A Posteriori Knowledge: From Ambiguous Knowledge or Undefined Information to Knowledge

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ABSTRACT

Incomplete worlds or ambiguous actions are topics that encourage research in theories about reasoning and action *e. g.* [5]. Driankov model-based approaches [8] propose to deal with the problem using Belnap's four valued logic. Incompleteness or ambiguity is clarified by postconditions that apply whenever persistence conditions are inhibited. In this paper, Driankov semantic is used in a formalisation that captures the dynamic character of knowledge and belief. *Ambiguous knowledge* is present in several kind of situations that formally correspond with knowing that a disjunction is true, but it is not known which element of the disjunction makes it true. We define an *a posteriori* knowledge operator that allows to extend knowledge from ambiguous knowledge or undefined information, being in the meanwhile, *potential* knowledge and/or belief. The expansion operation of AGM paradigm [9] is used to explore the possibilities from ambiguous information.

Keywords: Knowledge, *a posteriori* knowledge, potential knowledge, belief.

1 Introduction

For knowledge and belief, the semantics of possible worlds [16] has been attractive for the intuitive way in which description of present and possible worlds (states) can be established. Kripke's possible worlds are sets of interpreted formulas in classical semantics. The worlds are related through an algebraical, usually reflexive, symmetric and transitive relation. World w is possible for w_0 whenever w_0 and w satisfy the relation. Logics of knowledge and belief that use the semantics of possible worlds [12] [26], [10, 11]. consider knowledge in present world w_0 as the true formulas in all possible worlds of w_0. In addition, if formula φ is knowledge and ψ is a logical consequence of φ, then ψ is true in all states in which φ does, and is knowledge too. Thus, all logical consequence of initial knowledge is knowledge. This topic, known as *logical omniscience* is unintuitive to model *real* agents of knowledge and/or belief, because they should have unlimited time and computational resources [23].

Based upon Kleene's three valued logic [15], a formal tool that seems more flexible to model knowledge and belief is partial logic [3], that uses truth-values *true, false* and *undefined.* A partial interpretation of a set of formulas can to assign undefined truth-value to some formulas in the set, while complete interpretation assigns, only, *true* and *false* values. Process to assign *true* or *false* truth-values to *undefined* formulas is a refinement.

Possible worlds in partial logic are sets of formulas interpreted as *true*, *false* or *undefined.* World w is possible for present world w_0, when set of *true* formulas in w_0 is a subset of *true* formulas in w and set of *false* formulas in w_0 is a subset of *false* formulas in w. Semantically, partial interpretation of w is refinement of w_0 interpretation. Using partial interpretations, knowledge can be defined in accordance with real circumstance of incomplete present information and limited resources *e. g.* [1], [7], [8], [24].

However, although with partial logic is possible to deal with *undefined* information in flexible way, seems that there are *lacks* when dealing with incomplete or ambiguous situations, usually expressed by disjunctions. In some partial logics, a disjunction is *true* whenever, at least and necessarily, one literal is *true* [3], [1], [7]. However, there are several situations where knowledge is only a collection of alternatives and such that the true alternative is unknown. A formal expression of that corresponds with situations where we have a *true* disjunction with all literal truth-values *undefined* . In [8] a four valued logic that leads with *true* disjunction having no *true* literal is proposed. This logic is applied to incomplete worlds or ambiguous situations being completed or desambiguated through a postcondition.

In the present paper we consider possible epistemic states being monotone extensions of current state. Sintactically, expansion AGM operation [9] is used. Semantically, possible states are obtained by refinement of subsentences of actual state sentences, using a restriction of Driankov logic to values *true*, *false* and *undefined.* For a finite current state, there is a finite number of possible states that satisfy a informative partial order. Every possible state provides, at least, the information of that current state. The intuitive meaning of the partial order is that there are alternative strands to be analized when extending knowledge from actual knowledge.

Knowledge in present state is defined as the set of formulas interpreted as *true* in all possible states. Due to partiality of states interpretations, evolving of refinement process could generates *a posteriori* knowledge from ambiguous knowledge or undefined information. That process is accomplished gradually. *Potential* knowledge is the *true* or *undefined* formulas in all possible states while *belief* is every *true* formula in at least one possible state. Potential knowledge and belief can be considered the *meanwhile step* of *a posteriori* knowledge.

