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THE BLOCK GAUSS-SEIDEL METHOD IN SOUND TRANSMISSION PROBLEMS

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Abstract. *Sound transmission through partitions can be modelled as an acoustic fluid-elastic structure interaction problem. The block Gauss-Seidel iterative method is used in order to solve the finite element linear system of equations. The blocks are defined in a natural way, respecting the fluid and structural domains. The convergence criterion (spectral radius of iteration matrix smaller than one) is analysed and interpreted in physical terms by means of simple one-dimensional problems. This analysis highlights the negative influence on the convergence of a strong degree of coupling between the acoustic domains. A selective coupling strategy has been developed and successfully applied to problems with strong coupling (i.e. sound transmission through double walls).*

1 Introduction

Fluid-structure interaction is the key aspect of many acoustic problems of practical interest. This is the case, for instance, of sound propagation through partitions in buildings. Finite element discretisation of this coupled problem leads to a coupled linear system of equations. In the more widely used formulations, the unknowns are acoustic pressures and structural displacements. The diagonal blocks in the global matrix are typically symmetric and indefinite, but the off-diagonal blocks (which represent the coupling between the acoustic fluid and the elastic structure) break the symmetry of the global matrix.

For this reason, a monolithic solution approach requires the use of general solvers for unsymmetric and indefinite matrices, such as Crout factorisation or GMRES iterations with an appropriate preconditioner . Alternatively, block iterative solvers can be used. By doing so, the symmetry of the diagonal blocks can be exploited, and the storage requirements are decreased.

The block Gauss-Seidel iterative solver is considered here. The well-known convergence condition (spectral radius of iteration matrix smaller than one) is interpreted from a physical viewpoint, by considered simple, one-dimensional vibroacoustic models. This analysis shows the detrimental effect on the convergence of the iterative solver of *i)* the excitation frequency being close the an acoustic or structural eigenfrequency and *ii)* the level of coupling between the acoustic fluid and the structure. An outline of the paper follows. The block Gauss-Seidel solver and block iterative solvers in vibroacoustics are reviewed in Sections 2 and 3. The influence of the degree of coupling is discussed in Section 4. The convergence condition and its physical interpretation are covered in Sections 5 and 6. The application examples of Section 7 corroborate this interpretation, and motivate the selective coupling strategy presented in Section 8, which is applied to the problem of sound propagation through double walls. The concluding remarks of Section 9 close the paper.

2 The block Gauss-Seidel algorithm

The block Gauss-Seidel algorithm will be presented in matrix form. The coupled system of linear equations is

$$\begin{bmatrix} \mathbf{F} & \mathbf{C}_{SF} \\ \mathbf{C}_{FS} & \mathbf{S} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_F \\ \mathbf{x}_S \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_F \\ \mathbf{f}_S \end{Bmatrix} \tag{1}$$

The pressure-displacement formulation is taken as reference here. $\mathbf{F}$ is the flexibility matrix governing the fluid domain with unknown values $\mathbf{x}_F$ (typically pressures) and $\mathbf{S}$ is the stiffness matrix governing the structural domain with unknown values $\mathbf{x}_S$ (typically displacements and rotations). If FEM is used, $\mathbf{F}$ and $\mathbf{S}$ are typically sparse, symmetric and indefinite matrices. $\mathbf{f}_F$ and $\mathbf{f}_S$ are the forces acting in the fluid and structural domains. The coupling is taken into account by means of matrices $\mathbf{C}_{SF}$ and $\mathbf{C}_{FS}$. The forces acting on the structure due to the acoustic pressures in the fluid are

$$\mathbf{f}_{FS} = \mathbf{C}_{FS}\mathbf{x}_F \tag{2}$$

and the acoustic forces in the fluid contour caused by the structural vibrations are

$$\mathbf{f}_{SF} = \mathbf{C}_{SF}\mathbf{x}_S \tag{3}$$

$\mathbf{C}_{SF}$ is proportional to $\rho_F\omega^2$ and $\mathbf{C}_{FS}$ to the contact surface. The global matrix of Eq. (1) is non-symmetric for the more widely used formulations. The block Gauss-Seidel algorithm is summarised in Table 1.

The initial guess can be chosen as the solution of the uncoupled problems. The convergence is checked by means of the relative errors in the solution

$$e_F^{(i)} = \frac{\left|\left|\mathbf{x}_F^{(i)} - \mathbf{x}_F^{(i+1)}\right|\right|}{\left|\left|\mathbf{x}_F^{(i+1)}\right|\right|} \quad ; \quad e_S^{(i)} = \frac{\left|\left|\mathbf{x}_S^{(i)} - \mathbf{x}_S^{(i+1)}\right|\right|}{\left|\left|\mathbf{x}_S^{(i+1)}\right|\right|} \tag{4}$$

and the relative residual

$$r_F^{(i)} = \frac{\left|\left|\mathbf{F}\mathbf{x}_F^{(i)} + \mathbf{C}_{SF}\mathbf{x}_S^{(i)} - \mathbf{f}_F\right|\right|}{\left|\left|\mathbf{f}_F\right|\right|} \tag{5}$$

Choose an initial guess $\mathbf{x}_S^{(0)}$, $\mathbf{x}_F^{(0)}$
<u>for</u> $i = 1, 2, \ldots$
$\mathbf{F}\mathbf{x}_F^{(i+1)} = \mathbf{f}_F - \mathbf{C}_{SF}\mathbf{x}_S^{(i)}$
$\mathbf{S}\mathbf{x}_S^{(i+1)} = \mathbf{f}_S - \mathbf{C}_{FS}\mathbf{x}_F^{(i+1)}$
check convergence; continue if necessary
<u>end</u>

Table 1: The block Gauss-Seidel method

The divergence of the method can be controlled by comparing the errors in iterations i and $i-2$. If some measure of the error (Eqs. (4) and (5)) in iteration i is larger than in iteration $i-2$, the method is probably diverging. On the contrary, the algorithm can be converging even if the error in iteration i is larger than the error in iteration $i-1$.

The two systems of equations in Table 1 have to be solved several times with different force vectors but constant matrices $\mathbf{F}$ and $\mathbf{S}$. This has to be exploited for maximum efficiency. A first option is to use a direct solver (for small matrix dimensions) and save the factorisation of the matrices. Another possibility is to use the adequate iterative solver (GMRES, MINRES,... see (5) for more details) and save the preconditioner, which is calculated only once for $i = 0$ and can be reused for the successive iterations. Wave problems often require to perform calculations for successive frequencies or different types of force terms. Matrices are then very similar. The possibility of using the same preconditioner for several successive frequencies has also to be considered.

