out-of-band radiation it is advisable to use a filter in the transmitter, in this case the filter used is the FFT-IFFT filter. So, if the clipping operation is followed by the filter at the transmitter, this filter should also be included in the IDAR method at the receiver, Figure 3.12 shows a block diagram of the new method. In order to include the FFT filter in the IDARF method, the same number of zeros as in the transmitter are inserted into the estimated signal before converting it to time domain. Clip&filter is applied to the time domain signal which is then amplified like in the IDAR method. The zeros are removed from the signal after the amplifier; the signal is transformed to frequency domain, the zeros are removed and the signal is transformed back to time domain in order to calculate the error signal. The error signal is the difference between the estimated signal $\hat{x}_k$ (before adding the zeros) and the signal that suffered the clip&filter and amplifying process, $\hat{s}'_k$, after removing the zeros. The red boxes in Figure 3.12 show the operations that have to be added to the IDAR algorithm in order to include the FFT filter in the method.

All the methods explained in this chapter have been evaluated for different values of the variables that compose the system. The following chapter shows the results obtained from the simulations and comments these results.

Figure 3.12: IDARF algorithm block diagram

CHAPTER 4. SIMULATION RESULTS

After theoretically studying different PAPR reduction and noise mitigation techniques, these methods have been tested both in a flat fading channel and in a selective fading channel. The simulations have been made for the 16-QAM and the 64-QAM modulations and all them use 256 subcarriers, when analyzing the system for different CR, the SNR used varies depending on the modulation, this information is available in Table 4.3. The frequency selective channel parameters are shown in Table 4.1 and the number of iterations used for each method are shown in Table 4.2.

4.1. Flat fading channel

This section shows the performance of the BER of OFDM signals modulated with 16 and 64QAM and in a flat fading channel when varying the SNR and the CR .

Figures 4.1, 4.2 and 4.3 show the BER performance of a 16QAM modulation when varying the SNR from 0 to 18 dB. The simulations in Figure 4.1 show the performance of the DAR and IDAR methods when no filter is applied at the transmitter. The results show that the BER performance for DAR and for IDAR is very similar, being IDAR slightly better. When the FFT filter is used at the transmitter, IDARF shows clearly better performance than DAR and IDAR. Figure 4.3 shows that IDARF is also better than IDAR without filter at the transmitter, keeping the BER very close to that of the no clipping case.

Figures 4.4, 4.5 and 4.6 show the same simulations as the previous figures but this time holding the SNR at 15 dB and varying the CR from 2 to 14 dBs. For $CR > 4$ dB, the performance of DAR and IDAR in Figure 4.4, when no filter is applied, is approximately the same, for $CR < 4$ dB performs better than DAR. When the filter is applied, Figure 4.5, and for $CR < 8$ dB approximately, IDARF provides a significant better performance than DAR or IDAR and DAR provides no improvement compared to just clip&filter, also the difference between IDARF and DAR or IDAR without filter in the transmitter is very small, see Figure 4.6. As the CR decreases, the performance of all methods degrades, being DAR without filtering at the transmitter the one that faster degrades. From a CR of 8 dB the performance of all methods becomes flat and similar for all them.

Figures 4.7, 4.8 and 4.9 show the BER performance of a 64QAM modulation when varying the SNR from 0 to 30 dB and for a clipping ratio of 4 dB. In Figure 4.7, when no filter is applied at the transmitter, the IDAR method performs clearly better than DAR for

Table 4.1: Channel parameters

tap	delay	tap coeff.
h_0	0.0	0.8405
h_1	0.3	0.4726
h_2	1.0	0.2658

Table 4.2: number of iterations

clip&filter	1
DAR	2
IDAR	2
IDARF	2

Table 4.3: system parameters

number of subcarriers	256
CR	4 dB
SNR 16QAM	15 dB
SNR 64QAM	20 dB

Figure 4.1: DAR and IDAR methods with no filter in the transmitter, 16QAM modulation in flat fading channel, CR=4 dB, N=256 subcarriers

Figure 4.2: IDARF, DAR and IDAR methods with filter in the transmitter, 16QAM modulation in flat fading channel, CR=4 dB, N=256 subcarriers

Figure 4.3: DAR, IDAR and IDARF methods with and without filter in the transmitter, 16QAM modulation in flat fading channel, CR=4 dB, N=256 subcarriers

Figure 4.4: DAR and IDAR methods with no filter in the transmitter, 16QAM modulation in flat fading channel, SNR=15 dB, N=256 subcarriers

Figure 4.5: IDARF, DAR and IDAR methods with filter in the transmitter, 16QAM modulation in flat fading channel, SNR=15 dB, N=256 subcarriers

Figure 4.6: DAR, IDAR and IDARF methods with and without filter in the transmitter, 16QAM modulation in flat fading channel, SNR=15 dB, N=256 subcarriers

SNR > 12 dB approximately, for lower SNR their performance is nearly the same. For an SNR < 20 dB they behave like when no clipping is applied and after that they behave like when just clipping is applied. When the filter is used in the transmitter, Figure 4.8, the performance of IDARF for the whole SNR range is very close to that when no clipping is applied, on the other hand, there is nearly no difference between using IDAR and using just clip&filter. When comparing all methods in Figure 4.9, IDARF performs clearly better than DAR and IDAR without filter in the transmitter.

