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# Design of a system for measuring the weight and balance of sailplanes

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## Report

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## Aim

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The aim of this project is to design a system for the easy and accurate measurement of the weight and balance of sailplanes. The balance of a sailplane refers to the longitudinal location of its centre of gravity (CG), which critically affects its stability and manoeuvrability during flight.

The project will be focused on the method and the additional systems necessary in order to facilitate the measurement of weight and balance, i.e. measurement of distance between weighing scales, levelling of the sailplane, etc. A higher accuracy than that of existing systems will be a direct consequence of this design. Therefore, the specific design of the weighing scales is not the main goal of this project.

The final document will provide a study of alternatives and the design of the new system, as well as an assessment of the estimated measurement accuracy achievable with this design.

## Scope

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The scope of the project is shown below. The information gathered during the development of the project might call for slight changes.

- Determination of requirements:
  - Selection of sailplanes to be studied
  - Study of specifications:
    - Range of loads
    - Range of distances between scales
    - Size of the scales
    - Maximum height of the rear scale
    - Accuracy of the weight measurement
    - Accuracy of the measurement of the CG position
- Study of the sources of uncertainty:
  - Location of the point load on the surface of the scales
  - Distance between the scales
  - Distance between the sailplane's reference and the scales' reference
  - Alignment of the scales
  - Levelling of the sailplane
  - Weight measurements
- Design of the weight and balance measurement system:
  - Selection of the scales
  - Selection of the scales' reference

- Selection of a system to measure the distance between the scales
- Design of a system to align the scales
- Study of the different methods to measure sailplane levelling
- Design of a system to elevate the rear scale
- Assessment of the estimated achievable measurement accuracy

## Requirements

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The requirements of the new system are the following:

- To offer an easier weight and balance measurement process with respect to current systems
- To improve the accuracy of the measurements with respect to current systems
- Maximum measurable weight: 1,000 kg
- Weight measurement accuracy:  $\pm 1.5$  kg
- Range of distances between weighing points:
  - Minimum: 3 m
  - Maximum: 8 m
- Range of rear scale heights:
  - Minimum: 0 m
  - Maximum: 0.75 m

## Background

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When it comes to sailplanes, weight and balance are critical aspects. The weight of the aircraft must never exceed that stated by the manufacturer, both for flight behaviour and structural reasons. Locating the centre of gravity (CG) inside the limits stated by the manufacturer is not only important for an effective control of the aircraft, but also for reasons of safety. Two cases could endanger the pilot's life: a CG that is aft of the manufacturer's aft limit reduces stability and could prevent spin recovery, and a CG that is forward of the manufacturer's forward limit could prevent to reach a speed low enough for safe landing.

Every time a repair is done to a sailplane (as minor as it may be), a weight and balance measurement must be carried out. Routine maintenance tasks such as painting and waxing also require to repeat this measurement, and some manufacturers advise to weigh and balance every certain number of years.

In general, measuring the weight and balance of a sailplane is a difficult and rudimentary task. Two weighing scales are used, one under the main wheel and another one under the tail wheel.

The difficulties encountered by users include levelling the aircraft by elevating the rear scale, measuring the distance between the scales, setting a common reference for the aircraft and the scales, etc. Each of these problems add an uncertainty to the final calculation, which could result in a fatal accident. Therefore, the need for a system that solves or reduces the impact of these problems and improves accuracy is justified.



# 1 Determination of requirements

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In order to define the specific requirements of the system that will lead to the final design, it is necessary to study the needs of the weight and balance process as well as the relevant specifications of most sailplanes.

## 1.1 Selection of sailplanes to be studied

### 1.1.1 Selection criteria

The first step for determining the specific needs of the weight and balance measurement system is to study the maintenance manuals of representative sailplanes. By *representative* we understand those sailplanes which are more likely to require weighing and balancing. In general, new sailplanes do not require as many maintenance operations as old sailplanes do, and therefore they don't need weighing and balancing. Also, those intended for training tend to suffer from minor accidents more frequently, which results in a necessary repair followed by weighing and balancing.

Therefore, a number of sailplanes are going to be selected according to the following criteria:

- **Age:** old sailplanes are going to be favoured over modern ones, as they are more likely to require maintenance. As a general rule, the majority of sailplanes selected have to be introduced between 1960 and 1990 (it has to be taken into account that many sailplanes which are still being flown today were introduced during this time period). Some newer sailplanes are also going to be considered to ensure the validity of the system in the following decades.
- **Use:** training two-seater sailplanes are very likely to suffer from minor accidents once in a while, and therefore the needs of this specific type of glider deserve special attention. Single-seat sailplanes are also widely used in schools and clubs all over the world, and are also the most used among private users. Those sailplanes which are specially designed for competition will not be taken into account, since they tend to be manufactured in small numbers and have special characteristics which are not representative of the majority of models.
- **Popularity:** those sailplanes which are more popular will be included in the list. By doing so, it is ensured that the system will be able to be used by a large number of users. Also, more information about popular models will be available. Popular sailplanes are those which satisfy these two requirements: large number of built units and extended use in the present. Of course, these two factors will usually be closely bounded, but observing the first one will lead to finding those which satisfy the second one.
- **Other criteria:** in order to obtain information from varied sources, we will try to include different manufacturers in the selection. Also, different characteristics among the

selected sailplanes will be considered positive, e.g. differences in materials, shape, size, weight, etc.

### 1.1.2 Selection process and final list

First, we determined the major sailplane manufacturers in the present time and in the time period we had previously selected. Thanks to aircraft databases we were able to identify those models produced in larger numbers, and from this we identified the most popular ones and those which are widely used in flight schools. Finally, we selected the following sailplanes:

- **Schleicher ASK 13:** first flown in 1966, this is the precursor of the ASK 21 and features similar characteristics. It is a two-seater made of metal, wood, and fibreglass. Over 700 units were built and it is still being used for training.
- **Schleicher ASK 21:** this two-seater fibreglass sailplane first flew in 1979 and is intended for training. It is used in flight schools worldwide and by the Australian Air Force Cadets. With over 900 units built, it remains a very popular model today.
- **Schleicher ASW 20:** this fibreglass model was selected for variety because it is a 15 metre Class sailplane, as opposed to Standard and Two Seater Class ones. It first flew in 1977 and is a very popular model with over 900 units built. When introduced, it was highly successful in competitions, and it is still being used today for this purpose.
- **DG Flugzeugbau DG-1000S:** built by one of the major current manufacturers, this is a modern two-seater which first flew in 2000. It is used for training in the U.S. Air Force. Although no data about the number of units built was found, we selected this sailplane for being a representation of two-seater trainer design in the following years.
- **Grob G102 Astir CS:** this Standard Class single-seat sailplane was first flown in 1974. It features a composite construction of fibreglass and resin. Over 1,200 units were built, which makes it one of the most popular sailplanes in history. It is still active in the second hand market and used by pilots.
- **Grob G103 Twin II:** another remarkably popular trainer, the G103 Twin II is a two-seater from the 1980s. This high performance sailplane was designed for training and simple aerobatics. With 549 units built, it is widely used in flight schools and is used by the Argentinian Air Force, the Australian Air Force Cadets, the Belgian Air Component, the United Kingdom's Royal Air Force, and the Civil Air Patrol of the U.S.
- **Schleicher Ka 6E:** as the oldest sailplane in the list, the Ka 6 was selected because of its different construction materials: spruce and plywood with fabric covering. It is a single-seat sailplane first flown in 1955. Including all variants, over 1,000 units of this model were built. It is still being flown by private users and can be seen in Club Class competitions. The last version is the Ka 6E, from 1965.

## 1.2 Study of specifications

In this section, the sailplanes of the above list will be studied. By looking at the specifications contained in the different manuals of these aircraft, we will define the exact requirements of our system.

### 1.2.1 Weights

In the maintenance or flight manuals of all the sailplanes on the list, it is indicated that the weight and balance measurement has to be done with the aircraft empty, i.e. with all the necessary equipment to fly but without pilots, weights or water ballast. Therefore, the initial empty weight measurement performed by the manufacturer is the most relevant weight parameter. However, all manufacturers also indicate a maximum weight for the non-lifting parts, so this will be taken into account as well. Finally, there is the maximum all-up weight, which is expected to be calculated by the sum of the empty weight and all the weights added afterwards. However, there might be cases where the user suspects that the total weight is close to the limit and wants to make sure that this limit is not surpassed. In this case, a measurement of the total weight with this system becomes necessary and thus the maximum all-up weight is going to be considered.

The gathered weight data is summarised in the following list<sup>1</sup>:

- **Empty weight:** as expected, the two-seater sailplanes are the heaviest. Older sailplanes tend to be lighter (e.g. the ASK 13 is a two-seater that weighs only 295 kg).
  - **Maximum:** 415 kg (DG-1000S)
  - **Minimum:** 190 kg (Ka 6E)
  - **Average:** 310 kg
- **Maximum weight of non-lifting parts:** the same tendency of the empty weight is observed here.
  - **Maximum:** 469 kg (DG-1000S)
  - **Minimum:** 190 kg (Ka 6E)
  - **Average:** 324 kg
- **Maximum all-up weight:** in general, the maximum all-up weight is much higher than the empty weight because of the pilots' weight. This makes it necessary to have a high maximum weight limit in the system that is being designed.
  - **Maximum:** 750 kg (DG-1000S)
  - **Minimum:** 300 kg (Ka 6E)
  - **Average:** 516 kg

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<sup>1</sup> The specific values for each model can be found in the Annex.

As can be seen, the original requirement of maximum measurable weight (1,000 kg) fits the needs of all the selected sailplanes.

### 1.2.2 Weight distribution

The distribution of the sailplane's weight between the front and rear weighing points is important, as it determines the range of loads of each weighing scale and has an effect on the weight measurements accuracy. By using the data contained in the selected sailplanes' manuals, we have determined the weight distribution of each sailplane. It must be noted, however, that the weight distribution depends on the CG position, which has not a fixed value. Thus, a significant value of the CG position has been chosen in every case. This value is the middle value of the CG position range for the middle value of the empty weight range. Then, the following results have been obtained<sup>2</sup>:

- **Front weight:** the range of weights at the front is large, but it is possible to obtain a reasonable accuracy in such a range with weighing scales:
  - **Maximum:** 387.8 kg (DG-1000S)
  - **Minimum:** 174.6 kg (Ka 6E)
  - **Average:** 288.8 kg
- **Rear weight:** although the rear weights range is smaller than that of the front weights, the maximum to minimum weight ratio is much higher. Therefore, it will be more difficult to obtain a good accuracy for the rear weight measurement:
  - **Maximum:** 52.2 kg (DG-1000S)
  - **Minimum:** 4.6 kg (ASK 13)
  - **Average:** 26.9 kg

Then, the maximum and minimum values achievable with the combination between CG position and empty weight are the following:

- **Maximum front weight:** 430.2 kg (DG-1000S)
- **Minimum front weight:** 163.5 kg (Ka 6E)
- **Maximum rear weight:** 59.8 kg (DG-1000S)
- **Minimum rear weight:** 1.8 kg (ASK 13)

### 1.2.3 Distances

Regarding the distances, there are two relevant parameters: the distance between weighing points, L1, and the distance between the front weighing point and the datum (the reference vertical line with respect to which the CG position is calculated), L2. Both distances are important because they affect the accuracy of the measurements and dictate the requirements of the

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<sup>2</sup> The specific values for each model can be found in the Annex.

system. The distances are usually not provided by the manufacturer since one is expected to measure them when performing a weight and balance process. Therefore, these values are approximations calculated from the drawings in the manuals<sup>3</sup>:

- **Distance between weighing points:**
  - **Maximum:** 5.18 m (DG-1000S)
  - **Minimum:** 4.09 m (G102 Astir CS)
  - **Average:** 4.52 m
- **Distance between front weighing point and datum:**
  - **Maximum:** 0.00 m (ASW 20, Ka 6E)
  - **Minimum:** -0.59 m (ASK 21)
  - **Average:** -0.25 m

Therefore, the original requirements of the system satisfy the weight and balance needs of all the sailplanes on the list (the range of distances between weighing points was from 3 m to 8 m).

However, the main purpose of these specifications is to serve as a starting point for the study of uncertainties that will be performed later.

#### 1.2.4 Size of the scales

In order to estimate the size of the scales, we need to know the type of tyre used by each sailplane (from [1], [2]), as well as their inflation pressure. This data can be found in the Annex.

The contact area between the tyre and the scale is calculated by dividing the single wheel load by the tyre inflation pressure. The contact area can be modelled as an ellipse whose major axis is 1.6 times its minor axis [3]; then, the major and minor axes found will be used to obtain an estimation of the required size of the scale's weighing platform. The results can be seen in the following table:

Model	Contact area major axis (mm)	Contact area minor axis (mm)
ASK 13	162	101
ASK 21	170	106
ASW 20	142	89.0
DG-1000S	185	116
G102 Astir CS	140	88.0
G103 Twin II	175	109
Ka 6E	128	80.0

Table 1: Major and minor axes of the contact area of the main wheels.

<sup>3</sup> The specific values for each model can be found in the Annex.

Now, it is necessary to define the surface over which the wheel's CG (i.e. the centre of the elliptical contact area) will be able to be located during the weight and balance process. A reasonable value seems to be a square of side 25 cm; this allows some margin for the wheel to be off-centre with respect to the scale and to move forwards and backwards, as moving a sailplane is difficult. Then, considering the scale surface as a rectangle, its minimum longitudinal side is calculated by adding the 25 cm-sided square and the maximum major axis found:

$$\text{minimum longitudinal side} = 250 \text{ mm} + 185 \text{ mm} = \mathbf{435 \text{ mm}}$$

For the minimum lateral (or transversal) side, the same formula is used, but this time adding the maximum minor axis found:

$$\text{minimum lateral side} = 250 \text{ mm} + 116 \text{ mm} = \mathbf{366 \text{ mm}}$$

Of course, these are only minimum values, and the actual values will be defined during the design of the system.

#### 1.2.5 Rear scale height

In order to determine the maximum height up to which the rear scale should be able to be elevated, we have measured the height differences between the front and the rear weighing points for each sailplane in flight position:

Model	Rear weighing point height (m)
ASK 13	0.682
ASK 21	0.489
ASW 20	0.295
DG-1000S	0.659
G102 Astir CS	0.507
G103 Twin II	0.405
Ka 6E	0.227

Table 2: Heights of the rear weighing point with respect to the front weighing point.

As can be seen, all heights are well below 0.75 m, which is the maximum height originally defined in the requirements.

#### 1.2.6 Weight and balance procedure

The weight and balance measurement procedure indicated by the manufacturers is the same for the majority of the selected sailplanes:

- **Equipment, payload and pilots:** the measurements are made with the sailplane empty, i.e. with all the equipment necessary to fly but without pilots, parachutes, water ballast, or compensating weights.

- **Placement of the scales:** two weighing scales must be used. One is placed under the main wheel (or the front support, in some cases) and the other one is placed under the tail skid or wheel.
- **Longitudinal levelling:** the sailplane must be in flight position, i.e. the fuselage centreline is almost horizontal. In general, this means that the tail skid or wheel must be elevated. The levelling means is a wedge of appropriate dimensions placed on top of the rear fuselage (the length to height ratio of the wedge are indicated by the manufacturer and are specific to each model). The top side of the wedge must be horizontal when the flight position is achieved.
- **Lateral levelling:** only one manufacturer (DG Flugzeugbau) indicates that the sailplane must be laterally level, and that the wing must be held in such way that no load is applied (otherwise, the weight measurement of the scales would be false).
- **Datum (reference line):** the datum is the reference line with respect to which the CG position is measured. In general, it is the leading edge of the wing at the root or at a specific rib indicated by the manufacturer.
- **Distance measurements:** two measurements which directly affect the CG position measurement have to be made. The first one is the distance between the front weighing point (main wheel or front support, depending on the model) and the datum (L2), and the second one is the distance between the front and the rear weighing points (L1). These distances are usually not indicated by the manufacturers, since there are factors that could affect their values such as temperature, deflection of the main wheel because of the weight, manufacturing tolerances, etc.
- **Other considerations:** some manufacturers indicate that the weight and balance measurements have to be made in a closed space, so that wind gusts do not exert force on the sailplane.

