

where $Z = (R + j\omega L)$ and $Y = (G + j\omega C)$, then

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

and it is evident that if either R or G are non-zero, then γ will be complex and the real part α will be dependent on frequency. In fact, α increases with frequency, and therefore the gain decreases at higher frequencies, as depicted in Fig. 3.

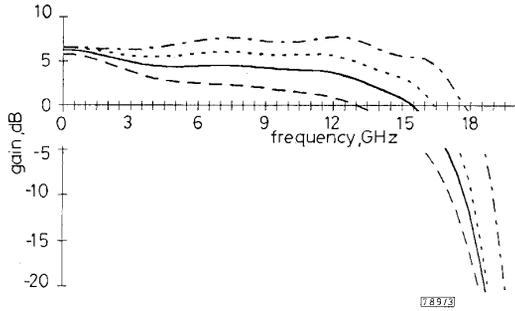


Fig. 3 Effect of lossy transmission lines 5 stage amplifier

Conclusions: This Letter has introduced a new wideband circuit simulation tool based on a modified form of transmission line matrix analysis, which is able to operate in both the frequency domain and fully-transient time domain. The approach is able to incorporate the effects of transmission line losses for inter-device connections, which is of particular significance for structures such as distributed amplifiers. Results for verification of the method have been presented, showing extremely close matching to previously published analytical results. Work is currently under way on applying the method to amplifiers with embedded signal shaping and to simulation of the transient behaviour of receivers intended for use in soliton systems. Also, a method for modelling the distributed amplifier in a two-dimensional environment is being considered, allowing the simulation to represent details of physical layout.

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Ill conditioning loci in noise parameter determination

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Indexing terms: Circuit noise, Noise measurement

An analysis of the problem of noise parameter determination is carried out. It is found that the mathematical problem is ill-posed if all the source reflection coefficients (Γ_s) used in the measurements fall onto a line, circle or arc of a circle in the Smith chart. This is used to expand on existing criteria for selecting Γ_s and to rule out sources of accuracy degradation.

Introduction: The dependence of the noise figure (F) of a linear two port on the source reflection coefficient (Γ_s) is fully determined by the so called noise parameters: a set of four real parameters which include the minimum noise figure achievable (F_{min}), its corresponding (complex) source reflection coefficient (Γ_o) and a fourth parameter (N) which dictates the rate of degradation in noise figure as PS deviates from Γ_o .

The problem of extracting the device noise parameters from measured data has been previously studied by various authors [1-4]. A number of different methods have been proposed, and some claims of enhanced performance have been made. All methods are based on a fit of the theoretical dependence between F and Γ_s to a set of experimental data consisting of pairs of a measured noise figure and its corresponding source reflection coefficient. Careless selection of the source reflection coefficients in that data set might lead to ill-conditioning in the equations used in the fit. A first study of the ill-conditioning problem was carried out in [2], where it was shown that the problem was ill-posed whenever the source reflection coefficients belonged to certain loci on the Smith chart. Even though not all ill-conditioning loci (ICL) were identified, a strategy for selecting Γ_s could be given.

On the other hand, most noise parameter extraction methods are based on a least-squares fit of the dependence between the noise figure (F) and the source reflection coefficient (Γ_s) or admittance (Y_s) to the measured data. This fit is performed by linearising the equation relating F to Γ_s (or Y_s), and solving an overdetermined system of equations. Claims have been made [3] indicating a degradation in accuracy due to that linearisation.

The purpose of this Letter is to provide a full description of the ICL on the Smith chart, expand on the criteria for selecting the source reflection coefficients, and point out that the linearisation step has no significant effect on the accuracy of the resulting noise parameters.