3 Review of block iterative solvers in acoustics

The block Gauss-Seidel method can be understood as a domain decomposition method. These methods base their efficiency in the splitting of the physical domain of the problem into smaller subdomains. The system of equations is then solved at two different levels. On the one hand each subdomain and on the other hand the continuity between them. These techniques have been mainly designed to be used in parallel computing machines. Each CPU deals with a single smaller domain using the more adequate solver for each region.

In (8) and (9) domain decomposition techniques have been used in order to solve scattering problems governed by the Helmholtz equations in big physical domains. The continuity of the pressure field (and its normal derivative) in the interface between regions is imposed by means of Lagrange multipliers. Moreover each subdomain has to be regularised by means of fictitious boundary conditions in order to avoid problems caused by artificial eigenfrequencies. The method has also been used for vibroacoustic problems. In (13) the partitions have been done in both the acoustic domains and the structure. Finally, in (10), (6) and (11), block Jacobi and block Gauss-Seidel algorithms have been used for vibroacoustic problems where the decomposition of the domain strictly respects the physical regions (fluid and structure). The only interface between subdomains is the fluid-structure boundary. Since the goal of the authors is to propose a general solver (also for strongly coupled problems) their discussion is focused in the convergence of the methods. It seems clear that using the physical interface conditions in order to transfer information between fluid and structural subdomains leads to divergence in a large number of situations. They propose relaxed coupling conditions that cause the block Gauss-Seidel algorithm to have fast convergence for all the analysed situations. However, the application examples shown using this method are rather poor (as well as the information of the physical data employed in the examples and the numerical parameters used to ensure convergence). The performance of the modified algorithms around the eigenfrequencies of the problem has not been analysed. Moreover, the use of the modified interface conditions require some modifications at finite element level.

4 Influence of the degree of coupling

It is important here to distinguish between the coupling understood as a physical phenomenon and the consequences of coupling for the block Gauss-Seidel algorithm or the organisation of the blocks in the algorithm. Based on these two concepts three different situations can be described. First the problems where the physical coupling is weak and decoupled solving strategies are adequate. This is a frequent situation in sound transmission problems. In those cases the interaction forces are small (at least the forces of the receiving acoustic domains over the structure). The decoupled strategy is also called chained approach and can be understood as a single iteration of the block Gauss-Seidel solver with the appropriate ordering of blocks. The global system is solved in a successive way. The domain with acoustic sources is firstly solved. The obtained pressure is imposed on the structure and finally the obtained displacement field is used to generate sound in the receiving acoustic domain. If the excitation is a mechanical force, the structural problem is firstly solved. The main problem of assuming weak coupling between the acoustic domains and the structure is that there is no practical criterion to check the validity of the hypothesis. It depends on too many factors (geometrical dimensions of the problem, physical data, analysed frequency). The second type of problems that can be distinguished are those where the physical coupling is important, the chained approaches lead to solutions with significant errors but the block Gauss-Seidel solver can be used and converges. Examples are shown in Section 7.3. Finally, there are also situations that are strongly coupled from a physical point of view and where a standard block Gauss-Seidel solver without modifications cannot be used, because it diverges. Typical examples are coupled problems with dense fluids (Section 7.2) and sound transmission problems with double walls (Section 8).

Our reasons for the use of the standard block Gauss-Seidel vs. a monolithic solver are:

1. **No assumptions on level of coupling.** Since the iterations are not stopped till

convergence is reached, the hypothesis of weak coupling is not necessary (however, if it is true, iterations are drastically reduced). As will be shown below, one iteration (like in chained approaches) or two are enough for weakly coupled situations. However, when necessary (due to strong coupling), the solver automatically iterates in order to reduce errors.

2. **Increase of efficiency and decrease of calculation times.** The calculations presented here have not been performed using parallel processing machines (which is one of the goals of domain decomposition methods). A single CPU has been used. However, it is more efficient to solve a vibroacoustic problem using a block Gauss-Seidel procedure than a solver considering the global matrix.

3. **Improvement of storage costs.** In the context of an analysis in the frequency domain where multiple frequencies have to be considered, the use of block Gauss-Seidel implies an improvement in the storage of coupling matrices. The coupling force vectors are obtained from a function where the coupling matrix is an input parameter. The pulsation of the problem or the density can be other input parameters. The outputs are $\mathbf{f}_{SF}$ and $\mathbf{f}_{FS}$. Only one coupling matrix (with basically geometrical information) is then required for each acoustic domain. On the contrary, for coupled problems this cost is multiplied by three: one matrix has to be stored to be used for other frequencies and the global system of equations includes two coupling matrices per acoustic domain.

5 Analysis of the block Gauss-Seidel method

As other stationary iterative methods, the block Gauss-Seidel algorithm converges if the spectral radius ρ (i.e. the maximum modulus of the eigenvalues) of the iteration matrix $\mathbf{G}$ is less than one, see (17). The algorithm in Table 1 can be rewritten as

$$\left\{ \begin{array}{c} \mathbf{x}_F^{(i+1)} \\ \mathbf{x}_S^{(i+1)} \end{array} \right\} = \left[\begin{array}{cc} \mathbf{0} & -\mathbf{F}^{-1}\mathbf{C}_{SF} \\ \mathbf{0} & \mathbf{S}^{-1}\mathbf{C}_{FS}\mathbf{F}^{-1}\mathbf{C}_{SF} \end{array} \right] \left\{ \begin{array}{c} \mathbf{x}_F^{(i)} \\ \mathbf{x}_S^{(i)} \end{array} \right\} + \left\{ \begin{array}{c} \mathbf{F}^{-1}\mathbf{f}_F \\ \mathbf{S}^{-1}\left(\mathbf{f}_S - \mathbf{C}_{FS}\mathbf{F}^{-1}\mathbf{f}_F\right) \end{array} \right\} \tag{6}$$

The iteration matrix $\mathbf{G}$ is the matrix in Eq. (6), so the convergence condition is

$$\rho\left(\mathbf{S}^{-1}\mathbf{C}_{FS}\mathbf{F}^{-1}\mathbf{C}_{SF}\right) < 1 \tag{7}$$

6 Physical interpretation of the convergence condition

The simplified model of Figure 1 will be used in order to understand and illustrate the phenomena of vibroacoustic coupling and the performance of the block Gauss-Seidel algorithm. A vibrating mass is coupled with an acoustic domain. Both can be excited: the mass by means of an exterior force $F(t) = \mathrm{Re}\left\{\varphi e^{\mathrm{i}\omega t}\right\}$ and the acoustic fluid cavity by an exterior imposed velocity $v_{\boldsymbol{n}}$. Note that the model is formulated for a unit surface.