Figures 4.10, 4.11 and 4.12 show how those methods behave when holding the SNR at 20 dB and varying the CR from 2 to 14 dB. Without filter in the transmitter, Figure 4.10, IDAR performs slightly better than DAR for CR < 5 dB, after that the performance is approximately the same. With filter in the transmitter, 4.11, IDARF performs much better than the other methods for CR < 9 dB and using DAR yields the same results as just clip&filter. Regarding the whole set of methods, Figure 4.12, IDARF performs better than IDAR and DAR for CR < 6 dB.

4.2. Selective fading channel

This section shows the results for the same simulations as the previous one but in a selective fading channel, where the parameters of the channel are shown in Table 4.1.

Figures 4.13, 4.14 and 4.15 show the BER performance of a 16QAM modulation when varying the SNR from 0 to 20 dB and for a CR of 4 dB. When no filter is used, Figure 4.13, IDAR and DAR show very similar performances, being IDAR slightly better than DAR. When the filter is used, Figure 4.14, IDARF is better than the other methods but still the

Figure 4.7: DAR and IDAR methods with no filter in the transmitter, 64QAM modulation in flat fading channel, CR=4 dB, N=256 subcarriers

Figure 4.8: IDARF, DAR and IDAR methods with filter in the transmitter, 64QAM modulation in flat fading channel, CR=4 dB, N=256 subcarriers

Figure 4.9: DAR, IDAR and IDARF methods with and without filter in the transmitter, 64QAM modulation in flat fading channel, CR=4 dB, N=256 subcarriers

Figure 4.10: DAR and IDAR methods with no filter in the transmitter, 64QAM modulation in flat fading channel, SNR=20 dB, N=256 subcarriers

Figure 4.11: IDARF, DAR and IDAR methods with filter in the transmitter, 64QAM modulation in flat fading channel, SNR=20 dB, N=256 subcarriers

Figure 4.12: DAR, IDAR and IDARF methods with and without filter in the transmitter, 64QAM modulation in flat fading channel, SNR=20 dB, N=256 subcarriers

Figure 4.13: DAR and IDAR methods with no filter in the transmitter, 16QAM modulation in selective fading channel, CR=4 dB, N=256 subcarriers

difference is small and when comparing all methods with and without filter in Figure 4.14, it can be seen that IDARF performs slightly better than IDAR without filter.

Figures 4.16, 4.17 and 4.18 show the BER performance of a 16QAM modulation when holding the SNR at 15 dB and varying the CR from 2 to 14 dB. Figure 4.16 shows the simulations without using a filter at the transmitter, for CR > 3 dB DAR shows better performance than IDAR and for CR < 4 dB the BER of them both increases very fast. When the filter is used, as shown in Figure 4.17, for CR < 8 dB approximately IDARF performs much better than the other methods and like in all the other simulations, DAR performs the same as just clip&filter. On the other hand, comparing all the methods in Figure 4.18, DAR when no filter is applied performs better than IDARF and all the other methods for CR > 4 dB.

Figures 4.19, 4.20 and 4.21 show the performance of the BER for a 64QAM modulation with a CR= 4 dB and varying the SNR from 0 to 30 dB. In this case, when no filter is applied, see Figure 4.19, IDAR shows better performance than DAR and both behave like when no clipping is performed. It can be observed from Figure 4.20 that when the filter is used, the performance of IDARF is very close to the case when no clipping is performed, and DAR and just clip&filter behave like when just clipping is used. In Figure 4.21 can be seen that IDARF performs, in the whole SNR range, better than IDAR and DAR when no filter is applied.

Figures 4.22, 4.23 and 4.24 show the performance of these methods when the SNR is 20 dB and the CR varies from 2 to 14 dB. Without filter in the receiver, Figure 4.22, IDAR performs better than DAR for CR < 6 dB approximately, although for CR < 4 dB both methods yield very bad BER. When using the filter, Figure 4.23, IDARF shows better performance than all other methods for CR < 8 dB and for CR < 3 dB its BER is also very bad. When all methods are compared together, as shown in Figure 4.24, for CR < 6 dB,

Figure 4.14: IDARF, DAR and IDAR methods with filter in the transmitter, 16QAM modulation in selective fading channel, CR=4 dB, N=256 subcarriers

Figure 4.15: DAR, IDAR and IDARF methods with and without filter in the transmitter, 16QAM modulation in selective fading channel, CR=4 dB, N=256 subcarriers

Figure 4.16: DAR and IDAR methods with no filter in the transmitter, 16QAM modulation in selective fading channel, SNR=15 dB, N=256 subcarriers

Figure 4.17: IDARF, DAR and IDAR methods with filter in the transmitter, 16QAM modulation in selective fading channel, SNR=15 dB, N=256 subcarriers

Figure 4.18: DAR, IDAR and IDARF methods with and without filter in the transmitter, 16QAM modulation in selective fading channel, SNR=15 dB, N=256 subcarriers

Figure 4.19: DAR and IDAR methods with no filter in the transmitter, 64QAM modulation in selective fading channel, CR=4 dB, N=256 subcarriers

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Figure 4.22: DAR and IDAR methods with no filter in the transmitter, 64QAM modulation in selective fading channel, SNR=20 dB, N=256 subcarriers

IDARF performs better than DAR and IDAR without filter.

