Validation of Equivalent Viscous Damping Methodologies

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Validation of Equivalent Viscous Damping Methodologies

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Foreword

The author chose to write this report in English for several reasons. First, because it is the working language in the European Space Agency. In addition, a copy of this report will remain in the Structures Section TEC-MSS archives so it can be consulted by any Structural Engineer in the section. Moreover, this document will also be presented as a Master's Thesis report at the Polytechnic University of Catalonia UPC-ETSEIB and at the French Aerospace University ISAE-SUPAERO so it needed to be written in a language that can be understood by all the parties.

It has been proposed that the author presents this work in the *European Conference on Spacecraft Structures, Materials & Environmental Testing* that will take place the 20-23 March 2012 at ESA/ESTEC as well as to write a paper on this project to publish it in the *Council of European Aerospace Societies* (CEAS) Journal.

Avant-propos

L'auteur choisit d'écrire ce rapport en Anglais pour plusieurs raisons. Tout d'abord, l'anglais est la langue de travail à l'ESA. De plus, une copie de ce rapport restera dans les archives de la Section Structures TEC-MSS de sorte qu'il puisse être consulté par les Ingénieurs Structure de la section. En revanche, ce document sera présenté comme Projet Fin d'Études à l'Institut Supérieur de l'Aéronautique et de l'Espace ISAE-SUPAERO ainsi qu'à l'Université Polytechnique de Catalogne UPC-ETSEIB. Il était donc essentiel qu'il soit compréhensible pour tout le monde.

De plus, l'auteur présentera ce travail à la conférence *European Conference on Spacecraft Structures, Materials & Environmental Testing* qui aura lieu du 20 au 23 Mars 2012 à l'ESA/ESTEC. Un article sur ce projet devrait aussi être publié dans le journal CEAS (*Council of European Aerospace Societies*).

Abstract

To obtain accurate predictions of the satellite dynamic environment it is therefore essential that the damping of the system is correctly defined and taken into account within the resolution methodologies for the Coupled Loads Analysis (CLA).

When working with Finite Element Models, the damping of the materials is characterized by structural damping ratios. In addition, most of the load cases present in the CLA are transient excitations. Consequently, the resolution of the equations of motion must be done in the time domain. Unfortunately, the transient analyses cannot be carried out using structural damping characterization since they cannot be used in the time domain. Therefore, a transformation from a structural to a viscous damping characterization needs to be performed.

Nevertheless, this transformation from structural damping to viscous damping models is not trivial. There exist many methodologies that aim at computing an Equivalent Viscous Damping Matrix of the system so it can be used in transient analyses.

This document describes the results obtained by the author as well as to evidence his contribution for the European Space Agency as an intern in the Structures Section TEC-MSS in the validation of the Equivalent Viscous Damping methodologies.

The project was born when the results on finite element models for one Vega propulsion stages where not correlated to the experimental data from the firing tests.

During the course of the internship, the limitations in the Equivalent Viscous Damping methodologies that are used in ESA's *Coupled Loads Analysis Toolbox* have been identified. The author implemented an enhanced methodology aiming at improving these limitations. The methodology that is currently implemented in the CLA Toolbox and the *enhanced method* have been compared to a Reference method for frequency response analyses. From this comparison, we could demonstrate that the Enhanced Method does indeed predict more reliable results than the methodology of the CLA Toolbox.

Moreover, the *enhanced method* has been implemented in transient analyses. We could conclude on the applicability of the *enhanced method* for a transient load case. In addition, it has been noticed an improvement in the reliability of the results for the *enhanced method* with respect to the method that is currently implemented in the CLA toolbox.

This enhancement is a great step forward on the the accurate characterization of the load environment at the Spacecraft's interface with the launcher in the Coupled Loads Analysis.

Therefore, a future perspective is **to implement the enhanced methodology in the ESA's Coupled Loads Analysis Toolbox**.

Moreover, this *enhanced method* could be tested in the **correlation of a real Solid Rocket Motor finite element model analysis with experimental data from firing tests**. Perhaps, new enhancements will need to be introduced for real cases.

Finally, the *enhanced method* shall also **be validated before performing the Coupled Loads Analysis between the launcher and the Spacecraft** with this method.

Résumé

Afin d'aboutir à une prédiction précise de l'environnement dynamique du satellite, il est essentiel entre autres que l'amortissement soit bien défini et proprement implémenté dans les méthodologies de résolution de l'analyse couplée.

Lorsque l'on travaille avec des modèles éléments finis, l'amortissement des matériaux est défini via des coefficients d'amortissement structural. En outre, les événements majeurs analysés dans les analyses dynamiques couplées (CLA) sont des excitations transitoires. Par conséquent, la résolution du système mécanique associé doit être menée dans le domaine temporel. Or, les analyses transitoires ne peuvent pas être conduites en utilisant des modèles d'amortissement structural qui ne sont pas applicables dans ce cadre. Ainsi, une transformation du modèle d'amortissement structural en un modèle d'amortissement visqueux est la voie que nous avons suivie.

Néanmoins, le passage d'un modèle d'amortissement structural vers un modèle d'amortissement visqueux n'est pas trivial. Il existe un grand nombre de méthodologies qui cherchent à calculer une matrice d'Amortissement Visqueux Equivalent du système de telle sorte qu'elle puisse être implémentée dans des analyses transitoires.

Ce document décrit les résultats et la contribution à l'Agence Spatiale Européenne de l'auteur comme stagiaire dans la Section Structures TEC-MSS sur la validation des méthodologies d'Amortissement Visqueux Equivalent.

Ce projet fut initié au vu des résultats éléments finis d'un des étages de propulsion du lanceur Vega qui n'étaient pas conformes aux résultats d'essais dits de "Firing Test".

Pendant le stage, les limitations dans les méthodologies d'Amortissement Visqueux Equivalent utilisées par l'ESA dans l'outil de calcul de l'analyse couplée *CLA Toolbox* ont été identifiées. L'auteur a développé une méthode qui cherche à améliorer ces limitations. La méthodologie implémentée dans le *CLA Toolbox* et la *méthode proposée* ont été comparées avec une méthode de Référence pour des analyses en fréquence. Les résultats de cette comparaison démontrent que la *méthode proposée* est celle qui aboutit aux résultats les plus fiables.

Ensuite, la *méthode proposée* a été appliquée au cas d'une excitation transitoire. On a pu conclure à l'applicabilité de la méthode proposée au cas d'une excitation transitoire. De plus, une amélioration sensible de la fiabilité des résultats de la *méthode proposée* comparativement aux méthodes actuelles de la *CLA Toolbox* a été constatée.

Cette amélioration est un saut important pour la prédiction de la matrice d'amortissement visqueux équivalent et, par conséquent, pour la prédiction des réponses transitoires.

Aussi, il sera intéressant **d'implémenter la méthodologie proposée dans l'outil de calcul de l'analyse couplée *CLA Toolbox*** basé sur l'utilisation du code NASTRAN.

De plus, la *méthode proposée* devra être **testée pour la corrélation des modèles éléments finis des moteurs à propulsion solide avec les essais au banc dits de "Firing Test"**. De nouvelles améliorations pourront être nécessaires pour les cas réels.

Finalement, la *méthode proposée* devra aussi **être validée avant de réaliser les analyses dynamiques couplées Lanceur/Satellites** avec cette méthode.

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Acronyms

Abbreviations

ATV	Automated Transfer Vehicle
CB	Craig-Bampton
CEAS	Council of European Aerospace Societies
CERN	European Council for Nuclear Research
CLA	Coupled Loads Analysis
DG	Director General
DMAP	Direct Matrix Abstraction Programming
DOF	Degree Of Freedom
EAC	European Astronaut Centre
ELDO	European Launch Development Organisation
EqVD	Equivalent Viscous Damping
ESA	European Space Agency
ESAC	European Space Astronomy Centre
ESOC	European Space Operations Centre
ESRIN	European Space Centre for Earth Observation
ESRO	European Space Research Organisation
ESTEC	European Space Research and Technology Centre
FEM	Finite Element Model
FRA	Frequency Response Analysis
FRF	Frequency Response Function
ID	Identifier
ISO	Infrared Space Observatory
MATLAB	MATrix LABoratory
MAC	Modal Assurance Criterion
NMA	Normal Modes Analysis
RBM	Rigid Body Mode
SDT	Structural Dynamic Toolbox
SOC	Scientific Operations Centre
SRM	Solid Rocket Motor
SSA	Space Situational Awareness
TF	Truncation Frequency
VILSPA	VILlafranca SPAIN satellite tracking station

Symbols

B	Viscous Damping Matrix
C	Modal Viscous Damping Matrix
	Total CB Generalized DOFs
c	Modal Viscous Damping coefficient for a single DOF system
D	Reduced Structural Damping Matrix
e	Thickness

F	Physical DOFs Force Vector
f	Frequency in Hz
	CB Generalized DOFs Force Vector
	Force Vector component
h	Height
I	Identity Matrix
	Total Number of Internal DOFs
i	Imaginary number $\sqrt{-1}$
J	Total Number of Interface DOFs
K	Stiffness Matrix
k	Stiffness of a single DOF
K_S	Structural Damping Matrix
M	Mass Matrix
	Size of the Normal Modes Basis for a CB Assembled System
m	Generalized Mass Matrix
	Mass of a single DOF
P	Total number of Modal DOFs
$P\#$	Total number of Modal DOFs for the CB Model number #
Q	Amplification/Quality Factor
q	CB Generalized DOFs Vector
	Modal DOF
R	Reaction Forces Vector
S	Total Number of CB Assembled System DOFs
t	Time variable
W	Energy Dissipation
x	Physical DOFs vector
α	Real part of the Complex Eigenvalue
β	Modal Viscous Damping Diagonal Sub-Matrix
	Imaginary part of the Complex Eigenvalue
ε	Error
η	Structural Damping Coefficient
Λ	Generalized Stiffness Matrix
λ	Complex Eigenvalue
Ω	Circular frequency Variable in rad/s
ω	Circular Frequency Variable in rad/s
	Eigenfrequency
ϕ	Constraint Modes
φ	Normal Modes
Ψ	Rayleigh-Ritz Transformation Matrix
ζ	Modal Viscous Damping Coefficient

Superscripts

$\cdot$	First Derivative
$\ddot{}$	Second Derivative
$\wedge$	Amplitude
-1	Inverse
$—$	Statically (Guyan) reduced
$==$	Projected to CB System Normal Modes Basis
<i>Decoupled</i>	Computed using the Decoupled EqVD Method
<i>Decoupled EqVD Method</i>	Computed using the Decoupled EqVD Method

<i>E</i>	Total number of Elements of a FEM
<i>EqVD Method</i>	Computed using the EqVD Method
<i>NASTRAN</i>	Value obtained using NASTRAN post-processing
<i>SDT</i>	Value obtained using SDT post-processing
<i>SOL107</i>	Value obtained using SOL107 Method
<i>SOL107 – SDT</i>	Computed using SOL107-SDT tool
<i>SOL110</i>	Value obtained using SOL110 Method
<i>System BCB Method</i>	Value obtained with the System BCB Method
<i>System CB Methods</i>	Value obtained for all System CB Methods
<i>System EqVD Method</i>	Value obtained with the System EqVD Method
<i>System SDCB Method</i>	Value obtained with the System SDCB Method
<i>T</i>	Transposed Matrix

Subscripts

<i>c</i>	Critical
<i>CB</i>	Craig-Bampton Condensed Matrix
<i>CB#</i>	Craig-Bampton Model #
<i>Component#</i>	Assembled Component number #
<i>dissip</i>	Associated to Energy Dissipation
<i>e</i>	Index for elements of a FEM
<i>ext</i>	External
<i>G</i>	Related to the Global DOFs of the Finite Element Model
<i>g</i>	Index for global DOFs
<i>i</i>	Index for internal DOFs
<i>int</i>	Internal
<i>j</i>	Index for interface DOFs
<i>k</i>	Index for the k^{th} retained CB Assembled System normal mode
<i>modal</i>	Modal Frequency Response Analysis solution
<i>p</i>	Index for the p^{th} retained normal mode
<i>p#</i>	Index for the p^{th} retained normal mode of CB Model number #
<i>sys</i>	Indicates that belongs to the CB Assembled System

Functions

<i>diag()</i>	Operator to extract diagonal values from a matrix
<i>D()</i>	Creates a diagonal matrix from given values
<i>H()</i>	Complex Frequency Response Transfer Function

Chapter 1

The European Space Agency

1.1 The birth of the European Space Agency

The first notion of a common European space agency dates from the late fifties. Many European scientists left Western Europe after the World War II heading to the United States or the Soviet Union seeking a job in the Aerospace domain. Even though the Western European countries could invest in research in space related activities at the 1950s, the European scientist realised that national projects would be unable to compete against the United States and the Soviet union space programs. Therefore, in 1958, two members of the Western European Scientific Community called Edoardo Amaldi and Pierre Auger met to discuss about the foundation of a common Western European space agency and suggested that European governments should set up a "purely scientific" joint organisation for space research taking the European Organisation for Nuclear Research (CERN) as a model. It is in 1964 when the European nations decided to have two different agencies: the European Launch Development Organisation (ELDO) and the European Space Research Organisation (ESRO). The former was aimed at developing a launch system and the latter at developing spacecraft.

Later on, the different ESRO centres were established around Europe. The first was ESRIN, placed in Frascati, Italy in 1966. ESRIN began to acquire data from environmental satellites in the 1970s. Currently, ESRIN is one of the five ESA specialised centres in Europe and is known as the ESA Centre for Earth Observation. One year later, in 1967, the city of Darmstadt in Germany was chosen to hold the European Space Operations Centre (ESOC) aiming at controlling ESA's satellites. In April 1968, ESA's Research and Technology Centre (ESTEC), the third of the five ESA centres, was finally established in Noordwijk, the Netherlands, after an initial period in Delft. In 1974, ESRO and Spain signed an agreement to create a satellite tracking station called Villafranca-Spain (VILSPA) and a Scientific Operations Centre (SOC) for the Infrared Space Observatory (ISO). In 2004 VILSPA became the European Space Astronomy Centre (ESAC) as it is currently known.

In 1975, ELDO and ESRO merged becoming the European Space Agency (ESA). The 10 founding members were: Belgium, Denmark, France, Germany, Italy, the Netherlands, Spain, Sweden, Switzerland and United Kingdom. Ireland joined the Agency later in the same year. After almost 40 years of ESA, the number of member states has grown from 10 to 19 with the incorporation of Austria, Czech Republic, Finland, Greece, Luxembourg, Norway, Portugal and Romania. Moreover, Estonia, Slovenia, Poland, Hungary, Cyprus, Latvia, Lithuania and the Slovak Republic have Cooperation Agreements. Bulgaria and Malta are negotiating Cooperation Agreements and Canada is also taking part in some programmes under a Cooperation Agreement.

Figure 1.1: *ESA Member states and Associates*

At present day, the European Space Agency has about 2200 staff members distributed among five establishments in Europe, the four aforementioned ESTEC, ESAC, ESRIN and ESOC plus the European Astronaut Centre (EAC) that was established in 1990 in Cologne, Germany. ESA Headquarters are situated in Paris, France. With a budget of 4 billion Euro in 2011, ESA has already designed, tested and operated in flight over 70 satellites, has 17 scientific satellites currently in operation and has developed six types of launcher. In February 2011, the 200th launch of Ariane took place.

Since 2003, Jean-Jacques Dordain is ESA's Director General and is assisted in his work by 11 Directors, each of whom is responsible for one of ESA's programmes or for administering part of the Agency. The organisation chart with all the Directorates is depicted in figure 1.2.

Figure 1.2: *Organisation chart of ESA's DG and Directorates*

1.2 ESA purpose and space programmes

As stated in the Article II of ESA Convention of establishment of a European Space Agency, the main purpose of the European Space Agency is to provide for, and to promote, for exclusively peaceful purposes, cooperation among European States in space research and technology and their space applications, with a view to their being used for scientific purposes and for operation space application systems.

To succeed in this purpose, ESA has become one of the few space agencies in the world to combine responsibility in nearly all areas of space activity which are space science, human spaceflight, exploration, earth observation, launchers, navigation, telecommunications, technology and operations.

Thus, the ESA's space programmes related to the aforementioned areas of space activity are listed below:

- Science & Robotic Exploration
- Human spaceflight
- Mission operations
- Space Situational Awareness (SSA)
- Earth observation
- Telecommunications & Integrated applications
- Navigation
- Launchers
- Space Technology

1.3 ESA's Research & Technology Centre (ESTEC)

ESA has sites in several European countries, but the European Space Research and Technology Centre (ESTEC) is the largest. With around 2700 employees, of which around 1400 are international ESA staff, ESTEC is the place where most ESA projects are born and where they are guided through the various phases of development. This is achieved by:

- Developing and managing all types of ESA missions related to any of ESA's programmes.
- Providing all the managerial and technical competences and facilities needed to initiate and manage the development of space systems and corresponding technologies.
- Operating an environmental test centre for spacecraft, with supporting engineering laboratories specialised in systems engineering, components and materials, and working within a network of other facilities and laboratories.
- Supporting European space industry and working closely with other organisations, such as universities, research institutes and national agencies from ESA Member States, and cooperating with worldwide space agencies.

The Structures Section (TEC–MSS)

As already mentioned in the Introduction chapter, this internship has been carried out in the Structures Section (TEC–MSS) at ESTEC. This section belongs to the Technical & Quality Management Directorate (TEC) which is divided in several Departments. The Mechanical Engineering Department (TEC–M) is the one that contains the Structures & Mechanisms Division (TEC–MS) which is split in two sections, the Mechanisms Section TEC–MSM and the aforementioned Structures Section TEC–MSS.

Figure 1.3: Organisation chart of the Structures and Mechanisms Division TEC-MS

The Structures Section is the centre of competence in all areas related to mechanical systems configuration engineering and to structural design, engineering and verification. It provides support to ESA projects, preparatory programmes and technology programmes and is responsible for:

1. Overall structural configuration studies and structural design, engineering and verification of manned and unmanned spacecraft, modules, payloads and equipment, including instrument and equipment concepts and accommodation on spacecraft, as well as launch vehicle to spacecraft structural interfaces;
2. Design of special purpose structures such as antennae, light weight mirrors, high or low temperature structures, ...;
3. Configuration analysis and evaluation of constraints such as mass budgets, mass balance, accessibility studies, ...;
4. Derivation of mechanical design loads associated with space flight, orbital and ground environment, as well as derivation of test loads and requirements;
5. Prediction of structural response and evaluation of test results, including correlation of test results with analytical predictions and updating of mathematical models;
6. Application of advanced manufacturing techniques and structures materials, including the definition of testing and characterisation requirements. This is done in cooperation with other D/TEC sections;
7. Definition and development of tools, methodologies and requirements for fracture control/damage tolerance of spacecraft and payload structures;
8. Definition and development of methods for active control of structures, and structure-control interaction modelling;
9. Definition and development of methods for passive and active noise and micro-vibration/jitter suppression and for control of micro-gravity environment;
10. Elaboration development and check of the Agency's mechanical systems configuration, design, verification and integrity approaches and policy;

11. Elaboration of advanced testing and test evaluation techniques, including modal identification, force limited testing;
12. Maintenance of a repository for launchers and basic Space Station configuration data.

1.4 The VEGA programme

One of the growing trends in the international satellite market at the beginning of the 21st century is a strong increase in interest in developing small and micro-satellites. In order to maintain its competitiveness on the world market and to ensure guaranteed access to space, Europe is developing a small launcher called Vega, which complements the mid-class Soyuz and heavy-class Ariane-5 launchers, as part of the European suite of launch services.

Vega is designed to cope with a wide range of missions in order to best respond to market needs. It will be launched from Europe's Spaceport in Kourou, French Guiana. The reference for the in-orbit capability of Vega is the placing of a 1500kg payload into a 700km-altitude polar orbit. Vega, that is depicted in figure 1.4, is a 30m high single-body launcher composed of three solid propellant stages and a liquid propellant upper module used for attitude and orbit control, and satellite release. The three Solid Rocket Motor (SRM) stages are the P80 (first stage, 88 tones of propellant), the Z23 (second stage, 24 tones of propellant) and the Z9 (third stage, 10 tonnes of propellant). The liquid propellant stage is called AVUM. The lift-off mass of the launcher is about 137 tonnes.