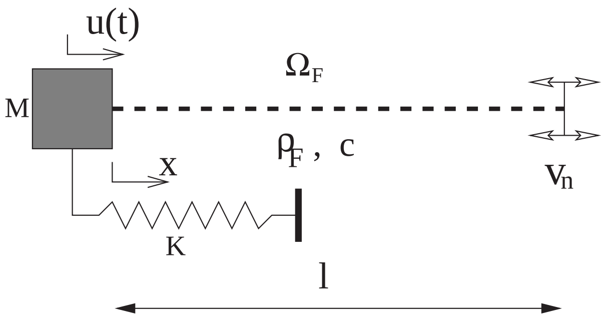


Figure 1: Simple one-dimensional coupled system with two degrees of freedom.

Thereby φ is the phasor of force per unit surface, and M and K the mass and stiffness per unit surface.

The interaction between the acoustic fluid and the single mass can be characterised by the pressure applied by the fluid on the mass and the displacement imposed by the mass at the acoustic contour.

The governing equation and boundary conditions for the fluid domain are

$$\frac{d^2\,p(x)}{d\,x^2} + k^2 p(x) = 0 \qquad x \text{ in } \Omega_F \tag{8}$$

$$\left.\frac{d\,p(x)}{d\,x}\right|_{x=0} = \rho_F \omega^2 u \tag{9}$$

$$\left.\frac{d\,p(x)}{d\,x}\right|_{x=\ell} = -\rho_F \mathrm{i}\omega v_{\boldsymbol{n}} \tag{10}$$

and if the frequency of the problem is a real value, the pressure field is

$$p(x) = C_1 \cos(kx) + C_2 \sin(kx) \tag{11}$$

where C_1 and C_2 are unknown complex constants. Taking into account the dynamic equilibrium of the single mass, a linear system with three equations results:

$$\left[\begin{array}{cc|c} \sin(k\ell) & -\cos(k\ell) & 0 \\ 0 & 1 & -\rho_F\omega c \\ \hline 1 & 0 & K-\omega^2 M \end{array}\right] \left\{\begin{array}{c} C_1 \\ C_2 \\ \hline u \end{array}\right\} = \left\{\begin{array}{c} \rho_F \mathrm{i} c v_{\boldsymbol{n}} \\ 0 \\ \hline \varphi \end{array}\right\} \tag{12}$$

This system is the particularisation for this simple one-dimensional example of Eq. (1). The convergence condition (7) leads to

$$\rho\left(\mathbf{G}\right) = \frac{\cos\left(k\ell\right)\rho_F\omega c}{\sin\left(k\ell\right)\left(K-\omega^2 M\right)} < 1 \tag{13}$$

A similar analysis can be done for the one-dimensional situation with two fluid domains studied in (14). The expression of the spectral radius is

$$\rho\left(\mathbf{G}\right)=\frac{\rho_F\omega c}{K-\omega^2 M}\left(\frac{\cos\left(k\ell_1\right)}{\sin\left(k\ell_1\right)}+\frac{\cos\left(k\ell_2\right)}{\sin\left(k\ell_2\right)}\right) \tag{14}$$

Several conclusions can be obtained from Eqs. (12), (13) and (14). The method is less efficient for denser fluids (i.e. larger density ρ_F) or fluids with higher wave speed c, because the coupling between the structure and the fluid increases.

The geometry of the problem is also important. For this one-dimensional case, the geometry is represented by terms $\cos\left(k\ell\right)$ and $\sin\left(k\ell\right)$. If $\sin(k\ell)\approx 0$ the method will not converge. This happens for the eigenfrequencies of the acoustic cavity, $k=n\pi/\ell$, but also when ℓ is very small (small fluid domains). If $\cos(k\ell)=0$ the method converges in one iteration. This is a very specific situation of the one-dimensional model and cannot be generalised to higher dimensions.

Note that the method also diverges for frequencies close to the structural eigenfrequency $\sqrt{K/M}$. Finally, Eqs. (13) and (14) also show that the performance of the iterative solver increases with the frequency.

A similar parameter ($\lambda=\rho_F c/\rho_S \mathrm{t}\omega$) has been defined by (1). t is the typical thickness of the structure and ρ_S its density. However, λ does not take into account the influence of the geometry nor the stiffness.

By condensing out the unknown C_2 and noting from Eq. (11) that C_1 is $p(x=0)$, system (12) can be recast as

$$\left[\begin{array}{cc}\sin(k\ell) & -\rho_F\omega c\cos(k\ell)\\ 1 & K-\omega^2 M\end{array}\right]\left\{\begin{array}{c}p(x=0)\\ u\end{array}\right\}=\left\{\begin{array}{c}\rho_F\mathrm{i}cv_{\boldsymbol{n}}\\ \varphi\end{array}\right\} \tag{15}$$

and $\rho\left(\mathbf{G}\right)$ can then be viewed as the ratio of stiffness of the fluid and the structure, including the effect of coupling:

$$\rho\left(\mathbf{G}\right)=\frac{\left(d\,p(x=0)/d\,u\right)_F}{\left(d\,p(x=0)/d\,u\right)_S} \tag{16}$$

The subscript F (S) means here derivative from the point of view of the fluid (structure).

The conceptual behaviour of the fluid-structure system has been plotted in Figure 2. The harmonic equilibrium is reached at the pair $\mathbf{x}_S^*-\mathbf{x}_F^*$. The acoustic pressure caused by the structural displacement is

$$\mathbf{x}_{SF}=-\mathbf{F}^{-1}\mathbf{C}_{SF}\mathbf{x}_S^* \tag{17}$$

and the displacement caused by the pressure is

$$\mathbf{x}_{FS}=-\mathbf{S}^{-1}\mathbf{C}_{FS}\mathbf{x}_F^* \tag{18}$$

They are caused by the coupling effect. Figure 3 shows a sketch of the convergence and divergence of the algorithm depending on the spectral radius.

