Break Point Analysis and Modelling in Subway Tunnels

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Abstract—The time constraints in network developments inside tunnels require a characterisation of canonical tunnels in terms of propagation path losses and developing a slope model which is usually represented as a two slope model. One of the problems arisen when the canonical model tries to predict the propagation path losses inside the tunnel is to determine the distance where the slope of the propagation path losses changes. This paper analyses two of the most common approaches to determine that distance and their behaviour in different scenarios in order to clarify the radio propagation mechanisms which define the break point distance.

I. INTRODUCTION

Models for the radio-propagation inside subway tunnels have to be a feasible and fast tools which allow to predict the propagation path losses along a track of certain tunnel. These models are usually presented in a two or three slope curve (see Fig. 1) [13] where the propagation path losses are predicted by two or three different expressions with a different slope parameter [10]. Therefore, it is necessary to predict the distance from the transmitter where the slope of the propagation path losses changes to other value different of the free space propagation behaviour. The literature usually calls this distance as the break point [6] [16] and in this paper the different models applied to obtain that distance will be discussed.

The literature [13] [12] [6] applies two different approaches or methods to analyse the radio-propagation inside a subway tunnel. The first method is the so called ray tracing modelling of the radio-propagation where the ray tracing algorithm presented at [12] is applied. This algorithm has been analysed deeply in [16] and [8]. The second method is based in the decomposition of the radio-propagation into the different wave-guide modes [1] for a dielectric circular or rectangular wave-guide. The references [6] [7] [5] [13] provide with a set of different modes propagating along a straight subway tunnel and obtain the final electric field at each point of the tunnel adding these modal contributions.

In this paper the two main approaches to characterise the radio-propagation are revised at Section II and the different results obtained by different authors [13] [17] will be discussed and compared with experimental results obtained from measurement campaigns [10] at Section III. At Section III the final discussion about the behaviour of the two approaches at different scenarios and environment will be presented.

II. THEORETICAL MODELS OF PROPAGATION PATH LOSSES INSIDE SUBWAY TUNNELS

In this section the two methods used to predict the radio-propagation inside the subway tunnel will be presented. These methods are described completely by the proposed literature [6] [12] [13] and they can be named as ray tracing method and modal analysis method.

Let us consider a canonical rectangular tunnel as is depicted at Fig. 2 excited by a punctual source in the center of the tunnel.

Fig. 1. Comparative of the real measurements inside certain tunnel with a two slope model

Fig. 2. Canonical rectangular cross section tunnel with different complex dielectric constant [9]
A. Ray Tracing Path Loss Prediction

The ray tracing method to propagation path loss prediction inside tunnels is based on the image algorithm presented at [3] and developed at [12] in order to apply it on subway environments. The basis of the image algorithm allow to calculate the distances and the grazing angles for each consider path as is depicted at Fig. 3. Once the distances are calculated, the algorithm applies the Eq. 1 [12].

\[
P_r = P_t \left( \frac{\lambda}{4\pi} \right)^2 \left( \frac{G_d(x)}{r} \right) + \sum_{i=1}^{N-1} G_R_i R_i \exp(j\varphi_i) \frac{r_i}{r_i} \]

(1)

where the parameters are defined bellow:
- \( P_r \):Received power.
- \( P_t \):Transmitted power.
- \( \lambda \):Free space wavelength.
- \( G_d \):Square root of transmit and receive antenna gain product in Line Of Sight (LOS) direction.
- \( G_r \):Square root of the antenna gain and receive antenna product in the direction of \( i^{th} \) ray.
- \( r \):Path length of LOS direction.
- \( r_i \):Path length of the \( i^{th} \) ray.
- \( R_i \):Reflection coefficient of the \( i^{th} \) ray given by Eq. 4 [12] or Eq.3 [12].
- \( \varphi_i \):Angle which correspond with Eq. 2 [12].