Figure 1.4: *Artist's view of Vega Launcher and topology*

The Structures Section TEC-MSS is in charge of the dynamic response correlation of the Solid Rocket Motor (SRM) firing tests data and analytical results extracted from dynamic analyses using Craig-Bampton (CB) condensed models. Using this correlated models, they perform the so-called launcher-spacecraft Coupled Loads Analysis (CLA) which aims at computing the dynamic environment of the satellite for the most severe load cases in flight. The team is composed of 3 Structural Engineers: Sebastiaan Fransen is in charge of the P80 analysis, Hermann Fischer works on the second stage Z23 analysis and Sylvain Germès analyses the third propulsion stage Z9. The author was under the supervision of Sebastiaan Fransen and Sylvain Germès performing the validation of the Equivalent Viscous Damping methodologies that are currently used at ESA to compute an Equivalent Viscous Damping matrix for CB condensed models that will be introduced in the CLA. The internship assignment is fully presented in Chapter 2.

Chapter 2

Internship Project's Description

An important step in the design and verification process of spacecraft structures is the Coupled Loads Analysis (CLA) with the launch vehicle in the low-frequency domain. When performing such analyses on Finite Element Models (FEMs) one may evidence that solving the Equations of Motion for the complete system may be infeasible or at least extremely expensive regarding costs of computation. In order to reduce these costs, the spacecraft model and launcher substructure models are often dynamically reduced. In the CLA, the reduction technique used is the so-called Craig-Bampton (CB) Condensation technique (see Appendix C and figure 2.1). The resulting condensed assembled system can subsequently be employed in transient or frequency response analysis.

Figure 2.1: *Graphical example of a CB Condensation assembly procedure*

The three first stages of Vega launcher are basically Solid Rocket Motors (SRMs). They are comprised mainly of a composite casing and a visco-elastic solid propellant. The first material exhibit low damping ratios ($\simeq 2\%$) whereas the visco-elastic propellant has a relatively high damping ($\simeq 20 - 40\%$). To obtain accurate predictions for the satellite dynamic environment, it is therefore essential that the damping of the system is defined in a representative way and properly taken into account within the resolution methodologies of the CB condensed system.

The damping characteristics of both materials are defined by Structural Damping coefficients which, for elastic materials, are easily characterized by performing simple dedicated experiments based on material test samples. However, it is not simple to carry out these kind of tests on the solid propellant since it is an explosive. Moreover, note that the structural damping of visco-elastic materials depends on many factors such as temperature, frequency, environmental conditions, deformation velocity, etcetera^[31]. As fully described in Appendix A, the main drawback of working with a Structural Damping characterization is that it cannot be used in transient analysis, but only for Frequency Response Analysis (FRA).

Unfortunately, many of the load cases to be analysed in the Coupled Load Analysis are time dependent and, consequently, transient analyses have to be carried out. Therefore, to be able to perform the transient analyses, a transformation of the Structural Damping Matrix of the CB system into an Equivalent Viscous Damping (EqVD) Matrix is required since, as also described in Appendix A, Viscous Damping models allow to perform both Frequency and Transient Response Analyses.

Nevertheless, this transformation from Structural Damping to Viscous Damping models is not trivial. There exist methodologies that estimate the Equivalent Viscous Damping matrix of the system accurately so it can be used in transient analyses.

The objective of the Internship is:

- to review the existent Equivalent Viscous Damping Methodologies;
- to implement them in a MATLAB environment;
- to perform a numerical validation and comparison on the basis of a few benchmark cases;
- to come up with improvements, modifications and/or new methodologies.

The work is related to a recently issued technology research proposal on EqVD methodologies^[19].

Software Employed

The validation procedure has been carried out using the following software:

- **MSC Patran** to built up the benchmark FEMs and perform the Post Processing analysis.
- **MSC Nastran**:
 - to perform the FEM analyses such as Normal Modes Analysis (NMA), direct/modal complex eigenvalue analysis and direct/modal frequency response analysis. All these analyses are theoretically introduced in Appendix B;
 - to execute DMAP routines^{[10],[11],[12],[13],[14]} which have been implemented by Sebastiaan Fransen and Sylvain Germès;
- **MATLAB** with the **Structural Dynamics Toolbox (SDT)**:
 - to post process the results computed with MSC Nastran;
 - to carry out the comparison analyses of the modal parameters computed with the equivalent viscous damping methodologies;
 - to implement a CB condensation routine and the equivalent viscous damping methodologies;
 - to execute a Frequency Response Analysis (FRA) and compare the responses for all the equivalent viscous damping methodologies;
 - to perform a transient analysis for the new methodology developed by the author.

In addition, the author had to get used to working with Unix interface since it was the easiest way to launch analyses with MSC NASTRAN using ESTEC servers, to modify the *.bdf* or *.dat* files to be launched with MSC NASTRAN so they were adapted to the DMAP routines as well as to verify the result files *.f06* or the *.f04* or *.log* files related to the computation procedure followed by MSC Nastran.

To conclude, the author wanted to outline the main troubles encountered during the first weeks of the internship which were basically the adaptation to the work environment in the Section and to get used to working with software such as MATLAB, SDT or Unix Interface, in which the author was basic user, or to understanding DMAP coding. Nevertheless, the adaptation period was short. On the other hand, the problems found within the EqVD Methodologies validation procedure listed above will be remarked in every chapter.

Chapter 3

Structural Damping to Viscous Damping Models for Transient Analyses

As already mentioned in the previous chapter, the Structural Damping characterization cannot be used in time dependent computations such as Transient Analysis. However, as can be seen in table 3.1, most of the load cases present in the Coupled Loads Analysis between the launcher Vega and the Spacecraft are time dependant excitations. Therefore, in order to demonstrate that the Launcher-Spacecraft Coupled Loads Analysis meet the specifications of the Spacecraft, the transient response analyses for all the time dependent load cases need to be implemented. For that reason, the Structural Damping Matrix of the System needs to be transformed into an Equivalent Viscous Damping Matrix.

Load Case Description	Type of Excitation	Type of Analysis
P80 Ignition – Lift-off	Transient	Transient Analysis
P80 Blastwaves	Transient	Transient Analysis
P80 Ignition + Blastwaves	Transient	Transient Analysis
Aerodynamic gust pitch – Mach 1	Transient	Transient Analysis
Aerodynamic gust yaw – Mach 1	Transient	Transient Analysis
Aerodynamic gust pitch – Qmax	Transient	Transient Analysis
Aerodynamic gust yaw – Qmax	Transient	Transient Analysis
P80 flight – Pressure Oscillations	Steady-State	Frequency Response Analysis
Z23 Ignition	Transient	Transient Analysis
Z9 Ignition	Transient	Transient Analysis

Table 3.1: Load cases during the launch of VEGA

Nevertheless, this transformation from Structural Damping to Viscous Damping models is not trivial. ESA is currently employing a methodology for Vega SRM models called Equivalent Viscous Damping (EqVD) Methodology to perform such transition from a Structural model to a Viscous model. This method is based on the distribution and combination of modal strain energy and structural damping^{[16],[17],[38]}. This first methodology will be referred in this report as the *Equivalent Viscous Damping Methodology* when working at CB component level.

Another EqVD Methodology for CB Components suggested by E.Balmès and J-M. Leclère^[2] is implemented in the MATLAB *Viscoelastic Vibration Toolbox*. This method is referred in this report as the *Decoupled Equivalent Viscous Damping Methodology*.

When working with CB Assembled Systems, ESA has implemented in the *CLA Toolbox* a methodology referred as the *System BCB Methodology*. This methodology makes a strong assumption which consist on neglecting the damping at the Interface DOFs.

Furthermore, a new methodology to compute the EqVD at CB Assembled System level aiming at improving the aforementioned *System BCB Methodology* by taking into account the Interface Damping has been implemented by Sebastiaan Fransen and Sylvain Germès^[20] and will be called from now on *System SDCB Methodology*.

Finally, during the course of the internship, the author has developed a third methodology to be used for CB Assembled Systems, called *System EqVD Methodology*^[40], which aims at improving the aforementioned methods. All these methodologies are presented in the Sections that follow.

To sum up, all these methodologies carry out the transformation from a Structural Damping Model to a Viscous Damping Model. Note that the aspects that these methodologies have in common are:

- They are based on Craig-Bampton condensed models so the Finite Element Models need to be CB Condensed. The mathematical procedure to perform a CB Condensation is fully described in Appendix C.
- They all assume that the Basile Hypothesis^[26] can be used so the Modal Viscous Damping Matrix C becomes a diagonal matrix.

Thus, the step where these Equivalent Viscous Damping Methodologies differ is in the computation of the Viscous Damping Matrix B . In the next sections, all the EqVD Methodologies that have been presented in the introductory paragraphs of this chapter are fully described.

Note that there are two different groups of EqVD Methodologies. First, the methodologies that are used for CB Components, which are the *Equivalent Viscous Damping Methodology* and the *Decoupled EqVD Methodology* and, finally, the methodologies that are used for CB Assembled Systems which are the *System BCB Methodology*, the *System SDCB Methodology* and the *System EqVD Methodology*. The theoretical description of the Assembly of CB Components can be found in Appendix C.

3.1 Methodologies for CB Condensed Components

3.1.1 The Equivalent Viscous Damping Methodology

The objective of the EqVD Methodology is to compute an Equivalent Viscous Damping Matrix for a CB Model^{[17],[18],[36]}. This EqVD Methodology has been developed by Sebastiaan Fransen and is implemented in the *CLA Toolbox* used at ESA. Thus, this section aims at presenting the mathematics behind this method. It is structured following the steps to be done in order to reach to the computation of the EqVD Matrix. First, the Structural Damping Matrix of the CB Model needs to be built up to subsequently proceed to the EqVD Matrix computation.

Structural Damping Matrix for a CB Model

If we suppose that the elements of the FEM have a structural damping defined, either globally and/or element wise, we can build the Structural Damping Matrix K_S as follows:

$$K_S = \eta_g K + \sum_{e=1}^E \eta_e K_e \quad (3.1)$$

The Structural Damping Matrix is introduced in the equations of motion as a complex stiffness. Both the formal and partitioned writing of the new equations of motion are shown below:

$$M\ddot{x} + (K + iK_S)x = F + R \quad (3.2)$$

$$\begin{aligned} \begin{bmatrix} M_{jj} & M_{ji} \\ M_{ij} & M_{ii} \end{bmatrix} \begin{Bmatrix} \ddot{x}_j \\ \ddot{x}_i \end{Bmatrix} + \left(\begin{bmatrix} K_{jj} & K_{ji} \\ K_{ij} & K_{ii} \end{bmatrix} + i \begin{bmatrix} K_{Sjj} & K_{Sji} \\ K_{Sij} & K_{Sii} \end{bmatrix} \right) \begin{Bmatrix} x_j \\ x_i \end{Bmatrix} = \\ = \begin{Bmatrix} F_j \\ F_i \end{Bmatrix} + \begin{Bmatrix} R_j \\ 0 \end{Bmatrix} \end{aligned} \quad (3.3)$$

Performing a CB reduction of the model using the coordinate transformation shown in equation (3.4) (and fully described in Appendix C) one yields to the following equations of motion:

$$\begin{Bmatrix} x_j \\ x_i \end{Bmatrix} = \Psi \begin{Bmatrix} x_j \\ q_p \end{Bmatrix} \quad (3.4)$$

$$\begin{aligned} \begin{bmatrix} \overline{M}_{jj} & M_{jp} \\ M_{pj} & m_{pp} \end{bmatrix} \begin{Bmatrix} \ddot{x}_j \\ \ddot{q}_p \end{Bmatrix} + \left(\begin{bmatrix} \overline{K}_{jj} & 0_{jp} \\ 0_{pj} & \Lambda_{pp} \end{bmatrix} + iD_{CB} \right) \begin{Bmatrix} x_j \\ q_p \end{Bmatrix} \\ = \begin{Bmatrix} F_j + \phi_{ij}^T F_i \\ \varphi_{ip}^T F_i \end{Bmatrix} + \begin{Bmatrix} R_j \\ 0 \end{Bmatrix} \end{aligned} \quad (3.5)$$

where D_{CB} is the fully populated reduced structural damping matrix which has the following shape:

$$D_{CB} = \Psi^T K_S \Psi = \begin{bmatrix} D_{jj} & D_{jp} \\ D_{pj} & D_{pp} \end{bmatrix} \quad (3.6)$$

where the sub-matrices are given by:

$$D_{jj} = K_{Sjj} + K_{Sji}\phi_{ij} + \phi_{ij}^T K_{Sij} + \phi_{ij}^T K_{Sii}\phi_{ij} \quad (3.7)$$

$$D_{jp} = K_{Sji}\varphi_{ip} + \phi_{ij}^T K_{Sii}\varphi_{ip} \quad (3.8)$$

$$D_{pp} = \varphi_{ip}^T K_{Sii}\varphi_{ip} \quad (3.9)$$

The Equivalent Viscous Damping Methodology

When performing frequency response analysis, both direct and modal, one can directly solve the equation (3.5) using the fully populated reduced structural damping matrix. However, for time dependent transient analysis this Structural Damping approach does not have any sense since the time dependent results will be complex values. It is for this reason that it is necessary to convert the complex structural damping matrix iD_{CB} into a real equivalent damping matrix that allows to carry out transient analyses.

The EqVD Methodology computes the EqVD Matrix by means of a frequency response analysis considering that the substructure is clamped and deducing the amplification factors at each resonant frequency of the fixed-interface modes of the CB Model. It turns out that in this case, when clamping the interface, the following equation of motion remains from equation (3.5) for the modal DOFs, assuming mass-normalized modes:

$$I_{pp}\ddot{q}_p + \Lambda_{pp}q_p + iD_{pp}q_p = \varphi_{ip}^T F_i = f_p \quad (3.10)$$

It is interesting to remark that the modal DOFs are not decoupled since the structural damping matrix D_{pp} is fully populated. Thus, to yield to the equations for a frequency response analysis a sinusoidal excitation is assumed with a driving frequency Ω and an associated sinusoidal response:

$$f_p = \hat{f}_p e^{i\Omega t} \quad (3.11)$$

$$q_p = \hat{q}_p e^{i\Omega t} \quad (3.12)$$

The substitution of both previous equations into equation 3.10 leads to the following solution and to the complex transfer function $\mathcal{H}_{pp}(\Omega)$:

$$\hat{q}_p = \mathcal{H}_{pp}(\Omega) \hat{f}_p \quad (3.13)$$

$$\mathcal{H}_{pp}(\Omega) = [-\Omega^2 I_{pp} + \Lambda_{pp} + iD_{pp}]^{-1} \quad (3.14)$$

The next step is to compute the quality/amplification factor Q_p as the amplitude of the dynamic response over the amplitude of the static response at the substructure eigenfrequencies ω_p ($\Omega = \omega_p$). This will be done mode by mode. For the p^{th} mode we take the p^{th} diagonal term from the complex transfer function $\mathcal{H}_{pp}$ written as $\mathcal{H}_{pp}(\omega_p)$ and lead to the following equation:

$$Q(\omega_p) = \frac{\hat{q}_{dynamic}}{\hat{q}_{static}} = \frac{\hat{q}_p(\omega_p)}{\frac{\hat{f}_p}{\omega_p^2}} = \frac{|\mathcal{H}_{pp}(\omega_p)|}{\frac{1}{\omega_p^2}} \quad (3.15)$$

Finally, the Equivalent Viscous Damping Coefficient for each mode can be computed as follows and the Equivalent Viscous Damping Matrix for a CB Model can be set-up. Note that it has been assumed no damping for the interface DOFs:

$$\zeta_p^{Equivalent} = \zeta(\omega_p) = \frac{1}{2Q(\omega_p)} = \frac{1}{2\omega_p^2 |\mathcal{H}_{pp}(\omega_p)|} \quad (3.16)$$

$$B_{CB} = \begin{bmatrix} 0_{jj} & 0_{jp} \\ 0_{pj} & \beta_{pp} \end{bmatrix} \quad (3.17)$$

where the sub-matrix β_{pp} is basically a diagonal matrix computed as follows if the mass-normalized assumption is made:

$$\beta_{pp} = \mathcal{D}(2m_p \omega_p \zeta_p^{Equivalent}) = \mathcal{D}(2\omega_p \zeta_p^{Equivalent}) \quad (3.18)$$

Now, the equation (3.5) can be rewritten replacing the complex reduced Structural Damping Matrix iD_{CB} by the Equivalent Viscous Damping Matrix B_{CB} yielding to the following equation of motion:

$$\begin{aligned} & \begin{bmatrix} \overline{M}_{jj} & M_{jp} \\ M_{pj} & m_{pp} \end{bmatrix} \begin{Bmatrix} \ddot{x}_j \\ \ddot{q}_p \end{Bmatrix} + \begin{bmatrix} 0_{jj} & 0_{jp} \\ 0_{pj} & \beta_{pp} \end{bmatrix} \begin{Bmatrix} \dot{x}_j \\ \dot{q}_p \end{Bmatrix} + \\ & + \begin{bmatrix} \overline{K}_{jj} & 0_{jp} \\ 0_{pj} & \Lambda_{pp} \end{bmatrix} \begin{Bmatrix} x_j \\ q_p \end{Bmatrix} = \begin{Bmatrix} F_j + \phi_{ij}^T F_i \\ \varphi_{ip}^T F_i \end{Bmatrix} + \begin{Bmatrix} R_j \\ 0 \end{Bmatrix} \end{aligned} \quad (3.19)$$

It is remarkable that equation (3.19) can be used now for both frequency and time dependent(transient) dynamic analysis whereas equation (3.5) can be used only for frequency response analysis. However, the equivalent viscous damping coefficient is determined mode by mode which may imply large costs of computation due to the matrix inversion that needs to be done. Finally, it should be noted that the interface damping has been neglected. This assumption can be executed provided that the number of internal DOFs is relatively high compared to the number of interface DOFs ($I \gg J$). When working on models where the interface damping cannot be neglected, the method needs to be enhanced by introducing the interface damping^{[17],[20]} and/or by enriching the Transformation Matrix with a set of residual vectors^{[15],[16],[33]}. The latter procedure is also used to compensate the Truncation Frequency influence on the results.

The Equivalent Viscous Damping calculated with this methodology depends on the modal truncation since it works with the fully populated matrix D_{pp} and considers the coupling or interactions between modes via the complex transfer function $\mathcal{H}_{pp}(\Omega)$.

3.1.2 The Decoupled Methodology

The *Decoupled EqVD Methodology* aims at computing as precise as possible the EqVD Matrix reducing the costs of computation with respect to the EqVD Methodology described in the previous section. In that way, the *Decoupled EqVD Method* goes straight from the Equation of Motion

for a CB Condensed Model with Structural Damping Matrix (equation (3.5)) to the Equation of Motion with the EqVD Matrix B_{CB} (equation (3.19)) avoiding the matrix inversion step ($\mathcal{H}_{pp}(\Omega)$) for the EqVD Method by creating the EqVD sub-matrix β_{pp} with a direct relation with the reduced fixed-modes Structural Damping sub-matrix D_{pp} .

Thus, the starting point is to assume the equivalence between Structural Damping and Viscous Damping Dissipative Forces (see also Appendix A, Section A.3) of the CB Condensed Model:

$$iD_{pp} = i\omega_p\beta_{pp} \quad (3.20)$$

where, D_{pp} is the Modal Sub-matrix of the *CB Reduced Structural Damping Matrix* (see Appendix C for the whole description of the CB Condensation procedure).

Assuming mass-normalized modes:

$$\beta_{pp} = \mathcal{D}(2m_p\omega_p\zeta_p^{Decoupled}) = \mathcal{D}(2\omega_p\zeta_p^{Decoupled}) \quad (3.21)$$

One leads to the modal Decoupled EqVD Coefficient when substituting equation (3.21) into equation (3.20):

$$\zeta_p^{Decoupled} = \frac{\text{diag}(|D_{pp}|)}{2\omega_p^2} \quad (3.22)$$

In equation (3.22), the term $\text{diag}(|D_{pp}|)$ consists on the module of the p^{th} diagonal term of the reduced fixed-modes Structural Damping sub-matrix.

Finally, after computing mode by mode all the modal EqVD Coefficients the EqVD Matrix can be built up:

$$B_{CB} = \begin{bmatrix} 0_{jj} & 0_{jp} \\ 0_{pj} & \beta_{pp} \end{bmatrix} \quad (3.23)$$

Note that this method is independent from the Truncation Frequency since it only works with the diagonal terms of fully populated reduced Structural Damping Matrix neglecting the coupling between modes. This is the reason why this method is referred in this report as the *Decoupled EqVD Methodology*.

