la actividad científica y técnica de este cuerpo militar en el siglo XIX. El cuerpo de artilleros sufrió durante el siglo XIX una serie de reformas, muchas de las cuales representaban su disolución temporal. Esto nos puede hacer ver la creciente importancia de la Artillería dentro del ejército. El Real Colegio de Artillería había sido creado en 1764 en Segovia, se convirtió en Academia de Artillería en 1867. En el Cuerpo de Artillería se formaron artilleros ingenieros, algunos de ellos responsables de las maestranzas, un sector industrial de mucho peso específico en algunas regiones españolas y, en general, en la industrialización del país. Medina también comenta los conflictos de competencias de los artilleros con otros ingenieros, principalmente, los industriales. La vocación científica y técnica del Cuerpo se refleja en la creación en 1844 de una publicación periódica, el *Memorial de Artillería*. Francisco Fernández González, de la Universidad Politécnica de Madrid, es autor de un estudio sobre el mundo de la navegación. Empieza haciendo una revisión de los problemas de la marina de guerra durante el siglo XIX, sumida en penurias, que no conseguirá superar a lo largo del siglo, lleno de convulsiones políticas y económicas. A continuación, el autor se refiere a la introducción de los primeros navíos de vapor y de acero en el país, la mayoría construidos en arsenales extranjeros. En 1815 se creó el cuerpo de ingenieros de Marina, con responsabilidades destacadas en los arsenales militares. A final de siglo se reguló la formación y encuadramiento de los maquinistas de la Armada. A continuación el autor hace una revisión de los colegios y academias de la Armada, destacando el debate en torno a la Escuela Naval que tuvo lugar en 1885. Al final del volumen encontramos un Índice de ilustraciones, un índice de cuadros y un resumen sobre disposiciones técnicas que facilita el manejo del volumen. Recordemos que en el volumen V se encuentra un complemento biográfico de los principales actores que intervienen en los volúmenes IV y V. Ahí hay que acudir para ampliar las noticias biográficas de los personajes citados en los capítulos, de manera que éstos se liberan de menciones biográficas demasiado extensas. Para terminar, quisiera destacar que este volumen, como los demás, sin ser un libro ilustrado, contiene un buen número de imágenes. Se trata, además, de una selección muy original de las imágenes, generalmente poco divulgadas, con un discurso complementario al del texto, de manera que, en muchos casos, las aportaciones en el terreno de las imágenes son tan interesantes como las del texto. Recordemos que uno de los capítulos es esencialmente visual, el que habla del reflejo del mundo de la técnica y de la ingeniería en la pintura española de la época.

ENGLISH ABSTRACTS

INTERNATIONAL CONGRESS "LEONHARD EULER (1707-2007) 300th ANNIVERSARY" AN INFORMAL PRESENTATION

Guillermo Lusa Monforte

This paper is a transcript of the inaugural speech for the congress organized by the Polytechnical University of Catalonia in Barcelona, devoted to Leonhard Euler. The author discusses the special place of Euler in the history of science, in an intermediate field between pure mathematics and engineering. The author recalls that Euler has been celebrated on several occasions in Barcelona and at the Polytechnical University of Catalonia. In 1993 a selection of texts by Euler was published by the University un collaboration with the Autonomous University of Barcelona. The paper also discusses the role of "great" characters in history of science, such as Leonhard Euler, outstanding figures the analysis of whom promotes the study of history of science.

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EULER TRANSGRESSING LIMITS: THE INFINITE AND MUSIC THEORY

Eberhard Knobloch

Leonhard Euler was one of the most creative mathematicians of all time. One of the characteristics of his creativity is his transgression or removal of limits. Four examples are discussed in order to illustrate this assertion. 1. Mathematical rigour: Euler claimed to have restored mathematical rigour in the analysis of the infinite. 2. Zeta-function for $s = 2$: Euler found the value of the series of the reciprocal square numbers by treating it as an algebraic equation of infinite degree. 3. Divergent series: He introduced a new notion of sum in order to avoid the use of divergent series. 4. Music theory: He abolished the distinction between consonances and dissonances.