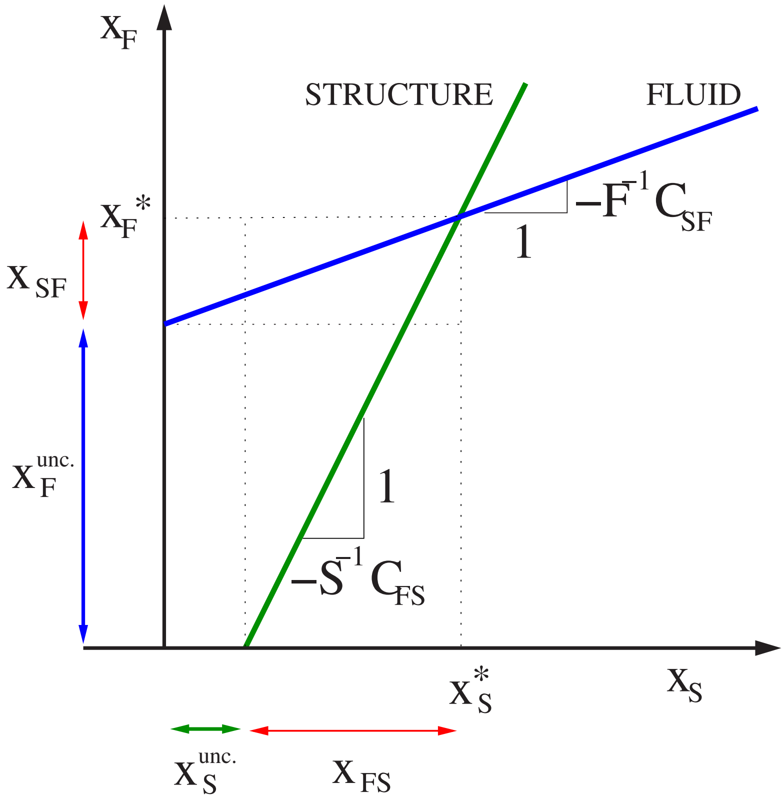


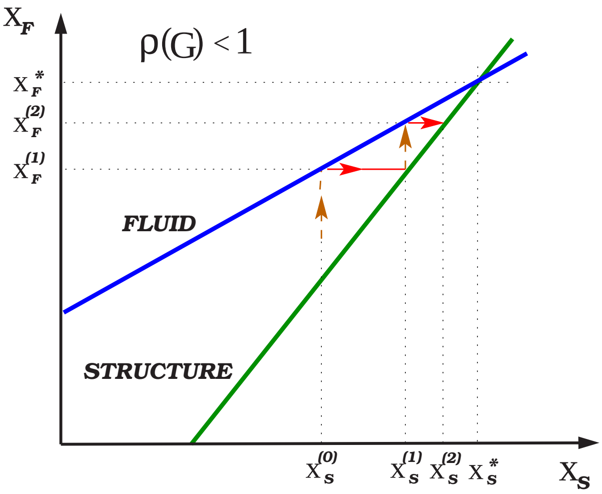
Figure 2: Conceptual behaviour of a coupled fluid-structure system.

7 Application examples

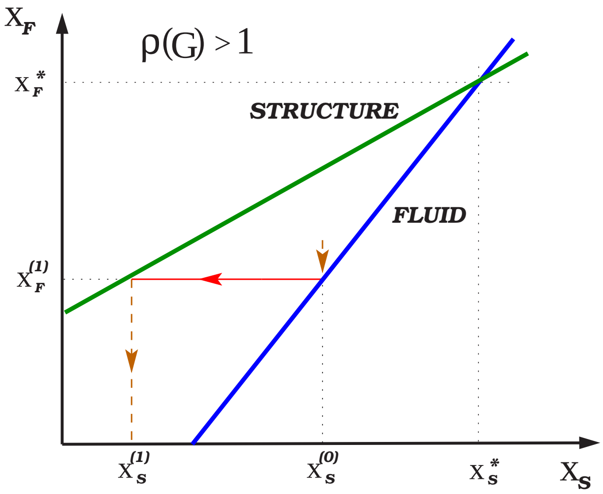
The performance of the block Gauss-Seidel method is illustrated here with various two-dimensional vibroacoustic problems (sound transmission through a single wall). The goal is to illustrate the influence on the convergence of the iterative solver of *i)* the damping, *ii)* the coupling between fluid and structure and *iii)* acoustic and structural eigenfrequencies. A FEM-FEM approach (i.e. finite elements for the fluid and acoustic domains) is used here.

7.1 Influence of damping

Two acoustic domains are separated by a single wall (represented in this two-dimensional setting by a concrete beam), see Figure 4. The acoustic excitation is a punctual sound source placed in the left bottom corner of the first domain, at a distance of 0.5 m to the contours. The room dimensions are 3×3 m^2 and 4×3 m^2. The material and geometrical parameters are summarised in Table 2. Note that we are dealing with air, which is a very light fluid. This is the typical situation where the method will have a very good behaviour. A relative tolerance of 10^{-9} is used in the stopping criteria defined in Eqs. (4)



(a)



(b)

Figure 3: Convergence of the block Gauss-Seidel algorithm: (a) Convergence for $\rho(\mathbf{G}) < 1$; (b) Divergence for $\rho(\mathbf{G}) > 1$.

and (5)

STRUCTURE

Meaning	**Symbol**	**Heavy**	**Lightweight**
Young's modulus	E	$2.943 \cdot 10^{10}$ N/m^2	$4.5 \cdot 10^9$ N/m^2
Poisson's ratio	ν	0.25	0.25
Wall density	ρ_S	2500 kg/m^3	913 kg/m^3
Wall thickness	t	0.10 m	0.013 m
Loss factor	η	$0 - 5$ %	$0 - 5$ %

FLUID

Meaning	**Symbol**	**Value**
Speed of sound	c	340 m/s
Density of fluid	ρ_F	1.18 kg/m^3
Source strength	Q	0.005i m^3/s
Acoustic absorption	α	$0 - 30$ %

