Technical Research Report

Study on behavioral impedance for route planning techniques from the pedestrian’s perspective: Part II – Mathematical approach

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Behavioral impedance for route planning techniques from the pedestrian’s perspective:
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Abstract

The theoretical foundations of the behavioral impedance domain are based on (1) a meta-model composed of analytical and mathematical approaches and (2) a taxonomy on the constraints involved in the decision-making process of a pedestrian during the route selection.

The goal of this technical report is to present the mathematical model of the behavioral impedance domain. The partial least squares approach has been used to validate the meta-model analytical approach and develop the proposed mathematical model.

This study contributes a mathematical model towards the implementation of behavioral impedance domain in geographic information systems for transportation through a constraint management module.

Keywords: Behavioral impedance domain, pedestrian’s perspective, constraint management module, geographic information systems for transportation, mathematical model
Behavioral impedance for route planning techniques from the pedestrian’s perspective:
Part II – Mathematical approach
Contents

1. Introduction 1
  1.1 Organization of the report 1

2. Methodological aspects 2

3. Geographic information systems for transportation 3

4. Previous work 5

5. Behavioral impedance 6
  5.1 Meta-model 6
  5.2 Analytical approach 6
  5.3 Revised taxonomy 7
  5.4 Validation of the BI domain 7
  5.5 Mathematical approach 17
  5.6 BI Constraint Management Module 21

6. Concluding remarks 22

Acknowledgements 22

References 22
Behavioral impedance for route planning techniques from the pedestrian’s perspective:
Part II – Mathematical approach
1. Introduction

Techniques and algorithms of route planning are one of the most used resources in geographic information systems (GIS) and GIS for transportation (GIS-T). Usually, distance and time are used as criteria of selection in route planning activities. However, several criteria such as topographic characteristics (i.e. an objective criterion) or comfort of the available transport modes (i.e. a subjective criterion), play a relevant role when the route selection process is based on pedestrian intentions and behavior.

In literature, there are few studies where the pedestrian’s constraint management is clearly pointed out. Within this context, Nogueira et al. (2001) and Pereira et al. (2001) provide a theoretical contextualization on the constraint identification and management in order to ascertain the behavioral impedance domain from the pedestrian’s perspective.

Using the theoretical foundations on the behavioral impedance (BI) domain as a starting point, the validation of the analytical approach is carried out through the partial least squares (PLS) approach. In addition, the mathematical approach of the BI meta-model has been developed and the constraint management module has been implemented.

1.1 Organization of the report

This technical report is organized as follows. Section 2 presents the methodological aspects of this research.

Section 3 presents a GIS and GIS-T overview.

Section 4 comments on previous work, which has made possible the implementation of the constraint management module.

Section 5 provides a review on the BI domain, in which the analytical approach of the BI meta-model is included. In addition, the mathematical approach development of the BI meta-model is presented.

Finally, Section 6 presents the concluding remarks.
2. Methodological aspects

The research on behavioral impedance for route planning techniques is organized in two big parts (Figure 1). The first part was presented in Nogueria et al. (2001) and Pereira et al. (2001), which represented a theoretical contextualization and was basically carried out taking into account qualitative research methods. On the other hand, Nogueria et al. (2002) and Pereira et al. (2002a) presented the partial results of the questionnaire and statistical analysis, which are placed in the second part.

Figure 1. Parts of the research on behavioral impedance domain.

In this technical report, the mathematical approach of the BI meta-model is developed start from the validation based on Partial Least Squares (PLS) method and the architecture of the constraint management module is also presented. In parallel, the implementation activities of the Behavioral Impedance Route Planning Algorithm (BIRPA) are being carried out and will be reported soon.
3. Geographic information systems for transportation

Nowadays, geographic information systems (GIS) are computer-based applications that are used to store, manage and analyze information related to geographic characteristics. Although several authors presented GIS definitions according to their research ideologies, terms such as environment, geo-referenced data, spatial data management, manipulation and analysis, and computer-based software are used in most of the GIS definitions. GIS have been applied in many science fields, but their use in the transportation area is a recent reality, thus not exhaustively researched yet. According to Vonderohe et al. (1993, p. 2), “GIS is much more than a tool to be applied case by case to a narrow set of specialized problems. Because of the inherent geographic nature of almost all transportation data, GIS concepts can and should serve as bases for the coherent organization of information structures and systems across the entire range of transportation applications. GIS provides a framework for moving from stand-alone isolated databases and applications to truly integrated information systems”.

On the other hand, several aspects such as pavement and bridge management, safety and security analysis, traffic analysis and control, travel demand analysis and cost analysis, not usually considered in conventional GIS, characterize the transportation world. Using the GIS definition structure as a starting point, transportation information systems (TIS) are applications that store, manage and analyze information related to transportation characteristics.

In this way, geographic information systems for transportation (GIS-T) are the result of the integration of transportation information systems and conventional geographic information systems (Thill, 2000) (Figure 1). According to Vonderohe, Travis, Smith and Tsai (1993, p. 11), “the necessary enhancement to existing TISs is the structuring of the attribute databases to provide consistent location reference data in a form compatible with the GIS, which in turn has been enhanced to represent and process geographic data in the forms required for transportation applications”.
As commented above, an interesting research area in geographic information systems for transportation is constraint management in route planning algorithms from the pedestrian’s perspective. In this context, the study on the behavioral impedance domain has been carried out in order to ascertain how the decision-making process in a pedestrian’s route selection is performed.

Within the GIS and GIS-T functional context, Vonderohe et al. (1993) presented six general categories of functions for data manipulation and spatial analysis: measurement, proximity analysis, raster processing, surface model generation and analysis, network analysis, and polygon overlay. Regarding constraint management in route planning and selection, this research is basically placed in the network analysis category, which is composed of two analysis functions: dynamic segmentation and network overlay. According to Pereira et al. (2002a), constraint management becomes more complex when route planning takes into account an integrated public transportation network (i.e. a multimodal network).

Constraint management in route planning algorithms from the pedestrian’s perspective is an issue for research. Moreover, constraint management becomes more complex when route planning takes into account an integrated public transportation network (i.e. a multimodal network). A study on the theoretical contextualization and taxonomy of a pedestrian’s behavioral impedance has been developed in order to improve the constraint management from the pedestrian’s perspective. A previous phase to reach the constraint management module is the building of a mathematical model. Therefore, some considerations on constraint management are presented in order to accommodate the behavioral impedance domain into GIS-T systems.
4. Previous work

Router 2.0 is an experimental application and works with a geographic information system for transportation. The main objective is to define the shortest path between two UPC buildings (origin and destination points) placed in the South and North campuses of the university zone of Barcelona. More details can be found in Pereira and Pérez (2000).