Figure 4.23: IDARF, DAR and IDAR methods with filter in the transmitter, 64QAM modulation in selective fading channel, SNR=20 dB, N=256 subcarriers

Figure 4.24: DAR, IDAR and IDARF methods with and without filter in the transmitter, 64QAM modulation in selective fading channel, SNR=20 dB, N=256 subcarriers

CHAPTER 5. CONCLUSIONS AND ENVIRONMENTAL IMPACT

This final chapter explains the conclusions that can be drawn from the simulations as well as an outlook of the future research work that can be done to this method. Also last section analyzes the impact that the implementation of the IDARF method would have in the environment.

5.1. Conclusions and outlook

The increment of high speed wireless applications has impulsed the development of bandwidth efficient air interface schemes. Wireless channels have some disadvantages like multipath fading that make it difficult to work with them. OFDM is a modulation that deals with those problems without high computation complexity, but it has also disadvantages, like a high PAPR, that make it not so simple to implement.

Research is being done in order to minimize the effects of high PAPR in OFDM systems, clipping is a simple solution that results from this research. Clipping is a non-linear operation that introduces intermodulation noise and out-of-band radiation in the OFDM signal. In order to mitigate the noise introduced by clipping some methods have been proposed, like filtering after clipping to reduce the out-of-band radiation and the DAR and IDAR methods to reduce the intermodulation noise. A variation of the IDAR method, called IDARF, has been proposed in this project in order to improve its performance. Some conclusions can be drawn from the simulations shown in Chapter 4.

When using the 16QAM modulation, DAR and IDAR have similar performance when no filter is applied at the transmitter, this performance is also very similar to that of IDARF when the FFT filter is applied at the transmitter, although IDARF performs slightly better. When the modulation used is 64QAM, IDAR performs better than DAR without filter at the transmitter and they both have similar behavior. IDARF, on the other hand, performs clearly better than DAR and IDAR and its behavior is closer to the case when no clipping is applied than to IDAR and DAR.

When the methods are analyzed for different clipping ratios, the simulations show that when the filter is used in the transmitter and for $CR < 7$ dB, IDARF performs better than all the other methods, for higher CR they all perform the same. When comparing all methods without filter in the receiver, IDARF performs better than IDAR and DAR for $CR < 4$ dB, for higher CR their performances are very similar.

The filter is an important element in OFDM systems since it reduces the out-of-band radiation and ICI caused by clipping. As seen in the simulations, when the filter is used in the transmitter, the method that better performs in the receiver is IDARF. Therefore, when clipping is applied, the better solution in order to mitigate the noise that it introduces is to use the FFT filter together with the IDARF method.

Some research can still be done regarding the IDARF method. When the filter was tested in the transmitter, the better results were obtained for one iteration of the filter and because of this all the simulations were carried out for one iteration of the filter, both in the transmitter and in the IDARF method. It would be interesting to evaluate the performance of the filter for more than one iteration in the transmitter and in the IDARF method and also for different number of iterations in each block. In this project the simulations have been done either using the 16QAM or the 64QAM modulations, but in both cases using the same modulation in all the subcarriers. It would be interesting to evaluate the performance of the system when different modulations are used in the different subcarriers. Regarding the code, it could be improved by making it faster.

5.2. Environmental impact

When the amplifier works in the saturation region, it consumes more power than when it works in the linear region. The high PAPR of OFDM systems causes the amplifier to work in the saturation region, making it to consume more power, but by clipping the high peaks are reduced avoiding this way the saturation region and reducing the power consumption. By using the filter, the out-of-band radiation is also reduced, causing less interference to the users of the frequencies nearby so that they do not need to increase the power of their transmissions.

GLOSSARY

AWGN	additive white Gaussian noise, 4
BER	bit error rate, 15
CCDF	complementary cumulative distribution function, 14
CR	clipping ratio, 17
DAC	digital-to-analog converter, 9
DAR	decision-aided reconstruction, 20
DFT	discrete Fourier transform, 7
DSP	digital signal processing, 4
FDM	frequency division multiplexing, 6
FFT	fast Fourier transform, 4
IBO	input backoff, 15
ICI	intercarrier interference, 11
IDAR	improved DAR, 23
IFFT	inverse FFT, 4
ISI	intersymbol interference, 6
OA	operational amplifier, 15
OFDM	orthogonal frequency division multiplexing, 4
PAPR	peak-to-average power ratio, 9
psd	power spectral density, 5
PSK	phase shift keying, 9
QAM	quadrature amplitude modulation, 9
SCNR	signal to clipping noise ratio, 18
SNR	signal-to-noise ratio, 20
VLSI	very large integrated circuits, 4

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