Table 2: Material parameters for the acoustic and structural domains

Two different situations have been analysed. On the one hand, an undamped problem (no acoustic absorption and no structural damping). On the other hand, the same problem

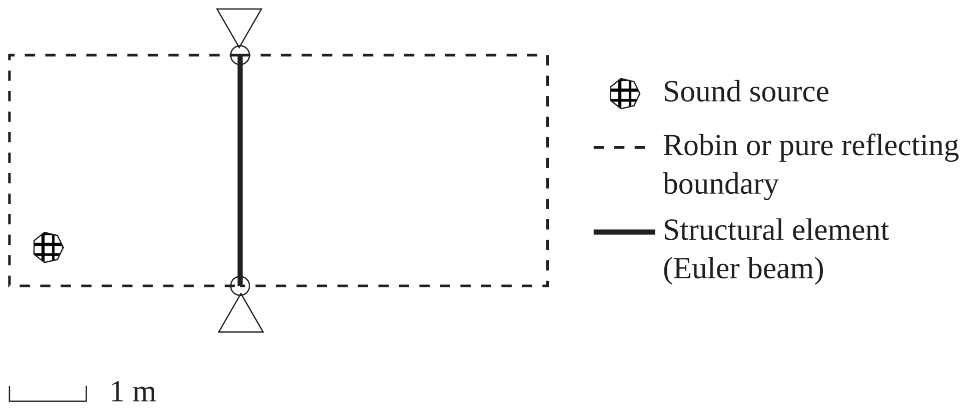


Figure 4: Sound transmission through a single wall

with an acoustic absorption of 30% at the boundaries (introduced by means of a Robin boundary condition) and hysteretic structural damping (5%). These are reasonable values, which have not been chosen for numerical convenience.

The results (number of iterations required) have been plotted in Figure 5. Note that damping considerably decreases the number of iterations required, especially near eigenfrequencies.

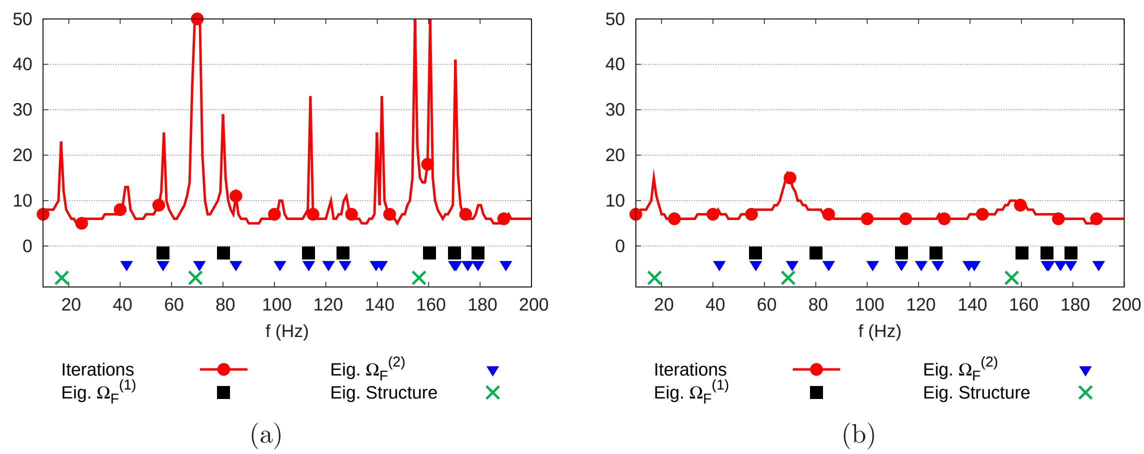


Figure 5: Iterations of the block Gauss-Seidel solver: (a) undamped problem; (b) damped problem (30 % acoustic absorption and 5 % structural damping). Eigenfrequencies of the sending and receiving domains, $\Omega_F^{(1)}$ and $\Omega_F^{(2)}$, and the structure are also shown.

7.2 Influence of the fluid density

The example of Section 7.1 (concrete wall and damping) is solved now for increasing values of the fluid density (ρ_F, $10\rho_F$ and $30\rho_F$) and a constant frequency of 100 Hz. The only parameter that has been changed besides the density is the admittance A of the acoustic contours, in order to keep a constant value of absorption (30 %). In view of the positive effect of damping pointed out in Section 7.1, this precaution is necessary for a fair comparison.

The convergence results (relative error vs. iterations) of Figure 6 show the expected behaviour. The convergence rates (i.e. absolute value of the slope) are 2.05, 0.92 and 0.43 for fluid density ρ_F, $10\rho_F$ and $20\rho_F$ respectively. When the density is increased by a factor of 10 (20), the convergence rate decreases by 1.13, (1.62), close to the theoretical value of $\log_{10} 10 = 1$ ($\log_{10} 30 = 1.47$).

As discussed in Section 6, taking a larger fluid density increases the coupling between the acoustic and structural domains. The eigenfrequencies are virtually unchanged by these modifications in fluid density and admittance, so 100 Hz is not close to an eigenfrequency for any of the three cases. Thus, changing ρ_F can be regarded as a 'clean' way of modifying the degree of coupling without affecting the frequency spectrum. If one plays with the wave speed c, on the contrary, the problem eigenfrequencies change.