For a summary of instructions for each model, please refer to the Annex.

### 1.2.7 CG position limits

The method for ensuring that the in-flight CG position limits are not exceeded is almost the same for all sailplane models. The CG position is only measured with the glider empty, and the range of allowed empty weight CG positions depends on the aircraft's empty weight, the weight of the pilots, and the water ballast. If the empty weight CG position is within the manufacturer's limits for a given combination of pilot weights and water ballast, the CG position will be within the safe range during flight. In the case of a pilot which has a weight under the minimum indicated by the manufacturer, compensating weights must be installed.

The empty weight CG position limits are indicated in different ways depending on the manufacturer. In some cases, a table with the limits as a function of the sailplane's empty weight

is provided, and then maximum and minimum weight limitations for the pilots are indicated. In other cases, the same is indicated using a graph. The CG position range increases as the empty weight increases, meaning that very high measurement accuracies are required for sailplanes near their minimum empty weight. This could happen, for example, if the user has changed original components for lighter ones.

The order of magnitude of the CG position range (the difference between the aft and the forward limits) is different for each glider. In some cases, like the ASK 21, the range can get to 0 mm with the minimum empty weight, and then increase to 5 mm with an empty weight just 5 kg above the minimum. The broader range for this model is 46 mm. Other older sailplanes, like the Ka 6, has a CG position range which goes from 102 to 121 mm. However, ranges from 10 to 50 mm are not unusual and therefore great effort is going to be put in achieving the highest possible accuracy.



## 2 Study of uncertainties

In order to determine which factors are the most important for enhancing the accuracy of the weight and balance calculations, a study of uncertainties will be carried out. On a first approach, we will consider a realistic uncertainty for each of the four basic measurements needed to obtain the CG position. Then, we will study the propagation of uncertainty to get an estimation of the achievable overall measurement accuracy and the impact each of the measurements has on the final uncertainty. Additional sources of uncertainty will also be considered.

### 2.1 Uncertainty of the CG position calculation

Consider the following diagram:

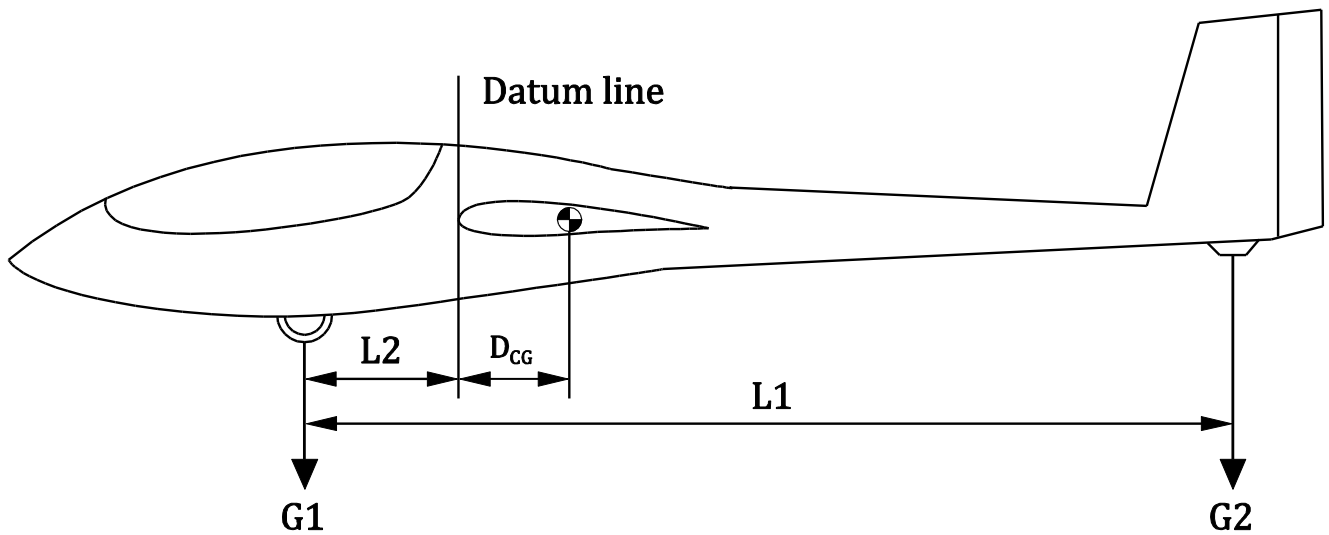


Figure 1: Sailplane distances, weights, and CG position definition.

Then, the four basic measurements needed to obtain the CG position ( $D_{CG}$ ) are the following<sup>4</sup>:

- **G1**: weight at the front (either at the front support or the main wheel)
- **G2**: weight at the rear (either at the tail skid or the tail wheel)
- **L1**: distance between weighing points
- **L2**: distance between front weighing point (G1) and datum (positive when the datum is aft from the front weighing point)

With the variables defined above, the longitudinal CG position with respect to the datum, defined positive aft from the datum, is calculated as follows:

$$D_{CG} = \frac{L1 \cdot G2}{G1 + G2} - L2$$

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<sup>4</sup> Values of G1, G2, L1 and L2 for each sailplane can be found in the Annex.

Let us define  $\epsilon_{G1}$ ,  $\epsilon_{G2}$ ,  $\epsilon_{L1}$ , and  $\epsilon_{L2}$  as the uncertainties of G1, G2, L1, and L2, respectively. Then, the total uncertainty of the CG position ( $\epsilon_{D_{CG}}$ ) can be calculated by propagation of uncertainties as a truncated Taylor series expansion:

$$\epsilon_{D_{CG}} = \left| \frac{\partial D_{CG}}{\partial G1} \right| \cdot \epsilon_{G1} + \left| \frac{\partial D_{CG}}{\partial G2} \right| \cdot \epsilon_{G2} + \left| \frac{\partial D_{CG}}{\partial L1} \right| \cdot \epsilon_{L1} + \left| \frac{\partial D_{CG}}{\partial L2} \right| \cdot \epsilon_{L2}$$

As can be seen, the dependency of  $\epsilon_{D_{CG}}$  is linear with respect to the uncertainties of each of the measurements. This means, for example, that reducing the uncertainty of the weight G1 by half will reduce its contribution to the uncertainty  $\epsilon_{D_{CG}}$  (but not the total uncertainty) by half as well.

For the sake of completeness, the partial derivatives are shown below:

$$\frac{\partial D_{CG}}{\partial G1} = -\frac{L1 \cdot G2}{(G1 + G2)^2}$$

$$\frac{\partial D_{CG}}{\partial G2} = \frac{L1 \cdot G1}{(G1 + G2)^2}$$

$$\frac{\partial D_{CG}}{\partial L1} = \frac{G2}{G1 + G2}$$

$$\frac{\partial D_{CG}}{\partial L2} = -1$$

The realistic uncertainties chosen for the study of uncertainties are shown in the following table. The weight uncertainties are based on the typical accuracy of sailplane-specific commercial scales. The length uncertainties are an estimation, as there is no actual data about the accuracy of the different length measurement methods.

Measurement	Uncertainty	Units
<b>G1</b>	$\pm 0.2$	kg
<b>G2</b>	$\pm 0.2$	kg
<b>L1</b>	$\pm 15$	mm
<b>L2</b>	$\pm 15$	mm

**Table 3: Initial estimation of weight and distance uncertainties.**

With these parameters, we have estimated the measurement uncertainties of each sailplane in order to cover a wide range of empty weights:

Model	Empty weight (kg)	CG position (mm)	CG position uncertainty (mm)
ASK 13	300	484.2	± 18.4
ASK 21	375	770.0	± 18.0
ASW 20	265	615.4	± 20.4
DG-1000S	440	724.5	± 19.1
G102 Astir CS	250	627.0	± 20.2
G103 Twin II	380	738.7	± 18.1
Ka 6E	200	570.0	± 21.4
Average	316	647.1	± 19.4

Table 4: Empty weights, CG positions, and CG positions uncertainties.

As can be seen, the estimated uncertainties are large in relation with the difference between the minimum and the maximum CG positions, which in many cases vary between 10 mm and 50 mm. The following table breaks down the uncertainty to show the contribution of each measurement:

Model	Contribution to uncertainty				Units
	G1	G2	L1	L2	
ASK 13	0.0	3.2	0.2	15	mm
	0.3	17.1	1.2	81.4	%
ASK 21	0.1	2.4	0.6	15	mm
	0.5	13.1	3.2	83.1	%
ASW 20	0.5	2.7	2.2	15	mm
	2.3	13.1	10.9	73.7	%
DG-1000S	0.3	2.1	1.8	15	mm
	1.5	10.8	9.3	78.4	%
G102 Astir CS	0.4	2.9	1.9	15	mm
	2.1	14.2	9.4	74.4	%
G103 Twin II	0.1	2.2	0.8	15	mm
	0.7	11.9	4.6	82.8	%
Ka 6E	0.6	3.9	1.9	15	mm
	2.7	18.3	8.9	70.1	%
Average	0.3	2.8	1.3	15	mm
	1.4	14.1	6.8	77.7	%

Table 5: Contribution to the CG position uncertainty of the weights and distances uncertainties.

In all cases, the greatest contribution to uncertainty (well above 70%) is that of the distance between the front weighing point and the datum (L2). Since this measurement appears in the formula for  $D_{CG}$  in the form of a direct addition, its uncertainty is added directly as well. Thus,

each unit of uncertainty reduced from this measurement will translate in a unit reduced from the total uncertainty. In consequence, this will be one of the primary focuses of our design.

The second most important contributor is the rear weight (G2); its contribution is about an order of magnitude higher than the contribution of the front weight (G1). The distance between weighing points (L1) is the third largest contributor, and its contribution is quite different for each sailplane (from 1 to 10 mm, approximately).

## 2.2 Additional sources of uncertainty

In this section, additional sources of uncertainty (those which are not directly used in the CG position calculation) will be analysed in order to estimate their impact on the uncertainty of the CG position.

### 2.2.1 Alignment of the weighing points

The alignment of the weighing points refers to whether or not such points are both on a straight line with respect to which the user takes the distance measurements. The following diagram illustrates a situation where the weighing points are misaligned (the magnitude of the misalignment is exaggerated for clarity purposes):

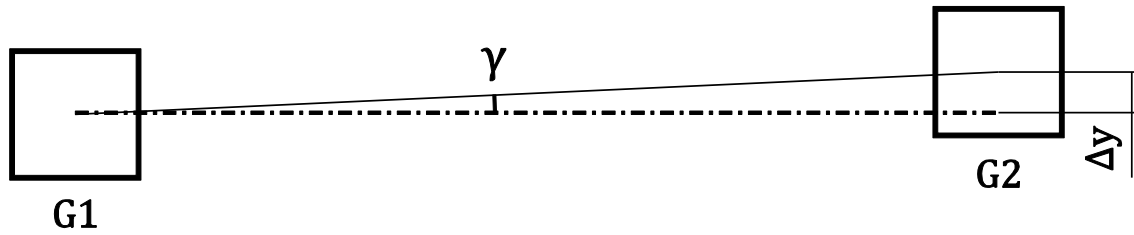


Figure 2: Scale misalignment (top view).

In such case, the distance measurements obtained would be smaller than the real distances by a factor of  $\cos \gamma$ . Therefore, the error on the CG position ( $D_{CG}$ ) can be calculated as follows:

$$\Delta D_{CG} = (L2 + D_{CG}) - (L2 + D_{CG}) \cdot \cos \gamma$$

At the same time,  $\cos \gamma$  can be approximated as:

$$\cos \gamma \approx \cos \left( \arctan \left( \frac{\Delta y}{L1} \right) \right)$$

Then, the added uncertainty to the CG position for different values of  $\Delta y$ , calculated for the selected sailplanes, is the following:

$\Delta y (\pm \text{ mm}) \rightarrow$ Model $\downarrow$	CG position uncertainty ( $\pm \text{ mm}$ )				
	20	40	60	80	100
<b>ASK 13</b>	0.0006	0.0025	0.0057	0.0101	0.0158
<b>ASK 21</b>	0.0017	0.0068	0.0152	0.0271	0.0423
<b>ASW 20</b>	0.0071	0.0284	0.0640	0.1138	0.1777
<b>DG-1000S</b>	0.0046	0.0183	0.0412	0.0733	0.1145
<b>G102 Astir CS</b>	0.0062	0.0247	0.0556	0.0989	0.1545
<b>G103 Twin II</b>	0.0025	0.0101	0.0228	0.0405	0.0633
<b>Ka 6E</b>	0.0057	0.0226	0.0509	0.0905	0.1413
<b>Average</b>	<b>0.0041</b>	<b>0.0162</b>	<b>0.0365</b>	<b>0.0649</b>	<b>0.1013</b>

Table 6: CG position uncertainties for different lateral misalignments of the scales.

As can be seen, for lateral misalignments of up to 100 mm, the error stays well below 1 mm. Therefore, we consider that the alignment of the weighing points is a negligible factor.

### 2.2.2 Longitudinal levelling

To estimate the uncertainty added by the longitudinal levelling, we have considered the following diagram:

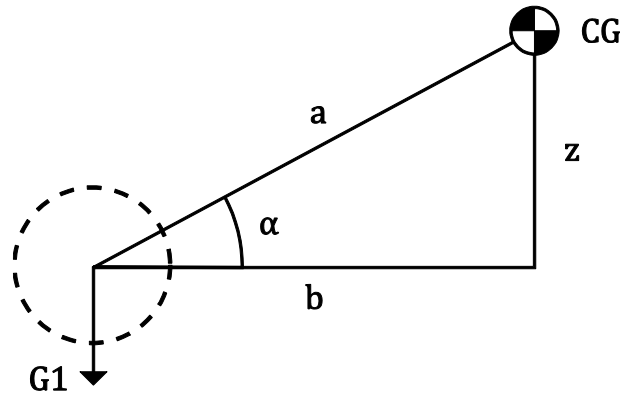


Figure 3: Longitudinal levelling effect on the CG position (side view).

The parts shown in the diagram are defined as follows:

- **Dashed circle:** main wheel's tyre
- **G1:** front weighing point, just under the main wheel
- **z:** vertical CG position, measured from the main wheel axis (front weighing point) with the sailplane in flight position
- **a:** a straight line joining the main wheel axis and the CG
- **b:** the projection of *a* on a horizontal reference (parallel to the ground)
- **α:** angle between *a* and *b*

Considering that, in general, the sailplane will be levelled by modifying the elevation of the rear weighing point, it is easy to see that any variation of the angle of the sailplane will make  $\alpha$  vary in exactly the same magnitude. This is so because the sailplane rotates as a rigid body around the main wheel axis. Therefore, it is possible to estimate the variation of the CG position (which equals to the variation of  $b$ ) with the variation of the angle of the sailplane, by calculating the variation of the projection of  $a$  with a variation of  $\alpha$ .

To obtain real values of the uncertainty for the selected sailplanes, the values of the following parameters must be obtained:

- **z**: the lack of data on the vertical location of the CG forces us to make an assumption. This parameter will be estimated as the distance between the main wheel axis and a horizontal line which passes approximately through the centre of the fuselage (not considering the vertical tail plane as a part of the fuselage).
- **b**: from Figure 1 and Figure 3, it can be seen that  $b$  is equal to the sum of  $D_{CG}$  and  $L2$ .
- **$\alpha$** : this angle will be calculated from the parameter  $z$  and the distance  $b$ .

Then, the uncertainty of  $D_{CG}$  caused by the levelling angle can be calculated as follows<sup>5</sup>:

$$\Delta D_{CG} = \Delta b = \pm |a \cdot \cos \alpha - a \cdot \cos (\alpha + \Delta\alpha)|$$

The variation of the uncertainty of  $D_{CG}$  as a function of the uncertainty of the angle ( $\Delta\alpha$ ) can be calculated as follows:

$$\frac{\partial \Delta D_{CG}}{\partial \Delta\alpha} = \pm a \cdot \sin(\alpha + \Delta\alpha)$$

Since this value depends on  $\Delta\alpha$ , an estimated value of  $1^\circ$  will be used.