The initial architecture of the Router 2.0 application was composed of three modules: management, map and route modules, which are responsible for the general application management, map control and shortest path calculus with Dijkstra's algorithm (Dijkstra, 1959), respectively. Figure 3 depicts the implemented modules in solid boxes and the future modules in outline boxes (i.e. behavioral impedance, animation and simulation modules).

The Router 2.0 application has been developed in ArcInfo Macro Language – AML® and C++ language. The management and map modules were developed in AML, and the route module was developed in C++. Router 2.0 application runs on a Windows NT® platform.

Currently, the Router application is being developed in Visual C++ 6.0 and Geomedia 4.0. The main reason for using these tools is political and commercial strategy. Many organizations and enterprises in Barcelona that work with GIS and GIS-T are using the Geomedia GIS platform.
5. Behavioral impedance: research overview

In a transportation network, behavioral impedance (BI) means a specific resistance imposed to the user on a part of or the whole transportation system. Thus, several objective (e.g. hard acclivities) and subjective (e.g. comfort) criteria will play an important role in the transportation system.

The BI domain has been developed using a meta-model as a starting point, which has allowed establishing the proposed taxonomy elements. Thus, the meta-model has allowed not only the classification and decomposition of the taxonomy elements, but also the identification of several constraints related to the BI domain (Pereira et al., 2001).

5.1 Meta-model

Taking into account that a meta-model represents the primary knowledge source that supports the identification of the appropriate constraints and dependency relations in a specific problem (Raghunathan, 1992), Pereira et al. (2001) have presented the BI meta-model structure (Figure 4). This meta-model is composed of analytical and mathematical approaches. The analytical approach of the meta-model is used to define the tree of the BI taxonomy. On the other hand, the mathematical approach is used to specify the constraint values that will be applied to the cost function mathematical model implemented in the BI constraint management module.

Figure 4. Meta-model of the Behavioral Impedance Domain. Source: Pereira et al. (2001).

The meta-model has also been organized as a hierarchy composed of four levels: Entity, State, Condition and Constraint. The analytical approach has the Entity, State and Conditions levels (i.e. three first levels). The Mathematical approach deals with the Constraints (i.e. fourth level).

5.2 Analytical approach

The analytical approach of the BI meta-model plays an important role in determining
the BI taxonomic tree elements. This determination considers all possible influence factors that could produce different solutions in the path selection by the user (i.e. pedestrian) of a public transportation system. Using an inductive process as starting point, five entities, thirteen states and thirty-six conditions have been identified in the analytical approach during the taxonomy design. After an exhaustive analysis, some conditions have been specialized in sub-conditions allowing a better identification of the constraints used in the mathematical model.

5.3 Revised taxonomy

The revised taxonomy consists of (1) some complex structural rules based on behavioral constraints of pedestrians; (2) a correspondence between the representation order from the meta-model's elements and the order observed in real occurrences; (3) a taxonomy scheme that avoids arbitrary elements; and (4) some validation criteria based on research findings from specialized literature and the results of a questionnaire submitted to a sample of people at the Technical University of Catalonia.

Pereira et al. (2001) presented the BI taxonomy. Using previous results from statistical analysis as a starting point, some changes have been carried out in the taxonomic tree. The reclassification of the condition “headway” of the network entity has been necessary. The new condition, “schedule system”, has been proposed. Consequently, “headway” becomes a sub-condition of the condition “schedule system”.

Figure 5 shows the revised taxonomy after the changes (Pereira et al., 2002a).

5.4 Validation of the BI domain

The partial least squares (PLS) approach has been used to validate the BI domain. The goal of this analysis technique is to obtain the weight of each factor in the pedestrian's decision-making process during route selection in a public transportation system. In this respect, it is necessary to determine a function that describes the pedestrian's behavior regarding the use of a transportation mode (i.e. the incidences of the factors taken into account in the final decision).

As a solution of the problem, the application of the PLS approach model proposed by Wold (1975, 1982, 1985) has been tried. This model allows estimating the latent variables from J groups of manifest variables (i.e. observed variables) on the same individuals (i.e. each group consists of an observable expression of a latent variable). On the other hand, the latent variables are linked by a set of structural relationships (Tenenhaus, 1998).

The PLS approach is characterized by its simplicity, since there are no probabilistic hypotheses (i.e. the data are directly modeled by means of a set of regressions). In addition, the latent variables are estimated at the individual’s level, allowing a simple geometric representation to be provided when the number of latent variables is reduced. In the BI domain context, in which it is also important to find a common structure for all variable blocks, the PLS approach is equivalent to multiple factorial analysis (Tenenhaus, 1999).
Behavioral impedance for route planning techniques from the pedestrian’s perspective:
Part II – Mathematical approach

Figure 5. Taxonomy of behavioral impedance domain. Source: Pereira et al. (2002a).
5.4.1 Origin of the analyzed data

The data used are obtained from a questionnaire applied to a specific sample (i.e. students and staff of the Technical University of Catalonia). In this questionnaire a set of assertions that describe each one of the BI domain factors were defined. The format of the scale of quantification of the factors employs a 5-point Likert scale with the following rates: 1=strongly disagree, 2=disagree, 3=not sure, 4=agree and 5=strongly agree. Moreover, the assertions for the factors begin with the same sentence: “I select a route in an urban public transportation system if...”.

The following description of each question is not only associated to the different factors but also to its average (Table 1).

Table 1. Question descriptions and them averages.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Indicators</th>
<th>Questions (questionnaire)</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topographic</td>
<td>• ... if the streets that I use to get the transportation mode have an upward slope.</td>
<td>2.82</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if the streets that I use to get the transportation mode are not under construction.</td>
<td>3.31</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if the ways are direct ones.</td>
<td>2.88</td>
<td></td>
</tr>
<tr>
<td>Weather</td>
<td>• ... if the weather is good.</td>
<td>3.47</td>
<td></td>
</tr>
<tr>
<td>Work Day</td>
<td>• ... if it is not peak hour in a work day.</td>
<td>3.83</td>
<td></td>
</tr>
<tr>
<td>Weekend (Saturday and Sunday)</td>
<td>• ... if it is not peak hour in the weekend (i.e. Saturday and Sunday).</td>
<td>3.19</td>
<td></td>
</tr>
<tr>
<td>Atypical Day</td>
<td>• ... if it is not peak hour in an atypical day.</td>
<td>3.13</td>
<td></td>
</tr>
<tr>
<td>Incidental</td>
<td>• ... although frequent interruptions occur due to the maintenance of the lines and equipments.</td>
<td>1.88</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if there are not frequent breakdown in the transport modes.</td>
<td>4.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if there are not frequent accidents related to pedestrian.</td>
<td>3.46</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if there are not terrorist attack scare.</td>
<td>3.94</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if there are not incidents in the travel zone.</td>
<td>3.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if there are not incidents outside of the travel zone.</td>
<td>2.90</td>
<td></td>
</tr>
<tr>
<td>Infrastructure</td>
<td>• ... if the bus, underground and rail stop are well connected.</td>
<td>4.54</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if there is not excessive number of change.</td>
<td>4.47</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if the changes are not difficult ones (e.g. long corridors).</td>
<td>4.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if the accessibility to the stop or station is good (e.g. there is elevator, mecanic stairways, etc.).</td>
<td>3.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if the headway in the stop or station is low.</td>
<td>4.51</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if the schedule of the transportation system is reliable.</td>
<td>4.19</td>
<td></td>
</tr>
</tbody>
</table>
Table 1. Question descriptions and them averages (cont.).