This paper is organized as follows: in section 2 an adapatation of Driankov's logic to values *true*, *false* and *undefined* for finite sets is developed. In section 3 a logic of knowledge and belief based upon logic of section 2 is defined. Some examples are given. In section 4 the proposed approach is compared with related works. Finally, natural work evolution is discussed and conclussions given.

2

In this section we develop a three-valued logic for finite sets. The logic can be considered a restriction of four-valued Driankov logic [8] to values t, f and u, but generalized to finite sets. We use partial interpretations instead of set-ups.

2.1 Logical and informative lattices

Consider a propositional language $\mathcal{L}$ defined in the usual way from a finite vocabulary, i.e. a set of propositional variables, $\text{Lit}(\mathcal{L}) = \{p_1, ..., p_n\}$, and the logical connectives $\vee$, $\wedge$, and $\neg$. Let ψ be a finite set of sentences and $Lit(\psi)$ the propositional variables from $\text{Lit}(\mathcal{L})$ which are used in the construction of formulas in ψ only. Then $\mathcal{L}(\psi)$ is the closed language from $Lit(\psi)$ under $\wedge, \vee$ and $\neg$.

On the semantics side, we consider the partial order $[L3; \leq]$ where $L3$ is the set of truth-values $\{u, t, f\}$ and $\leq$ is defined as follows: $f < u$, $f < t$, and $u < t$. Two operations defined for every two elements of $[L3; \leq]$ turn this partial order into a *logical lattice*. These are, $\vee$ and $\wedge$, defined as: $a \vee b = max(a, b)$, and $a \wedge b = min(a, b)$, where a and b belong to $L3$. Complement, denoted as $\neg$, is defined as: $\neg(t) = f$ $\neg(f) = t$ and $\neg(u) = u$. These lattice operations are used to define, respectively, the meaning of the logical connectives *and*, *or*, and *negation*, and thus are used to compute the truth-values of compound formulas given an assignment of truth-values to their propositional variables. The logical lattice obeys the following laws: *idempotency, commutativity, distributivity, associativity, absorption, consistency*, and the *De Morgan laws.*

On the information side, we consider the partial order $[I3; \sqsubseteq]$ where $I3$ is the set of truth-values $\{u, t, f\}$ and $\sqsubseteq$ is defined as follows: $u \sqsubset f$ and $u \sqsubset t$. Two operations defined for every two elements of $[I3; \sqsubseteq]$ turn this partial order into a *information lattice*. These are, $\sqcap$ and $\sqcup$, defined as: $a \sqcap b = min(a, b)$, and $a \sqcup b = max(a, b)$, where a and b belong to $I3$. The information lattice operations obey the following laws: *idempotency, commutativity, distributivity, associativity, absorption*, and *consistency.*

2.2 Partial interpretations

In this subsection partial interpretations of set expansions and partial orders among them are defined. Logical and informative operations are extended to them. Let $\mathcal{Q} = 2^{Lit(\psi)}$ and $\mathcal{P} \in \mathcal{Q}$. Any expansion from ψ uses as expansion set an element from $\mathcal{Q}$:

$$E = \psi^{+}_{\mathcal{P}}, \qquad \mathcal{P} \in \mathcal{Q}$$

The set of expansions from ψ is $\mathcal{E} = \{E_0 = \psi, E_1, \ldots E_n\}$; $\mathcal{E}$ is finite whenever ψ is finite. Let $\mathcal{F}$ be the set of $\mathcal{L}(\psi)$-formulas. The truth-value of formula α in E is defined in the usual inductive manner using the above introduced connectives and is denoted $I_E[\alpha]$

Definition (Partial interpretation)

A partial interpretation is a mapping I from $\mathcal{F}$ to $\{t, f, u\}$. An interpretation is complete whenever assigns truth values true or false to every $\mathcal{F}$-formula. A partial interpretation I is complete for a set E if assigns true or false to every formula in E. I is a *model* of a set E if and only if assigns truth-value true to every formula in E.