3.2 Methodologies for Systems of Assembled CB Components

3.2.1 The System BCB Methodology

The System BCB Method is the EqVD Methodology that is currently implemented in ESA's *CLA Toolbox* to be used in the transient analyses of the System VEGA-Spacecraft for the load cases shown in table 3.1. This method builds the CB System's Equivalent Viscous Damping Matrix by assembling the CB Component EqVD Matrices computed with the EqVD Method described in Section 3.1.1. Therefore, the energy dissipation of the CB Assembly Interface is not taken into account since the Viscous Damping associated to the interface DOFs j is zero.

$$B_{CBsys} = \begin{bmatrix} 0_{jj} & 0 & 0 \\ 0 & \beta_{1pp} & 0 \\ 0 & 0 & \beta_{2pp} \end{bmatrix} \quad (3.24)$$

where, when working with Mass-Normalized Eigenvectors:

$$\beta_{1pp} = \mathcal{D}(2m_{1p}\omega_{1p}\zeta_{1p}^{Equivalent}) = \mathcal{D}(2\omega_{1p}\zeta_{1p}^{Equivalent}) \quad (3.25)$$

$$\beta_{2pp} = \mathcal{D}(2m_{2p}\omega_{2p}\zeta_{2p}^{Equivalent}) = \mathcal{D}(2\omega_{2p}\zeta_{2p}^{Equivalent}) \quad (3.26)$$

Note that the assumption of zero damping at the interface DOFs is a measure that has been decided to taken to simplify the construction of the Viscous Damping Matrix but it has not any physical sense.

3.2.2 The System SDCB Methodology

The aim of this method is to improve the System BCB Methodology by introducing Energy dissipation at the interface DOFs by assigning them a viscous damping value, and to generate a fully populated Equivalent Viscous Damping Matrix B_{CBsys} . To do so, the CB System's Structural Damping Matrix D_{CBsys} is projected using a basis of M Normal Modes computed for the CB Assembled System, as described in equation (3.27) that follows:

$$\begin{bmatrix} -\omega^2 \cdot M_{CBsys} + K_{CBsys} \\ (S \times S) \quad (S \times S) \end{bmatrix} \cdot \varphi_{sys} = 0 \quad (3.27)$$

Each diagonal term of the projected D_{CBsys} matrix is taken and divided by its modal frequency to build up the System's Modal EqVD Matrix as shown in equation (3.28):

$$C_{CBsys} = \underset{(M \times M)}{diag} \left(\underset{(M \times S)}{\varphi_{sys}^T} \cdot \underset{(S \times S)}{D_{CBsys}} \cdot \underset{(S \times M)}{\varphi_{sys}} \right) \cdot \underset{(M \times M)}{diag} \left(\sqrt{\Lambda_{sys}^{-1}} \right) = \begin{bmatrix} 0_{(6 \times 6)} & & & \\ & \ddots & & \\ & & 2\omega_k \zeta_k & \\ & & & \ddots \end{bmatrix} \quad (3.28)$$

If the CB Assembled system is in Free-Free condition, the first six modal viscous damping coefficient values related to the Rigid Body Motion must be set to zero. Note that the diagonal terms of this Modal Viscous Damping Matrix C_{CBsys} are decoupled so they do not depend on the Modal Truncation performed in the Normal Modes Analysis for the CB Assembled System of equation (3.27).

Finally a back-transformation is performed to obtain the fully populated CB System's EqVD Matrix. The back-transformation is based on these two properties:

- The following triple product yields to identity:

$$\varphi_{sys}^T \cdot M_{CBsys} \cdot \varphi_{sys} = \varphi_{sys} \cdot \varphi_{sys}^T \cdot M_{CBsys} = M_{CBsys} \cdot \varphi_{sys} \cdot \varphi_{sys}^T = I \quad (3.29)$$

- The Modal damping corresponds to the assumption that the Viscous Damping matrix becomes diagonal when projected to the modal basis (Basile Hypothesis^[26]):

$$\underset{(M \times S)}{\varphi_{sys}^T} \cdot \underset{(S \times S)}{B_{CBsys}} \cdot \underset{(S \times M)}{\varphi_{sys}} = \underset{(M \times M)}{C_{CBsys}} = \begin{bmatrix} 0_{(6 \times 6)} & & & \\ & \ddots & & \\ & & 2\omega_k \zeta_k & \\ & & & \ddots \end{bmatrix} \quad (3.30)$$

where the first six zero terms of the matrix are related to the RBMs.

Using the properties just described one leads to the fully populated CB System EqVD Matrix:

$$B_{CBsys} = \underset{(S \times S)}{M_{CBsys}} \cdot \underset{(S \times S)}{\varphi_{sys}} \cdot \underset{(S \times M)}{C_{CBsys}} \cdot \underset{(M \times M)}{\varphi_{sys}^T} \cdot \underset{(M \times S)}{M_{CBsys}^T} \quad (3.31)$$

Finally, note that two Modal Truncations are carried out when using this method, one first truncation at Component level when performing the CB Condensation of the components and a second Frequency Truncation when computing the normal modes at system level.

3.2.3 The System EqVD Methodology

Finally, a third methodology for CB Assembled Systems called the System EqVD Methodology has been developed by the author during the course of the internship. This methodology executes a procedure that is similar to the EqVD Methodology at Component Level presented in Section 3.1.1 but for a CB Assembled System. The method aims at improving the accuracy of the System Modal Damping Values of the System SDCB Methodology that are computed using equation (3.28). First, the method projects the CB System Matrices using a basis of M System Normal Modes φ_{sys} :

– *System Normal Modes computation:*

$$\begin{bmatrix} -\omega^2 \cdot M_{CBsys} + K_{CBsys} \end{bmatrix}_{\substack{(S \times S) \\ (S \times S)}} \cdot \varphi_{sys} = 0 \quad (3.32)$$

– *System's Matrices Projection*

$$\overline{\overline{M_{sys}}} = \varphi_{sys}^T \cdot M_{CBsys} \cdot \varphi_{sys} = \overline{\overline{I_{sys}}} \quad (3.33)$$

$$\overline{\overline{K_{sys}}} = \varphi_{sys}^T \cdot K_{CBsys} \cdot \varphi_{sys} = \overline{\overline{\Lambda_{sys}}} \quad (3.34)$$

$$\overline{\overline{D_{sys}}} = \varphi_{sys}^T \cdot D_{CBsys} \cdot \varphi_{sys} \quad (3.35)$$

Afterwards, the Complex Transfer Function $\mathcal{H}_{sys}(\Omega)$ is computed as follows:

$$\mathcal{H}_{sys}(\Omega) = \left[-\Omega^2 \cdot \overline{\overline{I_{sys}}} + \overline{\overline{\Lambda_{sys}}} + i \cdot \overline{\overline{D_{sys}}} \right]^{-1} \quad (3.36)$$

Once the Complex Transfer Function is built, it is used to calculate the Quality/Amplification factor Q_k as the amplitude of the dynamic response over the amplitude of the static response at the system eigenfrequencies ω_k ($\Omega = \omega_k$). This calculation is done mode by mode. For the k^{th} mode we take the modulus of the k^{th} diagonal term from the Complex Transfer Function $\mathcal{H}_{sys}$, written as $\mathcal{H}_{sys,k}(\omega_k)$ leading to the following equation:

$$Q_k(\omega_k) = \frac{|\mathcal{H}_{sys,k}(\omega_k)|}{\frac{1}{\omega_k^2}} \quad (3.37)$$

Then, using this Amplification Factor Q_k , the Modal EqVD Coefficient is computed and the Modal EqVD Matrix is built as follows:

– *EqVD Coefficient Computation*

$$\zeta_k = \zeta(\omega_k) = \frac{1}{2 \cdot Q_k(\omega_k)} = \frac{1}{2 \cdot \omega_k^2 \cdot |\mathcal{H}_{sys,k}(\omega_k)|} \quad (3.38)$$

– *Modal EqVD Matrix*

$$C_{CBsys} = \begin{bmatrix} 0_{(6 \times 6)} & & & \\ & \ddots & & \\ & & 2\omega_k \zeta_k & \\ & & & \ddots \end{bmatrix} \quad (3.39)$$

Note that the EqVD Coefficient associated to the RBMs has been considered zero. Moreover, remark that for this System EqVD Methodology, the terms in the modal viscous damping

matrix C_{CBsys} depend on the modal truncation performed in the normal modes analysis at CB assembled system (equation (3.32)). Moreover, the Basile Hypothesis^[26] which states that the modal viscous damping matrix is diagonal has been again assumed.

Finally, considering the same properties described in equations (3.29) and (3.30) one yields to the fully populated System's EqVD Matrix B_{CBsys} :

$$B_{CBsys} = \underset{(S \times S)}{M_{CBsys}} \cdot \underset{(S \times M)}{\varphi_{sys}} \cdot \underset{(M \times M)}{C_{CBsys}} \cdot \underset{(M \times S)}{\varphi_{sys}^T} \cdot \underset{(S \times S)}{M_{CBsys}^T} \quad (3.40)$$

Note that as also mentioned for the System SDCB Methodology, when using this System EqVD Methodology, two modal truncations are carried out. One at component level to condense the FEM components and a modal truncation at assembled system level when projecting the system matrices into the system's modal basis.

Chapter 4

Validation of the Equivalent Viscous Damping Methodologies

This chapter aims at introducing the validation results of the Equivalent Viscous Damping Methodologies presented in Chapter 3. This validation has been based, first, on the the Modal Parameters comparison, i.e. Modal Frequencies and Modal Viscous Damping Coefficients, and, secondly, on the Frequency Response Analysis comparison. The steps followed, common for both validation procedures, have been the following:

1. Determination of a Reference Methodology
2. Comparison of the EqVD Methodologies with the Reference Methodology

To carry out the validation, several Benchmark Finite Element Models have been created. They are introduced in the next section.

4.1 Benchmark Models Description

4.1.1 Solid Rocket Motor

A simplified SRM Finite Element Model, depicted in figure 4.1, has been developed and consists basically on a cylindrical 8 mm thick Aluminium Case that is filled with Solid Propellant. This Solid Propellant is the element that introduces relatively high damping levels into the structure. The combustion chamber has also been designed by making a 1 m diameter hole in the solid propellant along the axial direction (Y axis). The SRM model is 9 m high and its external diameter measures 3 m.

Furthermore, so the SRM Model represents all the essential components of a real SRM, more components have been modelled:

- Two **Aluminium Cover Plates** representing the Domes of a real Vega Solid Rocket Motor propulsion stage. These plates are attached to the Aluminium Case and close the SRM. The aim of these two plates in the FEM is to increase the stiffness on the edges of the Case so the mode shapes of the SRM Model become more realistic. The diameter of the Cover Plates is of 3 m and they are 8 mm thick (the same as the Aluminium Case).
- Both **Aluminium Upper & Lower Skirts** in order to introduce a more realistic interface that will be useful for the future analyses where several SRM models will be assembled to reproduce the attachment of Vega SRM propulsion stages. Both skirts are 8 mm thick and their diameter is of 3 m. Both skirts are half a meter height which added to the 9 m height of both Solid Propellant and Aluminium Case makes the SRM 10 m height.

The material and geometry properties are resumed in tables 4.1 and 4.2 respectively. Finally, note that the SRM has been clamped on the free edge of the lower skirt (see figure 4.1) so the CB condensation can be executed.

MATERIALS		
	Aluminium	Solid Propellant
Young Modulus	70 GPa	40 MPa
Poisson Ratio	0,3	0,49
Density	2700 kg/m ³	1700 kg/m ³
Structural Damping Coefficient	2%	40%

Table 4.1: SRM – Material Properties

GEOMETRY		
Part	Material	Dimensions
Upper&Lower Skirts	Aluminium	$\phi = 3m$, $e=8mm$, $h=0.5m$
Forward&Aft Plates(Domes)	Aluminium	$\phi = 3m$, $e=8mm$
Case	Aluminium	$\phi = 3m$, $e=8mm$, $h=9m$
Propellant	Solid Propellant	$\phi_{ext} = 3m$, $\phi_{int} = 1m$, $h=9m$

ϕ =diameter
 e =thickness
 h =height

Table 4.2: SRM – Geometry Properties

Figure 4.1: SRM — FEM representation

4.1.2 System of Assembled Solid Rocket Motor

A Benchmark FEM formed by the Assembly of Enhanced SRM FEMs is used for the System Methodologies validation. This FEM, called *Non-Symmetric SRM System*, is created by assembling two SRM Finite Element Models with different heights so the symmetry with respect to the Assembly Interface is suppressed. This Benchmark model is depicted in figure 4.2. The material properties are the same shown in table 4.1. The geometrical properties are shown in table 4.3. Furthermore, some information about the CB Component Models is available in table 4.4.

Figure 4.2: View of Non-Symmetric SRM System Benchmark Model

GEOMETRY		
Part	Material	Dimensions
Upper&Lower Skirts	Aluminium	$\phi = 3m, e = 8mm, h = 0.5m$
Forward&Aft Plates(Domes)	Aluminium	$\phi = 3m, e = 8mm$
Case (Component 1)	Aluminium	$\phi = 3m, e = 8mm, h = 9m$
Case (Component 2)	Aluminium	$\phi = 3m, e = 8mm, h = 7m$
Propellant (Component 1)	Solid Propellant	$\phi_{ext} = 3m, \phi_{int} = 1m, h = 9m$
Propellant (Component 2)	Solid Propellant	$\phi_{ext} = 3m, \phi_{int} = 1m, h = 7m$

ϕ =diameter
 e =thickness
 h =height

Table 4.3: SRM Components Geometrical Properties

Craig-Bampton MODELS		
Model	Interface DOFs	Modal DOFs
Component 1	192	78
Component 2	192	67

Table 4.4: SRM Components CB Condensation Properties

4.2 Assessment of the Modal Parameters for CB Components

4.2.1 Selection of the Reference Methodology – The Complex Eigenvalue Methods

The aim of this section is to introduce the Complex Eigenvalue Analysis Methodologies compared as well as the procedure that has been followed to establish a Reference Methodology. The two Complex Methodologies compared are the *Direct Complex Analysis* (SOL107) and the *Modal Complex Analysis* (SOL110) and are presented below. The full mathematical description of these two methodologies can be found in Appendix B.

- **SOL107:** This methodology is the *Direct Complex Analysis* where the complete Equation of Motion (taking into account Structural&Viscous Damping dissipative forces) is exactly solved finding the exact Complex Eigenvalues (poles). The main drawback of this methodology is that, for big FEMs, the time of computation and memory space requirements are very large.
- **SOL110:** This methodology corresponds to the *Modal Complex Analysis*. In this case, the poles computation method differs from SOL107. Firstly, the method performs a Normal Modes Analysis (or Real Eigenvalue Analysis) of the FEM without considering damping effects. Afterwards, the matrices of the System are projected to the real modal basis, so the System's size is reduced to the size of the modal basis. Finally, a Direct Complex Analysis is carried out for the projected system. Note that with the Modal projection, the system size is considerably reduced so the costs of computation are reduced. Nevertheless, it is less accurate than SOL107 due to the Modal Truncation performed on the Normal Modes Analysis.

As stated in the introductory paragraph of this Chapter, the comparison analysis between the methodologies SOL107 and SOL110 is based on the difference in Modal parameters, i.e. EigenFrequency and Viscous Damping Coefficient extracted from the Complex Eigenvalues (poles). However, depending on the analysis tool used, these Modal Parameters can present significant differences. For that reason, one needs to identify the analysis tool that provides the exact values of the Modal Parameters. The author could choose between these two different tools:

- **MSC Nastran** itself by taking the Modal Frequency and Structural Damping Coefficient values displayed in the *.f06* file and converting them into a Viscous model using a MATLAB

function called *eta2xi*. It is known that Nastran effectuates a *low damping assumption*^{[22],[30]} (also described in Appendix A) when extracting the Modal Parameters. It is logical to think that when working with high damped models, NASTRAN Modal Parameters' values might be inaccurate.

- **MATLAB Structural Dynamics Toolbox SDT** using the *nasread* function to read the NASTRAN *Output2 (.op2)* file which contains the system's poles computed by NASTRAN. SDT calculates the Modal Parameters without making any assumption, so it is presumed that the values provided by this tool are exact.

Thus, a first analysis has been carried out using the SRM Model for the Direct SOL107 Methodology where the NASTRAN and SDT Modal values are compared. This analysis aims to evidence the impact of the *low damping assumption* carried out by NASTRAN on the modal parameters computation. The modal frequencies and modal viscous damping coefficients errors between NASTRAN and SDT are found using the following formulas:

- Modal Frequency Error:

$$\varepsilon_p^{NASTRAN/SDT}(\%) = 100 \cdot \frac{|f_p^{NASTRAN} - f_p^{SDT}|}{f_p^{SDT}} \quad (4.1)$$

- Modal Viscous Damping Coefficient Error

$$\varepsilon_p^{NASTRAN/SDT}(\%) = 100 \cdot \frac{|\zeta_p^{NASTRAN} - \zeta_p^{SDT}|}{\zeta_p^{SDT}} \quad (4.2)$$

The results of this comparison are shown in the following plots of figure 4.3

Figure 4.3: SRM — Modal frequency and Modal viscous damping coefficient comparison between SOL107 NASTRAN and SDT

We can observe that the errors between NASTRAN and SDT are not significant, reaching a maximum of 1.5% in frequency and a maximum of 3.5% in viscous damping coefficient. However, the author carried out the same analysis for a benchmark model coming from the automotive industry but it is not shown in this report. The errors for this model reached much higher values on high damped modes. Moreover, there are many references^{[22],[23]} which demonstrate that it is not recommended to use NASTRAN to extract modal values for high damped structures due to the *low damping assumption*. For this reason, **it has been decided to work always with SDT.**

SOL107 – SDT/SOL110 – SDT Comparison Analysis

Once the tool to compute the exact modal parameters has been identified (SDT), one can now deal with the aim of this chapter which is to identify a Reference Methodology to be used to evaluate the EqVD Methodologies by comparing modal frequencies and modal viscous damping coefficients.

Hence, in this section, the modal parameters computed with the Direct Complex Method SOL107 and the Modal Complex Method SOL110 will be compared for the SRM Model.

The parameters will be compared computing the errors as shown below:

$$\epsilon_p^{Frequency}(\%) = 100 \cdot \frac{|f_p^{SOL107} - f_p^{SOL110}|}{f_p^{SOL107}} \quad (4.3)$$

$$\epsilon_p^{Damping}(\%) = 100 \cdot \frac{|\zeta_p^{SOL107} - \zeta_p^{SOL110}|}{\zeta_p^{SOL107}} \quad (4.4)$$

The results of the evaluation are depicted in figures 4.4 and 4.5. Note that the highest modal viscous damping error between SOL107-SDT and SOL110-SDT is of 9% and the maximum frequency shifts are of 1.5%.

Figure 4.4: SRM SDT — Frequency comparison between SOL107 and SOL110

Figure 4.5: SRM SDT — Damping Coefficient comparison between SOL107 and SOL110

Moreover, the most important errors are not found in the high damped modes but in the intermediate damped modes. Hence, it seems that the Modal Complex Analysis has some problems when extracting the modal parameters of modes that are coupled with low and high damped dynamic behaviours. The author considers that errors of almost 10% cannot be assumed for a Reference methodology so, consequently, **the Modal Complex Analysis SOL110 is discarded.**

Conclusion

The main conclusion of this section is that the Direct Complex Analysis Computation SOL107 using SDT to post-process the computed poles **shall be used as a Reference** to validate, in terms of Modal Parameters comparison, the Equivalent Viscous Damping Methodologies since NASTRAN SOL107 computes the exact poles and SDT calculates the Modal Frequencies and Viscous Damping Coefficients using the exact pole definition^[22] (see Appendix A).

4.2.2 Decoupled & Equivalent Viscous Damping Methodologies evaluation

The evaluation procedure of the *EqVD Methodology* (Section 3.1.1) and the *Decoupled EqVD Methodology* (Section 3.1.2) is based on the correlation of the Modal Parameters, i.e. Modal Frequencies and Modal Viscous Damping Coefficients, computed with these two methodologies with the Reference Methodology SOL107-SDT. Thus, the Viscous Damping Coefficients computed with the *Decoupled EqVD Method* are compared to the ones provided by the Reference method SOL107-SDT. Moreover, to notice the differences between the *Decoupled EqVD Method* and the *EqVD Method*, the correlation results of the latter method with SOL107-SDT for Truncation Frequencies of 35 Hz, 60 Hz, 75Hz and 150Hz will be also presented in the same plots. Both the EqVD and the Decoupled EqVD methodologies obtain the same Modal Frequency values so the plots associated to the Frequency Values will be presented in a single plot.

The Frequency Range of analysis is 35 Hz and the Benchmark model used in the analysis is the SRM described in Section 4.1.1.

Note that when performing the CB Condensation of the SRM Model using the DMAP routine [10] developed by Sebastiaan Fransen, the Normal Modes can be filtered by their percent of effective mass. This options is really useful since there exist many singular propellant modes like the one shown in figure 4.6 which have a really low percent of effective mass. For this case of analysis, the filter value has been of $10^{-5}\%$.

Figure 4.6: *SRM Model — Mode-shape of a singular propellant mode*

After the filtering, only 17 main modes are left in the frequency range of 35 Hz. In figure 4.7, one can evidence that these 17 modes represent, in terms of Effective Mass, almost the 95% of the dynamic response of the SRM Model in translation motion for the three directions. In rotation motion, rotations along X and Z directions achieve almost the 100% of Cumulative Effective Mass and almost the 85% along the Y (axial) direction.

