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AN ELASTOPLASTIC MODEL FOR THE THM ANALYSIS OF FREEZING SOILS

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Summary. By employing a combination of ice pressure, liquid pressure and total stress as state variables, a new thermoplastic constitutive model has been developed that encompasses frozen and unfrozen behaviour within a unified effective-stress-based framework. It has been incorporated into a fully coupled thermo-hydro-mechanical (THM) finite element formulation that considers freezing and thawing in water-saturated soils and applied to a large-scale pipeline frost heave test.

1 INTRODUCTION

Freezing and thawing of pore fluid within soils involves complex thermal, hydraulic and mechanical processes that can have significant mutual geotechnical interactions. For example, phase changes of pore fluid caused by temperature variations modify the hydraulic regime of the soil, which in turn induces mechanical deformation. At the same time, any change in the hydraulic and mechanical conditions feeds back to the thermal processes via advection and changes in ice and water contents. Such Thermo-Hydro-Mechanical (THM) interactions underlie many cold-region geomorphological processes, such as solifluction and thermokarst formation, as well as geotechnical problems, such as frost heaving, foundation distress/settlement and slope instability. The rational analysis of this type of problems requires an adequate constitutive model to represent the mechanical behaviour of freezing soils and the development of an appropriate coupled THM formulation. Both are summarily presented herein.

2 THERMOPLASTIC MODEL FOR FROZEN AND UNFROZEN SOIL

Noting the close analogy between the physics of frozen-saturated and unfrozen-unsaturated soils, this study adopted an alternative two-stress variable constitutive relationship [1]. Adopting stress variables $p_n = p - \max\left(P_l, P_i\right)$ and $s = \max\left(P_i - P_l, 0\right)$ (in addition to the deviatoric stress, q) allows the Barcelona Basic Model (the BBM [2]) developed for unsaturated soil to be extended to provide a new constitutive model that describes the essential features of frozen and unfrozen behaviour. The variables p_n and s can be interpreted as the net stress representing external confinement and the suction, respectively whereas P_i and P_l are ice and liquid pressures respectively. The yield surface is expressed as

$$F = \left\{ p_n - \left(\frac{p_{n0} - ks}{2} \right) \right\}^2 + \frac{q^2}{M^2} - \left(\frac{p_{n0} + ks)}{2} \right)^2 \tag{1}$$

where

$$q = \sqrt{\frac{3}{2}(s_{ij} \cdot s_{ij})} \qquad s_{ij} = \sigma_{ij} - p\delta_{ij} \tag{2}$$

$$p_{n0} = \left(\frac{p_{n0}^{*}}{p^{c}} \right)^{\frac{\lambda(0)-\kappa}{\lambda-\kappa}} \qquad \lambda = \lambda(0)\left\{(1-r)\exp(-\beta s) + r\right\} \tag{3}$$

and M, k, $\lambda(0)$, κ, p^c, β and r are constants; see [2] for details.

Considering the $q - p_n$ plane, an associated flow rule is assumed, so the plastic potential is expressed by the same equation as (1). The hardening is defined by the plastic volumetric strain, ε_v^p as

$$\delta p_{n0}^{*} = \frac{1+e}{\lambda(0)-\kappa}\, p_{n0}^{*} \delta\varepsilon_v^p \tag{4}$$

where e is the void ratio. The yield surface defined by the above equations is illustrated in Figure 2. When unfrozen (i.e. s=0), the model reduces to an effective stress-based Critical State model similar to Modified Cam-Clay. As temperatures fall below the freezing point, s increases and the yield surface cross-section in the $q - p_n$ plane expands, giving the soil higher yield stress and strength.

It should be noted that the ice and liquid pressures are related through the Clasius-Clapeyron equation, derived from equilibrium of the chemical potentials between two phases:

$$dP_i = \frac{\rho_i}{\rho_l} dP_l - \frac{\rho_i l}{T} dT \tag{5}$$

where ρ_i and ρ_l are the ice and liquid densities, respectively and T is the absolute temperature.

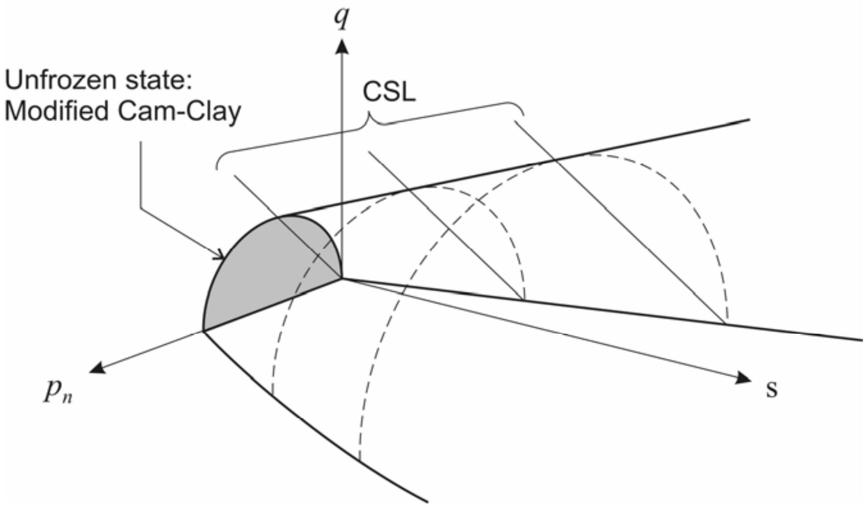

Figure 1: Three-dimensional view of yield surface for constitutive model for frozen and unfrozen soils