Then, the following values are obtained:

Model	$\frac{\partial \Delta D_{CG}}{\partial \Delta\alpha}$ (mm/°)
<b>ASK 13</b>	$\pm 9.7$
<b>ASK 21</b>	$\pm 9.1$
<b>ASW 20</b>	$\pm 7.1$
<b>DG-1000S</b>	$\pm 10.8$
<b>G102 Astir CS</b>	$\pm 8.0$
<b>G103 Twin II</b>	$\pm 7.2$
<b>Ka 6E</b>	$\pm 6.7$
<b>Average</b>	$\pm 8.4$

Table 7: Influence of the levelling angle uncertainty to the CG position uncertainty.

<sup>5</sup> Only positive values of  $\Delta\alpha$  will be considered, as the derivative of  $\cos \alpha$  is monotonically increasing in the considered range of angles. Therefore, the error will never be underestimated.

Since the variation of the uncertainty of  $D_{CG}$  with  $\Delta\alpha$  is not linear, the following table shows the calculated uncertainty for different values of  $\Delta\alpha$ :

Model	$\Delta D_{CG}$ (mm)		
	$\Delta\alpha = 0.5^\circ$	$\Delta\alpha = 1^\circ$	$\Delta\alpha = 2^\circ$
<b>ASK 13</b>	$\pm 4.8$	$\pm 9.7$	$\pm 19.3$
<b>ASK 21</b>	$\pm 4.5$	$\pm 9.0$	$\pm 18.1$
<b>ASW 20</b>	$\pm 3.5$	$\pm 7.0$	$\pm 14.2$
<b>DG-1000S</b>	$\pm 5.3$	$\pm 10.7$	$\pm 21.7$
<b>G102 Astir CS</b>	$\pm 4.0$	$\pm 7.9$	$\pm 16.1$
<b>G103 Twin II</b>	$\pm 3.6$	$\pm 7.2$	$\pm 14.5$
<b>Ka 6E</b>	$\pm 3.3$	$\pm 6.6$	$\pm 13.4$
<b>Average</b>	<b><math>\pm 4.1</math></b>	<b><math>\pm 8.3</math></b>	<b><math>\pm 16.8</math></b>

**Table 8: CG position uncertainty as a function of different longitudinal levelling uncertainties.**

As can be seen, the uncertainty caused by the longitudinal levelling rapidly increases to large values. When the levelling uncertainty reaches  $\pm 2^\circ$ , the CG position uncertainty can get larger than that of the previous four measurements combined. Therefore, it is clear that the reduction of the longitudinal levelling uncertainty must be a priority of this design.

## 3 State of the art

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Now that the basic method and requirements of the weight and balance process have been studied and we have a good estimation of the measurement uncertainty and its sources, it is time to evaluate the state of the art of this process. By studying the methods used by different entities and the products offered by specialised companies, we will obtain a starting point for our design.

### 3.1 Methods recommended by entities

#### 3.1.1 Federal Aviation Administration

The Federal Aviation Administration (FAA) of the United States provides indications about the weight and balance process for aircraft in its “Aircraft Weight and Balance Handbook” [4]. It must be noted that this handbook is not specific for sailplanes.

The recommendations of the FAA provide a similar level of accuracy to that of other methods. There are no specific remarks about the weighing scales for sailplanes. The distance measurements are obtained by dropping a plumb bob from the datum and marking its location on the floor, then marking the position of the weighing points on the floor, and calculating their distances on a line parallel to the aircraft’s longitudinal axis. No indications about how to find the exact location of the weighing points are provided.

#### 3.1.2 British Gliding Association

The British Gliding Association (BGA) describes the weight and balance process in its “Airworthiness and Maintenance Procedures” [5].

The general guidelines are to perform the measurements in a closed hangar in order to prevent wing lift forces, which might affect the weight readings. The BGA does not consider it necessary to use aircraft-specific scales, but the selected scales must have been checked within the last 12 months before the process. As for the procedure itself, it is exactly the same that has been described earlier in this report. The recommended method for measuring the distances is to drop a plumb bob from the weighing points and from the datum, but no information about the preferred instrument for performing the measurement is provided.

#### 3.1.3 Gliding New Zealand

In its “Advisory Circular AC 3-18” [6], Gliding New Zealand (GNZ) explains the weight and balance process.

The basic process described in this document does not differ from those already seen. In this case, the accuracy of the weighing scales is specified by the “Advisory Circular 43-2” [7] of the Civil Aviation Authority of New Zealand. The specified accuracy is  $\pm 0.2\%$  of the applied load or



$\pm 2$  kg, whichever is greater. Also, when applying the same load repeatedly, the deviation from the mean must not exceed 0.05% of the applied load. However, GNZ considers that  $\pm 2$  kg of uncertainty at the tail can add significant uncertainty, which we have already observed during the study of uncertainties. Therefore, they suggest an uncertainty lower than  $\pm 0.5$  kg.

For measuring the distances, this entity recommends dropping a plumb bob from each of the points (front and rear weighing points and datum), marking the location of the points on the floor, drawing a line joining the marks, and measuring on that line. The location of the weighing points is found by dropping the plumb bob from the axle of the wheel. If the rear weighing point is a skid, they advise to place under it a piece of round bar or angle transversely, so that the weighing point can be determined in an accurate manner.

## 3.2 Products by specialised companies

### 3.2.1 Intercomp

This company is specialised in weighing scales for different applications (aviation, racing, military, agriculture, mining, etc.). In its range of products for aviation, the low-profile platform scales are the only ones which could be used for sailplanes. The scale with the lowest maximum capacity can measure up to 1,250 kg with an accuracy of  $\pm 0.1\%$  [8]. When compared with the requirements of the weight and balance process for sailplanes, it can be seen that these products are far from providing optimal characteristics.

Intercomp also offers indicators which communicate with its scales and provide the weight readings and the CG position, but no additional features to increase the accuracy of the CG position calculation are provided.

### 3.2.2 HKM-Messtechnik

This company offers weighing scales for aircraft. In its range of products, we can find weighing scales with nominal loads of 100 kg, 500 kg, 1000 kg, and higher, with accuracies of  $\pm 0.2$  kg,  $\pm 0.4$  kg, and  $\pm 1$  kg, respectively [9]. These specifications are acceptable for the needs we have determined, even though the accuracies are not the highest on the market. These scales are portable and have a flat design, which makes them suitable for sailplane weight and balance. No other products for increasing the measurement accuracy are offered by this company.

### 3.2.3 Computerscales

Even though it is specialised in weighing scales for racing cars, this company offers products which come closer to satisfying the needs of sailplanes. For example, weighing scales with a maximum capacity of as low as 180 kg and an accuracy of  $\pm 0.1\%$  are offered. No additional systems for increasing the accuracy of the CG position calculation are offered.

It must be taken into account that the weighing scales offered by this company are not certified for nor recommended to use with any kind of manned aircraft, including sailplanes.

### 3.3 Conclusion

After researching the state of the art of the weight and balance process for sailplanes, it is clear that there is not any widespread method for improving the accuracy of the measurements. The indications of the manufacturers are the basis over which the different entities define their procedures and recommendations. The different measurements and preparations follow rudimentary methods, such as the plumb bob for measuring distances or the wedge and the level as a longitudinal levelling means.

Furthermore, manufacturers have not offered a definite solution yet. There are very few options when it comes to sailplane-specific weighing scales, and their accuracies do not allow to obtain accurate measurements of the CG position. Actually, some entities recommend the use of non-specific weighing scales provided they have been recently checked. We have not been able to find a product which is integrated into a weighing system in order to facilitate the process.

## 4 Design of the system

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The design of the weight and balance system for sailplanes will be made according to the results of the study of uncertainties. In consequence, this design will be oriented to reducing the total uncertainty of the CG position calculation by reducing the uncertainty of each of the following measurements, in order of importance:

1. Longitudinal levelling
2. Distance between front weighing point and datum (L2)
3. Rear weight (G2)
4. Distance between weighing points (L1)
5. Front weight (G1)

### 4.1 Longitudinal levelling

When performing the weight and balance process, levelling the sailplane longitudinally is a complicated process which is intrinsically inaccurate. This is due to the methods recommended by sailplane manufacturers, which require the use of a wedge of certain dimensions without providing any tolerance indications. Furthermore, the wedge is placed on top of the rear part of the fuselage, and then the level is placed on top of the wedge. All of this causes an addition to the uncertainty.

Also, elevating the rear weighing scale is something difficult for users without appropriate tools. In some cases, household or kitchen scales are used for the rear weighing point, and we have not found any evidence that the methods used for elevating the rear scale provide any accuracy to the longitudinal levelling.

In conclusion, two aspects need to be solved: the measurement per se and the means for elevating the rear scale.

#### 4.1.1 Longitudinal levelling measurement

It is clear that suppressing the interface between the sailplane and the levelling instrument is key both for reducing the uncertainty and simplifying the process. In this way, the user will not be forced to find or build a wedge of certain proportions. Also, the user will not have to worry about the wedge and the levelling instrument sliding off the fuselage and falling and will be able to be more focused on the task at hand.

The solution, then, consists of a levelling instrument that places directly on top of the fuselage and indicates the user when the sailplane is levelled. To do this, the levelling instrument needs to know at what angle the sailplane is correctly levelled. A digital level with the capacity to be tared to zero at an angle specified by the user could be used. As an example, imagine the user of an ASK 13, which is levelled with a 1000:55 wedge:

- First, the user would calculate the levelling angle  $\theta_0$ <sup>6</sup> corresponding to a 1000:55 wedge with the following formula:

$$\theta_0 = \arctan\left(\frac{55}{1000}\right) = 3.148^\circ \approx 3.1^\circ$$

- Then, the digital level would have to be tared to indicate zero at an angle of  $3.1^\circ$  with respect to the horizontal.
- The digital level is placed on top of the rear fuselage, as indicated by the manufacturer.
- When the level indicates an angle of  $0^\circ$ , the sailplane is longitudinally levelled.

#### 4.1.1.1 Estimation of the required instrument accuracy

Before deciding if a commercial digital level would be suitable for this system or a new digital level needs to be designed, it is necessary to assess the actual accuracy that could be achieved with this method. Consider that we want to set a maximum uncertainty of  $\Delta D_{CG} = \pm 3 \text{ mm}$  due to the measurement of the longitudinal levelling. This value has been chosen as it is a significant reduction with respect with the estimated uncertainty of existing methods, but is also reasonable. Then, using the formulas derived in the study of uncertainties, the accuracies required for each of the sailplanes whose levelling uncertainty was studied are the following:

Model	Required accuracy ( $\Delta\theta$ )
<b>ASK 13</b>	$\pm 0.311^\circ$
<b>ASK 21</b>	$\pm 0.333^\circ$
<b>ASW 20</b>	$\pm 0.434^\circ$
<b>DG-1000S</b>	$\pm 0.282^\circ$
<b>G102 Astir CS</b>	$\pm 0.379^\circ$
<b>G103 Twin II</b>	$\pm 0.419^\circ$
<b>Ka 6E</b>	$\pm 0.457^\circ$
<b>Average</b>	<b><math>\pm 0.374^\circ</math></b>

Table 9: Required longitudinal levelling accuracy for a CG position uncertainty of  $\pm 3 \text{ mm}$ .

It can be seen that these are reasonable values, as accuracies of  $\pm 0.2^\circ$  are easily found on commercial digital levels, even on the lower price range.

However, the resolution of the instrument is another aspect that has to be accounted for. Imagine that the wedge dimensions indicated by the manufacturer of a specific sailplane result in a tare angle of  $3.15^\circ$  (approximately, the value obtained for the ASK 13). Then, if the resolution of the digital level used is  $0.1^\circ$ , the user will have to round that value either upwards or downwards, which already adds another  $\pm 0.05^\circ$  of uncertainty to the measurement. Therefore, this uncertainty has to be taken into account when determining the required accuracy of the

<sup>6</sup> Refer to the Annex for the levelling angle values for each sailplane.

instrument. Let us take the sailplane for which we have obtained the highest required accuracy (i.e. the lowest value of uncertainty), the DG-1000S. Its required accuracy is  $\pm 0.282^\circ$ . If we subtract from that the addition  $0.05^\circ$  needed to compensate for the rounding of the angle, we obtain a required instrument accuracy of  $\pm 0.232^\circ$ .

If the same correction is performed for other resolutions which are commonly found in the market, the following instrument accuracies are found:

Resolution	Required instrument accuracy
<b>0.1°</b>	$\pm 0.232^\circ$
<b>0.05°</b>	$\pm 0.257^\circ$
<b>0.01°</b>	$\pm 0.277^\circ$

**Table 10: Required levelling instrument accuracy as a function of its resolution.**

#### 4.1.1.2 Instrument selection

Before considering the design of a new instrument for this purpose, we will examine the market in order to find an instrument which satisfies the determined requirements. Such requirements are the following:

- The possibility to have a tare angle manually introduced by the user
- A resolution of at least  $0.1^\circ$
- An accuracy equal or higher than the corresponding to the instrument resolution, according to Table 10

The commercial instruments which satisfy these requirements or that offer an alternative solution for some of the requirements are listed next.

#### ***Johnson 24" Waterproof Electronic Digital Level (model no. 1880-2400) [10]***

This model has the following specifications, which satisfy the determined requirements:

- **Minimum resolution (highest value):**  $0.1^\circ$ , 2 mm/m
- **Minimum accuracy (highest value):**  $\pm 0.1^\circ$
- **Cost:** \$155.98

However, this model does not have the ability to have the tare angle manually introduced by the user. As an alternative, the user will have to read the angle measured by the level, and change the elevation of the rear scale until the measured angle coincides with that calculated from the wedge dimensions.

Another alternative offered by this model is to show the measurement in mm/m, i.e. vertical millimetres divided by horizontal metres. The advantage of this kind of reading is that, in most cases, the user would not have to perform any calculations, reducing the chances of human

error. This is due to the fact that manufacturers tend to indicate the wedge dimensions as 1000:XX, where XX corresponds to an integer number. In this case, the sailplane would be levelled when the instrument indicates XX mm/m.

In order to assess the required accuracy ( $\Delta\text{Slope}$ ) when reading in units of mm/m, the following calculation has to be performed:

$$\Delta\text{Slope} = (\tan(\theta_0 + \Delta\theta) - \tan(\theta_0)) \cdot 1000 \frac{\text{mm}}{\text{m}}$$

This is so because the slope in mm/m does not depend linearly on the levelling angle  $\theta_0$ , and the required accuracy must be calculated for every sailplane. Then, the required accuracies for the previously selected sailplanes are:

Model	Required accuracy $\Delta\text{Slope}$ (mm/m)
<b>ASK 13</b>	$\pm 5.44$
<b>ASK 21</b>	$\pm 5.82$
<b>ASW 20</b>	$\pm 7.59$
<b>DG-1000S</b>	$\pm 4.92$
<b>G102 Astir CS</b>	$\pm 6.62$
<b>G103 Twin II</b>	$\pm 7.32$
<b>Ka 6E</b>	$\pm 8.04$
<b>Average</b>	$\pm 6.54$

**Table 11:** Required accuracy in mm/m for a CG position uncertainty of  $\pm 3$  mm.

Again, to the highest accuracy, corresponding to that required by the DG-1000S, we need to subtract the additional uncertainty caused by the rounding of the value. In the particular case of this Johnson digital level, the minimum resolution in mm/m is 2 mm/m. Therefore, the most unfavourable case is that where the value is an integer odd number, and the rounding would change the value by 1 mm/m (for example, if the slope for levelling the sailplane was 3 mm/m, the user would have to consider the sailplane levelled when the instrument indicates 4 mm/m). Then, the required instrument accuracy is:

$$\Delta\text{Slope} = \pm (4.92 - 1) \frac{\text{mm}}{\text{m}} = \pm 3.92 \frac{\text{mm}}{\text{m}}$$

The next step is to ensure that the instrument accuracy (i.e.  $\pm 0.1^\circ$ ) is enough to guarantee a minimum accuracy of  $\pm 3.92$  mm/m. To do this, we need to reverse the previous calculation by solving the next equation for  $\Delta\theta$ :

$$\Delta\text{Slope} = 3.92 \frac{\text{mm}}{\text{m}} = (\tan(\theta_0 + \Delta\theta) - \tan(\theta_0)) \cdot 1000 \frac{\text{mm}}{\text{m}}$$

Which gives:

$$\Delta\theta = 0.00392 \text{ rad} = 0.224^\circ,$$

which is a lower accuracy (higher value) than that offered by the instrument. In conclusion, the instrument's resolution and accuracy would be enough to guarantee the required accuracy in the longitudinal levelling measurement, either in units of degrees or in units of mm/m.