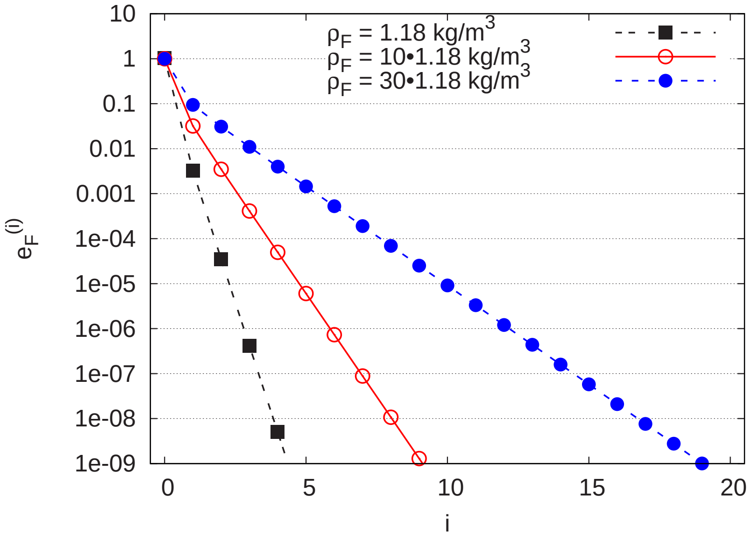


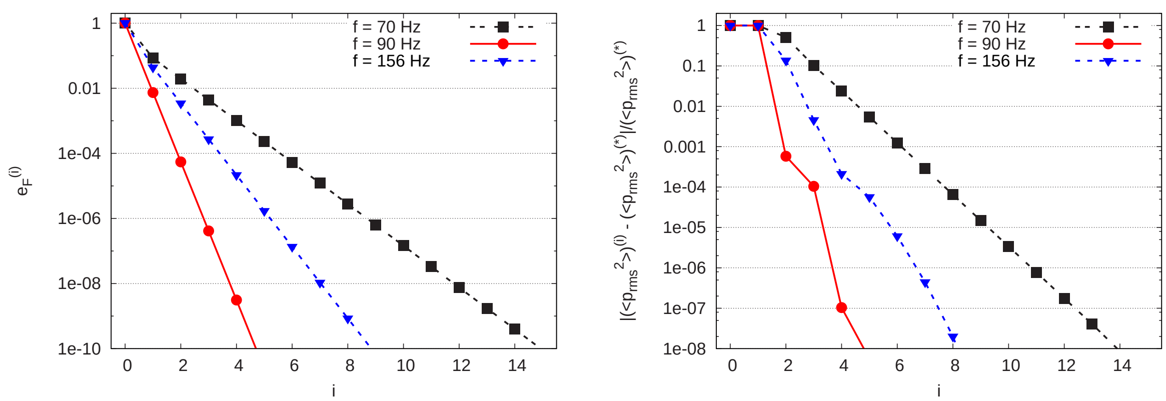
Figure 6: Influence of the fluid density in the convergence of the block Gauss-Seidel algorithm

7.3 Influence of particular eigenfrequencies

The performance of the block Gauss-Seidel solver for three particular frequencies has been analysed. The example of Section 7.1 has been considered (damped situation). The aim of the analysis is to show differences in the efficiency of the method depending on the type of eigenfrequencies that are close to the excitation frequency. The studied frequencies are: *i)* 70 Hz, which is close to uncoupled eigenfrequencies of the structure (70.83 Hz) and the receiving room (69.27 Hz); *ii)* 90 Hz, which is not close to any of the eigenfrequencies of the problem; *iii)* 156 Hz, which is close to an uncoupled eigenfrequency of the structure (156.22 Hz).

Results are presented in Figure 7. The better convergence is found for the case *ii)* that is not affected by any eigenfrequency of the problem. More iterations are required in situations *i)* and *iii)*. The eigenfrequencies of the problem increase the value of the spectral radius of the iteration matrix. This phenomenon has already been predicted in the one-dimensional model presented in Section 6, see Eqs. (13) and (14).

Results are relevant from an engineering point of view. The evolution of the relative error of $< p_{rms}^2 >$ (the most frequently used output in sound transmission problems) is shown in Figure 7 (b). For most of the excitation frequencies, the error is small (< 10 %) after the first effective iteration (it is the case of chained approach). However, for excitation frequencies that are close to the eigenfrequencies or for undamped problems, the error is larger.



(b)

Figure 7: Behaviour of block Gauss-Seidel solver for several particular eigenfrequencies: (a) Relative error in the sending domain; (b) Relative error of $< p_{rms}^2 >$ in the receiving domain.

8 The case of double walls: selective coupling of fluid domains

All the examples shown in Section 7 deal with single walls. For typical geometrical and material parameters, the acoustic domains (sending and receiving rooms) and the structure are weakly coupled, and thereby chained approaches have a good performance.

This is not the case, however, for double walls (consisting on two leaves separated by a cavity, either filled with an acoustic absorbing material or not), see Figure 8. For these applications, a chained approach or even the iterative block Gauss-Seidel method are not efficient or diverge. The reason is that the air cavity between walls is usually small (cavity thickness between 2 cm and 8 cm), and thereby, $k\ell$ is also small. As shown in Section 5, this increases the stiffness of the acoustic domain and causes the divergence of the block Gauss-Seidel algorithm. To overcome these difficulties, we present here a modification

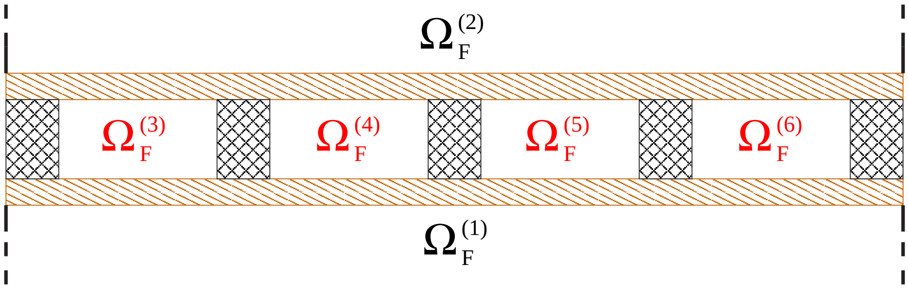


Figure 8: Sketch of a double wall. The sending and receiving rooms (1 and 2) are weakly coupled with the structure while the cavities (3, 4, 5 and 6) are strongly coupled. This information is used in the solver.

of the block Gauss-Seidel algorithm. The goal is to deal with situations where some of the fluid domains are strongly coupled to the structure. The matrices in Eq. (1) can be written in detail as

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}^{(1)} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{F}^{(2)} & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{F}^{(n)} \end{bmatrix} \tag{19}$$

and

$$\mathbf{C}_{SF} = \begin{bmatrix} \mathbf{C}_{SF}^{(1)} \\ \mathbf{C}_{SF}^{(2)} \\ \vdots \\ \vdots \\ \mathbf{C}_{SF}^{(n)} \end{bmatrix} \qquad \mathbf{C}_{FS} = \begin{bmatrix} \mathbf{C}_{FS}^{(1)} & \mathbf{C}_{FS}^{(2)} & \cdots & \cdots & \mathbf{C}_{FS}^{(n)} \end{bmatrix} \tag{20}$$

where n acoustic domains are assumed.