3 THM FORMULATION

The constitutive model has been incorporated into a fully coupled THM formulation that involves the simultaneous solution of the balance equations for water, energy and momentum:

$$\text{Water: } \frac{\partial}{\partial t}(\rho_l S_l \phi + \rho_i S_i \phi) + \nabla \cdot \left(\rho_l \mathbf{q}_l\right) = f^w \tag{6}$$

where ϕ is the porosity, S_l and S_i are degrees of liquid and ice saturation, respectively ($S_l + S_i = 1$), $\mathbf{q}_l$ is the liquid water flux vector and f^w is the sink/source term of mass.

$$\text{Energy: } \frac{\partial}{\partial t}(e_s \rho_s \left(1-\phi\right) + e_l \rho_l S_l \phi + e_i \rho_i S_i \phi) + \nabla \cdot (-\lambda \nabla T + \mathbf{j}_l^e) = f^e \tag{7}$$

where e_s, e_l and e_i are the specific internal energy of solid soil minerals, liquid water and ice, respectively, λ is in this case the overall thermal conductivity of the soil mass, $\mathbf{j}_l^e$ is the advective term of heat flux ($\mathbf{j}_l^e = e_l \rho_l \mathbf{q}_l$) and f^e is the sink/production term of energy.

$$\text{Momentum: } \nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = 0 \tag{8}$$

where $\boldsymbol{\sigma}$ are total stresses and **b** are body forces.

4 APPLICATION

The performance of the THM model and its numerical implementation were evaluated with reference to published pipeline frost heave experiments. Frost heave refers to ground expansion caused by water migration and accumulation in a frozen fringe (i.e. a transitional zone just behind a freezing front, where soil is partially frozen). The water migration is driven by cryogenic suction and is at the same time impeded by the reduced permeability developed in partially frozen soil. This phenomenon is most pronounced in silty soils, in which moderate to strong suction can be generated while retaining relatively high permeability. Frost heave is of particular concern in highway and pipeline engineering.

A benchmark series of in-situ tests conducted in Calgary [3] allow the model to be evaluated. Trenches were made in permafrost-free silty ground and full-scale 1.22m diameter steel pipelines installed, with the original soil being placed as backfill. Three different sections were prepared incorporating an initially 0.46m-high berm. The “Control” and “Gravel” sections had the pipeline invert set approximately 2m below the original ground surface, while a depth of 2.9m was adopted in a “Deep Burial” section. In the Gravel section, the silty soil found just beneath the pipeline invert was replaced by gravel to mitigate frost heave. The pipelines were chilled to -8.5 ^{o}C and the subsequent ground heave was monitored for 3 to 10 years.

The simulated pipeline heave developments are compared with the field measurements in Figure 2. The time-history of the heave in the Deep Burial section is well predicted over three years. The Control section prediction is also good up to the berm enlargement at day 430. From this point onwards, however, the simulated heave rates exceed the measurements. One plausible explanation is the site non-uniformity noted in [3]. In particular, the deeper ground may have been less frost heave-susceptible than the shallow material against which the model was calibrated. For the Gravel section, the simulation predicted substantial suppression of heaving while the freezing front advanced through the gravel.

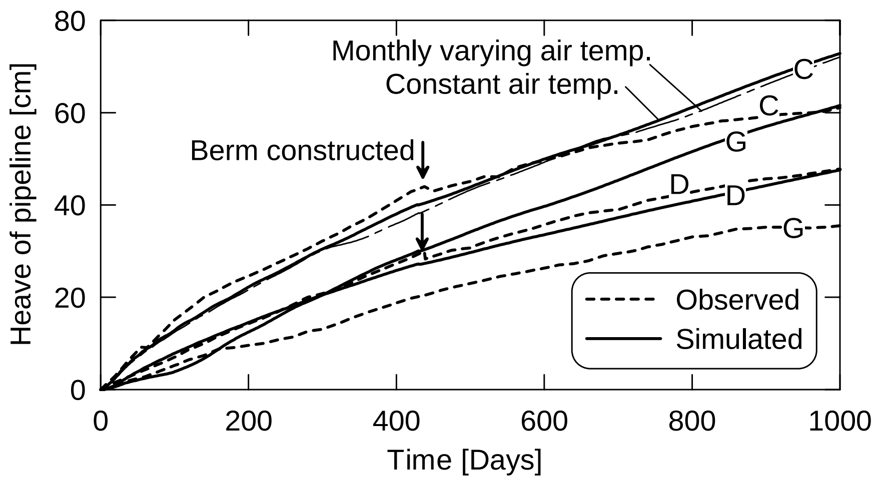

Figure 2: Simulated and observed heave of pipeline (Sections: C: Control, D: Deep burial, G: Gravel)

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