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THE INTEGRAL AS AN ANTI-DIFFERENTIAL. AN ASPECT OF EULER'S ATTEMPT TO TRANSFORM CALCULUS INTO AN ALGEBRAIC CALCULUS

Giovanni Ferraro

The integration of differential formulas was one of the main fields of Euler's activity. He wrote many papers on the subject. In this article, I will dwell upon an important aspect of Euler's work on integration: the notion of integration as anti-differentiation. I will show that this notion requires examination within the context of Euler's strategy, which was aimed at transforming integral calculus into an exclusively algebraic theory and which gave rise to several problems, the most important of which concerned the existence of the anti-differential and the nature of the functions involved in the operation of integration. I will also consider the role of general and particular integrals in Euler's theory and stress that the importance attributed to indefinite integration and general integrals was linked to the conception of analysis as the science that investigated mathematical objects in an abstract and general way. Finally, I will discuss the implication of Euler's concept in his treatment of elliptic integrals.

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CONSIDERATIONS ABOUT THE BETA FUNCTION IN THE WORK OF LEONHARD EULER (1707-1783)

Amadeu Delshams, M. Rosa Massa

According to Leonhard Euler (1707-1783), "the most important problem in the theory of progressions of numbers is to exhibit, for any given progression, its general term". In 1730, when looking for the general term of the famous factorial progression, he was led to the computations of integrals, nowadays known as the Beta function. The aim of this article is to analyse the analytic procedures found in Euler's early texts, to compute what is nowadays known as the Beta function. After describing these calculations, we subsequently present some relations between these integrals calculated by Euler. We also

analyze the construction of other tables of the Beta integral which were available in his time.

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EULER AND ANALITICS GEOMETRY

Pedro González Urbaneja

Euler exploits the analytic methods introduced by Fermat and Descartes and developed by van Schooten, De Witt, Wallis, La Hire, Newton and Leibniz. Euler stated that: "We should place out trust in algebraic calculus rather than in our own judgement", and indeed Euler took a great step forward by systematizing three-dimensional and plane Analytic Geometry. Although in his *Introduction* Euler deals with the equation of the straight line: $\alpha x+\beta y-a = 0$, in a general way, he pays little attention to the study of first-degree equations, since: "the Geometry of the straight line is already well-known". He gives a general analytical treatment of conic sections, free of references to the cone and even to diagrams, and states that "I will obtain the properties of conic sections by examining their equation and without recourse to other means". Euler writes the general equation of a conic section as a general quadratic with six terms, finds the centre and refers the conic section to its main axes, performs the classification of conic sections using the discriminate and identifies the main points, lines and ratios. Euler concludes his *Introduction* with an original appendix on three-dimensional Analytic Geometry in which he studies and graphically represents curves and surfaces by their equations. He provides the first formula for the translation and rotation of axes in three dimensions, henceforth known as Euler's formula. Euler writes the equation of the plane in the general form $\alpha x+\beta y+\chi z = a$, and studies intersections and angles with planes and co-ordinated axes. Euler introduces quadrics as a unitary family of second-degree surfaces by means of a general quadratic equation in ten terms; he considers the equation of the asymptotic cone and derives the general classification using transformations to five types of canonical forms: ellipsoid, hyperboloid of one sheet, hyperboloid of two sheets, hyperbolic paraboloid (discovered by him) and elliptical paraboloid. He does not include the degenerate quadrics. As a counterpoint to important and difficult problems of conic sections and conic sections and quadrics, Euler pays close attention to elements of Analytic Geometry concerning straight lines and planes; perhaps because, for Euler, equations of the straight line and the plane are functional relations, and therefore objects of interest for Infinitesimal Analysis rather than algebraic representations of straight lines and planes. This would explain why basic aspects of academic Analytic Geometry (midpoint, parallelism, perpendicularity, angles, slope, distances, areas, volumes, etc.), became objects of study after Euler.

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MILLS AND KITES BY MR. EULER LE FILS. MATHEMATICAL MODELS FOR HYDRAULIC MACHINES IN THE XVIII CENTURY

Juan Miguel Suay Belenguer

Throughout its history, the kite has crossed many geographical and cultural borders. Its shapes, its uses, its meaning and its symbology have undergone many changes to adapt to each of these different contexts. But perhaps one of the most significant transformations of this highly popular and traditional object of fun and leisure took place at the heart of Western culture when, at the height of the Enlightenment, the kite became a scientific instrument for physical experimentation, used by authors such as Benjamin Franklin for the study of atmospheric electricity, and shortly after as a device propelled by a fluid. In 1758, the Berlin Royal Academy of Sciences and Fine Arts published a volume containing memoires written in 1756. In the section devoted to Mathematics, three papers by Leonhard Euler (1707–1783) were published, one of them dealing with the study of windmills. In the same volume, his son, Johann Albert Euler (1734–1800) published a paper on kites (*Des Cerf- volants*). The formal structure of both articles is very similar as regards the definition of the geometric model of the device, the physical model they apply in order to explain how the kite works (impact theory), the mathematical development – algebraic – and the way their results are expressed. The purpose of my work is to analyze the ways in which the definition, the shape, the uses and the representation of the kite have changed in order to be included in further papers among those presented and published by the different

European Academies on hydraulic machines and naval theory throughout the 18th century.