***Stabila TECH Digital Electronic Level Type 196-2 (model no. 36514) [11]***

This digital level also satisfies the initial requirements, as shown below:

- **Minimum resolution (highest value):**  $0.01^{\circ}$
- **Minimum accuracy (highest value):**  $\pm 0.2^{\circ}$
- **Cost:** \$219.99

This model does not allow the user to manually introduce a tare angle, but it can show the measurement in different units, including mm/m. Nevertheless, the manufacturer does not provide technical information about the resolution in mm/m, which makes it impossible to guarantee that the accuracy satisfies the requirements in every case.

***Hammerhead 10" Digital Laser Level (model HLLT10) [12]***

Finally, this instrument is another one which satisfies the requirements:

- **Minimum resolution (highest value):**  $0.1^{\circ}$ , 0.1%
- **Minimum accuracy (highest value):**  $\pm 0.1^{\circ}$
- **Cost:** \$49.99

With respect to the previous two models, this one offers the advantage of being more compact (10" or 25.4 cm long), but has the disadvantage of not offering the mm/m reading. Instead, it shows the angle in degrees or the slope percent. When displaying the slope percent, the resolution is 0.1%, which corresponds to a resolution of 1 mm/m. As we have calculated before when describing the Johnson level, this is enough to satisfy our requirements. Furthermore, this Hammerhead level offers a higher resolution for the same accuracy, which provides an advantage when it comes to reducing the levelling uncertainty.

***Final selection***

After comparing the three instruments and assessing their accuracy and additional characteristics, it seems that the **Hammerhead HLLT10** is the best choice as a longitudinal levelling instrument, for the following reasons:

- **Resolution and accuracy:** this model offers the highest resolution and accuracy of the three ( $0.1^{\circ}$  and  $\pm 0.1^{\circ}$ , respectively), which makes it the best option for reducing the measurement uncertainty. Also, the display resolution is coincident with the accuracy, which makes it easier for the user to be aware of the actual measurement accuracy.
- **Size:** compared with the similar option in terms of accuracy (the Johnson level), the Hammerhead HLLT10 is much more compact (25.4 cm versus 61.0 cm), making it easier

to operate and store, and reducing the chances of the instrument being longer than the flat part of the fuselage.

- **Cost:** this model has a considerably lower cost than that of the other alternatives.

#### 4.1.1.3 Human error

When placing the digital level on top of the aircraft's fuselage, the user will not always be able to centre it appropriately, i.e. it might be slightly misaligned with respect to the longitudinal axis. In consequence, the slope measured by the instrument will not be exactly that of the top of the fuselage. To assess the loss in accuracy due to this human error, we will perform the corresponding calculations based on the following hypotheses:

1. For the majority of sailplanes, the aft part of the fuselage can be modelled as having a circular cross-section.
2. The aft part of the fuselage can be considered conical. In consequence, the cross-sectional radius reduction is linear and a straight object (like a levelling instrument) can be put in full contact with the surface of the fuselage.

Then, consider the following diagram, which represents the aft part of the fuselage. Notice how the slope of the dashed line is smaller than that of the solid line:

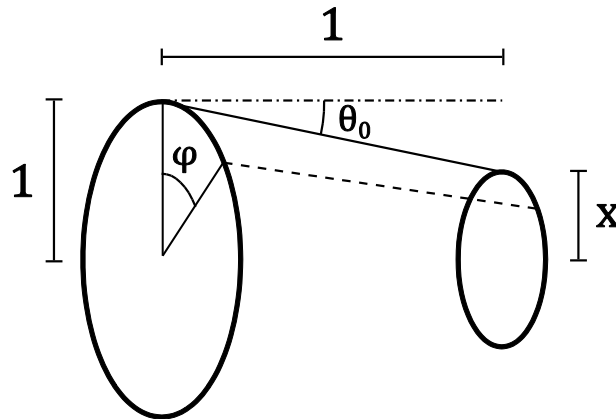


Figure 4: Effect of the levelling instrument displacement on the measured slope.

The levelling angle can be calculated as follows:

$$\theta_0 = \arctan(1 - x) \rightarrow (1 - x) = \tan \theta_0$$

Thus, if the digital level has rotated from the top of the fuselage by an angle  $\varphi$ , the angle indicated by such digital level becomes:

$$\theta = \arctan(1 \cdot \cos \varphi - x \cdot \cos \varphi) = \arctan(\cos \varphi \cdot (1 - x)) = \arctan(\cos \varphi \cdot \tan \theta_0)$$

And the error in the angle can be calculated as:

$$\Delta\theta = \theta - \theta_0$$



After obtaining the angle  $\theta_0$  for each sailplane from the wedge dimensions indicated in the manuals, we can calculate the error for a range of values of  $\varphi$ :

$\varphi \rightarrow$ Model $\downarrow$	$\Delta\theta$		
	$\pm 5^\circ$	$\pm 15^\circ$	$\pm 25^\circ$
<b>ASK 13</b>	-0.012°	-0.107°	-0.294°
<b>ASK 21</b>	-0.011°	-0.101°	-0.278°
<b>ASW 20</b>	-0.010°	-0.088°	-0.241°
<b>DG-1000S</b>	-0.007°	-0.064°	-0.177°
<b>G102 Astir CS</b>	-0.009°	-0.084°	-0.232°
<b>G103 Twin II</b>	-0.009°	-0.078°	-0.214°
<b>Ka 6E</b>	-0.019°	-0.174°	-0.480°
<b>Average</b>	<b>-0.011°</b>	<b>-0.099°</b>	<b>-0.274°</b>

Table 12: Levelling angle error as a function of different level rotations.

As can be seen, the error becomes significant for rotations equal or greater than  $\pm 15^\circ$ . This, however, has to be put in context for each sailplane. The following table shows the distance between the sailplane's axis and the digital level axis that corresponds to a displacement of  $\pm 15^\circ$ :

Model	Digital level distance from longitudinal axis (for $\varphi = \pm 15^\circ$ ) (mm)
<b>ASK 13</b>	$\pm 38.3$
<b>ASK 21</b>	$\pm 35.8$
<b>ASW 20</b>	$\pm 28.9$
<b>DG-1000S</b>	$\pm 29.4$
<b>G102 Astir CS</b>	$\pm 28.1$
<b>G103 Twin II</b>	$\pm 37.3$
<b>Ka 6E</b>	$\pm 41.0$
<b>Average</b>	<b><math>\pm 34.1</math></b>

Table 13: Lateral displacements of the levelling instrument for a rotation of  $\pm 15^\circ$ .

Therefore, displacements of between 30 mm and 40 mm can cause significant additional error in the measurement of  $\theta_0$ . It must be noted that displacements corresponding to more than  $\pm 15^\circ$  are very unlikely to happen. This is so because of two reasons: first, that this displacement corresponds to the lateral slope of the fuselage, and at slopes steeper than  $15^\circ$ , the user would notice the displacement because the level would slip due to gravity; and second, because they would be able to visually identify the displacement (a displacement of 30–40 mm is easy to

identify visually). Therefore, we will take  $\pm 15^\circ$  in our uncertainty assessment as the default maximum displacement of the digital level (due to human error).

#### 4.1.2 Mechanism for elevating the rear scale

Since a detailed design with a structural analysis of the mechanism is not the objective of this project, we are going to define only those parameters more relevant to the accuracy of the weight and balance process and the ease of use.

This mechanism has to accomplish the following:

- To allow the user to elevate the rear scale's platform up to 0.75 m higher than the front scale's surface (as defined in the requirements) in order to place the sailplane in flight position.
- To keep the scale's platform parallel to the ground.
- To allow the average person to elevate the maximum planned weight with relative ease (without exerting an excessive force).
- To facilitate the elevation of the scale with a precision such that a high levelling accuracy can be obtained.

Taking all of this into account, the best system seems to be a **scissor-type mechanical system**. A long threaded bar with a crank on its end would be used to elevate the platform, and it would be kept horizontal at all times.

#### 4.1.3 Uncertainty assessment

In this section, we will use the performed calculations, the hypotheses, and the technical data of the selected levelling instrument to assess the uncertainty of the CG position  $\Delta D_{CG}$  caused by the longitudinal levelling alone. As explained in 2.2.2 (Longitudinal levelling), the uncertainty of  $D_{CG}$  caused by the levelling angle can be calculated as follows:

$$\Delta D_{CG} = \Delta b = \pm |a \cdot \cos \alpha - a \cdot \cos (\alpha + \Delta \alpha)|$$

Since the sailplane moves like a solid body:

$$\Delta \alpha = \Delta \theta$$

And the uncertainty in the levelling angle  $\Delta \theta$  is composed by:

$$\Delta \theta = \text{Instrument accuracy} + \text{Rounding} + \text{Human error (displacement)}$$

In the case of the selected instrument, the accuracy is equal to  $\pm 0.1^\circ$ . The rounding will be taken as the greatest uncertainty that could be caused by the rounding to the resolution of the

instrument<sup>7</sup> (0.1°). When rounding to 0.1°, the greatest uncertainty is half of this value, i.e. 0.05°. The uncertainty on the levelling due to human error has already been calculated for each sailplane (it can be found in Table 12). The displacement angle will be considered to be  $\varphi = \pm 15^\circ$ . Then, we have:

$$\Delta\theta = \pm |0.1^\circ + 0.05^\circ + \text{Human error (displacement)}|$$

The following table shows the uncertainty of the levelling angle  $\Delta\theta$  and its contribution to the uncertainty of the CG position  $\Delta D_{CG}$ :

Model	Levelling angle uncertainty $\Delta\theta$	Contribution to CG position uncertainty (mm)
<b>ASK 13</b>	$\pm 0.257^\circ$	$\pm 2.5$
<b>ASK 21</b>	$\pm 0.251^\circ$	$\pm 2.3$
<b>ASW 20</b>	$\pm 0.238^\circ$	$\pm 1.6$
<b>DG-1000S</b>	$\pm 0.214^\circ$	$\pm 2.3$
<b>G102 Astir CS</b>	$\pm 0.234^\circ$	$\pm 1.8$
<b>G103 Twin II</b>	$\pm 0.228^\circ$	$\pm 1.6$
<b>Ka 6E</b>	$\pm 0.324^\circ$	$\pm 2.1$
<b>Average</b>	$\pm 0.249^\circ$	$\pm 2.0$

**Table 14: Final levelling angle uncertainties and their contributions to the CG position uncertainty.**

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<sup>7</sup> In the case of the selected digital level, the lowest resolution is found when measuring in units of degrees. Therefore, this represents the most unfavourable case and is the one we will use to perform the error assessment.

## 4.2 Weights measurement and point weight locations

Even though the distance  $L_2$  has already been defined as the most important for reducing the total uncertainty, it is clear that both  $L_1$  and  $L_2$  depend essentially on the accurate determination of three points with respect to a common reference system. These three points are the front weighing point, the rear weighing point, and the datum. From now on, these three points are going to be defined as follows:

- **$X_1$** : longitudinal location of the front point weight measured from the origin of coordinates of the reference system. The front point weight is defined as the exact location where the front weight of the sailplane is applied.
- **$X_2$** : longitudinal location of the rear point weight measured from the origin of coordinates of the reference system. The rear point weight is defined as the exact location where the rear weight of the sailplane is applied.
- **$X_D$** : longitudinal location of the datum (as defined by the manufacturer) measured from the origin of coordinates of the reference system.

Then, if  $X_1$ ,  $X_2$ , and  $X_D$  can be accurately determined with respect to the same reference system,  $L_1$  and  $L_2$  will also be accurately determined.

### 4.2.1 Determination of $X_1$ and $X_2$

First, we will start by finding a method to accurately determine the position of each point weight (separately) with respect to the corresponding scale. The positions of the front and rear point weights with respect to the front and rear scale will be called  $X_1'$  and  $X_2'$ , respectively.

When placing the sailplane over the front and rear weighing scales, the scales only display the weight that is over them, but not its exact location. When performing this operation, there is no immediate way of calculating this location from the weight measurement alone. In general, the vertical projection on the ground of the centre of the wheel's axis is considered to be the location of the weight, but since the tyre is flexible and has a non-negligible contact area with the scale, it is not possible to guarantee the accuracy of this hypothesis in every case.

There is, however, one way to accurately determine the weighing point with respect to a reference system attached to the weighing scale: to divide each weight measurement into two weight measurements whose location is known with a much higher accuracy. These two weight measurements would be made by two smaller weighing scales (which we would refer to as sub-scales, for the sake of simplicity) on the base of the scale's platform. Consider the following diagram<sup>8</sup>:

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<sup>8</sup> In this section, we will always refer to  $X_1'$  for convenience, but everything is true for  $X_2'$  as well.

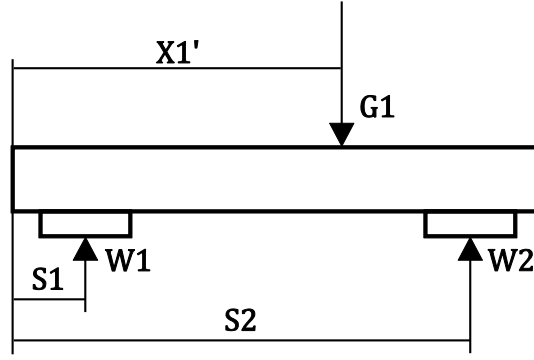


Figure 5: Schematic of the system for determining  $X1'$  (or  $X2'$ ) (side view).

Then,  $X1'$  can be determined using the following formula, in a way analogous to that of determining the sailplane's CG position:

$$X1' = \frac{W1 \cdot S1 + W2 \cdot S2}{W1 + W2}$$

Before continuing with more technical aspects of this method, it is necessary to assess its accuracy. In the same way that the uncertainty of the CG position has been estimated, we can calculate the propagation of uncertainties in this case:

$$\epsilon_{X1'} = \left| \frac{\partial X1'}{\partial W1} \right| \cdot \epsilon_{W1} + \left| \frac{\partial X1'}{\partial W2} \right| \cdot \epsilon_{W2} + \left| \frac{\partial X1'}{\partial S1} \right| \cdot \epsilon_{S1} + \left| \frac{\partial X1'}{\partial S2} \right| \cdot \epsilon_{S2}$$

The partial derivatives are:

$$\frac{\partial X1'}{\partial W1} = \frac{W2 \cdot (S1 - S2)}{(W1 + W2)^2}$$

$$\frac{\partial X1'}{\partial W2} = \frac{W1 \cdot (S2 - S1)}{(W1 + W2)^2}$$

$$\frac{\partial X1'}{\partial S1} = \frac{W1}{W1 + W2}$$

$$\frac{\partial X1'}{\partial S2} = \frac{W2}{W1 + W2}$$

For assessing the uncertainty in an adequate manner, some sample values need to be used both for measurements and their uncertainties. As a first approach, we will consider the following uncertainties for both the front and rear weighing points:

Concept	Uncertainty	Units
<b>W1</b>	$\pm 0.2$	kg
<b>W2</b>	$\pm 0.2$	kg
<b>S1</b>	$\pm 2$	mm
<b>S2</b>	$\pm 2$	mm

**Table 15: Estimation of the uncertainties of the two weight sub-divisions and the position of the sub-scales.**

The criteria used for selecting the uncertainties are the following:

- For the weights (W1 and W2), the common uncertainty of aircraft-specific scales previously mentioned is used.
- For the location of the two sub-scales of each scale (S1 and S2), a tolerance easily achievable by today's manufacturing techniques ( $\pm 2$  mm) is used.