The author would like to outline that when this Modal Parameters evaluation has been carried out, the Modes computed with both SOL107-SDT and both the EqVD and the Decoupled EqVD Methods were not computed in the same order so, consequently, a manual re-ordering of the modes needed to be done by comparing mode-shapes with PATRAN mode by mode.

Figure 4.7: SRM — Cumulative Effective Mass against Frequency

The correlation analysis of the Modal Frequency has been carried out using the following formulation:

$$\varepsilon_p^{EqVD \text{ Method}/SOL107-SDT}(\%) = 100 \cdot \frac{|f_p^{EqVD \text{ Method}} - f_p^{SOL107-SDT}|}{f_p^{SOL107-SDT}} \quad (4.5)$$

One can see depicted in figure 4.8 that the maximum Frequency Shifts between both the EqVD and the Decoupled EqVD Methodology and the Reference SOL107-SDT are of 3.5%.

Figure 4.8: SRM — Comparison of Decoupled & EqVD Method with SOL107-SDT

The correlation of the Modal Viscous Damping has been executed by calculating the errors between both the EqVD and the Decoupled EqVD Methodologies and the Reference SOL107-SDT using the equations that follow:

$$\varepsilon_p^{Decoupled \text{ EqVD Method}/SOL107-SDT}(\%) = \frac{|\zeta_p^{Decoupled \text{ EqVD Method}} - \zeta_p^{SOL107-SDT}|}{\zeta_p^{SOL107-SDT}} \quad (4.6)$$

$$\varepsilon_p^{EqVD \text{ Method}/SOL107-SDT}(\%) = \frac{|\zeta_p^{EqVD \text{ Method}} - \zeta_p^{SOL107-SDT}|}{\zeta_p^{SOL107-SDT}} \quad (4.7)$$

The plot of these Modal Viscous Damping Coefficient errors is depicted in figure 4.9 for Truncation Frequencies of 35 Hz, 60 Hz, 75 Hz and 150 Hz. The reason why these four TFs have been used is to evidence the evolution of the EqVD Methodology with the Frequency Truncation. Note that, as described in Section 3.1.2, the *Decoupled EqVD Method* is independent from the Frequency Truncation.

Figure 4.9: *SRM — Modal Viscous Damping Coefficient computed with the Decoupled EqVD Method and EqVD Method compared to SOL107-SDT*

One can observe in figure 4.9 that the errors in Damping Coefficient committed by the *Decoupled EqVD Method* achieve maximum values of 20% for mode 11-12 (mode (h) of figure D.1 in Appendix D) whereas the *EqVD Method* gets a maximum error of 9% for mode 11-12 with a TF of 150 Hz. Note that for most of the modes, the *Decoupled EqVD Method* commits higher errors than the *EqVD Methodology* for high TFs with respect to the Reference Methodology SOL107-SDT.

Moreover, it can be deduced from figure 4.9 that most of the modes (1-2, 3, 4-5, 6, 13 and 16-17 which correspond to modes (a),(b),(c),(d),(g) and (j) in figure D.1) are coupled with modes placed between 75Hz and 150Hz since the error in Modal Viscous Damping Coefficient for the *EqVD Methodology* at a TF of 150 Hz still decreased. On the other hand, on modes 7-8, 9-10, 11-12 and 14-15 (Mode-Shapes (e), (f), (h) and (i) in figure D.1), almost a constant error has been reached. Hence, these modes are coupled with much higher frequency modes ($> 150Hz$) or we converged to a base error committed by the simplifications effectuated in the *EqVD Methodology*.

Finally, for low TFs (such as 35 Hz), the *EqVD Method* gets close to the *Decoupled EqVD Method* values but when increasing the TF, the *EqVD Method* moves away from the *Decoupled EqVD Method* values to get closer to the Reference values computed with SOL107-SDT. A general plot of the Computed *EqVD Coefficients* versus Frequency is depicted below in figure 4.10

4.2.3 Conclusion

After this analysis one can conclude with several statements concerning the evaluation of the *EqVD Methodology* and the *Decoupled EqVD Methodology*:

- The first thing that can be inferred from these results is that the *Decoupled EqVD Method* and the Reference SOL107-SDT are the Boundaries where the *EqVD Methodology* moves in between from the less accurate values closer to the *Decoupled EqVD Method* towards more precise values when increasing the TF moving closer to the Reference SOL107-SDT.
- The higher the Modal Damping Coefficient is, the higher the frequency shifts between the Reference SOL107-SDT and both the *EqVD* and the *Decoupled EqVD Methodologies* are. Besides, there are also some frequency shifts that may be caused by the Modal Truncation when performing CB Condensation.

Figure 4.10: SRM — *Decoupled EqVD Method, EqVD Method and SOL107-SDT Modal Viscous Damping Coefficient against Frequency*

- The *Decoupled EqVD Methodology* is less accurate than the EqVD Methodology since the former commits much higher errors in the Modal Viscous Damping Coefficient computation than the latter with respect to the Reference Method SOL107-SDT. To be precise, the higher error committed by the *Decoupled EqVD Method* is of 20% for mode 11-12. The higher error committed by the EqVD Methodology is of 9% for the same mode 11-12.
- It has been demonstrated in figure 4.9 that the frequency truncation plays an important role on the EqVD Coefficient computation with the EqVD Methodology, as the Damping error is reduced as TF is increased. However, increasing the TF implies the increase of the costs of computation.
- Despite the *Decoupled EqVD Methodology* is worse than the EqVD Methodology in terms of Modal Viscous Damping coefficient prediction, it reduces significantly the costs of computation. As an example, in the case of the Enhanced SRM with a TF of 75Hz, the EqVD Method performs 971 times a $[1163 \times 1163]$ Matrix inversion whereas the *Decoupled EqVD Method* avoids this procedure. It might be interesting to use the *Decoupled EqVD Method* for preliminary analysis of Modal Damping Parameters of a model.
- The *Decoupled EqVD Methodology* saves computation costs but, owing to the high errors made in most of the modes compared to the Reference Method SOL107-SDT, it is not recommended to use it as it is much less accurate than the EqVD Methodology.
- Finally, it is interesting to affirm that the EqVD Methodology could be used to compute the Modal Parameters for SRM Models since the errors committed are relatively small (less than 10%). However, it is true that the SRM Finite Element Model represents a SRM Propulsion stage but it is still a very simplified model of a Real SRM. Perhaps this same analysis should be carried out again with a Real FEM of a Vega Propulsion Stage but this moves away from the Internship's task. However, it is expected that all conclusions apply to the real SRM Models, since all the essential components are modelled (Case, Domes, Propellant and Skirts).

4.3 Assessment of the Modal Parameters for CB Assembled Systems

This section aims at evaluation of the Modal Parameters, i.e. Modal Frequency and Modal Viscous Damping Coefficient obtained with the System BCB Methodology, the System SDCB Methodology and the System EqVD Methodology and compare them with the Reference Methodology SOL107-SDT. The Benchmark Model used for this assessment is the Non-Symmetric SRM System Model presented in Section 4.1.2.

4.3.1 Comparison of the System Methodologies with the Reference Methodology

Knowing that the System EqVD Methodology depends on the Modal Truncation at System Level, several cases of Frequency Truncation at Assembled System Level have been considered in this validation procedure in order to evidence the influence of this truncation on the aforementioned methodology. The components have been CB Condensed at a TF of 50Hz. The two cases of Modal Truncation at assembled System Level are 35 Hz and 50 Hz. The frequency range of analysis 35 Hz.

Note that the System EqVD Methodology is a new Methodology so the author needed to implement a MATLAB script which performs the Assembly of the CB Superelements followed by computation of the EqVD Matrix following the System EqVD Methodology steps presented in Section 3.2.3.

Remark also that the six first modes will not be analysed since they correspond to Rigid Body Motion and, consequently, their Modal Frequency and Modal Viscous Damping is zero.

The errors in Modal Frequency are computed with the equation that follows and they are depicted in figure 4.11, knowing that the three System Methods of Section 3.2 reach the same Modal Frequency values:

$$\varepsilon_{System\ CB\ Methods/SOL107-SDT}(\%) = 100 \cdot \frac{|f_k^{System\ CB\ Methods} - f_k^{SOL107-SDT}|}{\omega_k^{SOL107-SDT}} \quad (4.8)$$

From figure 4.11 one can observe that the maximum frequency shifts reach the 5%. Most of the

Figure 4.11: *Non-Symmetric SRM System – Plot of the Modal Frequency and Errors per mode*

Frequency Errors are located for modes which can be influenced by the Truncation Frequency of the Normal Modes Analysis.

On the other hand, the Modal Viscous Damping Coefficient Error is computed as follows:

$$\varepsilon_{System\ BCB\ Method/SOL107-SDT}(\%) = 100 \cdot \frac{|\zeta_k^{System\ BCB\ Method} - \zeta_k^{SOL107-SDT}|}{\zeta_k^{SOL107-SDT}} \quad (4.9)$$

$$\varepsilon_{System\ SDCB\ Method/SOL107-SDT}(\%) = 100 \cdot \frac{|\zeta_k^{System\ SDCB\ Method} - \zeta_k^{SOL107-SDT}|}{\zeta_k^{SOL107-SDT}} \quad (4.10)$$

$$\varepsilon_{System\ EqVD\ Method/SOL107-SDT}(\%) = 100 \cdot \frac{|\zeta_k^{System\ EqVD\ Method} - \zeta_k^{SOL107-SDT}|}{\zeta_k^{SOL107-SDT}} \quad (4.11)$$

The results for Modal Viscous Damping Coefficient, depicted in figure 4.12, show that the influence of the Truncation Frequency in the System EqVD Methodology is significant for some modes. Moreover, one observes that the System BCB Methodology commits a extremely high error for the first bending mode 7-8 (50% error) and that for the second and third bending modes (14-15 and 25-26) these errors reach 15%. One evidences also that the System EqVD Methodology predicts the best Modal Damping Values for most of the modes.

For modes 28, 30-31, 32-33, 44 and 46-47 the System BCB Methodology predicts better Modal Viscous Damping Coefficients than the other two System Methodologies.

There is also mode 38-39 where the Modal Viscous Damping Coefficient for the System SDCB Method is the best of the three Methods.

To sum up, the author wants to outline that this analysis has been also carried out on a Symmetric SRM System, where both components were identical, but it is not shown in this report. What has been remarked when comparing the results of the Symmetric SRM System with the Non-Symmetric SRM System results is that when suppressing the Symmetry of the Assembled Model, the maximum errors committed by the *System SDCB Method* and the *System EqVD Method* have been reduced from 22% to 17% for the *System SDCB Method* and from 17% to 13% for the *System EqVD Method* at a TF of 50 Hz.

Figure 4.12: *Non-Symmetric SRM System – Plot of the Modal Viscous Damping Coefficient and Errors per mode*

Finally, the mode-shapes of the modes that have been mentioned previously are depicted in figure 4.13.

Figure 4.13: *Non-Symmetric SRM System – Graphical representation of several Mode Shapes*

4.3.2 Conclusion

The general conclusions of this evaluation of the Modal Parameters for a CB Assembled System are:

- The comparison results show that the System EqVD Methodology, which has been developed by the author, is the most accurate methodology of the three methods for most of the modes analysed in a Frequency Range of 35 Hz. For the Non-Symmetric SRM System Model, the maximum error committed by this System EqVD Methodology does not reach the 15% for mode 28 and for the rest of the modes the errors are found below 10%.
- It has been proven that the System EqVD Methodology depends on the Truncation Frequency at System Level and that the higher the System TF is, the better the results are for almost all the modes in the Frequency Range of analysis.
- It has also been demonstrated that the System BCB Methodology, which is currently implemented in the ESA's *CLA Toolbox*, commits extremely high errors for low frequency modes, such as bending modes, but it improves as we move higher in Frequency. This inaccuracy at low frequencies might be attributable to the fact that the System BCB Method neglects any Energy Dissipation at the Interface DOFs when considering zero Interface Damping.
- The System SDCB Methodology compensates the low frequency errors committed by the System BCB Method by introducing Interface Damping values in the EqVD Matrix. However, since it does not take into account the coupling between modes, it loses accuracy as we move higher in frequency.

The author desires to outline that to carry out this comparison analysis, most of the time was spent on PATRAN correlating, mode by mode, the mode-shapes computed with the Reference SOL107 and the ones recovered with a DMAP Routine^[14] developed by Sebastiaan Fransen for the System Methodologies since the modes were not computed in the same order.

To conclude, the validation of EqVD Methodologies in terms of Modal Parameters comparison with a Reference Methodology SOL107-SDT has been completed with this validation analysis for Systems of Assembled SRM Models.

Therefore, the next step, described in the sections that follow, is to validate the *System BCB Method*, the *System SDCB Method* and the *System EqVD Method* in a Frequency Response Analysis for the same Non-Symmetric Assembled SRM Model.

4.4 Evaluation of System Methodologies in Frequency Response Analysis

4.4.1 Identification of the Reference Methodology

After the Eigenvalue Validation Analyses described in the previous sections, a Frequency Response Analysis Validation of System Equivalent Viscous Damping Methodologies is carried out. Following the same procedure than for the Eigenvalue Validation, one needs first to identify the Reference methodology to compare the EqVD Methodologies Frequency Responses.

Therefore, this section aims at comparing two FRA methods in order to choose the Reference. The methods compared are the *Direct Frequency Response Analysis SOL108* and the *Modal Frequency Response Analysis SOL111*. Both methods are post-processed using MATLAB SDT. The mathematical theory behind them is described on Appendix B but it is highlighted below.

The Frequency Response Analysis aims at solving the Equation of Motion for a Frequency Range of analysis. If the Structural Damping Matrix and/or the Viscous Damping Matrix are defined, the system to solve is the following:

$$[-\omega^2 M + i\omega B + K + K_S] \cdot \hat{x} = \hat{F} \quad (4.12)$$

where $\hat{x}$ is a vector that represents the DOFs displacements which are solution of the Frequency Response Analysis. If one desires to obtain accelerations, one needs to transform the displacement responses for each frequency to accelerations as follows:

$$\ddot{\hat{x}} = -\omega^2 \cdot \hat{x} \quad (4.13)$$

The benchmark model used in this identification analysis is the SRM Model presented in section 4.1.1. In addition, to carry out a Frequency Response Analysis a Load Case needs to be introduced. In this case, as depicted in figure 4.14, the SRM Model's clamped interface has been replaced by an axial Unit Load distributed along the 32 Nodes of the lower-skirt's free edge.

Once the load case is introduced, the next step is to select the Nodes in the FEM where the Frequency Response will be analysed. The selected nodes are presented in figure 4.14. Given the axial load case, the DOF that will be analysed in this Section for the selected nodes is the one referred to the axial Y direction. Nevertheless, the comparison plots for the other two DOFs X and Z directions are gathered in Appendix D.

Figure 4.14: SRM – Selected Nodes for the Frequency Response Analysis

Note that the Direct FRA SOL108 solves the Equation of Motion in equation 4.12 directly and, consequently, provides exact results. On the other hand, the Modal FRA SOL111 performs a modal decomposition before solving the Equation of Motion 4.12 so some differences from SOL108 are expected due to the modal truncation. In this analysis, the truncation frequency was set to 200 Hz and the frequency range of the comparison is 100 Hz.

Thus, depicted in figure 4.15, the Frequency Response of the selected nodes for both the *Direct FRA SOL108* and the *Modal FRA SOL111* is plotted. One can clearly observe that indeed the Modal SOL111 diverges from the exact solution SOL108 as the frequency increases for the Skirt,

Propellant and Case nodes as is shown in figures 4.15 c), d) and e).

On the other hand, for the response of Dome nodes RIGHT and LEFT, both the SOL108 and the SOL111 results are really similar, even for high frequencies.

It is quite evident that the differences between methodologies are due to the Modal Truncation effectuated in the Modal SOL110 Frequency Response Analysis. If we desire to correct the Frequency Truncation we could increase the Truncation Frequency of the Normal Modes Analysis or compensate the Truncation by adding to the Modal Basis a set of Residual Vectors of by using the mode acceleration method in the process of data recovery^[15].

Figure 4.15: SRM – Frequency Response in axial direction

Conclusion

From this analysis a general conclusion can be drawn. Considering that the aim of this work is to find a Reference Methodology to validate the EqVD Methodologies for Frequency Response Analysis, this Reference Method needs to be exact. Thus, despite it saves costs of computation, the Modal SOL111 Frequency Response Analysis is discarded since the errors that it commits due to the Modal Truncation cannot be assumed for the Validation process. Therefore, the chosen Methodology to be used for the Frequency Response Analysis Validation as the **Reference Methodology** is the **Direct SOL108 Frequency Response Analysis Method**.

4.4.2 System Methodologies in Direct Frequency Response Analysis

Once the Direct Frequency Response Analysis (FRA) Methodology SOL108 has been selected to be the Reference Methodology, the next step is to validate the System BCB Methodology, the System SDCB Methodology and the System EqVD Methodology (presented in Section 3.2) in a Frequency Response Analysis. To perform such validation, the Benchmark model used is the *Non-Symmetric SRM System Model* presented in Section 4.1.2.

Several DMAP routines developed by Sebastiaan Fransen to carry out the FRA of the CB Assembled System^[11] and to recover the Physical Response of the desired DOFs of the structure^[12] were used in this validation procedure.

The Frequency Response Analysis aims basically at computing the Response of a System in the Frequency Domain for a given Excitation/Load. Thus, the Load applied on the Non-Symmetric SRM System Model is a sinusoidal Unit load in the axial direction (Y direction) which is applied in the Assembly interface as shown in figure 4.16 a). The Nodes that have been selected to extract the Frequency Response results are shown in figure 4.16 b).

The reason why the loads have been applied on the Assembly Interface is to simplify the Load Definition for the CB Assembled System since the attachment interface is where the the Superelements/Components have been condensed and, consequently, it is not necessary to project the Force Vector to the Generalized DOFs Basis via the Matrix product $\Psi^T \cdot F$ (see Appendix C).

Figure 4.16: Graphical representation of the Load Case and Selected Nodes

Thus, the Equation of Motion that is solved in the FRA for these three methods is:

$$[-\omega^2 \cdot M_{CBsys} + \omega \cdot B_{CBsys} + K_{CBsys}] \hat{q}_{sys} = \hat{F} \quad (4.14)$$

where the matrix B_{CBsys} is the EqVD Matrix computed with the three aforementioned methodologies. Hence, equation (4.14) is solved three times, one for each methodology.

The Force Vector $\hat{F}$ represents an Axial sinusoidal Unit Load distributed on the 32 Interface Nodes. Its structure is simple and is shown below:

$$\hat{F} = \begin{Bmatrix} \hat{F}_j \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \quad (4.15)$$

where for the 32 Interface Nodes j :

$$\hat{F}_j = \{ f_1 \cdots f_j \cdots f_{32} \}^T \quad (4.16)$$

with

$$f_j = \{ 0 \quad 1/32 \quad 0 \quad 0 \quad 0 \quad 0 \} \quad (4.17)$$

From equation (4.14) we obtain the System Response in Frequency in terms of Generalized DOFs displacements $\hat{q}_{sys}$. To obtain the Generalized DOFs acceleration $\ddot{\hat{q}}_{sys}$ response one needs to use the following transformation for each frequency value:

$$\ddot{\hat{q}}_{sys} = -\omega^2 \cdot \hat{q}_{sys} \quad (4.18)$$

which is structured as follows:

$$\ddot{\hat{q}}_{sys} = \begin{Bmatrix} \ddot{\hat{x}}_j \\ \ddot{\hat{q}}_{p1} \\ \ddot{\hat{q}}_{p2} \end{Bmatrix} \quad (4.19)$$

On the other hand, since we are working in the Frequency Domain, we can forget about the System Methodologies and perform the FRA using the System's CB Reduced Structural Damping Matrix D_{CBsys} as is shown in the following Equation of Motion:

$$[-\omega^2 \cdot M_{CBsys} + K_{CBsys} + i \cdot D_{CBsys}] \hat{q}_{sys} = \hat{F} \quad (4.20)$$

Again, to obtain the Generalized DOFs accelerations, equation (4.18) must be used. This fourth Methodology with the System CB Reduced Structural Damping Matrix is also validated in this chapter and is referred in the plots as D_{CBsys} Matrix.