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THE *e* NUMBER IN SPANISH MATHEMATICAL TEXTBOOKS IN THE XVIII CENTURY

Juan Navarro Loidi

In books on Mathematics published in Spain from 1750, subjects dealing with differential and integral calculus began to appear. The influence of L. Euler was apparent in many of these texts; for example, P. Padilla Arcos cites Euler in his *Curso militar de mathemáticas* ("Military Course on Mathematics") (1753, II). In this paper, the treatment given in these treatises to the number ***e***, often associated with Euler, is traced. This number is commented on in these treatises in two different sections. First in the part devoted to logarithms or series, and later in the chapters devoted to integral and differential calculus. At the beginning it is defined as the base of Neperian logarithms or the number that has the Neperian logarithm 1. It is often calculated by means of series, although some authors such as T. Cerdá in his *Liciones de Matematica* (1758, I) confines themselves to finding its decimal logarithm M = $loge = 1/ln10$ = 0,43429448, which enables one to go from natural logarithms to vulgar logarithms. Most of those who manage to find its value give eight decimal figures (2,71828183), while B. Bails in his *Tabla de Logaritmos* (1787) arrives at 23 exact decimals (M gives it with 25 decimals). One observes that this constant still remains to be well founded, since while P. Giannini in his *Curso matemático* (1795, III) calls it the number **e**, Bails refers to it with ***n***, J. J. García in his *Elementos de Aritmetica Álgebra y Geometría* (1782) uses ***c*** and F. Villalpando in his *Tractatus praeliminaris* (1778) prefers the letter ***a***. In the section devoted to differential and integral calculus, the number **e** appears again. It is frequently mentioned when treating the derivation as that basis of logarithms that makes $dlog\ x = dx/x$, or as the basis of the exponentials that fulfilled $de^x = e^x dx$. It receives a similar treatment in the chapters devoted to integrals.

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EULER'S TREATISE "ON THE DOCTRINE OF NUMBERS, EXPOSED IN SIXTEEN CHAPTERS". A COMPARISON WITH GAUSS'S DISQUISITIONES ARITHMETICAE

Josep Pla

A detailed analysis of Euler's posthumous text "Tractatus de numerorum capita sedecim quae supersunt", a possible sketch for an arithmetical treatise, with some considerations on parallelisms with Gauss' work "Disquistiones Arithmeticae". Among its chapters we find intuitions of a more abstract form collected by Gauss in his work, as well as sentences and demonstrations of the results contained in Gauss' work. However, Gauss's work goes further. All this is the objective of my exposition.

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EULER, REY PASTOR AND THE SUMMABILITY OF SERIES

Luís Español González, Emilio Fernández Moral

Between 1926 and 1936 approximately, the Spanish-Argentinian mathematician Julio Rey Pastor researched into the summation of divergent series and drew up a general theory encompassing the different existing methods. He adopted as his basis a generalized definition of sum, drawing his inspiration directly from Euler. Euler's definition, necessarily imprecise given the period in which it was enunciated, was criticised around 1900 by authors such as Pringsheim or Borel, and again subsequently by Knopp, because of the lack of uniqueness of the sum proposed. Rey Pastor, on the other hand, proposes an appropriate interpretation of Euler's definition that overcomes such an objection and proves to be effective as a general abstract definition.

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ONE OF EULER'S WORKS ON CLASSICAL GEOMETRY: THE E135

Eduard Recasens

There are few works by Leonhard Euler in which he follows the methods of the classical geometers. This paper studies the content of the group of propositions collected under the title *Variae demonstrationes geometriae*, which were published in 1750 by the Academy of Sciences of Saint Petersburg. These propositions make reference to triangles and quadrilaterals and are demonstrated according to the style of classical geometers. One of these propositions demonstrates the relation that exists between the sum of the squares of the sides of a quadrilateral and the sum of the squares of the diagonals. Euler attributes the discovery of this result to himself, but contrary to what Euler thought, this same relation is enunciated and demonstrated in Josep Zaragozà's *Geometria Magna in Minimis*, a book which was published in Toledo in 1674. In the last part of this paper, Euler's demonstrations are compared with Zaragozà's.