<table>
<thead>
<tr>
<th>Factors</th>
<th>Indicators</th>
<th>Questions (questionnaire)</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>User Factors</td>
<td>Personal and Professional Use</td>
<td>I select a route in an urban public transportation system if...</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if the comfort of the transportation system is good. ..................................</td>
<td>4.13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if there are not emergency situations...........</td>
<td>3.70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if I do not have professional tasks to do...........</td>
<td>3.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• ... if the schedule of the transportation system is flexible..................................</td>
<td>4.06</td>
<td></td>
</tr>
<tr>
<td>Socio-politico-economic Factors</td>
<td>Safety</td>
<td>• ... if the traffic safety is good............................................................................</td>
<td>3.76</td>
</tr>
<tr>
<td></td>
<td>• ... if there is not congestion of vehicles.....................................................</td>
<td>3.99</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Security (Personal Security)</td>
<td>• ... if I do not travel through dangerous zones..............</td>
<td>3.62</td>
</tr>
<tr>
<td></td>
<td>Policy</td>
<td>Not considered</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fare System</td>
<td>• ... if the transportation fare system is integrated.............................................</td>
<td>4.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• ... if the price of fare is not excessive.......................................................</td>
<td>4.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• ... if the transportation fare system is subsidiary...........................................</td>
<td>4.07</td>
</tr>
</tbody>
</table>

5.4.2 Description of the model

The Principal Componet Analysis (PCA) of the variable groups has been carried out before the estimation of the model by PLS approach. When necessary, one factor has been divided into more according to the number of PCA needed to explain it (Table 2).

Table 2. New factors according to PCA method.

<table>
<thead>
<tr>
<th>Initial Factor</th>
<th>New Factors after PCA method</th>
<th>% of explained variability by the first component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment Factors</td>
<td>Environment Factor</td>
<td>44.78</td>
</tr>
<tr>
<td>Time of days factors</td>
<td>Time and hours factors</td>
<td>58.34</td>
</tr>
<tr>
<td>Network Factors</td>
<td>Network Incidence Factors</td>
<td>30.25</td>
</tr>
<tr>
<td></td>
<td>Network Infrastructure Factors</td>
<td>32.88</td>
</tr>
<tr>
<td></td>
<td>Network Schedule System Factors</td>
<td>59.17</td>
</tr>
<tr>
<td>User’s Factors</td>
<td>Travel reason1</td>
<td>24.94</td>
</tr>
<tr>
<td></td>
<td>Confidence Factor</td>
<td>54.57</td>
</tr>
<tr>
<td></td>
<td>Affability Factor</td>
<td>55.20</td>
</tr>
<tr>
<td>Socio-politico-economic Factors</td>
<td>Security and Safety Factor</td>
<td>43.72</td>
</tr>
<tr>
<td></td>
<td>Fare System Factor</td>
<td>43.89</td>
</tr>
</tbody>
</table>

The initial problem was defined with 5 factors, which evolved into a problem with 10 factors (Figure 5). Using the relationship between both the factors versions (i.e. 5 and 10 factors respectively) as a starting point, the latent variable has been determined by the blocks of the correspondent manifest variables (Table 3).

Table 3. Correspondence between the latent variables and manifest variables.

<table>
<thead>
<tr>
<th>Latent Variable</th>
<th>Definition</th>
<th>Manifest Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_1$</td>
<td>Environment Factors</td>
<td>$x_{11}$: ... if the streets that I use to get the transportation mode have an upward slope</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$x_{12}$: ... if the streets that I use to get the transportation mode are not under construction</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$x_{13}$: ... if the ways are direct ones</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$x_{14}$: ... if the weather is good</td>
</tr>
</tbody>
</table>

1 This factor has been added due to the fact that it is determinant for the user’s factor description.
Table 3. Correspondence between the latent variables and manifest variables (cont.).

<table>
<thead>
<tr>
<th>Latent Variable</th>
<th>Definition</th>
<th>Manifest Variables</th>
</tr>
</thead>
</table>
| \( \xi_2 \) | Time of days factors | \( x_{21} \): if it is not peak hour in a work day  
\( x_{22} \): if it is not peak hour in the weekend (i.e. Saturday and Sunday)  
\( x_{23} \): if it is not peak hour in an atypical day |
| \( \xi_3 \) | Network Incidence Factors | \( x_{31} \): although frequent interruptions occur due to maintenance of the lines and equipments.  
\( x_{32} \): if there are not frequent breakdown in the transport modes.  
\( x_{33} \): if there are not frequent accidents related to pedestrian  
\( x_{34} \): if there are not terrorist attack scare.  
\( x_{35} \): if there are not incidents in the travel zone.  
\( x_{36} \): if there are not incidents outside of the travel zone. |
| \( \xi_4 \) | Network Infrastructure Factors | \( X_{41} \): if the bus, underground and rail stop are well connected.  
\( X_{42} \): if the accessibility to the stop or station is good (e.g. there is elevator, mechanic stairways, etc.)  
\( X_{43} \): if there is not an excessive number of change.  
\( X_{44} \): if the changes are not difficult (e.g. long corridors) |
| \( \xi_5 \) | Network Schedule System Factors | \( X_{51} \): if the headway in the stop or station is low.  
\( X_{52} \): if the schedule of the transportation system is reliable. |
| \( \xi_6 \) | User’s Factor - Travel reason | \( x_{61} \): Personal travel reason (work day)  
\( x_{62} \): Professional travel reason (work day)  
\( x_{63} \): Personal travel reason (weekend and holiday)  
\( x_{64} \): Professional travel reason (weekend and holiday)  
\( x_{65} \): Personal and professional travel reason (weekend and holiday) |
| \( \xi_7 \) | User’s Factor (personal) | \( x_{71} \): if there are not emergency situations.  
\( x_{72} \): if I do not have professional tasks to do. |
| \( \xi_8 \) | User’s Factor (advantages) | \( x_{81} \): if the comfort of the transportation system is good.  
\( x_{82} \): if the schedule of the transportation system is flexible. |
| \( \xi_9 \) | Security and Safety Factor (SPE Factors) | \( x_{91} \): if I do not travel through dangerous zones.  
\( x_{92} \): if the traffic safety is good.  
\( x_{93} \): if there is not congestion of vehicles. |
| \( \xi_{10} \) | Fare System Factor (SPE Factors) | \( x_{101} \): if the transportation fare system is integrated.  
\( x_{102} \): if the price of fare is not excessive.  
\( x_{103} \): if the transportation fare system is subsidiary. |