However, with equations (4.14) and (4.20) we only obtain the Generalized DOFs displacements which are converted into Generalized accelerations with equation (4.18). To validate these accelerations with the Reference SOL108 a back-transformation to the Physical DOFs needs to be carried out. This back-transformation is done separately for each component using the following equations:

$$\ddot{\hat{x}}_{Component1} = \begin{Bmatrix} \ddot{\hat{x}}_j \\ \ddot{\hat{x}}_i \end{Bmatrix}_1 = \Psi_{Component1} \cdot \begin{Bmatrix} \ddot{\hat{x}}_j \\ \ddot{\hat{q}}_{p1} \end{Bmatrix}_1 \quad (4.21)$$

$$\ddot{\hat{x}}_{Component2} = \begin{Bmatrix} \ddot{\hat{x}}_j \\ \ddot{\hat{x}}_i \end{Bmatrix}_2 = \Psi_{Component2} \cdot \begin{Bmatrix} \ddot{\hat{x}}_j \\ \ddot{\hat{q}}_{p2} \end{Bmatrix}_2 \quad (4.22)$$

Thus, the Frequency Response Analysis has been carried out for the Non-Symmetric SRM System Model for the Load Case described in figure 4.16 and computed for the nodes in figure 4.16 b).

Since the Benchmark model is not Symmetrical, the Frequency Response for the two Components has been analysed separately and is shown, for the selected nodes, in figures 4.17 and 4.18. Note that only the axial response is depicted. The Response in the other two directions X and Z is shown in Appendix D.

For Component 1 (figure 4.17), one can see that for the Dome nodes LEFT and RIGHT, the Frequency Response predicted by the System Methodologies for the third resonance peak at 72 Hz which is more than twice lower than the Reference SOL108 peak. The *System EqVD Method* is the worst of the three System Methodologies in Component 1 node RIGHT. For both Dome Nodes, the *System BCB Method* is again the best of the three which is quite surprising since the other two System Methods were developed to improve the *System BCB Method*. Furthermore, in Component 1 node LEFT, both the *System EqVD Method* and the *System SDCB Method* reproduce an anti-resonance instead of the third resonance peak of 72 Hz.

Finally, also a frequency shift of the response that grows as the frequency increases is evidenced in figures 4.17 c) d) and e) for Component 1 nodes on the Skirt, the Propellant and the Case respectively.

On the other hand, the Structural Damping Method D_{CBsys} Matrix provides quite exact Responses with respect to the Reference SOL108 but it diverges from the Reference SOL108 for high frequency values due to the Modal Truncation executed in the component's CB condensation.

Figure 4.17: *Non-Symmetric SRM System – FRA for Component 1 in axial direction*

The Frequency Response of Component 2 is depicted in figure 4.18. Note that the same conclusions can be drawn from the results with respect to Component 1's. The Frequency Shifts present on the nodes of the Skirt, Propellant and Case are shown respectively in figures 4.18 c), d) and e), and they get higher as the frequency increases. Note that for Component's 2 Propellant note the high frequency response of the *System BCB Method* is completely erroneous.

For Dome Nodes LEFT and RIGHT shown in figures 4.18 a) and b), respectively, several things are remarked. It is quite clear that the responses get worse as the frequency increases. For node RIGHT, again the *System BCB Method* predicts the best response of the three System Methods and the *System EqVD Method* is not capable to reproduce the resonance peak at 70 Hz and it reproduces an anti-resonance instead. However, for node LEFT, the *System SDCB Method* predicts accurately the third resonance peak level. On the other hand, the *System EqVD Method* overrates the third resonance peak and the *System BCB Method* underrates it.

Finally, note that the fourth Methodology using the Structural Damping Matrix D_{CBsys} Matrix predicts exact results. Nevertheless, the effect of the Modal Truncation performed in the CB Condensation of the Superelements is noticed since the Frequency Response diverges a bit from the Reference SOL108 for high frequencies.

To sum up, the Response represented on the Finite Element Model for the resonance peaks shown in figures 4.17 and 4.18 is depicted in figure 4.19.

Figure 4.18: *Non-Symmetric SRM System – FRA for Component 2 in axial direction*

Figure 4.19: *Non-Symmetric SRM System – Modeshapes corresponding to the Resonance Peaks*

4.4.3 System Methodologies in Modal Frequency Response Analysis

Regarding the results shown in the previous section we suspected that the source of the inaccuracy of the *System SDCB* and the *System EqVD Methods* is due to the back-transformation from the System Modal Coordinates to the CB System Coordinates described in equation (3.40) on Chapter 3.

Thus, a new validation for Frequency Response Analysis has been carried out using a Modal formulation (see Appendix B) for both the System SDCB and the System EqVD Methods. The back-transformation of equation (3.40) is not executed in the Modal formulation since the Modal Viscous Damping Matrix C_{CBsys} is used instead of the CB System Viscous Damping Matrix B_{CBsys} .

The Equation of Motion that is solved in this case, for the System SDCB and System EqVD Methods is the following:

$$[-\omega^2 \varphi_{sys}^T \cdot M \cdot \varphi_{sys} + i\omega C_{CBsys} + \varphi_{sys}^T \cdot K \cdot \varphi_{sys}] \hat{q}_{modal} = \varphi_{sys}^T \cdot \hat{F} \quad (4.23)$$

where φ_{sys} is the System Normal Modes Basis computed using equation (3.32).

Then, once the modal displacements $\hat{q}_{modal}$ are found, we need to obtain the modal accelerations $\ddot{\hat{q}}_{modal}$ as follows:

$$\ddot{\hat{q}}_{modal} = -\omega^2 \hat{q}_{modal} \quad (4.24)$$

Finally, several back-transformations need to be carried out to obtain the physical accelerations. First, one needs to back-transform from the Modal coordinates to the CB System coordinates:

$$\ddot{\hat{q}}_{sys} = \varphi_{sys} \cdot \ddot{\hat{q}}_{modal} \quad (4.25)$$

and, finally, another back-transformation from the CB System coordinates to the Physical coordinates for each component is carried out as is shown in equations (4.21) and (4.22).

Thus, the Modal FRA has been carried out for the Non-Symmetric SRM System. The Frequency Response Analysis results in the axial direction are depicted on the next page in figures 4.20 and 4.21 for Components 1 and 2 respectively. The results in the other two directions can be found in Appendix D.

We can deduce from these pictures that the only differences in these responses compared to the Direct formulation responses of figures 4.17 and 4.18 are the high frequency response levels in some nodes due to the Modal Truncation carried out for the Modal FRA.

Therefore, from this Modal analysis we can conclude that the back-transformation of the Modal Viscous Damping Matrix C_{CBsys} to the CB system Viscous Damping Matrix B_{CBsys} using equation (3.40) is a correct procedure to go from the Modal coordinates to the CB system coordinates.

Conclusion

To conclude, it has been clearly seen that for the Frequency Response Analysis of CB Assembled systems, the best method to use is the CB Reduced Structural Damping Matrix D_{CBsys} since the few errors that are made are due to the CB Condensation Modal Truncation and can be solved by increasing the Components Modal Truncation Frequency or by adding Residual Vectors.

On the other hand, it has been also remarked that the *System BCB Method*, the *System SDCB Method* and the *System EqVD Method* predict inaccurate Frequency responses for some nodes as the Frequency increases. The author wants to outline that similar results have been observed for the P80, Z23 and Z9 Propulsion Stages of Vega in the Frequency Response Analyses carried out by Sebastiaan Fransen.

It has been demonstrated with the Modal FRA that the suspicion we had that the source of the inaccuracy of the *System SDCB* and the *System EqVD Methods* is due to the back-transformation from the System Modal Coordinates to the CB System Coordinates described in equation 3.40 is wrong since both Direct and Modal FRAs have similar shapes.

Figure 4.20: *Non-Symmetric SRM System – Modal FRA for Component 1 in axial direction* Figure 4.21: *Non-Symmetric SRM System – Modal FRA for Component 2 in axial direction*

On the other hand, we suspect that the inaccuracy of the *System BCB Method* resides on the consideration of zero Damping at the Interface DOFs.

4.5 General Conclusions

As stated in Chapter 2, the aim of this Internship is to perform a numerical validation and comparison of Equivalent Viscous Damping Methodologies on the basis of a few Benchmark Cases. Within this Chapter, different validation steps taken to achieve the Internship's objective have been described. First, one started with the Eigenvalue Analysis validation of Equivalent Viscous Damping Methodologies for Craig-Bampton Condensed Components/Superelements. Then, the Eigenvalue analysis validation was carried out for EqVD Methodologies for Assembled Systems of CB Superelements. Finally, these EqVD Methodologies for CB Assembled Systems have been also validated in Frequency Response Analyses.

Hence, this sections aims at presenting the general validation conclusion for each of the methodologies assessed within the chapter.

- *Equivalent Viscous Damping Methodology for CB Components*

This Equivalent Viscous Damping Methodology, developed by Sebastiaan Fransen^{[17],[18]}, has been analysed in Section 4.2.2 on the basis of Solid Rocket Motor model. Note that in this report, many validation analysis cases done and Benchmark Models used have been omitted. From the analyses carried out on a high damped Windscreen Benchmark model, the Eigenvalue validation analyses showed that this EqVD Methodology is less accurate when increasing the complexity and the damping of the Model. Nevertheless, for Solid Rocket Motor Models that have moderate Damping values, it has been demonstrated that the EqVD Methodology's accuracy to compute the modal parameters for CB Components is acceptable.

- *Decoupled Equivalent Viscous Damping Methodology*

This Methodology^[2] has been evaluated for CB Components in the Eigenvalue Analysis also presented in Section 4.2.2. It has been demonstrated that the assumptions made by this methodology yield to inaccurate Modal Viscous Damping values with respect to the Reference values for moderate damped structures such as SRMs.

- *System BCB Methodology*

The System BCB Methodology is the method that is currently implemented in ESA's *Coupled Loads Analysis Toolbox*. This method was developed to deal with the computation of the Viscous Damping Matrix for Systems of assembled CB Models so it can be used in Transient Analyses. In Section 4.3 one showed that this methodology commits really high errors in the computation of the Modal Viscous Damping Coefficient for the low frequency bending modes. However, regarding the Frequency Response Analysis described in Sections 4.4.2 and 4.4.3, one noticed that the System BCB Methodology Frequency Response is acceptable but it underrates level of the high frequency Dome Modes. Sebastiaan Fransen carried out the FRA on the Assembled Vega Launcher and obtained similar behaviours in the System BCB Method responses.

- *System SDCB Methodology*

The System SDCB Methodology for Assemblies of CB Components, developed by Sebastiaan Fransen and Sylvain Germès^[20], aims at enhancing the System BCB Methodology Viscous Damping Matrix by introducing damping in the interface DOFs. In the Eigenvalue analysis validation performed in Section 4.3 one can see that indeed the System SDCB Methodology is much more accurate than the System BCB Methodology in the Modal Viscous Damping Coefficient computation for the low frequency bending modes. However, when regarding the

Frequency Response Analysis of Sections 4.4.2 and 4.4.3, one could realise that the Frequency Response of the System SDCB Method was worse than the System BCB Method. Therefore, the fundamentals of this method need to be reviewed.

- *System EqVD Methodology*

During the course of the Internship, the author implemented this System EqVD Methodology^{[39],[40]} for Systems of Assembled CB Components, which aims at enhancing the System SDCB Method Modal Viscous Damping Matrix by taking into account the coupling between modes to compute its diagonal terms. It has been demonstrated for the Eigenvalue analysis in Section 4.3 that this System EqVD Methodology is the most accurate methodology of the three System Methodologies when computing the Modal Viscous Damping Coefficient. However, it has been also demonstrated in Sections 4.4.2 and 4.4.3 that the fundamentals of this method need to be reviewed since the Frequency Responses computed in the Frequency Response Analysis are really inaccurate for the high frequency Dome Modes.

Chapter 5

Enhancement of the System Equivalent Viscous Damping Methodologies

After the results commented in the previous chapter, it was evident that the System SDCB Method and the System EqVD Method needed to be reviewed in order to improve their accuracy for Frequency Response Analyses. Since it has been demonstrated that the back-transformation of equation (3.40) to obtain the CB Viscous Damping Matrix B_{CBsys} is correct, we realised that the point where both methodologies could be enhanced was on the construction of the Modal Viscous Damping Matrix C_{CBsys} . This matrix has been considered from the beginning as a diagonal matrix because the Basile Hypothesis^[26] was assumed.

The Basile Hypothesis states that the Modal Viscous Damping Matrix C can be considered as a Diagonal matrix when:

- The structure is globally low damped. This does not imply that the off-diagonal terms will be low with respect to the diagonal terms but they will have an insignificant influence on the frequency responses if the next point is verified;
- The Modal frequencies are not close to each other. For two neighbour modal frequencies f_k and f_l , the potential error over the responses at the frequency f_k due to the suppression of the off-diagonal term C_{kl} is proportional to $f_k^2/(f_l^2 - f_k^2)$.

Thus, in our case, for the Non-Symmetric SRM System, we computed 900 modes in a frequency range of 200 Hz which makes a ratio of one mode each 0,22 Hz. Taking two random neighbour frequencies corresponding to modes 502 and 503 we see that the potential error will be proportional to:

$$\begin{aligned} f_{502} &= 101.964 \text{ Hz} \\ f_{503} &= 101.9734 \text{ Hz} \\ \text{Potential_Error} &\propto \frac{f_{502}^2}{f_{503}^2 - f_{502}^2} = 5367 \end{aligned}$$

It is evident that for our case, none of the requirements of the Basile Hypothesis are met since our structure is highly damped and the modal frequencies are extremely close to each other.

Therefore, the enhancement introduced in the System SDCB Method and the System EqVD Method aims at converting the Modal Viscous Damping Matrix C_{CBsys} in a fully-populated matrix where the off-diagonal terms represent dissipation couplings between modes.

Thus, the enhancements for both methodologies are presented in the Sections that follow as well as their validation analyses in Frequency Response.

5.1 Enhanced System SDCB Method evaluation in Frequency Response Analysis

The original System SDCB Method presented in Section 3.2.2 computed the Modal Viscous Damping Matrix C_{CBsys} as follows:

$$C_{CBsys} = \underset{(M \times M)}{diag} \left(\underset{(M \times S)}{\varphi_{sys}^T} \cdot \underset{(S \times S)}{D_{CBsys}} \cdot \underset{(S \times M)}{\varphi_{sys}} \right) \cdot \underset{(M \times M)}{diag} \left(\sqrt{\Lambda_{sys}^{-1}} \right) = \begin{bmatrix} 0_{(6 \times 6)} & & & \\ & \ddots & & \\ & & 2\omega_k \zeta_k & \\ & & & \ddots \end{bmatrix} \quad (5.1)$$

where φ_{sys} is the System's Normal Modes Basis coming from solving the Eigenvalue problem shown below:

$$[-\omega^2 \cdot M_{CBsys} + K_{CBsys}] \cdot \varphi_{sys} = 0 \quad (5.2)$$

The resulting matrix of equation (5.1) is a diagonal matrix where the off-diagonal terms are zero. The enhancement introduced for the System SDCB Method is to enrich the C_{CBsys} Matrix by making it fully-populated. The diagonal terms are computed using equation (5.1) but the off-diagonal terms come from the following equation:

$$[C_{CBsys}]_{nm} = [C_{CBsys}]_{mn} = \frac{[\varphi_{sys}^T \cdot D_{CBsys} \cdot \varphi_{sys}]_{nm}}{\sqrt{\omega_n \cdot \omega_m}} \quad \text{for } n \neq m \quad (5.3)$$

where ω_k with $k \in [n; m]$ is the k^{th} modal eigenfrequency.

Thus, with this new procedure, the Modal Viscous Damping Matrix becomes fully-populated. The new CB Viscous Damping Matrix B_{CBsys} is computed using the same back-transformation shown in equation (3.40).

Once this Enhanced System SDCB Method has been implemented in MATLAB, a Direct Frequency Response Analysis has been carried out for the Non-Symmetric SRM System Model for the same load case presented in figure 4.16 a) of chapter 4 and the results in the axial direction are depicted in figures 5.1 and 5.2. The nodes where the response has been extracted are the same nodes used in the previous chapter and depicted in figure 4.16.

Remark that, for this Non-symmetric SRM System, the Enhanced System SDCB Method has significantly improved the System SDCB Method Frequency Responses for the Dome Nodes becoming almost equivalent to the Reference SOL108 values as shown in both figures 5.1 and 5.2 a) and b).

For the Skirt, Propellant and Case nodes we can state that the Enhanced System SDCB Method does not introduce significant changes. The frequency response shape is similar to the Reference SOL108 but the resonance levels diverge as the frequency increases. Note that for the Propellant node of Component 2 shown in figure 5.2 d) the Enhanced System SDCB Method does improve the frequency response levels with respect to the System SDCB Method in the frequency range between 50 Hz and 80 Hz.

Note that there are many references which demonstrate that including a set residual vectors on the modal bases improves significantly the errors committed due to the modal truncation that is carried out^{[15],[16],[24],[33],[34]}. We are mostly sure that, if we include residual vectors in the modal basis of the Enhanced SDCB Method, the divergence observed in the Skirt, Propellant and Case nodes responses will be corrected. The author did not have the time to implement residual vectors for the Enhanced SDCB Method since the filtering of modes made on the CB condensation complicated the procedure.

Figure 5.1: *Non-Symmetric SRM System – Enhanced System SDCB Method Component 1 FRA* Figure 5.2: *Non-Symmetric SRM System – Enhanced System SDCB Method Component 2 FRA*

5.2 Enhanced System EqVD Method evaluation in Frequency Response Analysis

Enhancing the System EqVD Method is not that easy. The System EqVD Method builds up the diagonal Modal Viscous Damping Matrix C_{CBsys} taking the modulus of the k^{th} diagonal term for each resonance frequency ω_k of the Complex Transfer Function $\mathcal{H}_{sys}(\omega)$:

$$[C_{CBsys}]_{kk} = \frac{1}{\omega_k \cdot |\mathcal{H}_{sys}(\omega_k)|_{kk}} \quad (5.4)$$

The first idea we had to include non-zero off-diagonal terms to the Modal Viscous Damping Matrix was to consider the viscous coupling between modes as follows:

$$[C_{CBsys}]_{nm} = \frac{1}{\sqrt{\omega_n \omega_m} \cdot |\mathcal{H}_{sys}(\omega_n)|_{nm}} \quad \text{for } n \neq m \quad (5.5)$$

where ω_k with $k \in [n; m]$ is the k^{th} modal eigenfrequency.

Note that this first approach of equation (5.5) yields to a non-symmetric matrix. Thus, one decided to impose the symmetry of the matrix as follows:

$$[C_{CBsys}]_{nm} = [C_{CBsys}]_{mn} = \frac{1}{\sqrt{\omega_n \omega_m} \cdot |\mathcal{H}_{sys}(\omega_n)|_{nm}} \quad \text{for } n \neq m \quad (5.6)$$

However, note that in equation (5.6) we assume that, while building the C_{CBsys} Matrix, we move only row or column wise. Depending on how we decide to move, we will obtain different off-diagonal values in the C_{CBsys} Matrix. Consequently, to avoid this inconvenience a last enhancement has been proposed:

$$[C_{CBsys}]_{nm} = \frac{1}{\sqrt{\omega_n \omega_m} \cdot |\mathcal{H}_{sys}(\omega_n)|_{nm}} + \frac{1}{\sqrt{\omega_n \omega_m} \cdot |\mathcal{H}_{sys}(\omega_m)|_{mn}} \quad \text{for } n \neq m \quad (5.7)$$

Using equation (5.7) we build symmetric fully populated Modal Viscous Damping Matrix that does not depend on the way we move to build the matrix (row or column wise).

Unfortunately, when comparing the fully populated C_{CBsys} Matrix from equations (5.4) and (5.7) to the C_{CBsys} Matrix from Enhanced System SDCB Method, the author noticed that the former did not take into account the sign of the off-diagonal terms. Furthermore, deducing the sign is not trivial since the terms of the matrix $\mathcal{H}_{pp}$ are complex values.

Therefore, the author decided to compute the off-diagonal terms in the same way they are computed for the Enhanced System SDCB Method and shown in equation (5.3). Thus, the Enhanced System EqVD Method C_{CBsys} Matrix is built up as follows:

– *Diagonal terms*

$$[C_{CBsys}]_{kk} = \frac{1}{\omega_k \cdot |\mathcal{H}_{sys}(\omega_k)|_{kk}} \quad (5.8)$$

– *Off-diagonal terms*

$$[C_{CBsys}]_{nm} = [C_{CBsys}]_{mn} = \frac{[\varphi_{sys}^T \cdot D_{CBsys} \cdot \varphi_{sys}]_{nm}}{\sqrt{\omega_n \omega_m}} \quad \text{for } n \neq m \quad (5.9)$$

From the Modal Viscous Damping Matrix C_{CBsys} , we can compute the CB System Viscous Damping Matrix B_{CBsys} using the back-transformation of equation (3.40).