The location of each of the sub-scales for both scales is shown in the following table. Notice that the position of one of the sub-scales is coincident with the main scale's reference system. The position of the other sub-scale has been chosen based on the approximate size of aircraft-specific scales:

Concept	Value	Units
<b>S1</b>	0	mm
<b>S2</b>	350	mm

**Table 16: Initial estimation of the position of the sub-scales.**

Then, the uncertainty in the calculation of  $X1'$  for the selected sailplane models is found:

Model	$X1'$ uncertainty (mm)
<b>ASK 13</b>	$\pm 2.3$
<b>ASK 21</b>	$\pm 2.2$
<b>ASW 20</b>	$\pm 2.4$
<b>DG-1000S</b>	$\pm 2.2$
<b>G102 Astir CS</b>	$\pm 2.4$
<b>G103 Twin II</b>	$\pm 2.2$
<b>Ka 6E</b>	$\pm 2.4$
<b>Average</b>	$\pm 2.3$

**Table 17: Initial estimation of the uncertainty of  $X1'$ .**

If the same sample values and uncertainties are used for the rear weighing point, the uncertainty in the calculation of  $X2'$  is found:

Model	X2' uncertainty (mm)
ASK 13	$\pm 17.3$
ASK 21	$\pm 6.8$
ASW 20	$\pm 3.8$
DG-1000S	$\pm 3.4$
G102 Astir CS	$\pm 4.3$
G103 Twin II	$\pm 5.4$
Ka 6E	$\pm 4.8$
Average	$\pm 6.5$

Table 18: Initial estimation of the uncertainty of X2'.

In fact, the uncertainties of both X1' and X2' depend on the front and rear weights, respectively. The lower the weight, the higher the uncertainty<sup>9</sup>, as shown in the following graph:

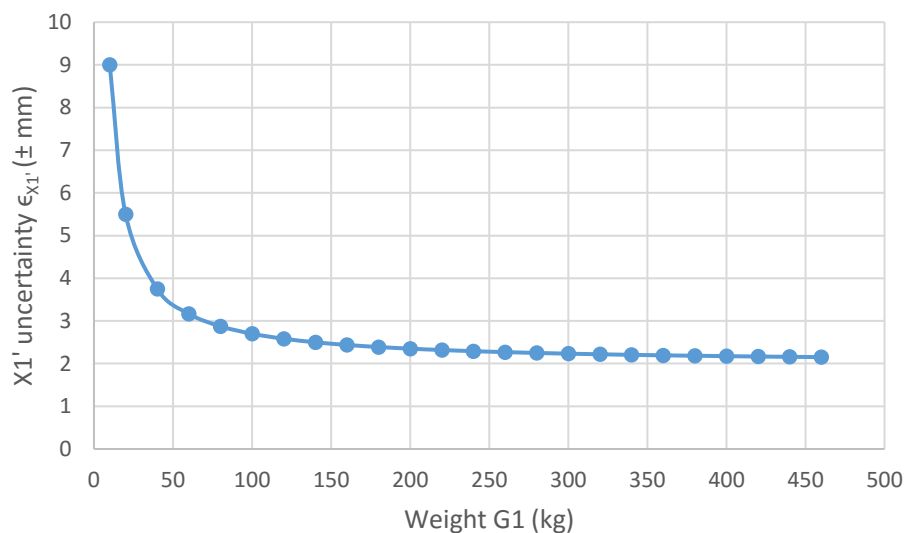


Figure 6: Initial estimation of the uncertainty of X1' (or X2') as a function of the weight G1 (or G2).

As can be seen, the uncertainty is close to  $\pm 2$  mm for weights larger than 100 kg, but it rapidly exceeds  $\pm 5$  mm for weights lower than 25 kg. This is especially problematic for the rear scale. For sailplanes that have the CG very close to the front weighing point, like the ASK 13, the rear weight becomes very low (under 10 kg) and the uncertainty in the calculation of X2' becomes too large to be acceptable. It is then clear that the accuracy of the sub-scales has to be much higher than  $\pm 0.2$  kg.

Furthermore, the technical implementation of a scale made of two sub-scales presents one major issue: stability. It is clear that the whole weight of one weighing point cannot be put over two supports alone, as the whole setup would tilt to one side. It is safer to use four support

<sup>9</sup> A mathematical proof can be found in the Annex.

points, i.e. four sub-scales for each of the two weighing points. The basic setup of the platform would consist of a solid surface supported by four points, each one having a sub-scale, as shown in the following diagram:

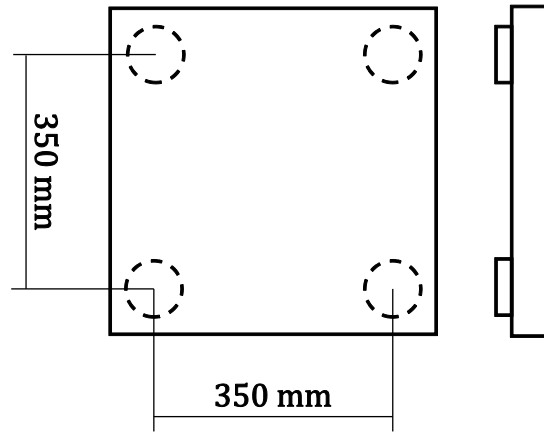


Figure 7: Basic schematic of the platform and location of the four sub-scales (top and front views).

#### 4.2.1.1 Rear weight sensors

The next step is to examine the market in order to find weight sensors suitable to use as sub-scales in this design. First, however, more specific requirements have to be defined. Consider the following table, which shows the maximum and minimum rear weights for each sailplane:

Model	Maximum G2 (kg)	Minimum G2 (kg)
<b>ASK 13</b>	7.9	1.8
<b>ASK 21</b>	15.9	10.6
<b>ASW 20</b>	46.2	34.4
<b>DG-1000S</b>	59.8	48.5
<b>G102 Astir CS</b>	36.5	24.9
<b>G103 Twin II</b>	22.6	17.1
<b>Ka 6E</b>	29.5	22.4

Table 19: Maximum and minimum rear weights.

As can be seen, the ASK 13 is an extreme, isolated case, and trying to obtain a high accuracy in the determination of  $X_2'$  for this sailplane is unrealistic. However, it is possible to aim for a high accuracy from a minimum G2 of 10 kg. We consider that a  $\epsilon_{X_2'} = \pm 5$  mm accuracy for  $X_2'$  is a realistic and technically acceptable value, especially considering that this accuracy will be significantly higher for the majority of sailplanes. In order to obtain this accuracy, a formula equivalent to that of  $X_1'$  is used:

$$\epsilon_{X_2'} = \pm \left( \frac{350 \text{ mm}}{G_2} \cdot |\epsilon_{W_1}| + 2 \text{ mm} \right),$$



where  $\epsilon_{W1}$  is the uncertainty of the measurement of each pair of sub-scales. Since two sub-scales will be used in the front and two more will be used in the rear of each weighing point,  $\epsilon_{W1}$  becomes the sum of the uncertainties of two sub-scales (the four sub-scales for each weighing point will be the same model). Then:

$$\epsilon_{W1} = 2 \cdot \epsilon_W,$$

where  $\epsilon_W$  is the uncertainty (or accuracy) of each sub-scale. The accuracy needed then becomes:

$$\epsilon_{x2'} = \pm \left( \frac{350 \text{ mm}}{G2} \cdot 2 \cdot |\epsilon_W| + 2 \text{ mm} \right)$$

$$\epsilon_W = \pm \frac{(|\epsilon_{x2'}| - 2 \text{ mm}) \cdot G2}{2 \cdot 350 \text{ mm}} = \pm \frac{(5 \text{ mm} - 2 \text{ mm}) \cdot 10 \text{ kg}}{2 \cdot 350 \text{ mm}} = \pm 0.0429 \text{ kg}$$

It is also necessary to define the range of weights which are going to be measured by each of the sub-scales. For this, we will define a square surface over which the point weight is allowed to move. The side of this square will be 25 cm (represented by a grey square in Figure 8), so that the point weight can move a minimum of 12.5 cm in any direction and the process is easier. Then, the maximum weight measured by one single sub-scale would correspond to the case where the point weight is placed on one vertex of the square.

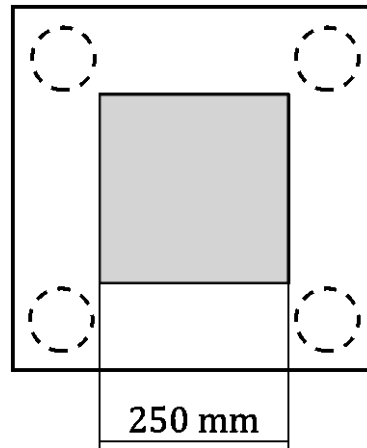


Figure 8: Point weight allowed area (in gray, top view).

The calculation is going to be made for the maximum G2 found, which equals to 59.8 kg of the DG-1000S. To this weight, we also need to add the weight of the platform over which the rear weight of the sailplane is going to be placed. Given that the design is not finished, we will consider an estimated weight of 2 kg for the rear scale platform. Taking all of this into account, the maximum weight that one single sub-scale would have to measure is:

$$W_{\text{Max}} = 43.9 \text{ kg} + \frac{2 \text{ kg}}{4} = 44.4 \text{ kg}$$

To sum up, we need to find a weight sensor which can be easily fit into a weighing platform, and which has the following specifications:

- **Accuracy:**  $\pm 0.0429$  kg or better
- **Capacity:** 44.4 kg or higher

With these criteria, we found the products described next.

***Omega LCAE Series Single Point Load Cell (model no. LCAE-45KG) [13]***

This single-point load cell<sup>10</sup> is compact and offers great accuracy, as seen in the following specifications:

- **Dimensions:** 150 x 40 x 25 mm
- **Capacity:** 45 kg
- **Combined accuracy**<sup>11</sup>:  $\pm 0.05\%$  FS<sup>12</sup> =  $\pm 0.0225$  kg
  - **Non-linearity:**  $\pm 0.015\%$  FS
  - **Hysteresis:**  $\pm 0.015\%$  FS
  - **Non-repeatability:**  $\pm 0.02\%$  FS
- **Cost:** \$241

***Loadstar Sensors RAP3 Single Point Load Cell (part no. RAP3-100S-A) [14]***

With similar characteristics to those of the Omega load cell, this one has the following specifications:

- **Dimensions:** 173.7 x 65.0 x 59.9 mm
- **Capacity:** 45.3 kg
- **Combined accuracy:**  $\pm 0.07\%$  FS =  $\pm 0.0317$  kg
  - **Non-linearity:**  $\pm 0.03\%$  FS
  - **Hysteresis:**  $\pm 0.02\%$  FS
  - **Non-repeatability:**  $\pm 0.02\%$  FS
- **Cost:** \$299

***Final selection***

Other products which satisfied the specifications were found, but their construction did not allow a solid and accurate implementation in a scale with our design. From the two options

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<sup>10</sup> A load cell is a type of transducer which generates an electrical signal directly proportional to the measured force.

<sup>11</sup> Although manufacturers use different methods for calculating the accuracy, we will define the combined accuracy as the sum of the three sources of error of the load cell (non-linearity, hysteresis, and non-repeatability), so that we obtain the accuracy for the most unfavourable case.

<sup>12</sup> FS refers to *Full Scale*, which means that the accuracy is the same for any measured value (in this case,  $\pm 0.07\%$  of the load cell capacity).

described, the **Omega LCAE-45KG** is the best option for its greater accuracy, more compact size, and lower cost.

#### 4.2.1.2 Front weight sensors

For selecting suitable load cells for the front weighing point, we must consider the range of front weights G1 which are going to be measured:

Model	Maximum G1 (kg)	Minimum G1 (kg)
<b>ASK 13</b>	318.2	272.1
<b>ASK 21</b>	389.4	334.1
<b>ASW 20</b>	251.6	202.6
<b>DG-1000S</b>	430.2	345.3
<b>G102 Astir CS</b>	249.5	196.9
<b>G103 Twin II</b>	382.9	337.4
<b>Ka 6E</b>	196.4	163.5

Table 20: Maximum and minimum front weights.

As can be seen, the minimum G1 is 163.5 kg and corresponds to the Ka 6E. If we perform the same calculation used for the rear weight to obtain the sub-scale accuracy needed to obtain a minimum accuracy of  $\pm 5$  mm in the calculation of X1', we obtain:

$$\epsilon_W = \pm \frac{(|\epsilon_{X1'}| - 2 \text{ mm}) \cdot G1}{2 \cdot 350 \text{ mm}} = \pm \frac{(5 \text{ mm} - 2 \text{ mm}) \cdot 163.5 \text{ kg}}{2 \cdot 350 \text{ mm}} = \pm 0.701 \text{ kg}$$

This, however, is an extremely large value; this is so because the uncertainty of the four sub-scales has to be added in order to find the uncertainty of G1, and we would have:

$$\epsilon_{G1} = \pm 4 \cdot \epsilon_W = \pm 4 \cdot 0.701 \text{ kg} = \pm 2.803 \text{ kg}$$

As specified in the requirements of this project, the maximum acceptable uncertainty of the weight measurement is  $\pm 1.5$  kg. Therefore, this criterion is more restrictive and is the one which we must follow. In order to keep the total weight uncertainty below  $\pm 1.5$  kg, the uncertainty of the rear weight G2 has to be considered as well. The uncertainty in the measurement of G2 is the sum of the uncertainties of the rear sub-scales, i.e.:

$$\epsilon_{G2} = \pm 4 \cdot 0.0225 \text{ kg} = \pm 0.09 \text{ kg}$$

And the uncertainty of the total weight of the sailplane  $\epsilon_G$  is equal to the sum of the uncertainties of G1 and G2:

$$\epsilon_G = \epsilon_{G1} + \epsilon_{G2}$$

$$\epsilon_{G1} = \epsilon_G - \epsilon_{G2} = \pm |1.5 - 0.09| \text{ kg} = \pm 1.41 \text{ kg}$$

Therefore, the maximum uncertainty of each front sub-scale is:

$$\epsilon_{W_{\text{Max}}} = \pm \frac{1.41}{4} \text{ kg} = \pm 0.3525 \text{ kg}$$

For determining the required capacity of the front load cells, a similar procedure to that used for the rear load cells is going to be used. The maximum measurable weight is going to be the greatest of the maximum G1 weights from Table 20 (430.2 kg of the DG-1000S). The maximum weight of the platform will be estimated as 2 kg, the same as the rear platform. The point load will also be able to move within a 25 cm-sided square. With these conditions, the maximum load that a single load cell will have to measure is:

$$W_{\text{Max}} = 316.1 \text{ kg} + \frac{2 \text{ kg}}{4} = 316.6 \text{ kg}$$

To sum up, we need to find a load cell which can be easily fit into a weighing platform, and which has the following specifications:

- **Accuracy:**  $\pm 0.3525 \text{ kg}$  or better
- **Capacity:** 316.6 kg or higher

With these criteria, we found the products described next.

***Omega LCCA Series “S” Beam Load Cell (model no. LCCA-750) [15]***

This “S” beam type load cell offers great accuracy and an adequate capacity for this purpose:

- **Dimensions:** 76.2 x 50.8 x 16.5 mm
- **Capacity:** 340.1 kg
- **Combined accuracy:**  $\pm 0.06\% \text{ FS} = \pm 0.205 \text{ kg}$ 
  - **Non-linearity:**  $\pm 0.03\% \text{ FS}$
  - **Hysteresis:**  $\pm 0.02\% \text{ FS}$
  - **Non-repeatability:**  $\pm 0.01\% \text{ FS}$
- **Accuracy according to manufacturer:**  $\pm 0.037\% \text{ FS} = \pm 0.126 \text{ kg}$
- **Cost:** \$390

***Omega LCEC Series Sealed Beam Load Cell (model no. LCEC-1K) [16]***

This sealed beam load cell is compact, has great accuracy and is designed for low-profile applications, like a weighing platform:

- **Dimensions:** 127.0 x 38.1 x 25.4 mm
- **Capacity:** 453.5 kg
- **Combined accuracy:**  $\pm 0.06\% \text{ FS} = \pm 0.2721 \text{ kg}$ 
  - **Non-linearity:**  $\pm 0.03\% \text{ FS}$
  - **Hysteresis:**  $\pm 0.02\% \text{ FS}$
  - **Non-repeatability:**  $\pm 0.01\% \text{ FS}$
- **Accuracy according to manufacturer:** N/A

- **Cost:** \$300

### ***Final selection***

Both load cells offer good characteristics and satisfy the requirements. However, the “S” beam load cell has a less adequate form factor since it is 76.2 mm tall in the measurement direction (vertical). The sealed beam load cell is only 38.1 mm tall in the measurement direction, which makes it more adequate for integration into a weighing platform. Even though the sealed beam load cell is less accurate, it still satisfies the requirements by a significant margin and offers a higher capacity and a lower cost. Therefore, the **Omega LCEC-1K** is the load cell chosen for the front weighing point.