Figure 5 shows the defined structural relationship. The latent variable \( \xi_0 \) represents the complete behavior of the user, which is related to the latent variables \( \xi_1 \) … \( \xi_{10} \) that represent the different factors that influence the final decision.
Behavioral impedance for route planning techniques from the pedestrian’s perspective: Part II – Mathematical approach

Ten blocks of variables ($J=10$) are observed $X_j = \{x_{j1}, \ldots, x_{jk}\}$. The linear combination of each block of manifest variables $X_j$ represents a latent variable $\xi_j$

$$\xi_j = \sum \lambda_{jh} \cdot x_{jh} + \delta_j, \ h = 1, \ldots, k_j$$ (1)

where $\lambda_{jh}$ is the contribution of each variable $x_{jh}$ to the formation of the latent variable $\xi_j$ and $\delta_j$ is the random term of average 0 not correlated with the manifest variables.

A new block $X$ is built, which takes into account the union of the $X_1, \ldots, X_J$ sets initially defined. This block can be summarized by the latent variable $\xi_0$ and represents a common structure among the $J$ defined blocks. The cause-effect relationship (i.e. structural relationship) is defined in order to relate the latent variable $\xi_0$ to the other latent variables $\xi_1, \ldots, \xi_{10}$. The Equation 2 presents the cause-effect relationship

$$\xi_0 = \beta_{1\xi_0} \xi_1 + \beta_{2\xi_0} \xi_2 + \beta_{3\xi_0} \xi_3 + \beta_{4\xi_0} \xi_4 + \beta_{5\xi_0} \xi_5 + \beta_{6\xi_0} \xi_6 + \beta_{7\xi_0} \xi_7 + \beta_{8\xi_0} \xi_8 + \beta_{9\xi_0} \xi_9 + \beta_{10\xi_0} + \upsilon$$ (2)

where $\beta_{\cdot\xi_0}$ is each parameter of the multiple linear regression and $\upsilon$ is the random term of average 0 not correlated with the explicative latent variables.

Figure 5 depicts the groups of the manifest variables that define each one of the latent variables. Variables that initially represented only a factor are shown as groups of two or three factors.
5.4.3 Results obtained

Some previous considerations are pointed out before the presentation of the comments on the main results obtained:

- The original variables with rank between 1 and 5 evolve into variables with values between 0 and 100 (i.e. minimum and maximum values respectively);
- In each iteration, normalized manifest and latent variables are used;
- The first component of the PCA analysis is selected as initial weigh;
- The centroid scheme is used to internally estimate the latent variables. In this framework, the relationship among the latent variables as a correlation symbol among estimated latent variables $Y_j (\hat{\xi}_j)$ and $Y_i (\hat{\xi}_i)$. Within the BI domain case, all relationships are considered positives a priori;
- The PLS approach procedure generated in MATLAB is used.

After 21 iterations, the results obtained are:

1. **Coefficients of correlation between the estimated latent variables $Y_j$ and the manifest variables $x_{jk}$**: These coefficients point out approximately what are the correlated variables taking into account each factor.
   a. Correlation between the estimated latent variable $Y_1$ and the environment factors (group $x_1$)
   
   $\begin{array}{cccc}
   x_{11} & x_{12} & x_{13} & x_{14} \\
   0.58 & 0.96 & 0.95 & 0.45 \\
   \end{array}$

   b. Correlation between the estimated latent variable $Y_2$ and the time of days factors (group $x_2$)
   
   $\begin{array}{ccc}
   x_{21} & x_{22} & x_{23} \\
   0.61 & 0.97 & 0.95 \\
   \end{array}$

   c. Correlation between the estimated latent variable $Y_3$ and the network factors – incidence (group $x_3$)
   
   $\begin{array}{ccccccc}
   x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\
   0.21 & 0.53 & 0.78 & 0.80 & 0.81 & 0.72 \\
   \end{array}$

   d. Correlation between the estimated latent variable $Y_4$ and the network factors – infrastructure (group $x_4$)
   
   $\begin{array}{cccc}
   x_{41} & x_{42} & x_{43} & x_{44} \\
   0.45 & 0.72 & 0.61 & 0.75 \\
   \end{array}$

   e. Correlation between the estimated latent variable $Y_5$ and the network factors – schedule system (group $x_5$)
   
   $\begin{array}{cc}
   x_{51} & x_{52} \\
   0.39 & 0.99 \\
   \end{array}$

   f. Correlation between the estimated latent variable $Y_6$ and the user’s factors – travel reason (group $x_6$)
   
   $\begin{array}{cccc}
   x_{61} & x_{62} & x_{63} & x_{64} & x_{65} \\
   0.60 & 0.18 & 0.38 & 0.65 & 0.32 \\
   \end{array}$

   g. Correlation between the estimated latent variable $Y_7$ and the user’s factors – flexibility (group $x_7$)
   
   $\begin{array}{cc}
   x_{71} & x_{72} \\
   0.83 & 0.69 \\
   \end{array}$

   h. Correlation between the estimated latent variable $Y_8$ and the user’s factors –
Behavioral impedance for route planning techniques from the pedestrian’s perspective:
Part II – Mathematical approach

1. **Correlation between the estimated latent variable** $Y_0$ **and the socio-politico-economic factors – security and safety (group $x_0$)**

   \[
   x_0 \quad x_2 \quad x_01 \\
   0.83 \quad 0.71 \\
   
   \]

2. **Correlation between the estimated latent variable** $Y_{10}$ **and the socio-politico-economic factors – fare system (group $x_{10}$)**

   \[
   x_{01} \quad x_{02} \quad x_{03} \\
   0.82 \quad 0.59 \quad 0.78 \\
   
   \]

2. **Structural relationship analysis**: This is the multiple regression among the estimated latent variables that takes into account the cause-effect relationship between $Y_0$ and the rest of the variables (Equation 3):

   \[
   Y_0 = 0.18 \cdot Y_1 + 0.21 \cdot Y_2 + 0.37 \cdot Y_3 + 0.18 \cdot Y_4 + 0.09 \cdot Y_5 + 0.07 \cdot Y_6 + 0.14 \cdot Y_7 + 0.17 \cdot Y_8 + 0.23 \cdot Y_9 + 0.21 \cdot Y_{10} \\
   (0.17,...,0.19) \quad (0.20,...,0.23) \quad (0.35,...,0.38) \quad (0.16,...,0.19) \quad (0.08,...,0.11) \quad (0.05,...,0.08) \quad (0.12,...,0.15) \quad (0.15,...,0.18) \quad (0.22,...,0.24) \quad (0.20,...,0.23) \\
   \]

   The coefficient of determination $R^2 = 0.9972$ points out a significant regression; in the same way, all parameters of the regression (Equation 3) are significative, with a significance level $\alpha = 0.05$, as shown in the confidence intervals indicated in Equation 3 in brackets.