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EULER AND THE JESUÏTS IN RUSSIA

Satnislav Južnič

The bicentenary of Jesuit General Gabriel Gruber's death is commemorated. The life and work of Gruber, the most famous Ljubljana Jesuit, are discussed. Special concern is given to his researches into hydro-mechanics and optics. Gruber's connections with other scientists of his time, especially with the Jesuit R.J. Bošković and L. Euler, are highlighted. The development of Gruber's navigation directorship in Ljubljana is researched. Gruber was the pioneer of the waterways used in the Sava River area. It was unfortunate that his efforts culminated just before the construction of the first railways, which completely changed the transport situation, and is probably the reason why his achievements are nowadays somewhat neglected. The documents about Gruber's Ljubljana navigation school are presented.

The evident signs of Gruber's ultimate fall in Ljubljana, which led to his escape to Russia, are brought to the reader's attention. His Russian scientific, pedagogical and political results are analyzed. We discuss Gruber's way of life and explain his Russian success. We mention political aspects of Gruber's work, paying due attention to his major contributions to the science and technology of his era. His position enabled him to use superior Western scientific achievements to prove the supposed superiority of the Catholic faith. Besides Gruber's abilities in mathematical sciences, his linguistic virtuosity is claimed to be the key for his diplomatic successes in Russia. He was able to manage Jesuit worldwide correspondence in many different languages. As a Slovene, he had a good command of Russian, which provided him with many advantages in comparison with other western diplomats. Furthermore, being a Slovene, Gruber was far more acceptable to the Russian authorities than Polish priests, who formed the majority among Russian Jesuits at that time. The new Jesuit provincial, Kareu, and two other Jesuits, stayed in St. Petersburg for two months. They visited some academics, among whom was Leonhard Euler, who by then was completely blind and whose home was again a center of St. Petersburg social life after his return from Berlin in 1766. Those social relations paved the way for Gruber's later scientific and political success in St. Petersburg. The Jesuit Gruber was able to display his White Russian scientific and technological achievements at the St. Petersburg Academy rooms, and he soon became the chief political advisor to the Emperor Paul. Like all great men, Gruber had powerful enemies. He was not always very polite towards them and his violent and tragic end was a result of the intolerant and impatient people with whom he coexisted. Gruber devoted his later life to the benefit of his religious order during the hardest times in the history of Jesuits. His amazing achievements proved the legendary abilities of the Slovene people to leave their mark on the world at large. General Gruber helped to spread the Jesuit organization all over the world, together with the fame of Slovene and Slavic scientific and technical talents.

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MATEMATICAL THEORY AND NAVAL PRACTICE DURING THE ENLIGHTMENT. SALVADOR JIMÉNEZ CORONADO, TRANSLATOR OF EULER'S WORK ON THE CONSTRUCTION AND MANOEUVER OF SHIPS

Marià Baig

In 1773 Leonhard Euler published his *Théorie complète de la construction et de la manoeuvre des vaisseaux mise à la portée de ceux qui s'appliquent à la navigation*. The work was taken as a text in many European naval academies and was translated into English, Italian and Russian. In 1782 Salvador Jiménez Coronado presented the Count of Floridablanca with a Spanish translation of Euler's work. After a report from the Academy of Marine Guards in Cádiz, the Admiralty authorized the publication of this translation, although it advised against its use in naval academies, as a result of which the translation remained unpublished. In this paper we include Jiménez Coronado's arguments in favour of the publication as well as the reply from the politicians of the period, revising the dichotomy between pure mathematics and naval practice during the Enlightenment.

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EULER AND THE DYNAMICS OF RIGID SOLIDS

Sebastià Xambó

After presenting an outline of the kinematics and dynamics of systems of particles in contemporary terms, with emphasis on the kinematics and dynamics of rigid bodies, we briefly consider the main historical developments leading to this understanding, and, in particular, the decisive role played by Euler. In our presentation the main role is not played by inertial (or Galilean) observers, but rather by observers who are allowed to move in an arbitrary (smooth, non-relativistic) way.