### **4.2.2 Uncertainty assessment**

After selecting suitable load cells for both the front and the rear weighing points, it is necessary to assess the uncertainty on the determination of  $X1'$  and  $X2'$ , as well as the total weight.

To do this, we will consider each pair of load cells (front pair and rear pair, for each weighing point) as one single unit. This can be done since each pair of load cells has the same longitudinal position. Then, the weight at each of these units is the sum of the weights measured by each pair of load cells and, therefore, the uncertainty is also the sum of the uncertainties of the load cells. The uncertainty of the position of the load cells, however, remains the same, i.e.  $\pm 2$  mm. This is so because we defined such uncertainty as the tolerance of the position of the load cell with respect to the scale; in consequence, the uncertainty on the location of the weight measured by each pair of load cells is not greater than  $\pm 2$  mm.

Then, using the same reasoning described in the beginning of 4.2.1 (Determination of  $X1$  and  $X2$ ) and with the specifications found, we can assess the uncertainty of this design. For the front weighing point ( $X1'$ ):

Concept	Value	Uncertainty	Units
<b>W1</b>	N/A	$\pm 0.5442$	kg
<b>W2</b>	N/A	$\pm 0.5442$	kg
<b>S1</b>	0	$\pm 2$	mm
<b>S2</b>	350	$\pm 2$	mm

**Table 21: Values and uncertainties of the two weight sub-divisions and the positions of the front load cells.**

Model	X1' uncertainty (mm)
ASK 13	± 2.7
ASK 21	± 2.6
ASW 20	± 2.9
DG-1000S	± 2.5
G102 Astir CS	± 2.9
G103 Twin II	± 2.6
Ka 6E	± 3.1
Average	± 2.8

Table 22: Uncertainties of X1' with the selected load cells.

Doing the same for the rear weighing point (X2'):

Concept	Value	Uncertainty	Units
W1	N/A	± 0.045	kg
W2	N/A	± 0.045	kg
S1	0	± 2	mm
S2	350	± 2	mm

Table 23: Values and uncertainties of the two weight sub-divisions and the positions of the rear load cells.

Model	X2' uncertainty (mm)
ASK 13	± 5.5
ASK 21	± 3.1
ASW 20	± 2.4
DG-1000S	± 2.3
G102 Astir CS	± 2.5
G103 Twin II	± 2.8
Ka 6E	± 2.7
Average	± 3.0

Table 24: Uncertainties of X2' with the selected load cells.

As can be seen, the uncertainties of X1' and X2' are quite low in the majority of cases, usually staying below ± 3 mm.

The uncertainty of the total weight is the sum of the uncertainties of each of the eight load cells:

$$\text{total weight uncertainty} = \pm |4 \cdot 0.2721 + 4 \cdot 0.0225| \text{ kg} = \pm 1.1784 \text{ kg}$$

#### 4.2.3 Basic dimensions of the scales

As determined in 1.2.4 (Size of the scales), the minimum dimensions of the platform of the front scale (the surface where the wheel will be placed) are 435 x 366 mm. For reasons of simplicity and to facilitate the process, we believe that a square surface of **450 x 450 mm** is best. For the

rear scale, even though the tail skid is usually smaller than the contact surface of a tyre, the same **450 x 450 mm** surface will be used, so that the point weight can stay within the limits when elevating the scale.

### 4.3 Measurement of L1

Now that we have found a way to determine the position of the two point weights of the sailplane X1' and X2' with respect to the scales, we must find a way to find the distance between such points. Of course, if we find a way to measure the distance between the scales, the distance between X1 and X2 will be found immediately.

Since we want a system that performs the weight and balance measurements and calculations with as little user contribution as possible, we must find a distance measurement method that is automatic, apart from accurate. The most relevant commercial solutions offered are the following:

- **Ultrasonic sensors:** commonly used for low-accuracy applications such as proximity sensors. They offer a limited range of measurable distances and are difficult to find with accuracies near  $\pm 10$  mm.
- **Laser sensors:** widely used in industrial and construction environments, laser sensors can offer great accuracies (as low as  $\pm 1$  mm) for either distance or displacement measurement, as well as a wide range of measurable distances.

Therefore, **laser sensors** seem like the best option. The implementation would consist on a laser sensor located in the front scale which would point to the rear scale. Let us perform a preliminary assessment of the uncertainty that we would obtain using such method, starting from the following hypotheses:

- The accuracy of the laser distance measurement  $\epsilon_{\text{laser}}$  is  $\pm 3$  mm. This value has been chosen as there are several commercial laser sensors with a similar accuracy, but higher accuracies can also be found in the market.
- The tolerance of the laser sensor position (with respect to the scale)  $\epsilon_{\text{LP}}$  is  $\pm 2$  mm, which is the same value chosen for the position of the load cells.

Then, the uncertainty of the distance measurement alone (L1') would be the sum of the two accuracies, that is:

$$\epsilon_{L1'} = \pm (3 + 2) \text{ mm} = \pm 5 \text{ mm}$$

To obtain the actual uncertainty of the distance measurement (L1), we have to add the uncertainty of the position of the point loads (X1' and X2'), which vary for each sailplane. The obtained values would be the following:



Model	L1 uncertainty (mm)	Contribution to CG position uncertainty (mm)
<b>ASK 13</b>	$\pm 13.2$	$\pm 0.2$
<b>ASK 21</b>	$\pm 10.7$	$\pm 0.5$
<b>ASW 20</b>	$\pm 10.3$	$\pm 1.6$
<b>DG-1000S</b>	$\pm 9.8$	$\pm 1.2$
<b>G102 Astir CS</b>	$\pm 10.4$	$\pm 1.4$
<b>G103 Twin II</b>	$\pm 10.4$	$\pm 0.6$
<b>Ka 6E</b>	$\pm 10.8$	$\pm 1.4$
<b>Average</b>	<b><math>\pm 10.8</math></b>	<b><math>\pm 1.0</math></b>

**Table 25: Initial estimation of the uncertainties of L1 and their contribution to the CG position uncertainties.**

As can be seen, even though the uncertainty of L1 is quite large, the truth is that its influence on the uncertainty of the CG position is very small.

In conclusion, this preliminary assessment shows that a laser accuracy of  $\pm 3$  mm and a position tolerance of  $\pm 2$  mm are enough to obtain a distance measurement that has an almost negligible influence on the uncertainty of the CG position.

#### 4.4 Measurement of L2

The measurement of L2 is the biggest challenge of this project. First, it requires an extremely high accuracy due to the fact that its uncertainty is directly added to the CG position uncertainty; and second, the determination of the datum point ( $X_D$ ) with respect to an arbitrary reference has to be done with rudimentary methods. As explained in 1.2.6 (Weight and balance procedure), the datum is usually the leading edge of the wing's root rib, although manufacturers do define other datums. It is clear that the determination of this point cannot be automatized.

The method that is most used among users and recommended by manufacturers is to drop a plumb bob from the datum point. Even though it is rudimentary, we believe that it is the best method for transporting the location of the datum to the ground. This is so because, if the datum is defined as the leading edge of the wing's root rib (as is usual), there is one clear lateral reference to support the plumb bob's string (the fuselage) and one clear longitudinal reference from which the string will hang down (the vertical tangent to the rib, i.e. the leading edge).

Once we have the projection of  $X_D$  on the ground, we still have to measure the longitudinal distance between  $X_1$  and  $X_D$ . To do this, a laser sensor could be used. There is, however, one main issue:  $X_D$  is at an unknown distance from the sailplane's longitudinal axis, and in consequence it is not possible to directly measure its distance with a laser sensor.

From here, there are two options for the measurement of L2, which are presented next.

#### 4.4.1 L2 measurement option A

As shown in Figure 9, this option consists on determining the three sides of a triangle, and then calculating L2 by mathematically solving the triangle. To do this, a minimum of two laser sensors must be used: one on the front scale and the other on the rear scale. The two sensors first measure the distance between the scales (L1'), and then rotate until they find the plumb bob (a and b), corresponding to X<sub>D</sub>.

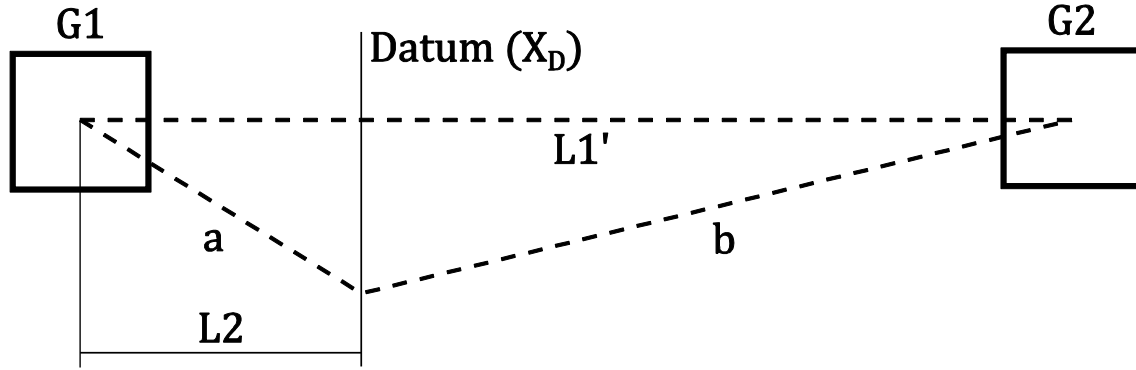


Figure 9: Option A setup (top view).

The necessary equipment for this option is the following:

- Two highly-accurate laser sensors
- Two servomotors to rotate the sensors
- Electronics and an algorithm to determine when the lasers are pointing to the plumb bob

Actually, by solving the triangle found with the laser sensor measurements, only the distance between the front scale laser sensor and the datum would be found; we will call this L2'. In order to find L2, we must add the distance between the point weight and the laser sensor (d<sub>L</sub>):

$$L2 = L2' + d_L$$

By solving the triangle, we find that L2' is:

$$L2' = \frac{a^2 - b^2 + L1'^2}{2 \cdot L1'}$$

In order to assess the accuracy achievable with this option, the uncertainties described below have to be taken into account for each distance measurement. Note that the triangle is defined with the front laser sensor as a reference; in consequence, the front laser position uncertainty does not have an effect on the determination of the triangle, but it does have an effect on d<sub>L</sub>:

- **Uncertainties of L1'** (as defined in 4.3 Measurement of L1):
  - Laser sensor measurement uncertainty (ε<sub>laser</sub>)
  - Rear laser sensor position uncertainty with respect to the scale (ε<sub>L,P</sub>)

- **Uncertainties of a:**
  - Laser sensor measurement uncertainty ( $\epsilon_{\text{laser}}$ )
  - Uncertainty of datum projection on the ground ( $\epsilon_{X_D}$ )
- **Uncertainties of b:**
  - Laser sensor measurement uncertainty ( $\epsilon_{\text{laser}}$ )
  - Uncertainty of datum projection on the ground ( $\epsilon_{X_D}$ )
  - Rear laser sensor position uncertainty with respect to the scale ( $\epsilon_{LP}$ )
- **Uncertainties of  $d_L$ :**
  - Point weight X1' position uncertainty, according to Table 22
  - Front laser sensor position uncertainty with respect to the scale ( $\epsilon_{LP}$ )

For consistency, we will choose estimated uncertainties similar to those used for the other preliminary assessments. However, since the laser sensor measurements a and b will not be parallel to the aircraft's longitudinal axis, the laser sensor position tolerances considered will be those corresponding to the most unfavourable case, a measurement at 45°. This is so because the tolerances admit the displacement of the laser sensor position inside a square. If the measurement is parallel to any sides of the square, the uncertainty is  $\pm$  half the side of the square, but if the measurement is at 45°, the uncertainty becomes  $\pm$  half the diagonal of the square, as shown in the following diagram:

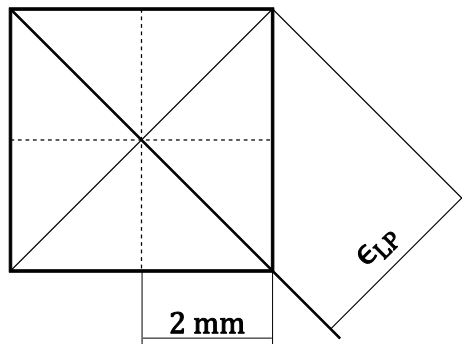


Figure 10: Maximum laser sensor position uncertainty calculation.

Considering the same position tolerances chosen before, the tolerance in the diagonal direction becomes:

$$\epsilon_{LP} = \pm \sqrt{2^2 + 2^2} \text{ mm} = \pm 2.9 \text{ mm}$$

The uncertainty of the datum projection on the ground is estimated considering the accuracy of the plumb bob method, which includes an uncertainty due to air movement, oscillation and human error. Then the estimated uncertainties are:

Concept	Value
$\epsilon_{laser}$	$\pm 3$ mm
$\epsilon_{LP}$	$\pm 2.9$ mm
$\epsilon_{X_D}$	$\pm 5$ mm

Table 26: Estimation of individual uncertainties for the preliminary assessment of option A.

Concept	Equals to	Value
$\epsilon_{L1'}$	$\epsilon_{laser} + \epsilon_{LP}$	$\pm 5.9$ mm
$\epsilon_a$	$\epsilon_{laser} + \epsilon_{X_D}$	$\pm 8$ mm
$\epsilon_b$	$\epsilon_{laser} + \epsilon_{X_D} + \epsilon_{LP}$	$\pm 10.9$ mm
$\epsilon_{d_L}$	N/A	depends on model <sup>13</sup>

Table 27: Estimation of total uncertainties for the preliminary assessment of option A.

The uncertainty of L2 is estimated by propagation of uncertainties<sup>14,15</sup>:

$$\epsilon_{L2} = \left| \frac{\partial L2}{\partial a} \right| \cdot \epsilon_a + \left| \frac{\partial L2}{\partial b} \right| \cdot \epsilon_b + \left| \frac{\partial L2}{\partial L1'} \right| \cdot \epsilon_{L1'} + \left| \frac{\partial L2}{\partial d_L} \right| \cdot \epsilon_{d_L}$$

Finally, the following values are obtained for each sailplane:

Model	L2 uncertainty (mm)
<b>ASK 13</b>	$\pm 25.2$
<b>ASK 21</b>	$\pm 25.8$
<b>ASW 20</b>	$\pm 23.3$
<b>DG-1000S</b>	$\pm 23.2$
<b>G102 Astir CS</b>	$\pm 23.8$
<b>G103 Twin II</b>	$\pm 25.4$
<b>Ka 6E</b>	$\pm 24.1$
<b>Average</b>	<b><math>\pm 24.4</math></b>

Table 28: Initial estimation of the uncertainty of L2 with option A.

As can be seen, the uncertainty of L2 becomes unacceptably large with this method. Let us introduce option B so that we can compare the accuracy of both options.

#### 4.4.2 L2 measurement option B

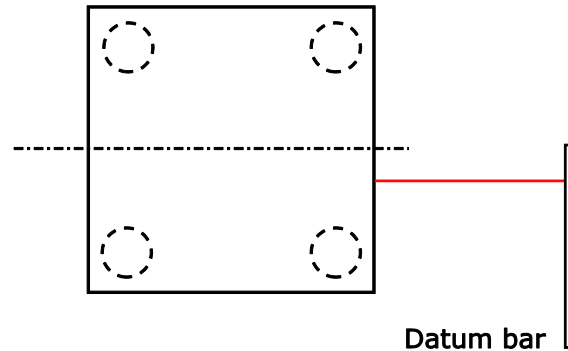
This option consists on using one single laser sensor to measure both L1 and L2 distances. To do this, the user would place a specially-designed bar (from now on, the **datum bar**) perpendicular

<sup>13</sup> These values can be found in the Annex.