3. **Correlation among the estimated latent variable (i.e. $Y_0$ or $\hat{Y}_0$) and the other**

   Table 4 presents these correlations.

   **Table 4. Correlation among the estimated latent variable (i.e. $Y_0$ or $\hat{Y}_0$) and the other.**

<table>
<thead>
<tr>
<th>Corr ($Y_0, Y_1$)</th>
<th>Corr ($Y_0, Y_2$)</th>
<th>Corr ($Y_0, Y_3$)</th>
<th>Corr ($Y_0, Y_4$)</th>
<th>Corr ($Y_0, Y_5$)</th>
<th>Corr ($Y_0, Y_6$)</th>
<th>Corr ($Y_0, Y_7$)</th>
<th>Corr ($Y_0, Y_8$)</th>
<th>Corr ($Y_0, Y_9$)</th>
<th>Corr ($Y_0, Y_{10}$)</th>
<th>Corr ($Y_0, Y_f$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.34</td>
<td>0.42</td>
<td>0.78</td>
<td>0.47</td>
<td>0.31</td>
<td>0.22</td>
<td>0.46</td>
<td>0.59</td>
<td>0.67</td>
<td>0.57</td>
<td></td>
</tr>
</tbody>
</table>

   As observed in Table 4, the higher correlation is identified between the estimated latent variable $Y_0$ and the network-incidence factor ($Y_3$) with a value around 0.8. In sequence is the correlation between $Y_0$ and the security and safety factor ($Y_9$). On the other hand, the latent variable less correlated is “reasons of public transportation” use ($Y_6$).

4. **Estimated latent variable** $Y_f$: For each individual of the sample, each latent variable is calculated. The calculus utility consists of the possibility of graphically representing the individuals in order to detect groupings or similar behaviors.

   **Table 5. Calculated values of the estimated latent variables.**

<table>
<thead>
<tr>
<th>$Y_0$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Y_4$</th>
<th>$Y_5$</th>
<th>$Y_6$</th>
<th>$Y_7$</th>
<th>$Y_8$</th>
<th>$Y_9$</th>
<th>$Y_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40</td>
<td>1.06</td>
<td>1.16</td>
<td>0.11</td>
<td>-1.08</td>
<td>0.82</td>
<td>-2.06</td>
<td>1.06</td>
<td>0.24</td>
<td>0.44</td>
<td>-0.07</td>
</tr>
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<td>-1.41</td>
<td>-1.22</td>
<td>-0.96</td>
<td>-0.85</td>
<td>-0.77</td>
<td>-0.21</td>
<td>-0.96</td>
<td>-1.39</td>
<td>0.39</td>
<td>0.05</td>
<td>-1.64</td>
</tr>
<tr>
<td>1.51</td>
<td>2.37</td>
<td>0.73</td>
<td>0.14</td>
<td>-0.60</td>
<td>-1.31</td>
<td>1.50</td>
<td>2.51</td>
<td>1.32</td>
<td>1.50</td>
<td>0.86</td>
</tr>
<tr>
<td>0.38</td>
<td>0.54</td>
<td>-0.96</td>
<td>0.26</td>
<td>1.12</td>
<td>0.82</td>
<td>1.30</td>
<td>0.24</td>
<td>-1.77</td>
<td>0.92</td>
<td>0.71</td>
</tr>
</tbody>
</table>
Table 5. Calculated values of the estimated latent variables (cont.).

| $Y_0$ | 1.48 | -1.00 | -1.49 | -2.05 | -0.08 | -1.37 | 1.21 | -0.18 | 1.00 | -0.30 | -1.03 | 0.55 | 0.37 | 0.24 | 0.05 | -0.25 | -0.21 | 0.39 | 0.85 | -2.78 | 0.64 | 0.24 | 0.66 | -1.84 | 0.78 | -1.59 | 1.58 | 0.09 | -2.04 | 0.22 | 0.57 | 1.43 | -2.22 | -1.73 | 1.04 | -0.40 | 0.24 | -0.11 | -0.61 | 0.53 | -0.14 | 0.81 | -0.51 | 0.05 | 0.02 | 0.75 | 0.23 | -0.37 | 0.39 |
|-------|------|-------|-------|-------|-------|-------|------|-------|------|-------|-------|------|------|------|------|------|-------|-------|------|-------|-------|------|-------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--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-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------------------
Table 5. Calculated values of the estimated latent variables (cont.).