To sum up, equations (5.8), (5.9) and (3.40) have been implemented in MATLAB and with the resulting B_{CBsys} Matrix, one carried out a Direct Frequency Response Analysis for the Non-Symmetric SRM System Model considering the Load Case of figure 4.16 a). When performing the

Figure 5.3: Non-Symmetric SRM System – Enhanced System EqVD Method Component 1 FRA

Figure 5.4: Non-Symmetric SRM System – Enhanced System EqVD Method Component 2 FRA

Frequency Response Analysis for the nodes depicted in figure 4.16 b), one yields to the following plots depicted in figures 5.3 and 5.4 for both Components.

We can see in figures 5.3 and 5.4 that the Dome Nodes Response is not properly characterised with the Enhanced System EqVD Method for the third resonance peak since sometimes it is extremely overrated (figures 5.3 b) and 5.4 a)) or it has not the proper shape (figure 5.4 b)) with respect to the Reference SOL108 results. For the other Nodes, some changes in the response shape and level are introduced by the Enhanced System EqVD Method with respect to the results for the System EqVD Methodology but they are not really significant.

5.3 Conclusion

First, the author wants to remark that the source of the errors of the System SDCB Method and the System EqVD Method in Frequency Response has been identified in the construction of the Modal Viscous Damping Matrix C_{CBsys} . Considering this matrix as a diagonal matrix (Basile Hypothesis^[26]) was a wrong assumption since the viscous couplings between modes represented by the off-diagonal terms needed to be taken into account.

When including the off-diagonal terms we could notice that, for the Enhanced System SDCB Method, the frequency response of the dome nodes become really accurate. Characterizing the Dome modes with such accuracy is an important achievement since the dome modes are the main modes in the frequency response of real Solid Rocket Motors such as P80, Z23 or Z9.

On the other hand, for the Enhanced System EqVD Method, we could realise that the frequency response of the system is really sensitive to the way the diagonal of the C_{CBsys} Matrix is computed. Remark that the only part that is different in the C_{CBsys} Matrix of both the Enhanced System SDCB and Enhanced System EqVD Methods is the diagonal, since the off-diagonal terms are computed in the same way. Thus, we could see that the enhancement introduced in the System EqVD Method has modified the Frequency Response levels but it has not really improved its accuracy. The author believes that maybe there exists a correlation between the way the diagonal and off-diagonal terms are computed.

To conclude, a Frequency Response comparison between both Enhanced System SDCB and EqVD Methods, the System BCB Method and the Reference SOL108 has been carried out for the Symmetrical and Non-Symmetrical SRM System Models and the results are depicted in figures 5.5 and 5.6.

Regarding these plots we can affirm that the Enhanced System SDCB Method has achieved its objective which was to improve the System BCB Method Frequency Response. Hence, **the Enhanced System SDCB Method could replace the System BCB Method in ESA's CLA Toolbox to carry out Transient Response Analyses.** On the other hand, the Enhanced System EqVD Method needs to be reviewed to improve the Frequency Response results.

Figure 5.5: *Non-Symmetric SRM System – FRA Comparison for Component 1 in axial direction* Figure 5.6: *Non-Symmetric SRM System – FRA Comparison for Component 2 in axial direction*

Chapter 6

Application of the Enhanced System SDCB Methodology in Transient Analysis

Once it has been demonstrated that the Enhanced System SDCB Method has introduced significant improvements in the frequency response analysis, it is time to test it in a transient analysis. In order to evidence their differences, the System BCB Method will be also analysed in the same transient analysis so their responses can be compared. Remember that the System BCB Method is currently implemented in ESA's CLA Toolbox, so it is interesting to check if the Enhanced SDCB Method can replace it in such toolbox.

To carry out a transient analysis, first we need to define a time excitation. The load chosen corresponds to a Ricker function^[21] that has the shape shown in figure 6.1 a). One of the reasons why the Ricker excitation function has been chosen is because we can select the frequency range that we desire to excite. In chapter 4 we computed the frequency response of the Non-Symmetric SRM system over a frequency range of 100 Hz. For that reason, we decided to establish the frequency cut of the Ricker function at 100 Hz so only the modes up to 100 Hz will be excited (see figure 6.1 b)). Hence, we expect that the transient response of the system will be mainly represented by its frequency response up to 100 Hz.

Figure 6.1: *Graphical representation of the Ricker function used for the excitation*

This force is applied in the axial direction at the assembly interface of the Non-Symmetric SRM System model in the same way that has been shown in figure 4.16 a) in chapter 4 so the transient responses can be contrasted to the frequency response.

To compute the transient response associated to both Enhanced System SDCB Method and System BCB Method, the CB viscous damping matrix B_{CBsys} computed with both methods will be implemented in the time dependant equation of motion that follows:

$$M_{CBsys}\ddot{x}(t) + B_{CBsys}\dot{x}(t) + K_{CBsys}x(t) = f_{Ricker}(t) \quad (6.1)$$

To solve equation (6.1) we used the Newmark method^[21] which is commonly used in transient computations. This Newmark methodology is not described in this report for several reasons. On one hand because to fully understand the mathematical procedure of the Newmark method, the author would have needed to devote lot of time we did not had since the transient computations were run in the last weeks. On the other hand because this chapter aims basically at showing the application of the Enhanced System SDB Methodology and its improvements and, therefore, focusing on describing a mathematical methodology moves away from the objective of the chapter.

The time response will be calculated up to 2 seconds and the responses computed will be accelerations. The author wants to outline that the modal truncation in the normal modes analysis for the Enhanced System SDCB Method has been done for 1000 Hz.

6.1 Computation of the Reference solution

This section is really delicate since in transient analyses we cannot obtain an exact reference methodology as it can be done in the frequency domain. Nevertheless, the other reason why we have chosen the Ricker excitation function is because it is easy to carry out its Fourier transformation to the frequency domain and to back-transform it with the Fourier inverse transformation.

Thus, the Reference transient response will come from the following procedure. First, the Frequency Response Function ($FRF(\omega)$) for an axial unit load applied at the assembly interface is computed on the Non-Symmetric SRM System FEM using the Direct Frequency Response computation. Note that there is a direct relationship between the total time of analysis T and the time step Δt with the maximum frequency of analysis F_{max} and the frequency step Δf which is shown below:

$$\Delta f = \frac{1}{T} \quad (6.2)$$

$$\Delta t = \frac{1}{2F_{max}} \quad (6.3)$$

Therefore, to compute the time response over 2 seconds, from equation (6.2) we deduce that the frequency step Δf must be of 0.5 Hz. On the other hand, after several computations we arrived to the conclusion that a time step of $2.5 \cdot 10^{-4}$ seconds is small enough to characterize the time response curves. Consequently, from equation (6.3) we deduce the that the frequency range of analysis must of 2000 Hz.

With the same T and Δt , the Ricker excitation function is transformed to the frequency domain via a Fast Fourier transformation. Then, using the Fourier properties, we can easily obtain the frequency response of the system $\ddot{\hat{X}}(\omega)$ for the Ricker excitation by multiplying the $FRF(\omega)$ with the Ricker excitation in the frequency domain $\hat{F}(\omega)$ as is shown below:

$$\ddot{\hat{X}}(\omega) = FRF(\omega) \cdot \hat{F}(\omega) \quad \text{with} \quad \hat{F}(\omega) = FFT(f_{Ricker}(t)) \quad (6.4)$$

Finally, the last step is to back-transform the frequency response $\ddot{\hat{X}}(\omega)$ to obtain the time response $\ddot{X}(t)$ via the inverse Fast Fourier transformation.

Note that when performing the Fourier transformations many assumptions are taken into account. For that reason, the time responses with this procedure will not be purely exact.

6.2 Transient Responses Comparison

This section aims at showing and comparing the results of the transient analysis performed on the Non-Symmetric SRM System model given the Ricker excitation at the assembly interface for the Enhanced System SDCB Method and the System BCB Method with the Reference response. Several nodes of the FEM which correspond to the points of analysis of chapter 4 and depicted in figure 4.16 b) are analysed. Note that the transient analysis has been performed up to 2 seconds but the plots will be shown in the time range where we see a response.

But first, it is interesting to observe the axial response for one of the assembly interface to observe influence on the response of the rigorous assumption made System BCB Methodology when it neglects the damping at the interface. The responses are depicted in figure 6.2.

Figure 6.2: *Time response comparison in axial direction for an Interface node*

One can see in figure 6.2 a) that the System BCB Method response presents a shift with respect to the Reference and it is less damped. Note that the Enhanced System SDCB Method predicts a really accurate response with respect to the Reference. Although we expected that the response of the System BCB Method at the interface would be more inaccurate, we can see that its response is not bad. This effect can be explained by the fact that excitation is applied at the interface so the interface nodes follow the excitation.

For the propellant node of Component 2, it is interesting to observe both the lateral and axial responses. First, in figure 6.3 we plot the lateral direction response.

Figure 6.3: *Time response comparison in lateral direction for the Propellant node of Component 2*

One observes that in the response of the System BCB Method there is a low damped mode which is excited but it should not exist since it does not appear in the Reference response. This is not surprising if we observe the FRF for this node depicted in figure D.22 d) in Appendix D. One sees in that figure that the System BCB Method FRF has a resonance peak at around 70 Hz that should not exist. Observing figure 6.3 a) we can easily calculate that the frequency of the low damped oscillation is also around 70 Hz. Apart from that oscillation, we can also observe that the amplitude of the main peaks is not well characterized.

On the other hand, we can say again that the Enhanced System SDCB Method predicts a really accurate response with respect to the Reference. Note that with the enhancement introduced in the method, the 70 Hz mode present in the System BCB Method response has been properly predicted so the oscillations do not appear.

For the axial direction responses depicted in figure 6.4 we can observe a similar behaviour. In figure 6.4 a) there is again a low damped response of 70 Hz that also appears in the FRF analysis of figure 5.6 d). However, in this direction the peak levels of the response are much better characterized.

The Enhanced System SDCB Method predicts a response which is almost equivalent to the Reference as can be seen in figure 6.4 b).

Figure 6.4: Time response comparison in axial direction for the Propellant node of Component 2

The next node to analyse is the Case node of Component 2. The results in the axial direction are depicted in figure 6.5 on next page. One can observe that for this node the System BCB Method obtains a good prediction of the response. The only remark to do is that the amplitude of the peaks differs slightly from the Reference. Almost the same conclusion can be drawn for the Enhanced System SDCB Method. However, the last peaks of the response are better characterized.

Another point of interest is the Skirt node of Component 1. The results in the axial direction are shown in figure 6.6 also on next page. The conclusions for this node are similar to the previous Case node. The System BCB Method predicts a good response but maybe the amplitude of the first oscillations differs from the Reference. On the other hand, one observes in figure 6.6 b) that the Enhanced System SDCB Method does not improve the amplitude of these oscillations.

Figure 6.5: Time response comparison in axial direction for the Case node of Component 2

Figure 6.6: Time response comparison in axial direction for the Skirt node of Component 1

The last nodes to analyse are the Dome nodes of the Component 1. Remember that on the Frequency Response Analysis we could observe that for an axial unit load at the interface we obtain a response in the domes which is characterized by three low damped modes at 8 Hz, 32 Hz and 72 Hz. It is expected that in this transient analysis these three modes will be excited and therefore present in the signal.

Figure 6.7: Time response comparison in axial direction for Dome node LEFT of Component 1

Several things can be observed in figure 6.7 corresponding to the Dome node LEFT. First of all that as expected there is a low damped mode that is excited. To be precise, the oscillations are around 32 Hz which corresponds to the second mode observed in the frequency response analysis. This is not surprising since, taking a look at the frequency spectrum of the Ricker excitation function of figure 6.1 b), the frequencies that are more excited are found around 30 Hz which means that this load is activating the second mode of oscillation of the domes.

Furthermore, we do not appreciate much differences between methodologies. The only appreciation to do is that the Enhanced System SDCB Method is much more accurate predicting the amplitude of the first oscillations.

Finally, the last plot depicted in figure 6.8 corresponds to the Dome node RIGHT of Component 1. We can see again that there is a low damped oscillation of 30 Hz which corresponds to the second Dome mode. On the other hand, we can see that the responses for each method are really different. In figure 6.8 a) we see that the response for System BCB Method presents a shift whereas in figure 6.8 b) the Enhanced System SDCB Method has corrected completely this shift.

Moreover, the amplitude of the first oscillations is badly characterized by the System BCB Method whereas the Enhanced System SDCB Method manages to predict almost equivalent values with respect to the Reference.

(a) System BCB Method and Reference

(b) Enhanced System SDCB Method and Reference

Figure 6.8: Time response comparison in axial direction for Dome node RIGHT of Component 1

Conclusions and Perspectives

In this chapter the conclusions for the transient analysis comparison as well as some general conclusions are drawn.

Concerning the transient analysis results one can state that the improvements on the response introduced by the Enhanced System SDCB Method are really significant. It is true that the values used as Reference are not exact but what is clear is that this Enhanced method corrects the deficiencies of the System BCB Method.

It is really important to remark that response of the Enhanced System SDCB Method is really sensitive to the Modal Truncation carried out for the System's Normal Modes Analysis. To evidence this sensitivity we decided to compute the transient response for an interface node using the B_{CBsys} Matrix of the Enhanced System SDCB Method for a modal truncation of 200 Hz depicted in figure 6.9 a), and compare the response to the results for a modal truncation of 1000 Hz in figure 6.9 b).

Figure 6.9: *Time response comparison in axial direction of the Enhanced System SDCB Method for an interface node for several Truncation Frequencies in the Normal Modes Analysis*

It is clearly seen that the modal truncation of 200 Hz is not enough to represent the response of the system. The shift between the Reference and the Enhanced System SDCB Method is big and there is an oscillation of about 280 Hz which appears. That means that, even if the Ricker excitation is defined only in the frequency range from 0 Hz to 100 Hz, we are exciting modes whose frequency is higher than our modal truncation. Hence, in our case, taking a factor 2 between the frequency range of excitation and the modal truncation was not enough to represent the response.

Note that there does not exist a rule to set this modal truncation to obtain representative results. It will depend mainly on the excitation. Hence, the author recommends, when possible, to set the modal truncation at system level to its maximum value to include all the modes of the CB system. Then, if the response is not representative enough, it shall be considered to enlarge the modal bases of the CB components.

Finally, to conclude on this report the author wants to outline that, thanks to the validation analysis of the Equivalent Viscous Damping Methodologies, one could evidence their advantages and disadvantages, to implement a new methodology and to enhance the existent methodologies leading to really accurate results. This enhancement is a great step forward on the the accurate characterization of the load environment at the Spacecraft's interface with the launcher in the Coupled Loads Analysis.

Thus, the next step shall be **to implement the Enhanced System SDCB Methodology in the ESA's Coupled Loads Analysis Toolbox**.

Moreover, the Enhanced System SDCB Methodology could be tested in the **correlation of a real Solid Rocket Motor finite element model analysis with experimental data from firing tests**. Perhaps, new enhancements will need to be introduced for real cases.

Finally, the Enhanced System SDCB Methodology shall also **be validated before performing the Coupled Loads Analysis between the launcher and the Spacecraft** with this method.

Moreover, the author also would like to remark that, due to lack of time, the Enhanced System EqVD Methodology could not be reviewed in order to achieve a better computation of the Equivalent Viscous Damping Matrix. Therefore, it is also interesting to put this **revision of the Enhanced System EqVD Methodology** as a future perspective.

Bibliography

- [1] R.Allemang, *The Modal Assurance Criterion – Twenty Years of Use and Abuse*, University of Cincinnati, Cincinnati, Ohio, Sound&Vibration Magazine, August 2003
- [2] E.Balmès, J-M.Leclère, *Viscoelastic Vibration Toolbox for Use with MATLAB*, User’s Guide Version 1.0, November 4, 2010
- [3] E.Balmès, *Modèles expérimentaux complets et modèles analytiques réduits*, Habilitation à diriger des Recherches, Chapter 2, 1997.
- [4] European Space Agency Website, *www.esa.int*
- [5] ESA Communications, *ESTEC – ESA’s Technical Heart*, European Space Agency, 2011
- [6] ESA Communications, *The European Space Agency*, Corporate Presentation June 2011, European Space Agency
- [7] ESA Communications, *All About ESA – Space for Europe*, European Space Agency, 2011
- [8] ESA Communications, *Annual Report 2008*, European Space Agency
- [9] ESA Publications, *VEGA – The European Small Launcher*, Brochure, November 2004
- [10] S.Fransen, *CB_OTM Version 9.5*, DMAP Routine, MSC/NASTRAN Version 2005 - SOL103 Normal Modes Analysis
- [11] S.Fransen, *CB_SYS_SOL108 Version 2.9*, DMAP Routine, MSC/NASTRAN Version 2005 - SOL108 Direct Frequency Response Analysis
- [12] S.Fransen, *OTM_Recovery Version 3.6*, DMAP Routine, MSC/NASTRAN Version 2005 - User DMAP for Recovery of Physical DOFs Frequency Responses
- [13] S.Fransen, *CB_SYS_SOL103 Version 3.1*, DMAP Routine, MSC/NASTRAN Version 2005 - SOL103 Normal Modes Analysis for CB Systems
- [14] S.Fransen, *CB_SYS_MODESHAPE_SOL103 Version 2.4*, DMAP Routine, MSC/NASTRAN Version 2005 - SOL103 Normal Modes Analysis Mode Shape Recovery from CB Systems
- [15] S.Fransen, *Data Recovery Methodologies for Reduced Dynamic Substructure Models with Internal Loads*, AIAA Journal Vol. 42, No.10, October 2004
- [16] S.Fransen, *Methodologies for Launcher-Payload Coupled Dynamic Analysis*, European Space Agency, TEC-MSS Structures Section.
- [17] S. Fransen, *Methodologies for Launcher-Payload Coupled Dynamic Analysis*, PhD thesis, TU-Delft, 2005
- [18] S.Fransen, H.Fischer, S. Kiryenko, D. Levesque, T. Henriksen, *Damping Methodology for Condensed Solid Rocket Motor Structural Models*, European Space Agency-ESTEC
- [19] S. Fransen, S. Germès, *Statement of Work – Equivalent Viscous Damping Methodology for Condensed Solid Rocket Motor Finite Element Models in Launcher Coupled Loads Analysis*, TEC-MSS/2011/178/ln/BF, March 2011

- [20] S.Fransen, S.Germès, *An Enhanced Equivalent Viscous Damping Technique for Systems of Condensed Models in Transient Analysis*, TEC-MSS/2011/179/lm/BF, March 2011
- [21] S.Germès, *Propagation et dissipation des efforts mécaniques entrant dans les ossatures de caisses automobiles*, Doctor Thesis for École Central Paris, July 2000
- [22] S.Germès, *Calcul des fréquences de résonance et des amortissements modaux d'une structure vibrante à partir d'un calcul d'analyse modale complexe*, PSA Peugeot Citroën, Technical Note 04NT084, April 2004
- [23] S.Germès, *Méthodologie d'extraction des amortissements modaux d'une structure à amortissement localisé - Application à un pare-brise simplifié*, PSA Peugeot Citroën, Technical Note 04NT040, April 2004
- [24] S.Germès, S.Fransen, *A residual vector method for improved damping characteristics of Craig-Bampton Models*, TEC-MSS/2011/183/lm/SG, March 2011
- [25] A.Girard, *Dynamique des structures avancée*, Unité de formation Structures et matériaux, ISAE-SUPAERO cursus 2010/11
- [26] A.Girard, N.Roy, *Dynamique des structures industrielles*, Hermes Science publications 2003
- [27] T.K.Henriksen, *Structures & Mechanisms Division*, 2011 Presentation, Structures & Mechanisms Division, TEC-MS
- [28] N.Merlette, *Amortissement des Caisses Automobiles par des Films Minces Viscoélastiques pour l'Amélioration du Confort Vibratoire*, Doctor Thesis for École Centrale de Lyon, November 2005
- [29] MSC Software, *MD Nastran 2011 & MSC Nastran 2011 Quick Reference Guide*, MSC Software Corporation, Revision 0, March 30, 2011
- [30] MSC Software, *MD Nastran 2011 & MSC Nastran 2011 Dynamic Analysis User's Guide*, MSC Software Corporation, Revision 0, March 28, 2011
- [31] A.Nashif, D.Jones, J.Henderson, *Vibrations Damping*, John Wiley & Sons, 1985
- [32] A.Rittweger, S.Dieker, K.Abdoly, J.Albus, *Coupled Dynamic Load Analysis with different component damping of the Substructures*, IAC-08-C2.3.3
- [33] N.Roy, A.Girard, *Impact of Residual Modes in Structural Dynamics*, European Conference on Spacecraft Structures, Materials & Mechanical Testing 2005, ESA-ESTEC
- [34] Top Modal, *Amortissement Structural avec vecteurs résiduels*, Technical Note
- [35] X.Vaquer Araujo, *Report of the Benchmark models Analysis*, TEC-MSS, European Space Agency-ESTEC, April 2011
- [36] X.Vaquer Araujo, *Comparison between an Equivalent Viscous Damping Methodology and a Direct Complex Analysis performed on two Benchmark Models*, TEC-MSS, European Space Agency-ESTEC, May 2011
- [37] X.Vaquer Araujo, *Comparison between an Equivalent Viscous Damping Methodology and a Direct Complex Analysis performed on an Enhanced Solid Rocket Motor Model*, TEC-MSS, European Space Agency-ESTEC, May 2011
- [38] X.Vaquer Araujo, *An Equivalent Viscous Damping Methodology and a Decoupled Equivalent Viscous Damping Methodology comparison analysis performed on two SRM Finite Element Models [Reviewed Version]*, TEC-MSS, European Space Agency-ESTEC, June 2011
- [39] X.Vaquer Araujo, *Validation of a CB Condensation MATLAB Routine and a Comparison Analysis of several Methodologies used to compute the Equivalent Viscous Damping Matrix for Assembled Systems of Solid Rocket Motors*, TEC-MSS, European Space Agency-ESTEC, July 2011

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- [40] X.Vaquer Araujo, *Equivalent Viscous Damping Methodology for Assembled Systems of CB Condensed Solid Rocket Motors*, TEC-MSS, European Space Agency-ESTEC, July 2011
- [41] X.Vaquer Araujo, *Personal Notes from Advanced Structural Dynamics courses*, ISAE-SUPAERO, Year 2010/11

Appendixes

Appendix A

Structural/Viscous Damping Models

This appendix aims at describing different ways to characterise the Energy Dissipation of a system. First the Structural Damping model is presented. Then the Viscous Damping characterisation is described and finally, a relation between both models is introduced. Furthermore, the way to extract both Structural and Viscous Modal Parameters from the Complex Eigenvalues is also presented in this appendix.