A selective coupling strategy will be used. The m problematic fluid domains will now be solved together with the structure. They are in general the smaller fluid domains (i.e. the air cavities inside the double wall, see Figure 8), which are strongly coupled with the structure. A new matrix for the structural part of the problem including these coupled acoustic domains can be written as

$$\mathbf{S}^* = \begin{bmatrix} \mathbf{F}^{(1)} & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{C}_{SF}^{(1)} \\ \mathbf{0} & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{F}^{(m)} & \mathbf{C}_{SF}^{(m)} \\ -\mathbf{C}_{FS}^{(1)} & \cdots & \cdots & -\mathbf{C}_{SF}^{(m)} & \mathbf{S} \end{bmatrix} \tag{21}$$

The double wall in Figure 8 is a typical situation where this selective coupling is very efficient. The sending room $\Omega_F^{(1)}$ and the receiving room $\Omega_F^{(2)}$ are weakly coupled with the structure, so the matrices describing this part of the problem are considered as independent blocks. On the contrary, the cavities between leaves (acoustic domains $\Omega_F^{(3)}$, $\Omega_F^{(4)}$, $\Omega_F^{(5)}$ and $\Omega_F^{(6)}$) are strongly coupled with the structure, so their related matrices are solved in the same block as the structural part of the problem, in matrix $\mathbf{S}^*$.

Due to this coupling, matrix $\mathbf{S}^*$ loses the symmetry of matrix $\mathbf{S}$, so an unsymmetric solver is required. However, the coupled acoustic domains are small and the increase in the size of the matrix is moderate. Apart from the definition of matrix $\mathbf{S}^*$, the rest of the iterative process remains unchanged.

8.1 Validation: one-dimensional example

The selective coupling strategy has been used in order to solve the one-dimensional problem for layered partitions presented in (14). The example with data in Table 3 has been used to illustrate the performance of selective coupling, as compared to the standard block Gauss-Seidel algorithm. Note that there is no damping.

In Figure 9(a) the spectral radius of the iteration matrix for several values of the cavity thickness is shown. The frequency of the problem is 100 Hz, not close to any eigenfrequency. The standard algorithm only converges for wide air cavities, whereas selective coupling converges for all the thickness range.

Figure 9(b) shows the evolution of the spectral radius with frequency for the case of a 7 cm thick cavity. Again, only selective coupling is convergent for all the range of interest (problems around eigenfrequencies are caused by the lack of damping).

8.2 Application: two-dimensional example

Selective coupling has also been used for two-dimensional problems. The example of Figure 4 is solved again, but replacing the single wall by heavy and a lightweight double wall. The thickness of each leave is 0.03 m (heavy) and 0.013 m (lightweight) with the material data of Table 2. The cavity between leaves is 0.07 m thick. Two cases have been considered: air cavity and absorbing material (resistivity $\varrho = 10^4$ N/ $(s \cdot \mathrm{m}^4)$). The acoustic absorption is 30% and the structural damping 5%.

The two larger acoustic domains have been considered as independent blocks while the acoustic cavity between leaves and the structural matrix, as well as the coupling matrices

Meaning	Symbol	Value
Density of leave 1	ρ_{wall}	913 kg/m^3
Thickness of leave 1	t_{wall}	0.013 m
Density of leave 2	ρ_{wall}	809 kg/m^3
Thickness of leave 2	t_{wall}	0.009 m
Length of domain 1	ℓ_1	3 m
Length of domain 2	ℓ_2	4 m
Air gap length	ℓ_3	0.07 m
Surface of the wall	S	3 m^2
Stiffness of the single mass	K	0 N/m
Structural damping (C = 2 $\beta\omega_{nat}$ M)	β	0 N s/m kg
Normal velocity	$v_{\boldsymbol{n}}$	$7.5 - 2.5 \cdot 10^{-3}$i m/s
Admittance	A	0 m^3/Ns

Table 3: Geometrical and material data for a lightweight double wall.

between them, have been assembled together in $\mathbf{S}^*$. The number of iterations required vs. the frequency has been plotted in Figure 10, for both the standard block Gauss-Seidel and the selective coupling strategies.

It has to be noted that an iteration of the selective coupling strategy is computationally more expensive because of the cost of solving the linear system with matrix $\mathbf{S}^*$ (as compared to solving a system with matrix $\mathbf{S}$ and other small systems with matrices $\mathbf{F}^{(j)}$).

However, this fact is more than compensated by the better convergence behaviour of selective coupling. The convergence is quickly reached in all the frequency range; eigenfrequencies do not drastically increase the number of iterations. In fact, the only situation of non-convergence takes place for the mass-air-mass resonance of the wall. This is an eigenfrequency that couples all the subdomains of the problem (for more details, see (7)). For the case of the heavy double wall studied here it is 36.3 Hz and for the lightweight double wall it is 91.2 Hz. This only happens for double walls without absorbing material placed in the cavity.

In contrast, when the problem of a heavy double wall is solved by means of a totally uncoupled procedure (i.e. standard block Gauss-Seidel), the method diverges for frequencies under 60 Hz. For higher frequencies, the number of iterations required is considerably larger than with selective coupling. For the case of lightweight double walls with and without absorbing material, convergence is rarely reached if selective coupling is not used.

9 Concluding remarks

A block Gauss-Seidel solver has been used for vibroacoustic problems. The division of the system matrix has been done in a physical way, distinguishing the blocks generated

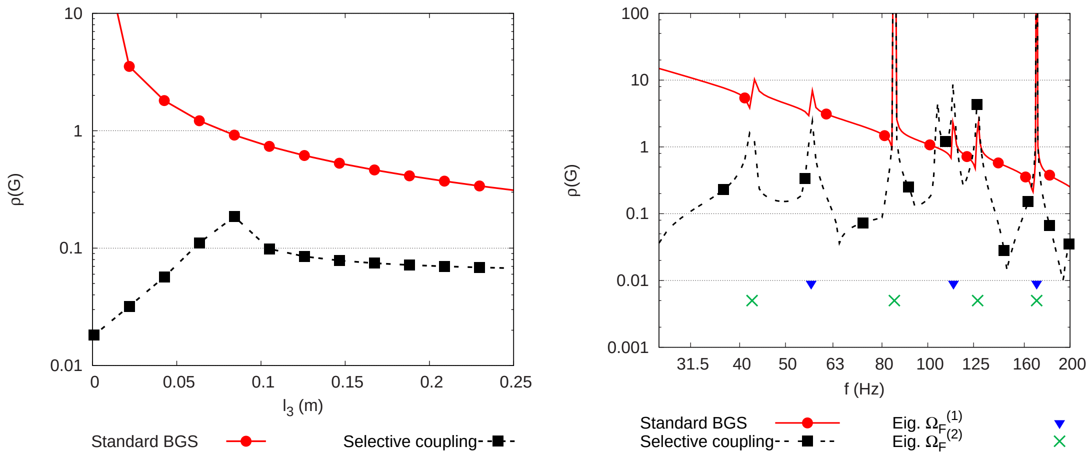


Figure 9: Selective coupling strategy applied to a one-dimensional model for a double wall. Evolution of the spectral radius of the Gauss-Seidel iteration matrix: (a) vs. cavity thickness, for a constant frequency of 100 Hz; (b) vs. frequency, for a constant thickness of 0.07 m.