Linearised, least squares formulation: If the dependence of F with Γ_s is chosen, the equation to linearise is [5]

$$F = F_{min} + 4N \frac{|\Gamma_s - \Gamma_o|^2}{(1 - |\Gamma_s|^2)(1 - |\Gamma_o|^2)} \quad (1)$$

and the linearisation results in [2]

$$F = x_1 + x_2 \frac{1}{1 - \rho_s^2} + x_3 \frac{\rho_s \cos \theta_s}{1 - \rho_s^2} + x_4 \frac{\rho_s \sin \theta_s}{1 - \rho_s^2} \quad (2)$$

with $\Gamma_s = \rho_s e^{j\theta_s}$, $\Gamma_o = \rho_o e^{j\theta_o}$ and

$$\begin{aligned} x_1 &= F_{min} - 4N \frac{1}{1 - \rho_o^2}; & x_2 &= 4N \frac{1 + \rho_o^2}{1 - \rho_o^2} \\ x_3 &= -8N \frac{\rho_o \cos \theta_o}{1 - \rho_o^2}; & x_4 &= -8N \frac{\rho_o \sin \theta_o}{1 - \rho_o^2} \end{aligned} \quad (3)$$

Eqn. 2 leads naturally to an overdetermined system of equations with separate equation for each of the m measured data pairs (F, Γ_s), and can be written in matrix form as $\mathbf{Ax} \approx \mathbf{b}$, where \mathbf{A} is a $4 \times m$ matrix and \mathbf{x} , \mathbf{b} are column vectors:

$$j = 1 \dots m$$

$$\mathbf{A}(1, j) = 1; \quad \mathbf{A}(2, j) = \frac{1}{1 - \rho_{s-j}^2};$$

$$\mathbf{A}(3, j) = \frac{\rho_{s-j} \cos \theta_{s-j}}{1 - \rho_{s-j}^2}; \quad \mathbf{A}(4, j) = \frac{\rho_{s-j} \sin \theta_{s-j}}{1 - \rho_{s-j}^2};$$

$$\mathbf{x} = (x_1, x_2, x_3, x_4)^T; \quad \mathbf{b} = (F_1, F_2, \dots, F_m)^T \quad (4)$$

The least squares problem is then reduced to solving $\mathbf{Ax} = \mathbf{b}$ which can be done by straight differentiation of $\|\mathbf{Ax} - \mathbf{b}\|^2$ with respect to the components of \mathbf{x} [2] or by any other standard numerical technique [6]. The device noise parameters are then extracted from the components of \mathbf{x} through eqn. 3.

Ill conditioning loci: When the problem is well-posed, the overdetermined system $\mathbf{Ax} = \mathbf{b}$ has a unique solution \mathbf{x} satisfying $\|\mathbf{Ax} - \mathbf{b}\|^2 \rightarrow \min$. However, a unique solution does not exist unless the rank of \mathbf{A} is maximum (i.e. 4 in our case) [6] or equivalently, if the columns of \mathbf{A} are not linearly independent. A linear dependence of these columns only occurs when there are four real numbers α , β , γ , δ satisfying:

$$\alpha + \beta \frac{1}{1 - \rho_s^2} + \gamma \frac{\rho_s \cos \theta_s}{1 - \rho_s^2} + \delta \frac{\rho_s \sin \theta_s}{1 - \rho_s^2} \quad (5)$$

which, by making $x = \text{Re}[\Gamma_s]$, $y = \text{Im}[\Gamma_s]$ can be rewritten as:

$$\left(x - \frac{\gamma}{2\alpha}\right)^2 + \left(y - \frac{\delta}{2\alpha}\right)^2 = 1 + \frac{\beta}{\alpha} + \frac{\gamma^2 + \delta^2}{4\alpha^2} \quad (6)$$

When $\alpha \neq 0$, this equation can be fitted to any circle enclosed in the Smith chart or arc of a circle crossing it. For $\alpha = 0$, the equation is reduced to that representing a straight line, with any slope and any intersection with the axis. As a consequence, any line, circle or arc of a circle in the Smith chart is an ill-conditioning loci for the problem of noise parameter extraction. Thus, if all the source reflection coefficients were used when measuring the noise figure of the device happen to be on (or very close to) a line, circle, or arc of a circle in the Smith chart, a great dependence of the error in the results (noise parameters) with the experimental errors (in F and Γ_s) can be expected.