The objective of the Damping characterisation is to introduce Dissipative Forces into the Equation of Motion of a non-damped system B.1 as shown below:

$$M\ddot{x}(t) + Kx(t) + F_{dissip}(t) = F(t) \quad (\text{A.1})$$

where the term $F_{dissip}(t)$ is a vector related to the internal and external dissipative forces of the system. The expression of this term depends on the Damping Model assumed. The most used models are the Structural Damping model and the Viscous Damping model described below.

A.1 Structural Damping Model

The Structural Damping characterisation states that the Dissipative Forces F_{dissip} are proportional to the strain. Hence, in the frequency domain, the Structural Damping forces are represented by a complex term proportional to displacements:

$$F_{dissip}(\omega) = iK_S x(\omega) \quad (\text{A.2})$$

where if we suppose that the elements of the FEM have a structural damping defined, either globally and/or element wise, the Structural Damping Matrix K_S is built up as follows:

$$K_S = \eta_G \cdot K + \sum_{e=1}^E \eta_e \cdot K_e \quad (\text{A.3})$$

K is the Stiffness Matrix of the system, η_G represents the *Global Structural Damping Coefficient* and η_e is the *Element Structural Damping Coefficient*.

Thus, the Equation of Motion proportional to displacements in the frequency domain considering the Structural Damping forces is the following:

$$[-\omega^2 M + (K + iK_S)] x(\omega) = F(\omega) \quad (\text{A.4})$$

The physical basis for the phenomenon of the imaginary stiffness is explained by figure A.1. On one hand, the force due to a stiffness matrix term, K , is plotted against the displacement of the point. On the other hand, as the displacements cycle sinusoidally, the introduced imaginary

stiffness, K_S , causes a phase lag in the force response resulting in an elliptical path. Thus, the area enclosed by the curve is equal to the Dissipated Energy.

Figure A.1: View of the Equivalent Hysteresis path for Structural Damping

For a steady-state sinusoidal displacement, if a Structural Damping Matrix is added to the Stiffness Matrix, the resulting forces are as shown below:

$$F(\omega) = (K + iK_S) x_0 e^{i\omega t} \quad (\text{A.5})$$

The Energy Dissipation per cycle results as follows:

$$W = \pi K_S x_0^2 \quad (\text{A.6})$$

To conclude, the Equation of Motion for a Structural Damping Model shown in equation A.4 can be used in Frequency Response Analysis but not in Transient Analyses since the results in time-dependent analyses become complex when introducing the imaginary term of Structural Damping.

A.2 Viscous Damping Model

The Viscous Damping model is based on introducing Dissipative Forces which are proportional to velocity. Thus, the Damping Force can be represented by the following equation:

$$F_{dissip}(t) = B\dot{x}(t) \quad (\text{A.7})$$

where B is the Viscous Damping Matrix. This Damping model is mostly used since it can be used in both Frequency Response Analysis and Transient Analysis.

Then, one leads to the Equation of Motion for a Viscous Damping characterisation as is shown below:

$$M\ddot{x}(t) + B\dot{x}(t) + Kx(t) = F(t) \quad (\text{A.8})$$

or, in frequency domain,

$$[-\omega^2 M + i\omega B + K] x(\omega) = F(\omega) \quad (\text{A.9})$$

A.3 Comparison between both Structural&Viscous Models

To resume Sections A.1 and A.2, the following table has been developed:

	Dissipative Force	Frequency Response	Transient Response
Structural Model	$F_{structural}(\omega) = iK_S x(\omega)$	YES	NO
Viscous Model	$F_{viscous}(t) = B\dot{x}(t)$	YES	YES

The main drawback of working with damped models is that Structural Damping ratios are the materials' damping value given by the industry since they can be easily recovered from specific tests performed on samples. Hence, if a Transient Response analysis needs to be run, a transformation from a Structural Model to a Viscous model needs to be done. Nevertheless, this transformation is not trivial.

Suppose that we characterize a system with a global constant Structural Damping η , the Equation of Motion of a single DOF system can be formally written as:

$$m\ddot{x}(\omega) + (1 + i\eta) kx(\omega) = F(\omega) \quad (\text{A.10})$$

or

$$[-\omega^2 m + (1 + i\eta) k] x(\omega) = F(\omega) \quad (\text{A.11})$$

In the same way, the Equation of Motion of the single DOF system for a Viscous Damping model is written as follows

$$m\ddot{x} + c\dot{x} + kx = F \quad (\text{A.12})$$

or

$$[-\omega^2 m + i\omega c + k] x(\omega) = F(\omega) \quad (\text{A.13})$$

If we desire to equivalence both models, equations A.11 and A.13 should be equivalent which leads to imposing that the Damping forces must be equivalent:

$$[-\omega^2 m + (1 + i\eta) k] x(\omega) = F(\omega) \iff [-\omega^2 m + i\omega c + k] x(\omega) = F(\omega) \quad (\text{A.14})$$

$$\eta k = \omega c \quad (\text{A.15})$$

Equation A.15 leads to the plot depicted in figure A.2 where both Viscous and Structural Dissipative force distributions in frequency are plotted.

From equation A.15 one can compute the circular frequency ω_c where both Dissipative models are equivalent:

$$\omega_c = \frac{\eta k}{c} \quad (\text{A.16})$$

Figure A.2: *Viscous/Structural Dissipative Forces Comparison*

Now, starting again with the equivalence of Dissipative Models of equation A.15 for a single DOF system and knowing that the *Viscous Damping Coefficient* is given by the following relation:

$$\zeta = \frac{c}{c_{critical}} = \frac{c}{2\sqrt{mk}} = \frac{c}{2m\omega_p} \quad (\text{A.17})$$

Combining equations A.15 and A.17, one leads to:

$$\zeta = \frac{\eta}{2} \cdot \frac{\omega_p}{\omega} \quad (\text{A.18})$$

At the Resonance Frequency ω_p , thanks to equation A.18, one obtain the so-called *Modal Equivalent Viscous Damping Coefficient* which values:

$$\zeta_p = \frac{\eta}{2} \quad (\text{A.19})$$

For a multi DOF system, such as a FEM, the Viscous Damping Matrix can be defined similarly to equation A.15 if the Structural Damping is defined global and constant:

$$B = \frac{\eta}{\omega} K \quad (\text{A.20})$$

In equation A.20, the Viscous Damping Matrix B is frequency dependent. By definition, this matrix should be independent from the frequency, so a frequency value ω_1 is chosen:

$$B = \frac{\eta}{\omega_1} K \quad (\text{A.21})$$

Projecting equation A.21 to the Mass-Normalized system modes one obtains:

$$\mathcal{D}(2\zeta_p\omega_p) = \frac{\eta}{\omega_1} \mathcal{D}(\omega_p^2) \quad (\text{A.22})$$

On the other hand, equation A.17 for a multi DOF system is written as follows:

$$\zeta = \frac{c}{c_{critical}} = \frac{\varphi_p^T B \varphi_p}{2\omega_p} \quad (\text{A.23})$$

Combining equations A.22 and A.23 one yields to the definition of the modal viscous damping for a Multi DOF system:

$$\zeta_p = \frac{\eta}{2} \cdot \frac{\omega_p}{\omega_1} \quad (\text{A.24})$$

As a consequence of equation A.24, the modal viscous damping profile is defined linearly, as shown in figure A.3. From this figure, one can see that for the mode whose Eigenfrequency is ω_p , its viscous damping coefficient is exactly extracted and values $\zeta_1 = \eta/2$. On the other hand, the damping of the system's modes whose Eigenfrequency ω_p is below the chosen frequency ω_1 ($\omega_p < \omega_1$) is underrated, whereas the damping of the modes for which $\omega_p > \omega_1$ is overrated.

Figure A.3: Viscous Damping for multi DOF systems

A.4 Structural/Viscous Modal Parameters extraction from Complex Eigenvalues

Considering the Eigenvalue problem with a Structural Damping Matrix:

$$[-\lambda^2 M + (K + iK_S)] \varphi = 0 \quad (\text{A.25})$$

$$\det(-\lambda^2 M + (K + iK_S)) = 0 \quad (\text{A.26})$$

or with a Viscous Damping Matrix:

$$[-\lambda^2 M + i\lambda B + K] \varphi = 0 \quad (\text{A.27})$$

$$(-\lambda^2 M + i\lambda B + K) = 0 \quad (\text{A.28})$$

In both cases, the solution of the Characteristic Equation, A.26 or A.28, is a set of Complex Eigenvalues which have the following structure:

$$\lambda_p = \alpha_p + i \cdot \beta_p \quad (\text{A.29})$$

where p refers to the p^{th} Complex Eigenvalue.

From equation A.29, one can deduce the Modal Frequency and Modal Structural/Viscous Damping Coefficient for each Complex Eigenvalue with the equations presented in the following sections.:

A.4.1 Structural Damping Model

The Structural Damping characterisation states that the p^{th} Complex pole is given by:

$$\lambda_p^2 = -\omega_p^2 (1 + i\eta) \quad (\text{A.30})$$

From equation A.30, one can deduce that the Modal Parameters using a Structural Damping characterisation are given by:

$$\omega_p = \sqrt{\beta_p^2 - \alpha_p^2} \quad (\text{A.31})$$

$$\eta_p = -\frac{2\alpha_p\beta_p}{\beta_p^2 - \alpha_p^2} \quad (\text{A.32})$$

NASTRAN Low Damping Assumption^[22]

As stated, and demonstrated, in Chapter 3, NASTRAN performs a *low damping assumption*^[22]. Equalling equations A.29 and A.30 one yields to:

$$\alpha_p + i\beta_p = \sqrt{-\omega_p^2 (1 + i\eta_p)} \quad (\text{A.33})$$

or

$$\alpha_p + i\beta_p = i |\omega_p| \sqrt{(1 + i\eta_p)} \quad (\text{A.34})$$

The procedure implemented in NASTRAN considers that:

$$\eta_p \ll 1 \quad (\text{A.35})$$

Hence, carrying out a first order reduction of equation A.34, NASTRAN leads to the following Modal Parameters:

$$\omega_p \approx \beta_p \quad (\text{A.36})$$

$$\eta_p \approx -\frac{2\alpha_p}{\omega_p} \quad (\text{A.37})$$

Note that for values of $\alpha > 0$ the Modal Structural Damping Coefficient η_p becomes negative which does not have any physical sense.

A.4.2 Viscous Damping Model

The Viscous Damping Model states that the Complex Eigenvalue A.29 structure is as shown below:

$$\alpha_p + i\beta_p = \omega_p \left(-\zeta_p + i\sqrt{1 - \zeta_p^2} \right) \quad (\text{A.38})$$

From equation A.38, one yields to the Modal Frequency and Modal Viscous Damping coefficient:

$$\omega_p = \sqrt{\alpha_p^2 + \beta_p^2} \quad (\text{A.39})$$

$$\zeta_p = \frac{1}{\sqrt{1 + \left(\frac{\beta_p}{\alpha_p} \right)^2}} \quad (\text{A.40})$$

Note that this is the procedure implemented in MATLAB SDT.

Appendix B

Eigenvalue and Frequency Response Analysis Methodologies

The objective of this Appendix B is to mathematically introduce the different Analysis methodologies employed in the Validation of Equivalent Viscous Damping Methodologies. Five methodologies are presented, the first three of them are related to the Eigenvalue Computation and the two last are related to the Frequency Response Analysis.

B.1 Real Eigenvalue Analysis

Normal Modes Analysis SOL103

When working with Finite Element Models, the Equation of Motion of a non-dissipative system of G Degrees of Freedom is represented as follows:

$$M\ddot{x}(t) + Kx(t) = F(t) \quad (\text{B.1})$$

where M is System's Mass Matrix, K the Stiffness Matrix, $\ddot{x}(t)$ and $\dot{x}(t)$ the displacements and acceleration vectors of the nodes of the FEM and, finally, $F(t)$ is the force vector.

Equation B.1 can also be written in frequency domain supposing that:

- The Force is sinusoidal of circular frequency ω :

$$F(t) = \hat{F}e^{i\omega t}$$

where $\hat{F}$ is a Force Vector no time-dependent.

- The System's response will also be sinusoidal of the same circular frequency ω :

$$x(t) = \hat{x}e^{i\omega t}$$

and

$$\ddot{x}(t) = -\omega^2\hat{x}e^{i\omega t}$$

where $\hat{x}$ is a no time-dependant displacement vector.

Thus, the system from equation B.1 can be written in Frequency Domain as follows:

$$[-\omega^2 M + K] \hat{x} = \hat{F} \quad (\text{B.2})$$

The Normal Modes Analysis aims at computing the system's dynamic behaviour with no applied loads, leading to solve the Equation of Motion for undamped free vibration:

$$[-\omega_p^2 M + K] \hat{x} = 0 \quad (\text{B.3})$$

To solve equation B.3, one assumes a harmonic solution of the form:

$$x(\omega) = \varphi_g \sin(\omega t) \implies \hat{x} = \varphi_g \quad (\text{B.4})$$

where φ_g is the so-called Eigenvector or Mode Shape and ω is the circular natural frequency. Equation B.3 becomes:

$$[-\omega^2 M + K] \varphi_g = 0 \quad (\text{B.5})$$

where the non-trivial solutions of the Eigenvalue Problem are found by solving the following system:

$$\det(-\omega_p^2 M + K) = 0 \quad (\text{B.6})$$

The non-trivial solutions ω_p of equation B.6 are called Eigenvalues. The number of Eigenvalues ω_p is P with $P \leq G$ which means that there are as Eigenvalues as system's Degrees of Freedom.

For each Eigenvalue, one can find the associated solution of equation B.6 called Eigenvector φ_g which represents a vector that contains the System's DOFs displacements at the Eigenfrequency ω_p . Hence, there will be also as Eigenvectors, or Mode Shapes, as Degrees of Freedom.

Orthogonality properties of the Normal Modes

The Eigenvectors φ_g have the special property of being orthogonal between them. Considering φ_{gp} the matrix containing the P Eigenvectors φ_g , also called *Modal Basis*, one can evidence their orthogonality properties with the system's matrices M and K since they become diagonal matrices when performing the following triple matrix products:

$$\varphi_{gp}^T \cdot M \cdot \varphi_{gp} = \begin{bmatrix} \ddots & & \\ & m_p & \\ & & \ddots \end{bmatrix}_{(P \times P)} \quad (\text{B.7})$$

and

$$\varphi_{gp}^T \cdot K \cdot \varphi_{gp} = \begin{bmatrix} \ddots & & \\ & k_p & \\ & & \ddots \end{bmatrix}_{(P \times P)} \quad (\text{B.8})$$

where m_p and k_p are the so-called *Generalised Mass* and *Generalised Stiffness*. Moreover, if the Normal Modes φ_g are *mass-normalised*, matrices B.7 and B.8 become:

$$\varphi_{gp}^T \cdot M \cdot \varphi_{gp} = I_{pp} \quad (\text{B.9})$$

and

$$\varphi_{gp}^T \cdot K \cdot \varphi_{gp} = \mathcal{D}(\omega_p^2)_{pp} = \Lambda_{pp} \quad (\text{B.10})$$

with

$$\omega_p^2 = \frac{k_p}{m_p} \quad (\text{B.11})$$

B.2 Complex Eigenvalue Analyses

The Complex Eigenvalue Analysis is necessary when the System's Matrices contain unsymmetric terms, damping effects or complex numbers where real modes analysis cannot be used.

The equation of motion of a Structural and/or Viscous damped system written in the frequency domain is as follows:

$$M\ddot{x}(\omega) + B\dot{x}(\omega) + (K + iK_S)x(\omega) = F(\omega) \quad (\text{B.12})$$

In the case of an unforced system ($F(\omega) = 0$), the free vibration motion of the system of equations B.12 can be expressed as the sum of the motion of its eigenvectors $(\varphi_g)_p$:

$$x(\omega) = \text{Real} \left(\sum (\varphi_g)_p \sin(\lambda_p(\omega) \cdot t) \right) \quad (\text{B.13})$$

where $\lambda_p(\omega)$ is the p^{th} Complex Eigenvalue described in Appendix A equation A.29 which has the following structure:

$$\lambda_p = \alpha_p + i \cdot \beta_p \quad (\text{B.14})$$

Leading to the Complex Eigenvalue problem:

$$[-\lambda_p^2 M + i\lambda_p B + (K + iK_S)] \varphi_{gp} = 0 \quad (\text{B.15})$$

Depending on the Complex Methodology used, equation B.15 is solved in a different way.

B.2.1 Direct Complex Eigenvalue Analysis SOL107

The Direct Complex Analysis methodology aims at solving the full equation B.15. Thus, it solves the following system of equations finding the Complex Eigenvalues λ_p that will be introduced in equation B.15 to find the Complex Eigenvectors, or Complex Mode Shapes, φ_{gp} :

$$\det(-\lambda_p^2 M + i\lambda_p B + (K + iK_S)) = 0 \quad (\text{B.16})$$

Then, as described in Section A.4 of Appendix A, depending on the Damping characterisation employed, the Modal Parameters of Modal Frequency and Modal Damping Coefficient will be extracted from α and β parameters of equation B.14 in different ways.

The main drawback of this method is that the bigger the FEM is, the bigger the costs of computation are.

B.2.2 Modal Complex Eigenvalue Analysis SOL110

The Modal Complex Analysis aims at reducing the costs of computations when solving equation B.15 by reducing the size of the System's Matrices. To achieve this reduction, the System's Matrices are projected to the Normal Modes Basis φ_{gp} which is computed by solving the Normal Modes Eigenvalue Problem shown in equation B.5.

Thus, the new Complex Eigenvalue Problem's size will be set by the size of the Normal Modes Basis as is shown below:

$$\det \left(-\lambda_p^2 \underbrace{\varphi_{gp}^T \cdot M_{gg} \cdot \varphi_{gp}}_{(P \times P)} + i\lambda_p \underbrace{\varphi_{gp}^T \cdot B_{gg} \cdot \varphi_{gp}}_{(P \times P)} + \underbrace{\varphi_{gp}^T \cdot (K_{gg} + iK_{Sgg}) \cdot \varphi_{gp}}_{(P \times P)} \right) = 0 \quad (\text{B.17})$$

where P is the size of the Modal Basis and G the size of the System with $P \ll G$.

Hence, once the system has been reduced, a Direct Complex Method is used to solve equation B.17. Note that the costs of computation will be considerably reduced but the accuracy of the method is worse than SOL107 due to the Modal Truncation that SOL110 carries out.

B.3 Frequency Response Analyses

Another type of dynamic analysis is the Frequency Response Analysis which aims at computing the dynamic behaviour of the system under a Load case in the frequency domain. Hence the equation that is solved in this case is the full Equation of Motion B.12 that is rewritten below:

$$M\ddot{x}(\omega) + B\dot{x}(\omega) + (K + iK_S)x(\omega) = F(\omega) \quad (\text{B.18})$$

Two ways of computing the Frequency Response of the system are described. First, the Direct Frequency Response Analysis that calculates the response from the full equation B.18 and, finally, the Modal Frequency Response Analysis which first performs a Modal Decomposition of the System in order to reduce the size of the system of equations B.18.