<table>
<thead>
<tr>
<th>$Y_0$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Y_4$</th>
<th>$Y_5$</th>
<th>$Y_6$</th>
<th>$Y_7$</th>
<th>$Y_8$</th>
<th>$Y_9$</th>
<th>$Y_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>-2.19</td>
<td>-0.07</td>
<td>1.64</td>
<td>0.71</td>
<td>-0.82</td>
<td>-0.96</td>
<td>1.06</td>
<td>0.39</td>
<td>1.02</td>
<td>0.13</td>
</tr>
<tr>
<td>-0.04</td>
<td>-2.37</td>
<td>-0.14</td>
<td>1.00</td>
<td>-0.09</td>
<td>0.82</td>
<td>-0.02</td>
<td>-0.57</td>
<td>-0.54</td>
<td>-0.15</td>
<td>0.71</td>
</tr>
<tr>
<td>-1.18</td>
<td>0.00</td>
<td>-0.95</td>
<td>-2.78</td>
<td>-0.83</td>
<td>0.03</td>
<td>-0.96</td>
<td>0.00</td>
<td>0.51</td>
<td>0.00</td>
<td>0.69</td>
</tr>
<tr>
<td>1.36</td>
<td>0.00</td>
<td>1.54</td>
<td>0.85</td>
<td>1.53</td>
<td>0.82</td>
<td>0.32</td>
<td>1.06</td>
<td>0.81</td>
<td>-0.06</td>
<td>0.69</td>
</tr>
<tr>
<td>-1.06</td>
<td>-1.62</td>
<td>1.54</td>
<td>-0.29</td>
<td>-1.59</td>
<td>0.76</td>
<td>-0.96</td>
<td>-0.48</td>
<td>1.32</td>
<td>-0.65</td>
<td>-3.17</td>
</tr>
<tr>
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<td>-0.11</td>
<td>1.54</td>
<td>1.53</td>
<td>0.52</td>
<td>-0.16</td>
<td>1.30</td>
<td>1.06</td>
<td>1.32</td>
<td>0.72</td>
<td>0.17</td>
</tr>
<tr>
<td>2.63</td>
<td>1.26</td>
<td>1.54</td>
<td>1.96</td>
<td>1.53</td>
<td>0.82</td>
<td>-0.96</td>
<td>1.69</td>
<td>1.32</td>
<td>1.50</td>
<td>1.24</td>
</tr>
<tr>
<td>-0.16</td>
<td>-0.45</td>
<td>-1.82</td>
<td>0.08</td>
<td>-0.60</td>
<td>0.82</td>
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<td>-0.39</td>
<td>-1.62</td>
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<td>1.32</td>
<td>0.83</td>
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<td>0.71</td>
<td>0.82</td>
<td>-0.96</td>
<td>2.51</td>
<td>-0.54</td>
<td>0.32</td>
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</tr>
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<td>-0.43</td>
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<td>-1.79</td>
<td>0.46</td>
<td>0.72</td>
<td>-1.16</td>
<td>1.50</td>
<td>0.00</td>
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<td>-0.11</td>
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<td>-0.07</td>
<td>0.00</td>
<td>0.72</td>
<td>0.03</td>
<td>0.32</td>
<td>0.00</td>
<td>-1.05</td>
<td>0.00</td>
<td>-0.67</td>
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<td>-0.94</td>
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<td>-0.14</td>
<td>-0.83</td>
<td>-1.94</td>
<td>-2.22</td>
<td>1.50</td>
<td>-0.57</td>
<td>0.39</td>
<td>-0.36</td>
<td>-0.69</td>
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<tr>
<td>0.23</td>
<td>0.21</td>
<td>1.54</td>
<td>-1.38</td>
<td>-2.20</td>
<td>0.82</td>
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<td>1.32</td>
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<td>0.85</td>
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<td>-0.96</td>
<td>1.06</td>
<td>-1.08</td>
<td>1.50</td>
<td>0.06</td>
</tr>
<tr>
<td>-1.11</td>
<td>-0.88</td>
<td>-0.95</td>
<td>-1.35</td>
<td>0.31</td>
<td>0.82</td>
<td>-0.96</td>
<td>-2.12</td>
<td>1.32</td>
<td>-1.33</td>
<td>0.13</td>
</tr>
<tr>
<td>1.02</td>
<td>0.21</td>
<td>0.01</td>
<td>0.00</td>
<td>0.72</td>
<td>0.03</td>
<td>0.32</td>
<td>0.00</td>
<td>1.32</td>
<td>1.50</td>
<td>1.24</td>
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<tr>
<td>-0.54</td>
<td>0.21</td>
<td>0.00</td>
<td>0.03</td>
<td>-0.61</td>
<td>-1.19</td>
<td>0.32</td>
<td>0.00</td>
<td>-0.12</td>
<td>0.33</td>
<td>-2.01</td>
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<tr>
<td>-0.46</td>
<td>-1.64</td>
<td>0.71</td>
<td>-0.29</td>
<td>-0.19</td>
<td>-1.13</td>
<td>0.32</td>
<td>0.00</td>
<td>-0.54</td>
<td>0.10</td>
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<tr>
<td>-0.84</td>
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<td>-0.52</td>
<td>0.50</td>
<td>-2.36</td>
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<td>-0.96</td>
<td>1.69</td>
<td>-1.08</td>
<td>-1.92</td>
<td>-0.25</td>
</tr>
<tr>
<td>-0.03</td>
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<td>-0.41</td>
<td>0.03</td>
<td>0.32</td>
<td>0.00</td>
<td>-0.03</td>
<td>0.00</td>
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<tr>
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<td>-0.03</td>
<td>0.90</td>
<td>0.19</td>
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<td>-0.96</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>-0.33</td>
<td>0.23</td>
<td>0.00</td>
<td>0.40</td>
<td>-0.54</td>
<td>-1.19</td>
<td>0.32</td>
<td>0.00</td>
<td>0.00</td>
<td>-1.23</td>
<td>-0.19</td>
</tr>
<tr>
<td>0.11</td>
<td>-0.09</td>
<td>0.33</td>
<td>0.17</td>
<td>0.40</td>
<td>0.03</td>
<td>-0.96</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.40</td>
<td>0.10</td>
<td>1.54</td>
<td>-0.35</td>
<td>0.31</td>
<td>0.82</td>
<td>-0.96</td>
<td>1.45</td>
<td>-0.15</td>
<td>-0.06</td>
<td>-0.10</td>
</tr>
<tr>
<td>-1.47</td>
<td>-2.00</td>
<td>1.10</td>
<td>-1.16</td>
<td>-1.45</td>
<td>-2.11</td>
<td>-0.96</td>
<td>1.69</td>
<td>-0.54</td>
<td>-0.76</td>
<td>-1.81</td>
</tr>
<tr>
<td>0.75</td>
<td>1.94</td>
<td>-0.01</td>
<td>1.32</td>
<td>0.71</td>
<td>-3.08</td>
<td>0.32</td>
<td>1.06</td>
<td>0.78</td>
<td>-0.66</td>
<td>-0.27</td>
</tr>
<tr>
<td>-0.32</td>
<td>-0.28</td>
<td>1.10</td>
<td>-0.30</td>
<td>-0.81</td>
<td>-0.21</td>
<td>-0.96</td>
<td>-0.57</td>
<td>0.00</td>
<td>-0.65</td>
<td>0.34</td>
</tr>
</tbody>
</table>

5.4.4 Validation of the model

The cross validation technique has been used to verify the quality of the proposed model. From that, 10 different estimations have been obtained from the structural regression parameters and the correlation coefficients. Using these estimations as a starting point, the confidence intervals have been established (95 %) from the different parameters of the model. Thus, there is evidence that all parameters are significative. In particular, with respect to the structural relationship, the confidence intervals are as follows:

Table 6. Confidence intervals with respect to structural relationship.