According to this definition of a partial interpretation it can be shown that there is a unique model for any conjunction in ψ and at least one model for any disjunction. In [8], in order to reduce the number of models for a disjunction, the notion of an *informationally minimal model* of a formula α is introduced —by defining partial order between set-ups. We extend this definition to partial interpretations of any finite set.

Let now $\mathcal{I}$ be the set of all partial interpretations for all elements in $\mathcal{E}$, $\mathcal{I} = \{I_{E_i} : E_i \in \mathcal{E}\}$. In general there is a finite set of I_{E_i} for each E_i. The relation $\sqsubseteq$, defined initially on $I3$ can be extended to the elements of $\mathcal{I}$ in the following manner:

- $\forall E_1, E_2 \in \mathcal{E}$, $I_{E_1} \sqsubseteq I_{E_2}$ iff $\forall \alpha \in E_1 \cap E_2$, $I_{E_1}[\alpha] \sqsubseteq I_{E_2}[\alpha]$.

It can be shown that $[\mathcal{I};\sqsubseteq]$ is a partial order, and operations, $\sqcap$ and $\sqcup$, defined between partial interpretations, make it a lattice:

- $\forall \alpha \in E_1 \cap E_2$, $(I_{E_1} \sqcap I_{E_2})[\alpha] = I_{E_1}[\alpha] \sqcap I_{E_2}[\alpha]$, and
  $\forall \alpha \notin E_1 \cap E_2$, $(I_{E_1} \sqcap I_{E_2})[\alpha] = I_{E_1}[\alpha]$ if $\alpha \in E_1$, or
  $(I_{E_1} \sqcap I_{E_2})[\alpha] = I_{E_2}[\alpha]$ if $\alpha \in E_2$.
- $\forall \alpha \in E_1 \cap E_2$, $(I_{E_1} \sqcup I_{E_2})[\alpha] = I_{E_1}[\alpha] \sqcup I_{E_2}[\alpha]$, and
  $\forall \alpha \notin E_1 \cap E_2$ $(I_{E_1} \sqcup I_{E_2})[\alpha] = I_{E_1}[\alpha]$ if $\alpha \in E_1$, or
  $(I_{E_1} \sqcup I_{E_2})[\alpha] = I_{E_2}[\alpha]$ if $\alpha \in E_2$.

Now we can define IM_{E_1} as this model of E_1 such that there does not exist another model of E_1, $M^*_{E_1}$, such that $M^*_{E_1} \sqsubseteq IM_{E_1}$. Here again, if the set of formulas contains only conjunctions it has a unique informationally minimal model which is equal to its model . However, in the case of a set of formulas with disjunctions, there is a number of informationally minimal models which can not be ordered amongst themselves with respect to $\sqsubseteq$ on $\mathcal{I}$, and from which the notion of an *epistemic state* is derived.

2.3 Epistemic states

In general, an epistemic state ES_ψ of a set ψ of formulas, is a set of partial interpretations of ψ. The truth-value of α in ES_ψ, denoted as, $ES_\psi[\alpha]$, is obtained as:

- Let $ES_\psi = \{I_1, ..., I_n\}$.
  $ES_\psi[\alpha] = I_1[\alpha] \sqcap \ldots \sqcap I_n[\alpha]$,

A *model epistemic state* of ψ, denoted as MES_ψ, is this epistemic state in which the truth value of the formulas in ψ is t. Let $\mathcal{ES}$ be the set of all epistemic states. The partial order $\sqsubseteq$ on $\mathcal{I}$ can be extended to all sets in $\mathcal{ES}$ as follows:

- $\forall\alpha,\beta \in \mathcal{F}$, $ES_{E_1} \sqsubseteq ES_{E_2}$ iff $E_1 \cap E_2 \neq \emptyset$, and $\forall I_{E_2} \in ES_{E_2}$, $\exists\, I_{E_1} \in ES_{E_1}$, such that $I_{E_1} \sqsubseteq I_{E_2}$.