<sup>14</sup> Being this a preliminary assessment, the partial derivatives have been left out of the Report and can be found in the Annex.

<sup>15</sup> For the estimation of the error, we will obtain the values of L1', a, and b as if the laser sensors were located on the point weight, due to the fact that the design is not yet complete. Please note that the difference in the error is negligible.

to the longitudinal axis just under the datum point, which would extend close enough to the aircraft's longitudinal axis to be detected by the laser sensor, as shown in the next diagram:



**Figure 11: Option B setup (top view). The red line represents the laser sensor beam.**

This presents two challenging problems:

- Locating the bar exactly perpendicular to the sailplane's longitudinal axis is impossible to do without special tools. Even if the bar was tilted at a very small angle, the distance measured by the laser sensor would change noticeably. As an example, if the datum was at 0.43 m from the sailplane's axis<sup>16</sup> and the laser sensor was on the sailplane's axis, the uncertainty caused by this method with a bar tilted  $\pm 1^\circ$  with respect to the perpendicular would become  $0.43 \text{ m} \cdot \sin(\pm 1^\circ) = \pm 7.5 \text{ mm}$ .
- Even though the datum is usually located forward of the front weighing point, it can also be aft of such point. Therefore, the laser sensor must be able to measure datums which are forward and aft from the weighing point.

However, there are ways to overcome these two challenges which will be presented, if necessary, after the initial uncertainty estimation. Let us first describe the basic necessary equipment for this option:

- One highly-accurate laser sensor
- One specially-designed straight bar (the datum bar)
- Basic electronics

It can already be seen that the setup is much simpler than that of option A, which reduces cost and increases reliability.

For the uncertainty estimation, the calculations are very simple and only implicate the following values:

- Laser sensor measurement uncertainty ( $\epsilon_{\text{laser}}$ ).
- Laser sensor position uncertainty with respect to the front scale ( $\epsilon_{\text{LP}}$ ).

<sup>16</sup> Specific values of the lateral location of the datum ( $Y_D$ ) can be consulted in the Annex.

- Point weight X1' position uncertainty, according to Table 22.
- Uncertainty due to the tilt of the bar with respect to the lateral axis ( $\epsilon_T$ ). For the moment, we will consider a tilt of  $\pm 1^\circ$  and a laser sensor on the aircraft's axis, although there are ways to significantly reduce this uncertainty.

Then, the uncertainties are:

Concept	Value
$\epsilon_{\text{laser}}$	$\pm 3 \text{ mm}$
$\epsilon_{\text{LP}}$	$\pm 2 \text{ mm}$
$\epsilon_{X_D}$	$\pm 5 \text{ mm}$
$\epsilon_T$	$\pm Y_D \cdot \sin(1^\circ)$

Table 29: Estimation of individual uncertainties for the preliminary assessment of option B.

And the total uncertainty is the sum of the individual uncertainties:

$$\epsilon_{L2} = \epsilon_{\text{laser}} + \epsilon_{\text{LP}} + \epsilon_{X_D} + \epsilon_T$$

Finally, we obtain the following uncertainties of L2 for the selected sailplanes:

Model	L2 uncertainty (mm)
<b>ASK 13</b>	$\pm 20.9$
<b>ASK 21</b>	$\pm 17.0$
<b>ASW 20</b>	$\pm 15.5$
<b>DG-1000S</b>	$\pm 16.4$
<b>G102 Astir CS</b>	$\pm 15.7$
<b>G103 Twin II</b>	$\pm 16.6$
<b>Ka 6E</b>	$\pm 20.8$
<b>Average</b>	$\pm 17.6$

Table 30: Initial estimation of the uncertainty of L2 with option B.

It is clear that the uncertainties of L2 are significantly lower using this method, especially considering that there are ways to reduce this uncertainty.

#### 4.4.3 Option choice

After performing initial uncertainty estimations of both options, it is easy to see that option B is the more accurate method. Furthermore, it is significantly cheaper and less complex, which allows reducing the project's budget, increasing reliability and simplifying the manufacturing process. Therefore, **option B** will be our method of choice for measuring L2.

#### 4.4.4 Development of the selected option

In this section, the system for measuring L2 is going to be developed. For an explanation of the basic principle of this system, refer to section 4.4.2 (L2 measurement option B).

The first consideration for designing the system for measuring L2 is the angle of the bar that transports the datum position to the laser sensor measuring line. To do this, we first need something to use as a longitudinal reference. A good option is a laser pointer (not to be confused with a laser sensor), which projects a light beam in a perfectly straight line. If we can reflect the laser beam with a flat, shiny surface so that the light illuminates the same source that generates it, we know that such surface is exactly perpendicular to the laser beam. Therefore, if the datum bar has a flat, shiny surface, a laser pointer can be used in this way to place the bar perfectly perpendicular to the longitudinal reference. The only sources of uncertainty would then be the following:

- The alignment of the laser pointer with the longitudinal reference<sup>17</sup>
- The quality of the surface of the datum bar
- Human error

Since determining the possible surface quality of a bar and its relation to the cost is out of the scope of this project, specific requirements are going to be defined instead:

- **Alignment of the laser pointer with the longitudinal reference:**  $\pm 0.05^\circ$ . This value can be easily obtained if the design includes a fine-calibration method to modify the angle of the laser pointer. The calibration method would consist on pointing the scale to a wall that is about 15 m away. Then, the user would measure the distance between the laser sources (pointer and sensor); this distance should be the same distance between the laser dots projected on the wall. At 15 m, only an accuracy of  $\pm 13$  mm for the distance between laser sources and laser dots is needed to obtain an alignment of  $\pm 0.05^\circ$ , as it can be seen:

$$\text{calibration accuracy} = \pm \arctan\left(\frac{13}{15,000}\right) = \pm 0.05^\circ$$

- **Maximum angle of datum bar's surface, at a given point, with respect to its axis:**  $\pm 0.1^\circ$ . This refers to the bar's surface quality, as well as the manufacturing tolerances. In other words, a ray of light which hits the bar's surface perpendicularly cannot be reflected with an angle higher than  $0.1^\circ$  with respect to the perpendicular.
- **Maximum tilt due to human error:**  $\pm 0.3^\circ$ . This parameter depends on two other parameters: the minimum distance between the laser pointer and the datum bar, and the width of the stripe where the reflection of the laser pointer would have to be located. The minimum distance between the laser pointer and the datum bar will be 150 mm (the method for achieving this is explained later), and the width of the stripe will be

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<sup>17</sup> The longitudinal reference is the beam of the laser sensor, since it is the line over which all distance measurements will be taken.

3 mm. The reason for this is the fact that the laser beam has to travel from the source to the datum bar and back, and then the maximum distance between the laser source and the reflection becomes:

$$2 \cdot 150 \text{ mm} \cdot \sin(\pm 0.3^\circ) \approx \pm 1.5 \text{ mm}$$

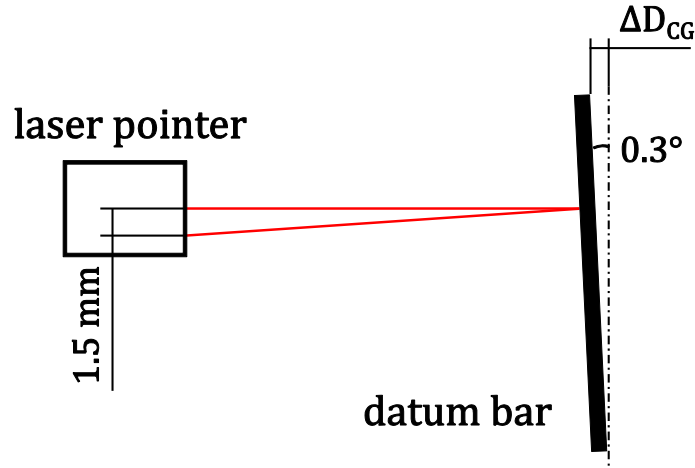


Figure 12: Effect of the datum bar's tilt.

Therefore, the maximum tilt of the bar with respect to the perpendicular to the longitudinal axis would be:

$$\epsilon_{T_{\text{Max}}} = \pm (0.05 + 0.1 + 0.3) \text{ mm} = \pm 0.45^\circ$$

Again, this is a maximum value and it will be smaller for the majority of sailplanes, especially those with larger values of L2.

#### 4.4.4.1 Mechanism for changing the laser sensor's measuring direction

This mechanism has a double purpose. On one hand, it allows the sensor to rotate 180°, so that it can measure in both directions. On the other hand, it ensures that the distance between the laser sensor and the datum bar is larger than 150 mm (this value was chosen for guaranteeing a minimum accuracy and to comply with the laser sensor's minimum measurable distance).

The basic principle of this mechanism is simple: a rigid arm which rotates with respect to a point outside the surface of the scale and which has the laser sensor attached to its end. To ensure that datum positions very close to the front point weight can be measured, the rotating arm will be flat in shape and will be attached to the scale from above, allowing the datum bar to be placed under the mechanism<sup>18</sup>. The following diagram shows the described mechanism:

<sup>18</sup> This is necessary as many sailplanes have the datum point at a distance close to zero from the front weighing point, but such distance still needs to be accurately measured. Therefore, in many cases the datum bar will be placed under the rotating arm, which means that appropriate clearance has to be guaranteed.



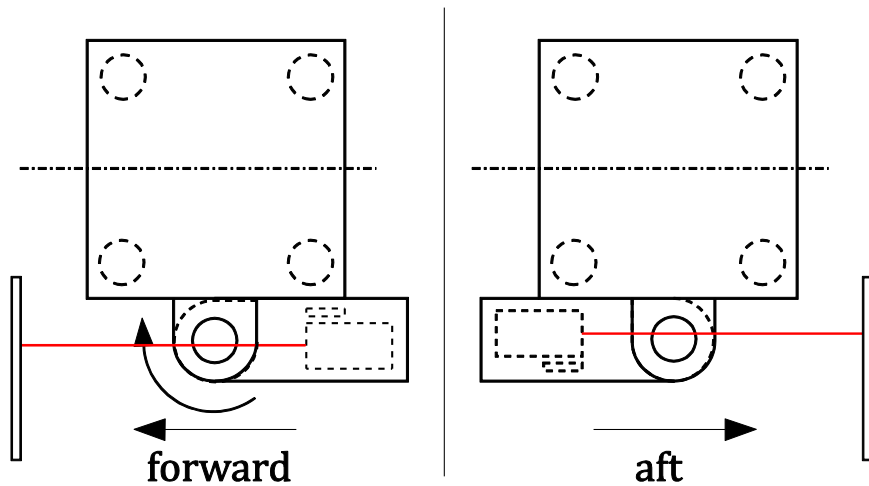


Figure 13: Schematic of the rotating arm mechanism (top view). The red lines represent the laser sensor beam.

Of course, the specific dimensions of the mechanism depend on the chosen laser sensor, but one of the dimensions is immediately defined from our requirements: the distance between the rotation centre of the arm and the reference of the laser sensor must be equal or greater than 150 mm.

Let us examine the market for laser sensor options, in order to present them and justify our choice.

#### 4.4.4.2 Laser sensor selection

For choosing the laser sensor, we will look for the following criteria:

- **Minimum accuracy:**  $\pm 3$  mm
- **Measuring range:** 150–10,000 mm (or better)
- Ability to read the measurements from an electronic system (like a microcontroller)
- Adequate dimensions for integration into the scale

The options found are presented next:

##### ***Dimetix DLS-C15 Laser Distance Sensor (part no. 500601) [17]***

This laser sensor has the following characteristics:

- **Accuracy:**  $\pm 1.5$  mm
- **Measuring range:** 0.05–65 m = 50–65,000 mm
- **Dimensions:** 150 x 80 x 55 mm
- Can be read from an electronic system with the following interfaces: RS-232, RS-422, and Profibus.
- **Cost:** approx. €1,405

#### ***Acuity AR1000 Laser Distance Sensor (model no. N/A) [18]***

This sensor also satisfies our requirements, as it can be seen below:

- **Accuracy:**  $\pm 3$  mm
- **Measuring range:** 0.1–30 m = 100–30,000 mm
- **Dimensions:** 195 x 96 x 50 mm
- Can be read from an electronic system with the following interfaces: RS-232, RS-422, or analog signal.
- **Cost:** \$1,250

#### ***Final selection***

After analysing the commercial options available, we believe that the **Dimetix DLS-C15** is the best option in terms of accuracy, measuring range (especially for the minimum measuring distance) and dimensions, all of which are important factors for integrating the sensor into our design.

##### **4.4.4.3 Laser pointer selection**

The other device that is going to be attached to the rotating arm is the laser pointer, whose laser beam will be parallel to that of the laser sensor. The requirements of such laser pointer are very simple:

- Compact dimensions
- A light point small enough to be precise but large enough to be seen by the user

With these criteria, we have found the following product, which is going to be integrated in our design.

#### ***LaserLyte V5D (model no. 5200-00) [19]***

This laser pointer is designed for alignment applications such as this one. It offers one of the smallest diameter laser beams in the market, as well as a low divergence (which means that the diameter of the dot is only slightly increased with distance). It can also be focused from a distance of 100 mm for enhanced accuracy and has adequate dimensions:

- **Spot size over working distance:** 0.25 mm at 0.5 m, 2.50 mm at 5.0 m (linear)
- **Focus range:** from 100 mm to infinity
- **Dimensions:** 69 x 14 mm (cylindrical)
- **Cost:** €297.79

##### **4.4.4.4 Laser sensor and laser pointer locations**

Now, it is time to define where the laser sensor will be placed with respect to the scale. The criteria followed for the lateral locations are the following:

- It must be able to detect the rear scale and the datum bar. Of course, the system will first measure the distance to the rear scale (L1), then it will ask the user to place the datum bar, and then it will measure the distance to the datum bar (L2).
- It must be as close to the datum point as possible, in order to reduce the uncertainty due to the tilt of the datum bar.

To start with, it is clear that the laser sensor has to be outside of the scale's platform, i.e. at a minimum of 225 mm from the scale's longitudinal axis. From here, the best location for the laser sensor is the one that is closer (laterally) to the datum. Since the lateral datum location is different for each sailplane, we will consider the average of such value, which is 431 mm from the longitudinal axis. Therefore, the laser sensor will be located somewhere between 225 mm and 431 mm from the longitudinal axis (or the centre of the scale).

Consider the following diagram, where the laser sensor is labelled as "S" and the laser pointer is labelled as "P":

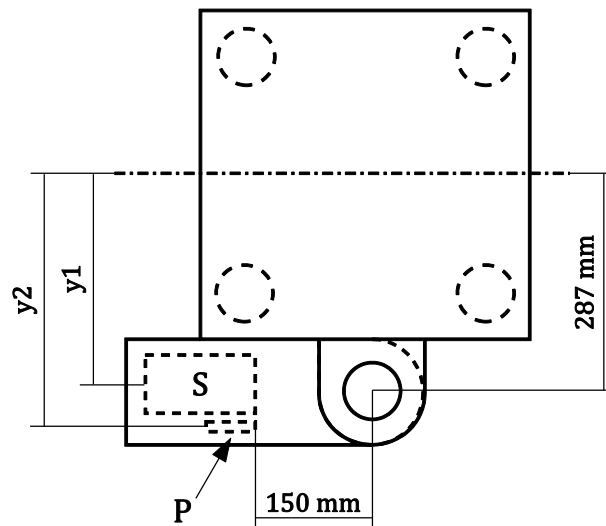


Figure 14: Basic schematic of the laser sensor and pointer locations (top view).

Taking into account the dimensions of both the laser sensor and the laser pointer, the optimal location of such sensors for maximum accuracy and reasonable size of the system, are the following:

- **y1:** 275 mm
- **y2:** 332 mm

With these dimensions, the laser sensor beam will be located at **275 mm** from the scale's axis when it is measuring aft, and at **299 mm** from the same axis when it is measuring forward (this information is provided as it has an effect on the accuracy of the measurement of L2).

