On Four Intuitionistic Fuzzy Topological Operators

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Abstract

Four new operators, which are analogous of the topological operators “interior” and “closure”, are defined. Some of their basic properties are studied.

Their geometrical interpretations are given.

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Initially, we shall introduce some basic definitions, related to the Intuitionistic Fuzzy Sets (IFSs), following [1].

Let a set $E$ be fixed. An IFS $A$ in $E$ is an object of the following form:

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle | x \in E \},$$

where the functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let an universe $E$ be given and let the figure $F$ in the Euclidean plane with Cartesian coordinates be given (see Fig. 1).

![Figure 1](image-url)
Let the IFS $A$ be fixed. Then we can construct a function $f_A$ from $E$ to $F$, such that if $x \in E$, then

$$p = f_A(x) \in F,$$

the point $p$ has coordinates $(a, b)$ for which:

$$0 \leq a + b \leq 1,$$

and these coordinates are such that:

$$a = \mu_A(x),$$
$$b = \nu_A(x).$$

Therefore the function $f_A$ is a surjection (see [2]).

For every two IFSs $A$ and $B$ a lot of operations, relations and operators are defined (see, e.g. [1,3]), the most important of which, related to the present research, are:

$$A \subset B \iff (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x));$$
$$A \supset B \iff B \subset A;$$
$$A \sim B \iff (\forall x \in E)(\mu_A(x) - \mu_B(x) \& \nu_A(x) - \nu_B(x));$$
$$\bar{A} = \{(x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))) | x \in E\};$$
$$A \cup B = \{(x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))) | x \in E\};$$
$$\Box A = \{(x, \mu_A(x), 1 - \mu_A(x)) | x \in E\};$$
$$\Diamond A = \{(x, 1 - \nu_A(x), \nu_A(x)) | x \in E\};$$
$$C(A) = \{(x, K, L) | x \in E\};$$
$$I(A) = \{(x, k, l) | x \in E\}.$$

where

$$K = \max_{y \in E} \mu_A(y), \quad L = \min_{y \in E} \nu_A(y)$$

and

$$k = \min_{y \in E} \mu_A(y), \quad l = \max_{y \in E} \nu_A(y).$$

The geometrical interpretations of the above operations and operators (and of the other ones) are discussed in [2]. Here we shall show only the geometrical interpretations of the last two operations about the IFS $A$ from Fig. 2. - see Fig. 3 and 4.
Up to now, these two operators, which are in some sense analogous of the
topological operators "closure" (C) and "interior" (I), transform a given IFS to a new
one, all elements of which have equal degrees of membership and non-membership.
This situation really correspond to the sense of the two topological operators, but
this correspondences are in an extremally powerful forms.
Now, we shall modify these two operators. They will be again analogous of the two topological operates "closure" and "interior" (I), but now their forms will be not so powerful. Their definitions are the following:

\[
C_{\mu}(A) = \{ (x, \mu_{\min}(1 - K, \nu_A(x))) | x \in E \};
\]

\[
C_{\nu}(A) = \{ (x, \mu_A(x), L) | x \in E \};
\]

\[
I_{\mu}(A) = \{ (x, k, \nu_A(x)) | x \in E \};
\]

\[
I_{\nu}(A) = \{ (x, \min(1 - l, \mu_{\min}(1 - K, \nu_A(x))), l) | x \in E \},
\]

where \( K, L, k, l \) have the above forms.

The geometrical interpretations of the new operators about the IFS \( A \) from Fig. 2 are shown on Fig. 5 - 8.

Obviously, for every IFS \( A \):

\[
I(A) \subset I_{\mu}(A) \subset I_{\nu}(A) \subset A \subset C_{\nu}(A) \subset C_{\mu}(A) \subset C(A).
\]

**Theorem 1.** For every IFS \( A \):

(a) \( C_{\mu}(C_{\nu}(A)) = C_{\nu}(C_{\mu}(A)) \subset C(A) \),

(b) \( I_{\mu}(I_{\nu}(A)) = I_{\nu}(I_{\mu}(A)) \subset I(A) \),

(c) \( C_{\mu}(I_{\mu}(A)) = I_{\mu}(C_{\mu}(A)) \),

(d) \( C_{\nu}(I_{\mu}(A)) = I_{\nu}(C_{\nu}(A)) \),

(e) \( \Box C_{\mu}(\Box A) \subset C_{\mu}(\Box A) \),

(f) \( \Box C_{\mu}(A) \subset \Box C_{\mu}(\Box A) \),

(g) \( \Box C_{\nu}(A) \subset \Box C_{\nu}(\Box A) \),

(h) \( \Box C_{\nu}(A) \subset \Box C_{\nu}(\Box A) \),

(i) \( \Box I_{\mu}(A) \subset I_{\mu}(\Box A) \),

(j) \( \Box I_{\nu}(A) \subset I_{\nu}(\Box A) \),

(k) \( \Box I_{\nu}(A) \subset I_{\nu}(\Box A) \).
(l) $\text{Int} (A) = I_\mu (\text{Int} A)$,
(m) $\overline{\text{Int} (A)} = I_\mu (A)$,
(n) $I_\mu (\overline{A}) = C_\nu (A)$.

Fig. 5.

Fig. 6.

Fig. 7.
\textbf{Theorem 2.} For every two IFSs $A$ and $B$:

(a) $C_p(A \cup B) = C_p(A) \cup C_p(B)$,
(b) $C_p(A \cap B) = C_p(A) \cap C_p(B)$,
(c) $C_p(A \cap B) = C_p(A) \cap C_p(B)$,
(d) $C_p(A \cap B) = C_p(A) \cap C_p(B)$.

The validities of the above assertions follow directly from the definitions.

\textbf{References}