by each fluid domain and each structure. The method has been used in order to solve two-dimensional and three-dimensional vibroacoustic problems (already shown in (16) and (15)).

A physical interpretation of its convergence has been done by means of a one-dimensional model. Analytical expressions of the spectral radius have been obtained.

For the analysed situations, the method is more efficient and fast than solving the global system of equations with a solver that does not consider the block structure of the matrix. The convergence rate of the method highly depends on the ratio of stiffnesses between the structure and the fluid as well as how are the domains coupled. The best convergence rate is obtained for weakly coupled situations.

The method is not efficient around the eigenfrequencies for undamped situations. However, it is significantly improved by using typical values of acoustic absorption or structural damping required in a realistic model.

Two particular situations have been analysed. On the one hand, a case where the solution is very conditioned by the resonance of some part of the problem. It is a case where a chained approach would provide a solution with error and the block Gauss-Seidel strategy iterates till convergence. On the other hand, the case of a double wall. Since the fluid cavity between leaves is narrow, the coupling of this part of the problem is strong. A semi-coupled strategy has been used for these situations where the fluid cavities have been solved in the same block than structure. The improvement allows the resolution of the problem by means of a block strategy.

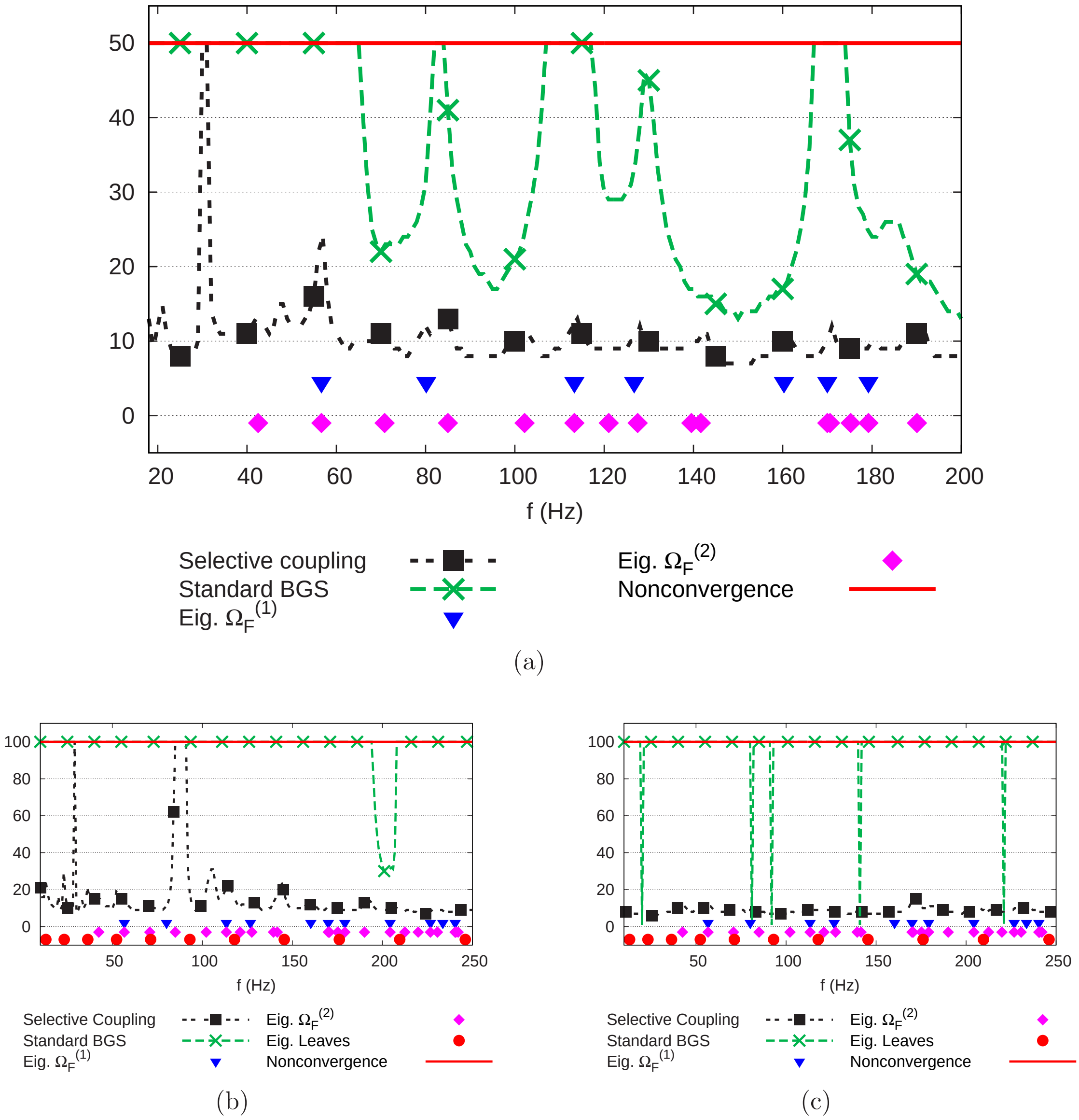


Figure 10: Application of the selective coupling to a problem of sound transmission through a double wall. Iterations required for each algorithm. Tolerance: $= 10^{-9}$; maximum number of iterations : 50 and 100. (a) Heavy double wall (b) Lightweight double wall with air cavity (c) Lightweight double wall with absorbing material

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