As for the vertical locations, the distance of the laser beams from the ground will be defined as the sum of the following distances<sup>19</sup>:

- **Clearance of the rotating arm from the ground:** 15 mm
- **Distance from the arm's lower wall to the laser sensor beam:** 28 mm

Therefore, we get:

$$\text{laser beams height} = (15 + 28) \text{ mm} = \mathbf{43 \text{ mm}}$$

#### 4.4.4.5 Design of the datum bar

The design of the datum bar consists on determining the following characteristics:

- Material
- Dimensions
- Surface

During the next lines, each of these characteristics is going to be defined along with a proper justification.

##### **Material**

Since weight is not an important factor, we will focus on materials with a low thermal expansion coefficient (to maintain the accuracy), high stiffness, easy machinability and low cost. There are three main options:

- **Aluminium:** it has a thermal expansion coefficient of around  $22 \text{ K}^{-1}$ . It has an adequate stiffness, an easy machinability, and a medium cost.
- **Stainless steel:** its thermal expansion coefficient varies between 10 and  $17 \text{ K}^{-1}$ . It is very stiff and there is wide experience in manufacturing at a low cost.
- **Titanium:** this material offers the best characteristics, with a thermal expansion coefficient of  $8.6 \text{ K}^{-1}$ , and acceptable ease of manufacturing. It has good mechanical properties and a low weight, but it has a high cost.

It seems that **stainless steel** offers the best combination of characteristics: it is low in cost, easily available, and has good properties.

##### **Dimensions**

The first thing that needs to be determined is the length of the bar. The bar must be long enough to cover the lateral distance between the datum and the laser sensor beam. Although the average of the lateral positions of the datum is 431 mm from the sailplane's longitudinal axis, we must only look at the most restricting value. This corresponds to the ASK 13, whose datum

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<sup>19</sup> As a more in-depth design of the structure of the system is out of the scope of this project, the defined distances are estimations which could be optimised in a further design. Nevertheless, these are not critical dimensions, and thus these estimations will suffice for the objectives of this project.

is at 622 mm from the longitudinal axis. However, to make sure that our design covers the needs of most sailplanes, we will apply a safety factor of 1.25 to this distance. To the obtained value, we have to subtract the minimum distance of the laser sensor beam from the longitudinal axis, which is 275 mm, and add 20 mm so that the laser sensor is not measuring right on the edge of the bar. Therefore:

$$\text{datum bar length} = (622 \cdot 1.25 - 275 + 20) \text{ mm} = \mathbf{522.5 \text{ mm}}$$

The next dimension is the bar's height. There are two limiting factors in this case: the lower bound is the laser sensor beam's distance from the ground (to ensure the sensor's visibility of the bar), and the upper bound is the rotating arm's upper wall distance from the ground (to ensure clearance). The first is 43 mm, and the second is 69 mm. Therefore, we will choose a **height of 56 mm**, which is the middle value between both bounds. This ensures a good balance between the sensor's visibility and clearance from the rotating arm.

Finally, there is the bar's thickness. The bar has to be thin enough to allow its precise location under the plumb bob, but thick enough to avoid deformation (and thus inaccuracy). As a first approach, a **thickness of 10 mm** is a reasonable value for stainless steel, especially when no loads have to be applied over the bar.

### **Surface**

The datum bar only needs one special surface: the area where the laser pointer beam will be reflected. This area must be polished to a mirror-like finish and must be perfectly parallel to the bar's longitudinal axis. Taking into account the positions of the lasers, this surface will extend **from 67 to 87 mm** from the edge of the bar, and it will cover the whole height of the bar.

The only restriction to the rest of the surface is that it cannot be polished to a mirror-like finish, as this would cause an error in the laser sensor (it would receive an excessively strong reflection). Other than that, the manufacturer of the sensor states that the measurement accuracy does not depend on the target's surface.

## 4.4.5 Uncertainty assessment

Now that the methods for measuring both L1 and L2 have been presented, the tolerances have been defined, and the commercial products needed have been chosen, it is time to assess the uncertainty of this design.

### 4.4.5.1 Uncertainty of L1

Recall from 4.3 (Measurement of L1) that the total uncertainty of L1 is calculated as follows:

$$\epsilon_{L1} = \epsilon_{\text{laser}} + \epsilon_{LP} + \epsilon_{X1'} + \epsilon_{X2'}$$

With our design choices, the individual uncertainties are defined as:

Concept	Value
$\epsilon_{\text{laser}}$	$\pm 1.5 \text{ mm}$
$\epsilon_{\text{LP}}$	$\pm 2 \text{ mm}$
$\epsilon_{X1'}, \epsilon_{X2'}$	depend on model <sup>20</sup>

Table 31: Final values of the individual uncertainties of L1.

Then, the uncertainty of L1 for each sailplane and its contribution to the CG position uncertainty become:

Model	L1 uncertainty (mm)	Contribution to CG position uncertainty (mm)
ASK 13	$\pm 11.7$	$\pm 0.2$
ASK 21	$\pm 9.2$	$\pm 0.4$
ASW 20	$\pm 8.8$	$\pm 1.3$
DG-1000S	$\pm 8.3$	$\pm 1.0$
G102 Astir CS	$\pm 8.9$	$\pm 1.1$
G103 Twin II	$\pm 8.9$	$\pm 0.5$
Ka 6E	$\pm 9.3$	$\pm 1.2$
Average	$\pm 9.3$	$\pm 0.8$

Table 32: Final uncertainties of L1 and their contribution to the CG position uncertainties.

As can be seen, the contribution of the uncertainty of L1 to the uncertainty of the CG position is quite low, around  $\pm 0.8 \text{ mm}$ .

#### 4.4.5.2 Uncertainty of L2

In 4.4.2 (L2 measurement option B) we defined the total uncertainty of L2 as:

$$\epsilon_{L2} = \epsilon_{\text{laser}} + \epsilon_{\text{LP}} + \epsilon_{X_D} + \epsilon_T$$

The following table shows the final values of each individual uncertainty:

Concept	Value
$\epsilon_{\text{laser}}$	$\pm 1.5 \text{ mm}$
$\epsilon_{\text{LP}}$	$\pm 2 \text{ mm}$
$\epsilon_{X_D}$	$\pm 5 \text{ mm}$
$\epsilon_T$	depends on model <sup>21</sup>

Table 33: Final values of the individual uncertainties of L2.

Then, the uncertainty of L2 for each sailplane and its contribution to the CG position uncertainty becomes:

<sup>20</sup> These values can be found in Table 22 and Table 24.

<sup>21</sup> These values can be found in the Annex.

Model	L2 uncertainty (mm)	Contribution to CG position uncertainty (mm)
<b>ASK 13</b>	$\pm 9.8$	$\pm 9.8$
<b>ASK 21</b>	$\pm 8.9$	$\pm 8.9$
<b>ASW 20</b>	$\pm 8.7$	$\pm 8.7$
<b>DG-1000S</b>	$\pm 8.9$	$\pm 8.9$
<b>G102 Astir CS</b>	$\pm 8.7$	$\pm 8.7$
<b>G103 Twin II</b>	$\pm 8.8$	$\pm 8.8$
<b>Ka 6E</b>	$\pm 11.0$	$\pm 11.0$
<b>Average</b>	$\pm 9.3$	$\pm 9.3$

Table 34: Final uncertainties of L2 and their contribution to the CG position uncertainties.

## 5 Final design and weight and balance procedure

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In this section, the final result of the design process is going to be clearly presented, as well as the procedure for measuring a sailplane's weight and balance with this system. For the assessment of the accuracy of the system, refer to section 6 (Final uncertainty assessment).

### 5.1 Parts of the system

#### 5.1.1 Front scale

The front scale is used to measure the front weight of the sailplane (G1) and the distances (L1 and L2), as well as interacting with the user<sup>22</sup>. It includes the following components:

- 4 Omega LCEC-1K load cells
- 1 Dimetix DLS-C15 laser distance sensor
- 1 LaserLyte V5D laser pointer
- 1 electronics module and user interface
- 1 rotating arm for changing the distance measurement direction

#### 5.1.2 Rear scale

The rear scale is used to measure the rear weight of the sailplane (G2) and to elevate its rear part to set it into flight attitude. It includes the following components:

- 4 Omega LCAE-45KG load cells
- 1 scissor-type mechanical elevation system
- 1 electronics module

This scale is connected to the front scale by means of a wireless system, in order to send the weight data to the front scale.

#### 5.1.3 Datum bar

The datum bar consists of a straight stainless steel bar. It measures 522.5 x 56 x 10 mm and includes a highly reflective area for reflecting the laser pointer beam. It is used to transport the datum point location to the line of sight of the laser distance sensor.

#### 5.1.4 Digital level

The digital level is a commercial model, the Hammerhead HLLT10. It has been chosen to provide the highest possible accuracy for levelling the sailplane, as well as ease of use.

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<sup>22</sup> The interaction of the system with the user requires an electronics module and an interface, like an LCD display and buttons. For this system, an Arduino microcontroller would probably be enough, and its software would not need to be excessively complex. However, since this is well out of the scope of this project, the development of this sub-system is not detailed.



## 5.2 System specifications

The specifications and limits of the system are listed next:

- **Maximum total measuring weight (off-centre<sup>23</sup>):** 677.1 kg
  - **Maximum front measuring weight (off-centre):** 616.5 kg
  - **Maximum rear measuring weight (off-centre):** 60.5 kg
- **Maximum total measuring weight (centred):** 1,990 kg
  - **Maximum front measuring weight (centred):** 1,812 kg
  - **Maximum rear measuring weight (centred):** 178 kg
- **Total weight measurement accuracy:**  $\pm 1.179$  kg
  - **Front weight measurement accuracy:**  $\pm 1.089$  kg
  - **Rear weight measurement accuracy:**  $\pm 0.090$  kg
- **Allowed range of distances between scales:** 0–65 m
- **Allowed range of distances between front scale and datum:** 0–65 m (in any direction)
- **Point weight location limits:** 250 x 250 mm square centred on scale's platform
- **Datum lateral location limits:** 225.0–777.5 mm
- **Rear scale elevation range:** 0–0.75 m

## 5.3 Procedure and safety

For performing a weight and balance measurement, the user should follow the steps described next:

1. **Location of the scales:** place the scales on the floor. The distance between the centres of the scales should be as close as possible to the distance between the weighing points of the sailplane (a measuring tape is enough).
2. **Alignment of the scales:** make sure that the scales are aligned by measuring the distances between the two pairs of facing ends. Both distances should be the same.
3. **Positioning of the sailplane:** place the main wheel of the sailplane over the platform of the front scale, as close to its centre as possible. Then, place the rear wheel or skid over the platform of the rear scale.
4. **Levelling of the sailplane:** look for the CG position calculation section in the sailplane's manual. There, the levelling means is indicated. In most cases, the levelling means is an X:Y wedge located on top of the rear part of the fuselage. With the aid of a calculator, perform the following calculation:

$$\theta_0 = \tan^{-1} \left( \frac{Y}{X} \right)$$

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<sup>23</sup> *Off-centre* refers to the situation where the point weights are over the point weight location limits. *Centred* refers to the situation where the point weights are right in the centre of the scales' platforms.

Then, place the digital level along the top of the rear part of the fuselage. By rotating the crank of the rear scale, elevate the rear of the sailplane until the digital level indicates exactly the angle  $\theta_0$ .

5. **Weight measurement:** the display of the front scale now indicates the total weight of the sailplane. If necessary, take note of the weight. Then, press the button in the front scale.
6. **Measurement of L1:** make sure that there is nothing obstructing the line of sight of the laser sensor, which should be pointing towards the rear scale. Also, make sure that you see two red laser dots on the rear scale (laser sensor and pointer). Then, press the button in the front scale and the system will measure L1.
7. **Measurement of L2:** if the datum is aft of the front weighing point, keep the lasers pointing aft. If the datum is forward of the front weighing point, rotate the arm counter-clockwise by  $180^\circ$ . Drop a plumb bob from the datum point (as defined by the manufacturer). Place the datum bar just under the plumb bob, perpendicular to the sailplane's longitudinal axis. The beam of the laser pointer must be reflected on the reflective area of the bar; proceed to rotate the bar until the reflection of the laser beam is inside the white stripe that is placed over the laser pointer. Check again that the datum bar is just under the plumb bomb. Now, press the button on the front scale and the system will measure L2.
8. **CG position measurement:** now the system will display the CG position with respect to the datum (considered positive aft of the datum).

**Safety notice:** the lasers used in this system are Class 2 laser products. Do not stare into the laser beams, as this could cause injury.

**Note:** the described procedure is valid for the great majority of sailplanes. However, the user should always check the sailplane's flight or maintenance manual to ensure that the CG position can be calculated in this way.

## 6 Final uncertainty assessment

After the design phase has been finished, it is time to assess the final uncertainties of the CG positions:

Model	CG position uncertainty (mm)
<b>ASK 13</b>	$\pm 14.2$
<b>ASK 21</b>	$\pm 13.1$
<b>ASW 20</b>	$\pm 15.3$
<b>DG-1000S</b>	$\pm 14.6$
<b>G102 Astir CS</b>	$\pm 15.2$
<b>G103 Twin II</b>	$\pm 12.5$
<b>Ka 6E</b>	$\pm 19.1$
<b>Average</b>	<b><math>\pm 14.9</math></b>

Table 35: Final CG position uncertainties.

As can be seen, the uncertainties are slightly lower than those estimated during the study of uncertainties. However, the uncertainties of Table 35 include the uncertainty of the longitudinal levelling, which is one of the largest sources of uncertainty. This means that, in reality, the reduction of the uncertainty has been significantly larger. In the following table, we have included a longitudinal levelling uncertainty of  $\pm 1^\circ$  into the initial estimation of the CG position uncertainty, so that the actual increase in accuracy can be appreciated:

Model	Initial CG position uncertainty estimation (mm)	Final CG position uncertainty (mm)	Absolute uncertainty reduction (mm)
<b>ASK 13</b>	$\pm 28.1$	$\pm 14.2$	13.9
<b>ASK 21</b>	$\pm 27.0$	$\pm 13.1$	13.9
<b>ASW 20</b>	$\pm 27.4$	$\pm 15.3$	12.1
<b>DG-1000S</b>	$\pm 29.8$	$\pm 14.6$	15.2
<b>G102 Astir CS</b>	$\pm 28.1$	$\pm 15.2$	12.9
<b>G103 Twin II</b>	$\pm 25.3$	$\pm 12.5$	12.8
<b>Ka 6E</b>	$\pm 28.0$	$\pm 19.1$	8.9
<b>Average</b>	<b><math>\pm 27.7</math></b>	<b><math>\pm 14.9</math></b>	<b>12.8</b>

Table 36: CG position uncertainties reduction (with respect to the initial estimation).

## Conclusions

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The overall evaluation of the project is positive, as all the requirements have been satisfied. The system allows for an easier weight and balance process and an enhanced accuracy, especially considering that the calculated uncertainties have never been underestimated. This means that, in reality, the final uncertainties are maximum values, and in most cases the accuracy will be higher than estimated. All calculations can be automatized and the user does not have to measure the distances L1 and L2. The system presents virtually no restrictions on the values of L1 and L2, and the maximum measurable weight of 1,000 kg has also been guaranteed, with the only restriction that the wheels have to be adequately centred on the platforms.

As for the economic feasibility of the project, the estimated final price of the system (€5,411.62, approximately) is clearly too high to be of any interest to private pilots. The only potential customers would be soaring clubs or flight schools with a large number of affiliates, which would have both the economical means and the need to perform frequent weighing and balancing of their aircraft.

## Future perspectives

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The continuity of the project would have to start, first of all, for the design of the electronics. A microcontroller for each scale should have to be chosen, and it would have to be evaluated if the load cells and the laser sensor can be read directly from the microcontroller. This task could take around 100 h of design. Then, a structural analysis would lead to the final design of the structure which, in turn, would have to accommodate the electronics. The structural analysis would include the choice of the material and it could be done using the finite element method. To complete the analysis, about 300 h would be needed, approximately.

In order to improve the system, the use of only two load cells for each scale should be considered, provided the stability of the scales can be guaranteed. This could reduce the final cost by around €1,000 whilst keeping a similar accuracy. Also, the uncertainty of the position of the laser sensor could be reduced by using an adequate calibration procedure. This, in turn, could allow to use looser tolerances, reducing the cost and the manufacturing requirements.

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