B.3.1 Direct Frequency Response Analysis SOL108

The Direct Frequency Response Analysis solves the following system of equations for the full Model:

$$[-\omega^2 M + i\omega B + K]x(\omega) = F(\omega) \quad (\text{B.19})$$

where the matrix K is the Complex Stiffness Matrix which contains the structural Damping matrix $K = K + iK_S$.

Given the Frequency Range of Analysis and the Frequency Step desired, the Direct FRA solves the system of equations B.21 for each frequency given by the frequency step until the Frequency Range is reached.

As can be noticed, the costs of computation using this methodology can be extremely elevated for big FEMs.

B.3.2 Modal Frequency Response Analysis SOL111

The Modal Frequency Response Analysis aims at reducing the costs of computation in a FRA with respect to the Direct methodology. The way to reduce these costs is by reducing the size of the system. Hence, this method performs a Modal Analysis to use the mode shapes of the structure to reduce the size of the system's matrices so the numerical solution becomes more efficient.

The Modal Analysis performed is the Normal Modes Analysis explained in Section B.1. Then, the Equation of Motion B.21 is projected to the Modal Basis. If there is no Damping on the system, the modal equations of motion become uncoupled and the FRA is easy to solve. However, if a Structural and/or Viscous damping is considered, then the orthogonality properties of the modes does not, in general, diagonalise the System's Damping Matrix and the System's Complex Stiffness Matrix.

The coordinate transformation from the generalised system to the modal basis is the following:

$$x(\omega) = \varphi_{gp} \cdot q_p(\omega) \quad (\text{B.20})$$

where q_p is the vector of modal coordinates. Introducing this coordinate transformation in equation B.21 and pre-multiplying it by the transposed of the Modal Basis Matrix one leads to the system solved by the Modal Frequency Response Analysis:

$$\left[\underbrace{-\omega^2 \varphi_{gp}^T M \varphi_{gp}}_{\text{diagonal}} + i\omega \underbrace{\varphi_{gp}^T B \varphi_{gp}}_{\neq \text{diagonal}} + \underbrace{\varphi_{gp}^T K \varphi_{gp}}_{\neq \text{diagonal if } K=K+iK_S} \right] q_p(\omega) = \varphi_{gp}^T F(\omega) \quad (\text{B.21})$$

Appendix C

Craig-Bampton Condensation Methodology

When performing coupled dynamic analysis on Finite Element Models (FEMs) one may observe that solving the Equations of Motion for the complete system can be infeasible or at least extremely expensive in matter of costs of computation. In this way, system reduction methodologies have been developed and the one described in this Appendix chapter is the Craig-Bampton Method which consist on condensing the FEM by using a set of P fixed-interface normal modes and J constraint modes to describe the displacements of the model.

The equations of motion for a FEM with G Degrees of Freedom (DOFs) in total are written as follows :

$$M\ddot{x} + Kx = F + R \quad (C.1)$$

One can partition equation (C.1) by differentiating the J interface DOFs and the I internal DOFs with $G = J + I$:

$$\begin{bmatrix} M_{jj} & M_{ji} \\ M_{ij} & M_{ii} \end{bmatrix} \begin{Bmatrix} \ddot{x}_j \\ \ddot{x}_i \end{Bmatrix} + \begin{bmatrix} K_{jj} & K_{ji} \\ K_{ij} & K_{ii} \end{bmatrix} \begin{Bmatrix} x_j \\ x_i \end{Bmatrix} = \begin{Bmatrix} F_j \\ F_i \end{Bmatrix} + \begin{Bmatrix} R_j \\ 0 \end{Bmatrix} \quad (C.2)$$

As described in the introductory paragraph, the main purpose of the CB Method is to reduce the model dynamically considering P fixed-interface normal modes and J constraint modes to describe the displacements of the FEM. To do so, a Rayleigh-Ritz coordinate transformation using the constraint modes and the fixed-interface normal modes was proposed by Craig and Bampton(1968):

$$x = \Psi q \quad (C.3)$$

$$\begin{Bmatrix} x_j \\ x_i \end{Bmatrix} = \Psi \begin{Bmatrix} x_j \\ q_p \end{Bmatrix} \quad (C.4)$$

where Ψ is the Rayleigh-Ritz Transformation Matrix that has the following shape :

$$\Psi = \begin{bmatrix} I_{jj} & 0_{jp} \\ \phi_{ij} & \varphi_{ip} \end{bmatrix} \quad (C.5)$$

and ϕ_{ij} are the so-called *Constraint Modes* and φ_{ip} is the *Normal Modes* basis.

In equation (C.3) the vector q has $S = J + P$ generalized DOFs, where $S \ll G$. Considering the Enhanced FEM of a SRM used from Chapter 4 in this report, the total G DOFs of the model were 22860 whereas after the CB condensation the S generalized DOFs were 622 for a Truncation Frequency (TF) of 200Hz. It is evident that the condensation executed by the CB Method is significant and that it will have a direct impact on the reduction of the costs of computation .

The terms present on equation (C.5) refer to the constraint modes ϕ_{ij} due to unit displacements I_{jj} of the interface DOFs and to the fixed-interface 0_{jp} normal modes φ_{ip} .

The constraint modes ϕ_{ij} can be computed by solving the second row of the static equilibrium equation (C.6) shown below :

$$\begin{bmatrix} K_{jj} & K_{ji} \\ K_{ij} & K_{ii} \end{bmatrix} \begin{bmatrix} I_{jj} \\ \phi_{ij} \end{bmatrix} = \begin{bmatrix} R_{jj} \\ 0_{ij} \end{bmatrix} \quad (\text{C.6})$$

$$K_{ij}I_{jj} + K_{ii}\phi_{ij} = 0 \quad (\text{C.7})$$

where finally

$$\phi_{ij} = -K_{ii}^{-1}K_{ij} \quad (\text{C.8})$$

On the other hand, the fixed-interface normal modes can be calculated by solving the following eigenvalue problem:

$$[-\omega^2 M_{ii} + K_{ii}] \varphi_{ii} = 0 \quad (\text{C.9})$$

The solution of equation (C.9) has, at the most, a number of eigenvalues ω^2 and eigenvectors φ_{ii} equal to the total number of internal DOFs I . Thus, φ_{ip} is basically a subset of the total modal basis of eigenvectors φ_{ii} .

Finally, when effectuating the coordination transformation seen in equation (C.3) into equation (C.2) and also pre-multiplying equation (C.2) with the transpose of the transformation matrix shown in equation (C.5), one obtains the following reduced equation of motion :

$$\Psi^T M \Psi \ddot{q} + \Psi^T K \Psi q = \Psi^T F + \Psi^T R \quad (\text{C.10})$$

$$\begin{bmatrix} \overline{M}_{jj} & M_{jp} \\ M_{pj} & m_{pp} \end{bmatrix} \begin{Bmatrix} \ddot{x}_j \\ \ddot{q}_p \end{Bmatrix} + \begin{bmatrix} \overline{K}_{jj} & 0_{jp} \\ 0_{pj} & \Lambda_{pp} \end{bmatrix} \begin{Bmatrix} x_j \\ q_p \end{Bmatrix} = \begin{Bmatrix} F_j + \phi_{ij}^T F_i \\ \varphi_{ip}^T F_i \end{Bmatrix} + \begin{Bmatrix} R_j \\ 0 \end{Bmatrix} \quad (\text{C.11})$$

where if the modes are mass-normalized, the generalized mass matrix m_{pp} will be the identity:

$$m_{pp} = \varphi_{ip}^T M_{ii} \varphi_{ip} = I_{pp} \quad (\text{C.12})$$

The mass matrix partition $\overline{M}_{jj}$, called the statically reduced or Guyan reduced mass matrix, is given by:

$$\overline{M}_{jj} = \phi^T M \phi = M_{jj} + M_{ji}\phi_{ij} + \phi_{ij}^T M_{ij} + \phi_{ij}^T M_{ii} \phi_{ij} \quad (\text{C.13})$$

The mass coupling matrix M_{jp} is called the modal mass participation matrix and is given by:

$$M_{jp} = \phi^T M \varphi = M_{ji}\varphi_{ip} + \phi_{ij}^T M_{ii} \varphi_{ip} \quad (\text{C.14})$$

For the stiffness matrix partitions, the stiffness matrix $\overline{K}_{jj}$ is called the statically reduced or Guyan reduced stiffness matrix and can be computed as follows:

$$\overline{K}_{jj} = \phi^T K \phi = K_{jj} + K_{ji}\phi_{ij} \quad (\text{C.15})$$

The coupling between the interface DOFs and generalized DOFs is zero which is proved below:

$$K_{jp} = \phi^T K \varphi = [K_{ji} + \phi_{ij}^T K_{ii}] \varphi_{ip} = \left[K_{ji} + (K_{ii}^{-1} K_{ij})^T K_{ii} \right] \varphi_{ip} = 0_{jp} \quad (\text{C.16})$$

and, finally, the generalized stiffness matrix Λ_{pp} will be:

$$\Lambda_{pp} = \varphi_{ip}^T M_{ii} \varphi_{ip} = \mathcal{D}(\omega_p^2) \quad (\text{C.17})$$

If there exist Structural and/or Viscous Damping matrices, they can also be CB reduced. For the Structural Damping Matrix, the fully populated Reduced Structural Damping Matrix D is computed as follows:

$$D = \Psi^T K_S \Psi = \begin{bmatrix} D_{jj} & D_{jp} \\ D_{pj} & D_{pp} \end{bmatrix} \quad (\text{C.18})$$

where each partition of the Reduced Structural Damping Matrix are computed as shown below:

$$D_{jj} = K_{Sjj} + K_{Sji}\phi_{ij} + \phi_{ij}^T K_{Sij} + \phi_{ij}^T K_{Sii}\phi_{ij} \quad (\text{C.19})$$

$$D_{jp} = K_{Sji}\varphi_{ip} + \phi_{ij}^T K_{Sii}\varphi_{ip} \quad (\text{C.20})$$

$$D_{pp} = \varphi_{ip}^T K_{Sii}\varphi_{ip} \quad (\text{C.21})$$

The same triple matrix product should be performed if there is a Viscous Damping Matrix B yielding to the B_{CB} condensed Viscous Damping Matrix:

$$B_{CB} = \Psi^T B \Psi = \begin{bmatrix} B_{CBjj} & B_{CBjp} \\ B_{CBpj} & B_{CBpp} \end{bmatrix} \quad (\text{C.22})$$

where each partition of the matrix is computed as seen below:

$$B_{CBjj} = B_{jj} + B_{ji}\phi_{ij} + \phi_{ij}^T B_{ij} + \phi_{ij}^T B_{ii}\phi_{ij} \quad (\text{C.23})$$

$$B_{CBjp} = B_{ji}\varphi_{ip} + \phi_{ij}^T B_{ii}\varphi_{ip} \quad (\text{C.24})$$

$$B_{CBpp} = \varphi_{ip}^T B_{ii}\varphi_{ip} \quad (\text{C.25})$$

Hence, the full CB Reduced Equation of Motion considering Structural and/or Viscous Damping dissipative forces can be written as follows:

$$\begin{aligned} \begin{bmatrix} \overline{M_{jj}} & M_{jp} \\ M_{pj} & m_{pp} \end{bmatrix} \begin{Bmatrix} \ddot{x}_j \\ \ddot{q}_p \end{Bmatrix} + B_{CB} \begin{Bmatrix} \dot{x}_j \\ \dot{q}_p \end{Bmatrix} + \left(\begin{bmatrix} \overline{K_{jj}} & 0_{jp} \\ 0_{pj} & \Lambda_{pp} \end{bmatrix} + iD_{CB} \right) \begin{Bmatrix} x_j \\ q_p \end{Bmatrix} \\ = \begin{Bmatrix} F_j + \phi_{ij}^T F_i \\ \varphi_{ip}^T F_i \end{Bmatrix} + \begin{Bmatrix} R_j \\ 0 \end{Bmatrix} \end{aligned} \quad (\text{C.26})$$

Assembly of CB Models

Once the Superelements/Components of a system have been condensed using a CB Condensation Method, the next step is to assemble them to obtain the CB Condensed Model of the Assembled System. An example of an assembly of two CB Condensed Superelements is mathematically described in this section.

The assembly of CB Models is done by joining the interface DOFs as it is graphically depicted in figure C.1. The mathematical procedure is really simple and consists in assembling the CB Matrices as presented in the table that follows:

Figure C.1: Graphical description of the Assembly of CB Condensed Superelements

Model 1	Model 2
$q_{CB1} = \left\{ \begin{array}{c} x_j \\ q_{p1} \end{array} \right\}_{(C_1 \times 1)}$ <p style="text-align: center;">with $C_1 = J + P1$</p> $M_{CB1} = \begin{bmatrix} \overline{M}_{1jj} & M_{1jp} \\ M_{1pj} & m_{1pp} \end{bmatrix}_{(C_1 \times C_1)}$ $K_{CB1} = \begin{bmatrix} \overline{K}_{1jj} & 0_{jp} \\ 0_{pj} & \Lambda_{1pp} \end{bmatrix}_{(C_1 \times C_1)}$ $D_{CB1} = \begin{bmatrix} D_{1jj} & D_{1jp} \\ D_{1pj} & D_{1pp} \end{bmatrix}_{(C_1 \times C_1)}$ $B_{CB1} = \begin{bmatrix} B_{1jj} & B_{1jp} \\ B_{1pj} & B_{1pp} \end{bmatrix}_{(C_1 \times C_1)}$	$q_{CB2} = \left\{ \begin{array}{c} x_j \\ q_{p2} \end{array} \right\}_{(C_2 \times 1)}$ <p style="text-align: center;">with $C_2 = J + P2$</p> $M_{CB2} = \begin{bmatrix} \overline{M}_{2jj} & M_{2jp} \\ M_{2pj} & m_{2pp} \end{bmatrix}_{(C_2 \times C_2)}$ $K_{CB2} = \begin{bmatrix} \overline{K}_{2jj} & 0_{jp} \\ 0_{pj} & \Lambda_{2pp} \end{bmatrix}_{(C_2 \times C_2)}$ $D_{CB2} = \begin{bmatrix} D_{2jj} & D_{2jp} \\ D_{2pj} & D_{2pp} \end{bmatrix}_{(C_2 \times C_2)}$ $B_{CB2} = \begin{bmatrix} B_{2jj} & B_{2jp} \\ B_{2pj} & B_{2pp} \end{bmatrix}_{(C_2 \times C_2)}$

where J is the size of the Interface DOFs and $P1$, $P2$ the size of the Modal Basis of Components 1 and 2 respectively. Vectors q_{CB1} and q_{CB2} represent the Generalised DOFs where x_j are the Physical Interface DOFs and q_{p1} / q_{p2} are the Modal DOFs of Components 1 and 2 respectively.

Thus, the CB Assembled System's Generalised Coordinate vector has the following shape:

$$q_{sys} = \left\{ \begin{array}{c} x_j \\ q_{p1} \\ q_{p2} \end{array} \right\}$$

Finally, using the matrices of both CB Models 1 and 2, one leads to the matrices of the CB Assembled System as follows:

$$M_{CBsys} = \sum_{(S \times S)} M_{CB\#} = \begin{bmatrix} \overline{M_{1jj}} + \overline{M_{2jj}} & M_{1jp} & M_{2jp} \\ M_{1pj} & m_{1pp} & 0 \\ M_{2pj} & 0 & m_{2pp} \end{bmatrix}_{(S \times S)} \quad (C.27)$$

$$K_{CBsys} = \sum_{(S \times S)} K_{CB\#} = \begin{bmatrix} \overline{K_{1jj}} + \overline{K_{2jj}} & 0 & 0 \\ 0 & \Lambda_{1pp} & 0 \\ 0 & 0 & \Lambda_{2pp} \end{bmatrix}_{(S \times S)} \quad (C.28)$$

$$D_{CBsys} = \sum_{(S \times S)} D_{CB\#} = \begin{bmatrix} D_{1jj} + D_{2jj} & D_{1jp} & D_{2jp} \\ D_{1pj} & D_{1pp} & 0 \\ D_{2pj} & 0 & D_{2pp} \end{bmatrix}_{(S \times S)} \quad (C.29)$$

$$B_{CBsys} = \sum_{(S \times S)} B_{CB\#} = \begin{bmatrix} B_{1jj} + B_{2jj} & B_{1jp} & B_{2jp} \\ B_{1pj} & B_{1pp} & 0 \\ B_{2pj} & 0 & B_{2pp} \end{bmatrix}_{(S \times S)} \quad (C.30)$$

where S is the size of the CB Assembled System with $S = J + P1 + P2$.

To sum up, the equation of motion of the CB condensed assembled system can be written as follows:

$$M_{CBsys}\ddot{q}_{sys} + B_{CBsys}\dot{q}_{sys} + (K_{CBsys} + iD_{CBsys})q_{sys} = \Psi_{sys}^T F_{sys} \quad (C.31)$$

where

$$\Psi_{sys} = \begin{bmatrix} I_{jj} & 0_{jp1} & 0_{jp2} \\ \phi_{i1j} & \varphi_{i1p1} & 0 \\ \phi_{i2j} & 0 & \varphi_{i2p2} \end{bmatrix} \quad (C.32)$$

and the vector F_{sys} is structured as follows:

$$F_{sys} = \begin{Bmatrix} F_j \\ F_{i1} \\ F_{i2} \end{Bmatrix} \quad (C.33)$$

Appendix D

Figures and Analysis Plots

This appendix document contains all the figures and analysis plots that have been omitted from the main report to reduce its size and make it easier to read. All the images are distributed by the Chapters and Sections to which they belong so the reader can easily associate them to the main document.

D.1 Chapter 4 — Validation of the Equivalent Viscous Damping Methodologies

D.1.1 Decoupled & Equivalent Viscous Damping Methodologies evaluation

Figure D.1: SRM — EqVD Coefficient distribution against frequency and Mode Shapes

D.1.2 Identification of the Reference Methodology

Figure D.2: *SRM – Frequency Response in lateral X direction*

Figure D.3: *SRM – Frequency Response in lateral Z direction*

D.1.3 System Methodologies in Direct Frequency Response Analysis

Figure D.4: Non-Symmetric SRM System – FRA for Component 1 in lateral X direction

Figure D.5: Non-Symmetric SRM System – FRA for Component 1 in lateral Z direction

Figure D.6: Non-Symmetric SRM System – FRA for Component 2 in lateral X direction

Figure D.7: Non-Symmetric SRM System – FRA for Component 2 in lateral Z direction

D.1.4 System Methodologies in Modal Frequency Response Analysis

Figure D.8: *Non-Symmetric SRM System – Modal FRA for Component 1 in lateral X direction* Figure D.9: *Non-Symmetric SRM System – Modal FRA for Component 1 in lateral Z direction*

Figure D.10: *Non-Symmetric SRM System – Modal FRA for Component 2 in lateral X direction* Figure D.11: *Non-Symmetric SRM System – Modal FRA for Component 2 in lateral Z direction*

D.2 Chapter 5 – Enhancement of the System Equivalent Viscous Damping Methodologies

D.2.1 Enhanced System SDCB Method evaluation in FRA

Figure D.12: *Non-Symmetric SRM System – Enhanced System SDCB Component 1 FRA lateral X direction*

Figure D.13: *Non-Symmetric SRM System – Enhanced System SDCB Component 1 FRA lateral Z direction*

Figure D.14: Non-Symmetric SRM System – Enhanced System SDCB Component 2 FRA lateral X direction

Figure D.15: Non-Symmetric SRM System – Enhanced System SDCB Component 2 FRA lateral Z direction

D.2.2 Enhanced System EqVD Method evaluation in FRA

Figure D.16: *Non-Symmetric SRM System – Enhanced System EqVD Component 1 FRA lateral X direction*

Figure D.17: *Non-Symmetric SRM System – Enhanced System EqVD Component 1 FRA*

Figure D.18: Non-Symmetric SRM System – Enhanced System EqVD Component 2 FRA lateral X direction

Figure D.19: Non-Symmetric SRM System – Enhanced System EqVD Component 2 FRA lateral Z direction

D.2.3 Conclusion

Figure D.20: Non-Symmetric SRM System – Enhanced Methods FRA for Component 1 in lateral X direction

Figure D.21: Non-Symmetric SRM System – Enhanced Methods FRA for Component 1 in lateral Z direction

Figure D.22: *Non-Symmetric SRM System – Enhanced Methods FRA for Component 2 in lateral X direction*

Figure D.23: *Non-Symmetric SRM System – Enhanced Methods FRA for Component 2 in lateral Z direction*

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