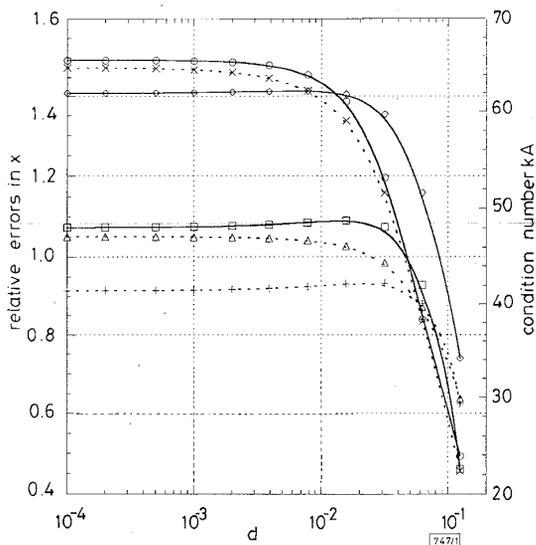


Fig. 1 Condition number $K(\mathbf{A})$ and relative error in x against distance d for three types of ICL

Condition numbers $K(\mathbf{A})$ and relative errors in solution vector \mathbf{x} ($\|\mathbf{x}_{\text{perturbed}} - \mathbf{x}\| / \|\mathbf{x}\|$) against distance to the ICL of one of the points in the constellation.

Eight points (Γ_s, F) have been placed on an ICL and a small perturbation with $\sigma_F = 0.03$ and $\sigma_r = 3\%$ has been applied to them, simulating the effect of experimental errors. The distance d is then progressively reduced, while the relative error in \mathbf{x} and the condition number $K(\mathbf{A})$ are calculated.

Different realisations of the perturbation used yield curves with the same shape but somewhat different values, although the scale of the figure is a realistic range of $K(\mathbf{A})$ and relative error in \mathbf{x} for the values of σ_F and σ_r indicated.

- err-circle
- err-arc
- ◇— err-line
- ×— $K(\mathbf{A})$ -circle
- +— $K(\mathbf{A})$ -arc
- Δ— $K(\mathbf{A})$ -line

Numerical simulations: To evaluate the consequences of an improper selection of Γ_s , the process of noise parameter extraction was simulated under varying degrees of ill-conditioning. Eight

source reflection coefficients were placed on an ICL and a ninth one at a small distance d . These reflection coefficients and their corresponding noise figures were then perturbed, simulating the measurement inaccuracies of a network analyser and a noise figure meter. The least-squares fit was repeated as a function of d . The resulting condition number $K(\mathbf{A})$ and relative error in x as a function of the distance d are shown in Fig. 1 for the three types of ICL possible. It is clear from this figure that relative errors and condition numbers follow the same trend, and that both are maximum when the constellation of Γ_s falls onto an ICL. Note that the plateaux in these curves are reached when d is comparable to the perturbations introduced in Γ_s .

In this study, the noise parameters were also calculated by alternative methods which use a linearisation of $F(Y_s)$ [1, 4] instead of $F(\Gamma_s)$ [2]. Even though the system of linear equations involved in the two approaches is completely different, the resulting noise parameters are identical. This happens even for the smallest values of d , where the effect of experimental errors is supposed to be strongest, pointing out the irrelevance of the linearising step in the final result.

Possible strategies for selection of Γ_s : The criteria for selection of Γ_s given in [2] can be extended with the generalised description of the ICL given in this work:

1. Ill conditioning will be avoided whenever the Γ_s are distributed among two different ICL (lines, circles or arcs of a circle in the Smith chart).
2. At least three different values of Γ_s must be placed on a given ICL, in order to guarantee that it is unambiguously determined.
3. Low error sensitivity is possible whenever the constellation of Γ_s can be placed close to Γ_o , so that small values of noise figure are involved in the measurement and fitting process [2].

Conclusion: A complete description of the ill conditioning loci for Γ_s in the problem of noise parameter determination has been found. This description allows an extension of the previously given criteria for selecting Γ_s . The performance of the least squares method has been checked, even under ill conditioning, and no degradation due to a linearisation step has been found.

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