<table>
<thead>
<tr>
<th>Estimated latent variable</th>
<th>Confidence intervals</th>
<th>Estimated latent variable</th>
<th>Confidence intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_7$</td>
<td>(0.13, ..., 0.25)</td>
<td>$Y_6$</td>
<td>(0.03, ..., 0.16)</td>
</tr>
<tr>
<td>$Y_2$</td>
<td>(0.14, ..., 0.26)</td>
<td>$Y_5$</td>
<td>(0.06, ..., 0.18)</td>
</tr>
<tr>
<td>$Y_3$</td>
<td>(0.31, ..., 0.47)</td>
<td>$Y_4$</td>
<td>(0.11, ..., 0.21)</td>
</tr>
<tr>
<td>$Y_9$</td>
<td>(0.09, ..., 0.22)</td>
<td>$Y_8$</td>
<td>(0.19, ..., 0.27)</td>
</tr>
<tr>
<td>$Y_5$</td>
<td>(0.05, ..., 0.14)</td>
<td>$Y_{10}$</td>
<td>(0.17, ..., 0.25)</td>
</tr>
</tbody>
</table>

5.4.5. Some Considerations on the PLS Approach

Table 7 details the correlations among the factors and the estimated latent variables.
Table 7. Factors and correlated variables.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Variable more correlated</th>
<th>Variable less correlated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environmental factors</td>
<td>Streets without construction</td>
<td>Good weather conditions</td>
</tr>
<tr>
<td>Time and hours factors</td>
<td>It is not rush hour in weekend</td>
<td>It is not rush hour in weekend</td>
</tr>
<tr>
<td>Network-incidence factors</td>
<td>No breakdown</td>
<td>No interruptions due to</td>
</tr>
<tr>
<td>Factors related to</td>
<td>Good accessibility on the stop</td>
<td>The bus, subway and train stops</td>
</tr>
<tr>
<td>accessibility and connections</td>
<td>or station</td>
<td>have good connections</td>
</tr>
<tr>
<td>Factors related to</td>
<td>Reliability in the schedule</td>
<td>Low headway</td>
</tr>
<tr>
<td>schedule system of the</td>
<td>system</td>
<td></td>
</tr>
<tr>
<td>transportation network</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factors related to</td>
<td>Personal use – work days</td>
<td>Professional use – work days</td>
</tr>
<tr>
<td>the reasons of public</td>
<td>Personal use – holiday</td>
<td></td>
</tr>
<tr>
<td>transportation use</td>
<td></td>
<td></td>
</tr>
<tr>
<td>User’s factors – flexibility</td>
<td>No emergency situations</td>
<td></td>
</tr>
<tr>
<td>User’s factors – affability</td>
<td>Transportation comfort</td>
<td></td>
</tr>
<tr>
<td>Security and safety factors</td>
<td>Good safety road</td>
<td>Without traffic jam</td>
</tr>
<tr>
<td>Fare systems factors</td>
<td>There is fare integration</td>
<td>The fare is not expensive</td>
</tr>
</tbody>
</table>

The study on the structural relationship indicates that the factors (i.e. latent variables) that most contribute to the formation of the latent variable $\xi_0$ are:

- Network – incidence factor ($\xi_3$)
- Security and safety factor ($\xi_9$)
- Fare system factor ($\xi_{10}$)
- Time of days factor (peak hour) ($\xi_2$)

On the other hand, the factors that less contribute to the formation of the latent variable $\xi_0$ are:

- User’s factor (Travel reason) ($\xi_6$)
- Network factor (Schedule system) ($\xi_5$)

Regarding the correlations between the latent variable $\xi_0$ and the rest of the estimated latent variables, the strongest correlation is related to the network factor (incidence) ($\xi_3$) with an approximate value of 0.8; the next correlation is related to the security and safety factor ($\xi_9$). On the other hand, the latent variable less correlated is the user’s factor (travel reason) ($\xi_6$).

5.5 Mathematical approach

Constraint management from the pedestrian’s perspective should improve the route planning techniques that take into account an integrated public transportation network. In this way, it is important to mathematically model the BI domain, allowing its accommodation to the GIS-T.
In the analytical approach of the BI meta-model, thirty-six conditions have been identified. Each one generates a variable set of constraints (i.e. elements of the mathematical approach), which compose the BI mathematical model of the cost function. Within the BI meta-model, the mathematical approach is composed of the constraints that represent the numerical coefficients of the impedance.

The building process of the mathematical model was divided into three parts. First, a questionnaire, commented on Section 4.1.4.1, has been designed. Pereira et al. (2002a) have presented some results of the previous statistical analysis, which have allowed to carry out the PLS approach. The next step has been the elaboration of a function related to the BI conditions in order to obtain the constraints used in the mathematical model. Finally, using the obtained results as a starting point, the identified coefficients have been assigned to the BI table allowing for the determination of the cost function through of empirical analysis.

Assuming a transportation network is a directly connected graph $G = [N, A]$, let $N$ be the set of nodes (i.e. nodal points that correspond to the origin and destination points of the pedestrian’s path, and intersection points) and let $A$ be the set of arcs representing direct sub-paths between two locations (i.e. nodes), and $n = |N|$ and $m = |A|$, where the symbol $||$ indicates the cardinality of the sets of nodes and arcs respectively, $J$ is defined as the set of indices of all paths, so that for each path there is a function $g$ that defines the arcs of a path and there is an index $I$ that indicates the quantity of arcs of a path, that is:

$$\forall Cam_j \in J, \exists g \land \exists I \in \{1,2,...,m\}$$

such that $g : J \rightarrow A$

$$g(Cam_j) = \{\text{link}_{1Cam_j}, \text{link}_{2Cam_j}, ..., \text{link}_{ICam_j}\} \subset A$$

Using the definition presented in Equation 4, the cost function for a path $j$ in an instant $t$ taking into account the BI weigh table is given by:

$$f_{Cost}(Cam_j)_t = \sum_{i=1}^{I} \left(C_{LinkCam_j}_i \cdot \left[1 + f\left(BI_{LinkCam_j}_i\right)\right]\right)$$

where $f_{Cost}(Cam_j)_t$ represents the cost function of the path $j$ in an instant $t$, $C_{LinkCam_j}_i$ represents the cost (i.e. a cost measure) in an arc $i$ in an instant $t$, $i$ represents an arc in the composition of the path represented by $Cam_j$, $j$ represents the possible paths, and $t$ represents a specific instant of the travel.