The parameters \( \varphi_i \) and \( R_i \) are described conveniently:

\[
\varphi_i = \frac{2\pi \Delta l_i}{\lambda}
\]

(2)

where \( \Delta l \) is the distance difference between LOS and \( i^{th} \) path.

\[
R_h_i = \frac{\cos(\varphi_i) - \left( \epsilon - \sin^2(\varphi_i) \right)^{\frac{1}{2}}}{\cos(\varphi_i) + \left( \epsilon - \sin^2(\varphi_i) \right)^{\frac{1}{2}}}
\]

(3)

\[
R_v_i = \frac{\epsilon \cos(\varphi_i) - \left( \epsilon - \sin^2(\varphi_i) \right)^{\frac{1}{2}}}{\epsilon \cos(\varphi_i) + \left( \epsilon - \sin^2(\varphi_i) \right)^{\frac{1}{2}}}
\]

(4)

where \( R_{h_i} \) is the reflection coefficient for a horizontal polarised impinging wave and \( R_{v_i} \) is the reflection coefficient for a vertical polarised impinging wave.

One example of different simulations for a straight tunnel where \( w = 4m \) and \( h = 7.5m \) is is described at [12] where the two slope behaviour of the propagation path losses is confirmed at 900MHz and the dependence of the attenuation or slope parameter with the roughness of the walls is explained. The mentioned simulation has been performed with a \( \epsilon_r = 10 \) and \( \sigma = 0.01 \frac{S}{m} \). Following the same procedure the proposed literature [16] obtain different results for different kinds of tunnels and frequencies. This analysis has led to a general equation for the break point estimation described by Eq. 5.

\[
b_p = \frac{w^2}{\lambda}
\]

(5)

The Eq. 5 has been used in [11] [4] [14] with a good behaviour of the approach [17] at L-band for different scenarios. Therefore, the Eq. 5 predicts the distance where the slope of the propagation path losses change from the free space behaviour to the tunnel behaviour inside a rectangular straight tunnel at L-Band frequencies.

B. Modal Analysis

The modal analysis for subway and underground tunnels has been developed since 1970 [6] [11]. These studies use the dielectric wave-guide approach to solve the field equations inside the subway tunnel [9] in some canonical scenarios such as a rectangular or circular straight tunnel. This method has been completed during the last decade [7] [5] [13] studying different ways to excite the allowed electromagnetic modes inside different tunnels.

Let us consider the rectangular cross section straight tunnel depicted at Fig. 2 where \( w = 2b \) is the y-axis dimension and \( h = 2a \) is the x-axis dimension. Let us consider different complex dielectric constant \( \epsilon_a \) for the top and bottom walls and \( \epsilon_b \) for the right and left walls (see Fig. 4 [9]). Only the circular canonical tunnel allows closed field lines in the cross...
section without the necessity of a longitudinal component. The rectangular canonical tunnel only allows hybrid modes $EH_{mn}$. Therefore, following the constraints presented at [9] and [13], the X-polarised hybrid modes have an approximate propagation constant given by the Eq. 6 and the Eq. 7 [9].

$$Im(kg) = -\frac{1}{a} \left( \frac{m\lambda}{4a} \right)^2 Re \left[ \frac{\epsilon_a}{(\epsilon_a - 1)^{\frac{1}{2}}} \right]$$

$$Re(kg) = \frac{2\pi}{\lambda} \left[ 1 - \frac{1}{2} \left( \frac{m\lambda}{4a} \right)^2 - \frac{1}{2} \left( \frac{n\lambda}{4b} \right)^2 \right]$$

Where $\epsilon_a$ and $\epsilon_b$ are the normalised complex dielectric constants by the free space dielectric constant $\epsilon_0$.

The Y-polarised approximate propagation constant is given by the Eq. 8 and the Eq. 9 [9].

$$Im(kg) = -\frac{1}{a} \left( \frac{m\lambda}{4a} \right)^2 Re \left[ \frac{1}{(\epsilon_a - 1)^{\frac{1}{2}}} \right]$$

$$Re(kg) = \frac{2\pi}{\lambda} \left[ 1 - \frac{1}{2} \left( \frac{m\lambda}{4a} \right)^2 - \frac{1}{2} \left( \frac{n\lambda}{4b} \right)^2 \right]$$

The results presented at [13] conclude that the break point change its position with frequency and the dependence of the break point distance with the source and its position inside the tunnel.

