APPENDIX A: Source term formulation in SWAN

Formulae of the physical processes of generation, dissipation and non-linear wave-wave interaction implemented in SWAN, as described in its manual:

**Wind input \((S_{in})\)**

Wave growth by wind is described by:

\[ S_{in}(\sigma, \theta) = A + BE(\sigma, \theta) \]

The model is driven by wind speed at 10 m elevation \(U_{10}\), although the computations use friction velocity \(U^*\). The transformation from one to the other is as follows (for the WAM Cycle 3 approach developed by the WAMDI group, 1988):

\[ U^*_2 = C_D U_{10}^2 \]

Where \(C_D\) is the drag coefficient from Wu (1982):

\[ C_D(U_{10}) = \begin{cases} 1.2875 \times 10^{-3} & \text{for } U_{10} < 7.5 \text{ m/s} \\ (0.8 + 0.065U_{10}) \times 10^{-3} & \text{for } U_{10} \geq 7.5 \text{ m/s} \end{cases} \]

\(B\) is governed by the following expression, formulated by Komen et al. (1984):

\[ B = \max \left[ 0, 0.25 \frac{\rho_a}{\rho_w} \left( \frac{28 U}{c_{ph}} \cos(\theta - \theta_a) - 1 \right) \right] \sigma \]

Where:
- \(c_{ph}\): phase speed \([\text{m/s}]\)
- \(\rho_a\): air density \([\text{kg/m}^3]\)
- \(\rho_w\): water density \([\text{kg/m}^3]\)

**Dissipation \((S_{ds})\)**

**Whitening \((S_{ds,w})\)**

Hasselman’s model (1974) is based on the pulse is the one used, albeit reformulated, in terms of wave number (rather than frequency) so as to be applicable in finite water depth (cf. the WAMDI group, 1988):

The waves’ steepness controls whitening. In third-generation wave models such as SWAN, whitening formulations rely on a pulse-bade model (Hasselmann, 1974), adapted by the WAMDI group (1988) as:
\[
S_{dr,w}(\sigma, \theta) = -\Gamma \tilde{\sigma} \frac{k}{k} E(\sigma, \theta)
\]

Where \( \tilde{\sigma} \) and \( \tilde{k} \) are, respectively, the mean frequency and wave number and the coefficient \( \Gamma \) depends on the wave steepness. Its value The value of \( \Gamma \) was estimated by Komen et al. (1984) by closing the energy balance of the waves in fully developed conditions (thus, it depends on the formulation used for wind input). As given by the WAMDI group (1988) and adapted by Günther et al. (1992):

\[
\Gamma = \Gamma_{kj} = C_{ds} \left( (1 - \delta) + \delta \frac{k}{k} \left( \frac{s}{s_{PM}} \right) \right)^p
\]

Which is reduced to the exact WAMDI expression if the tuneable coefficient \( \delta = 0 \). \( C_{ds} \) is also a tuneable coefficient, whereas \( \tilde{s} \) is the overall wave steepness which is defined below and \( \tilde{s}_{PM} = \sqrt{3.02 \cdot 10^{-3}} \) is the value of \( \tilde{s} \) for the Pierson-Moskowitz spectrum (1964).

\[
\tilde{s} = \tilde{k} \sqrt{E_{tot}}
\]

The mean frequency \( \tilde{\sigma} \), the mean wave number \( \tilde{k} \) and the total wave energy \( E_{tot} \) are defined as:

\[
\tilde{\sigma} = \left( E_{tot}^{-1} \int_0^{2\pi} \int_0^{\infty} \frac{1}{\sigma} E(\sigma, \theta) d\sigma d\theta \right)^{-1}
\]

\[
\tilde{k} = \left( E_{tot}^{-1} \int_0^{2\pi} \int_0^{\infty} \frac{1}{k} E(\sigma, \theta) d\sigma d\theta \right)^{-2}
\]

\[
E_{tot} = \int_0^{2\pi} \int_0^{\infty} E(\sigma, \theta) d\sigma d\theta
\]

The tuneable coefficients \( C_{ds} \) and \( \delta \) and exponent \( p \) have the values suggested by Komen et al. (1984) when closing the energy balance of the waves in idealised wave growth conditions (both for growing and fully developed seas) for deep water. This implies that coefficients in the steepness dependent coefficient \( \Gamma \) depend on the wind input formulation being used. For the wind input of Komen et al. (1984; corresponding to WAM Cycle 3; the WAMDI group, 1988): \( C_{ds} = 2.36 \cdot 10^{-5}, \delta = 0 \) and \( p = 4 \).

**Bottom friction**

Bottom friction in SWAN follows the empirical model of JONSWAP (Hasselmann et al. 1973), the drag model of Collins (1972) and the eddy viscosity model of Madsen et al. (1988), which can be commonly expressed as:

\[
S_{dr,b}(\sigma, \theta) = -C_{bottom} \frac{\sigma^2}{g^2 \sinh^2(kd)} E(\sigma, \theta)
\]
With the bottom friction coefficient ($C_{bottom}$) depending on the bottom orbital motion ($U_{rms}$):

$$U_{rms} = \sqrt{\frac{2\pi}{g}} \int_{0}^{\infty} \int_{0}^{2\pi} \frac{\sigma^2}{\sinh^2(kd)} E(\sigma, \theta) d\sigma d\theta$$

Madsen et al. (1988) derived a formulation where the bottom friction factor is a function of the bottom roughness height and the wave conditions:

$$C_{bottom} = f_w \frac{g}{\sqrt{2}} U_{rms}$$

Where the non-dimensional friction factor $f_w$ is estimated with Johnson’s formulation (1966; cf. Madsen et al., 1988):

$$\frac{1}{4\sqrt{f_w}} + \log_{10} \left( \frac{1}{4\sqrt{f_w}} \right) = m_f + \log_{10} \left( \frac{a_b}{K_N} \right)$$

In which $m_f = -0.08$, $K_N$ is the bottom roughness length scale and $a_b$ represents the near bottom excursion amplitude:

$$a_b^2 = 2\int_{0}^{2\pi} \int_{0}^{\infty} \frac{1}{\sinh^2(kd)} E(\sigma, \theta) d\sigma d\theta$$

If $a_b / K_N$ is smaller than 1.57 the friction factor $f_w$ is 0.30 (Johnson, 1980).

**Depth-induced wave braking**

The mean rate of energy dissipation per unit of horizontal area due to wave breaking ($D_{tot}$) is formulated according to the model of Battjes and Janssen (1978):

$$D_{tot} = -\frac{1}{4} \alpha BJ Q_b \left( \frac{\sigma}{2\pi} \right) H_n^2$$

Where $\alpha_{BJ} = 1$ in SWAN and the $Q_b$ is the fraction of breaking waves:

$$\frac{1 - Q_b}{\ln Q_b} = -8 \frac{E_{tot}}{H_n^3}$$

With $H_n$ being the maximum wave height possible at the given depth and the mean frequency ($\bar{\sigma}$) is defined as:

$$\bar{\sigma} = E_{tot}^{-1} \int_{0}^{2\pi} \int_{0}^{\infty} \sigma E(\sigma, \theta) d\sigma d\theta$$
Extending the expression of Eldeberky and Battjes (1995) to include the spectral direction, the dissipation for a spectral component per unit time is calculated in SWAN with:

\[ S_{\text{tot},\theta}(\sigma, \theta) = D_{\text{tot}} \frac{E(\sigma, \theta)}{E_{\text{tot}}} \]

**References**