As shown in Equation 5, the travel cost is calculated in a multiplicative fashion between the sum of the default costs of each arc $i$ ($C_{LinkCam_j}_i$) (i.e. distance or time cost) and the BI calculated value imposed to the path $j$ ($Cam_j$) in an instant $t$ is represented
by \( 1 + f(BI_{Link,Cam_j}) \); the resultant products are summed for all \( I \) arcs of the path \( j \). In this way, there are two premises with respect to the BI function for a arc \( i \) of a path \( j \) in an instant \( t \):

1. If \( \exists BI \Rightarrow f(BI_{Link,Cam_j})_i \rightarrow 1 \)
2. If \( \forall BI \Rightarrow f(BI_{Link,Cam_j})_i = 0 \)

Assuming the BI values are saved in a table composed of \( k+1 \) attributes (i.e. the quantity of BI conditions plus an identifier attribute), the follows notations are defined:

- \( K' \): is the set of all BI attributes predefined by the transportation system through the BI table (\( K' = \{ h_{i1}, h_{i2}, ..., h_{ik} \} \));
- \( k = |K'| \): is the cardinality of the set of BI attributes of the BI table;
- \( h_i \): represents each attribute of the BI table;
- \( U' \): is the set of BI values defined by the user (\( U' = \{ h_{u1}, h_{u2}, ..., h_{uQ} \} \));
- \( h_{ui} \): represents each BI attribute defined by the user; and
- \( Q \): represents an indicator of the quantity of BI attributes changed by users (\( Q = |U'| \leq k \)).

Thus, the function that calculates the BI value from the data defined by the transportation system (i.e. BI table of the database GIS-T) and/or by the user, for each arc \( i \) of a path \( j \) in an instant \( t \), it is given by:

\[
 f(BI_{Link,Cam_j})_i = \frac{1}{k} \left[ \sum_{h \in (K' \cap \Omega)} \left( \sigma_{hLink,Cam_j} \right)_i + \sum_{h \in U'} \left( \sigma_{hLink,Cam_j} \right)_i \right]
\]

(6)

where \( \sum_{h \in (K' \cap \Omega)} \left( \sigma_{hLink,Cam_j} \right)_i \) represents the sum of the variables determined by the transportation system through the BI table, \( \sum_{h \in U'} \left( \sigma_{hLink,Cam_j} \right)_i \) represents the sum of the values provided by the users of the transportation system, \( \Omega \) represents the resultant set of the intersection of the sets \( K' \) and \( U' \) (\( K' \cap U' \)), \( K' \setminus \Omega \) represent the difference between the sets \( K' \) and \( \Omega \), and \( \sigma_{hLink,Cam_j} \) represents the values of the attributes \( h \) for the arc \( i \) of a path \( j \).

The BI value, for each arc \( i \) of a path \( j \) in an instant \( t \), is calculated through of the arithmetic average of the BI values obtained from BI table. However, if the user changes the predetermined values of the transportation system during route selection, these values are used in the BI function calculation.
The values of $\sigma_{hLink_{Cam}}$ for the BI attributes that pertain to the BI table (i.e. $h \in \left( K \setminus \Omega \right) / \Omega = \emptyset$) are initially determined by data collection tools (e.g. questionnaire and/or interviews) and maintained by the spatial and geographic database management system. The variables that represent these data such as meteorological and topographical data, information about rush hour, schedule system of the transportation lines, dangerous zones, information about transportation network incidents, fare systems and strategic and operating information can be automatically updated by GIS-T.

On the other hand, the values of $\sigma_{hLink_{Cam}}$ for the BI attributes defined by the user (i.e. $h \in U$), are basically determined by the users of the transportation system during route selection. These values are provided considering a qualitative and subjective analysis from the user’s preferences and converted into quantitative values used in the calculus of the BI cost function. Some of these values are personal data (i.e. user’s profile), travel reason, user’s demand (i.e. tolerance in meters to walk between two points), emergency situations and schedule system associated to professional activities.

As commented above, Equation 6 calculates the BI value of each arc $i$ of a path $j$ in an instant $t$. Therefore, the BI value sum of all arcs of the path $j$ in an instant $t$, is represented as:

$$f \left( BI_{Cam_j} \right)_t = \sum_{i=1}^{T} f \left( BI_{Link_{Cam_i}} \right)_t$$

(7)

Regarding the set of possible paths during the selection of a route in a transportation network, there are three kinds of paths according to the cost criteria defined by the user. The first one is the shortest path in which the distance criterion is fixed. The second one is the fastest path, which takes into account the time criterion. The third one is the best path, which not only takes into account distance and time criteria, but also other criteria defined by users. In this way, the values identified in the BI domain are considered as criteria that will support the best path calculus.

Assuming $p_o$ as the origin point and $p_d$ as the destination point, $J' \subset J$ is the set of possible path identifiers between $p_o$ and $p_d$, $d_{min}$ and $d_{max}$ are respectively the minimum and maximum travel distances in $J'$, and $t_{min}$ and $t_{max}$ are respectively the minimum and maximum travel times in $J'$, the following equation represents the general formula to select the optimum path taking into account the BI domain:

$$Cam^* = \text{Min}_{j \in J'} \left[ f_{Cost} \left( Cam_j \right)_t \right]$$

subject to: $d_{min} \cdot f \left( BI_{Cam_j} \right)_t \leq f_{Cost} \left( Cam_j \right)_t \leq d_{max} \cdot f \left( BI_{Cam_j} \right)_t$; $\forall j \in J'$; and $t_{min} \cdot f \left( BI_{Cam_j} \right)_t \leq f_{Cost} \left( Cam_j \right)_t \leq t_{max} \cdot f \left( BI_{Cam_j} \right)_t$; $\forall j \in J'$. 

Technical Report 20 May, 2002
where $Cam^*$ represents the best path, and $f_{cost}(Cam_j)$, represents the cost function of the path $j$ in an instant $t$.

5.6 BI Constraint Management Module

Using the mathematical model of the BI cost function presented above as a starting point, the constraint management module architecture (Figure 4) is proposed to be attached to the owner applications that work with geographic data as well as GIS-T.

Figure 4 – BI Constraint management module architecture

The constraint management module architecture is composed of a main engine characterized by the behavioral impedance route planning algorithm (BIRPA). BIRPA algorithm has two primary data providers (i.e. GIS-T database and users of the transportation system) and two secondary data providers (i.e. weight table of the BI constraints and data from the theory of planned behavior).

More details about the BI constraint management module can be found in [Pereira et al. 2002b]. The authors also comment on the BIRPA algorithm and its performance results.
6. Concluding remarks

In this technical report, the development of the BI meta-model’s mathematical approach has been presented. Using the data obtained from the application of a questionnaire as a starting point, the PLS approach (i.e. method of statistical analysis) has been used to verify the factors initially defined in BI meta-model’s analytical approach and validate the BI domain. The BI factors (i.e. BI conditions) are important during the route selection by pedestrians within a public transportation system.

Starting from the mathematical model used in the BI cost function, the constraint management module has been implemented in order to contribute to route planning techniques for pedestrians. This module can be attached to owner applications that work with geographic data as well as to commercial GIS-T.

Some future activities are proposed such as (1) simulation with of the BIRPA algorithm considering different user’s profiles (e.g. cyclists) and (2) improving the calculus time of the BIRPA algorithm because of graphs that represent medium or big cities can require a large amount of processing.

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References


Técnicas, p. 277-283.