### III. Break Point Estimation Discussion

In this section a discussion about the results obtained from the models presented at Section II will be presented. Let us consider the semi-arched tunnel depicted at Fig. 5 in the L2 of the Barcelona Metro Network. Let us consider the measurement conditions applied in [10] with a C-Band working frequency and directive vertical polarised antenna at the transmitter. Therefore, there are two important differences between the experiments presented at the literature [15] [17] which are the directivity of the transmitting antenna and the working frequency.

Applying the models given at Section II the break point its placed at 11 km for the ray tracing approach and at 500–300 m for the modal analysis. However the Fig. 6 shows a break point distance about 200–250 m for the proposed tunnel. The results presented at section above 8 lead to the following conclusions.

1) The break point distance has a strong dependence with the working frequency. The approach derived from the ray tracing analysis allows to estimate the break point in the L-band range of frequencies but has a bad behaviour for the C-band range of frequencies.

2) The break point distance has a dependence with the way used to excite the tunnel. The results lead to conclude that the transmitting antenna has a strong influence on the break point distance. Therefore, the analysis of different transmitting antenna configuration is necessary to model the radio propagation inside the tunnel.

3) The modal analysis presents a complexity level which does not allow to predict the break point distance in a fast way. There is not any approach to predict the break point distance as in the ray tracing analysis which allows to estimate that distance faster than applying the complete modal decomposition. Usually, the modal analysis assume several modes before the distance estimated by the Eq. 5 and the dominant mode after that distance.

The presented conclusions lead to develop a new model for the break point estimation which will be able to use at C-band frequencies. That model must be capable to analyse different excitation methods inside the tunnel and it must be capable to work at different frequencies. Therefore, the authors propose a new method to estimate the break point distance inside subway straight tunnels based on the Spectral Iterative Technique (SIT) [2].

Let assume a canonical circular tunnel with diameter $h$ and a punctual source at the center of the tunnel. For a given
distance \(d\) (Fig. 7) the angle \(\theta_n\) defines the boundary between the components of the PWS which do not suffer any reflection, denoted by direct PWS, and those that at least suffer one reflection. This boundary in the PWS domain corresponds to a circumference centered at the origin and radius related with the distance \(d\) and the diameter \(h\). The PWS produced by the punctual source inside the tunnel is uniform; therefore the area of the direct PWS is proportional to the power associated to the direct components of the PWS which shows \(1/d^2\) behaviour (Fig. 10). In a tunnel with a given roughness diffusive reflection occurs when a wave impinges in the tunnel wall (Fig. 8), giving origin to a new set of waves. Some of those new waves are outside the main PWS and therefore will be quickly attenuated by multiple reflections. However, there is a set of waves which propagate inside the direct PWS area. The evaluation of the respective areas (Fig. 9) in the PWS domain along the distance gives the break point situation in the stretch (Fig. 10).

Fig. 7. Subway tunnel geometry used to define the \(\theta_n\) angle

![Fig. 7](image1.png)

Fig. 8. Dispersive cone of the first reflection inside a canonical circular tunnel, a longitudinal cut

![Fig. 8](image2.png)

IV. CONCLUSIONS

Present break point distance prediction methods either give wrong estimates at high frequencies or are cumbersome to compute. A PWS model that allows to include physical characteristics of the tunnel as its dimensions and surface roughness has been proposed.

Fig. 9. PWS Domain translation of the geometry presented in Fig. 8 with the influence of the dispersive PWS over the total PWS where \(k d\) is related with \(\theta_d\)

![Fig. 9](image3.png)

Fig. 10. PWS area comparative for a canonical tunnel with an arbitrary angle of dispersion

![Fig. 10](image4.png)

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