Moreover, it is still possible to extend the $\sqcap$ and $\sqcup$ operations, defined on $\mathcal{I}$ to elements of $\mathcal{ES}$. This is done as follows:

- Let $ES_{E_1} = \{S^1_\alpha, ..., S^n_\alpha\}$, $ES_{E_2} = \{S^1_\beta, ..., S^m_\beta\}$.
  $ES_{E_1} \sqcap ES_{E_2} = \{S^i_\alpha \sqcap S^j_\beta \mid i = 1, ..., n; j = 1, ..., m; S^i_\alpha \in ES_{E_1}, S^j_\beta \in ES_{E_2}\}$
  $ES_{E_1} \sqcup ES_{E_2} = \{S^i_\alpha \sqcup S^j_\beta \mid i = 1, ..., n; j = 1, ..., m; S^i_\alpha \in ES_{E_1}, S^j_\beta \in ES_{E_2}\}$

Now we can define an *informationally minimal model epistemic state*, denoted as $IMES_{E_1}$, as this model epistemic state of E_1 such that there does not exist another model epistemic state of E_1, $MES^*_{E_1}$, such that $MES^*_{E_1} \sqsubseteq IMES_{E_1}$. Here again, for a set E that contains only conjunctions there is only one informationally minimal model epistemic state which is equal to its unique model. In the case of a set containing only a disjunctions α of n literals in E_1, it can be shown that $IMES_{E_1}$ can only be constructed as follows:

- Let $Lit(\psi) = \{p_1, ..., p_n\}$ be the literals of α.
  $IMES_{E_1} = \{IMS^1_{E_1}, ..., IMS^n_{E_1}\}$, where
  $IMS^i[p_i] = t$ if $l(p_i) \in Lit^+(\alpha)$, or
  $IMS^i[p_i] = f$ if $l(p_i) \in Lit^-(\alpha)$, and
  $\forall p \neq p_i, IMS^i[p] = u$.

The important thing about $IMES_{E_1}$ is that it has the following property:

- $IMES_{E_1}[p] = u$, $\forall p \in Lit(\psi)$, while $IMES_{E_1}[\alpha] = t$

This construction for one disjunction is generalized for sets with any number of disjunctions taking the union of informationally minimal model epistemic state of each disjunction:

$$\bigcup_{\alpha \in E_1} IMES_\alpha.$$

The above representation of an informationally minimal model epistemic state captures the intuitions about the ambiguous information conveyed by a disjunction in a very economic way: the number of models that are members of $IMES_{E_1}$ is equal to the number of literals in α. In addition each other model epistemic states of E_1, MES_{E_1}, that has the above property as well, satisfies that $IMES_{E_1} \sqsubseteq MES_{E_1}$. This semantical property is very desirable to deal with ambiguous knowledge in the epistemic approach developed in the next section.

In the particular case of epistemic states obtained by expansion, let $\mathcal{I}(\mathcal{E})$ be the set of $IMES_E$'s. Because expansion is monotone, the informative order $\sqsubseteq$ over $\mathcal{I}(\mathcal{E})$ corresponds with the order over expansions set $\mathcal{E}$: whenever E' is expansion of E, $IMES_E \sqsubseteq IMES_{E'}$ is satisfied. The model epistemic state for E_i in which any formula not being in E_i is undefined, is the minimal model epistemic state $IMES_{E_i}$:

$$\forall\, \beta \notin E_i, \;\; IMES_{E_i}[\beta] = u.$$

Example 1
For $\psi = \{\alpha = a \vee b\}$, $IMES_\psi = \{(t,u),(u,t)\}$; $IMES_\psi[a] = u$, $IMES_\psi[b] = u$, but $IMES_\psi[\alpha] = t$.

Formal structure defined at follows provides framework for epistemic definition based on the above semantic.

Definition (Model Frame)
A model frame is an ordered tupla $\mathcal{M} = \langle \mathcal{I}, I_\psi, \sqsubseteq \rangle$, where I_ψ is the set of partial interpretation that satisfies ψ and $\mathcal{I}$ is the set of I_{E_i}, for $i = 0, \ldots, n$. For any $I' \in \mathcal{I}$ we assume that $I \sqsubseteq I'$, for every $I \in I_\psi$.

Definition (Satisfaction)
An interpretation $I \in \mathcal{I}$ in the model frame $\mathcal{M}$ satisfies a sentence $\varphi \in \mathcal{L}$, $I \models_{\mathcal{M}} \varphi$, if and only if $I(\varphi) = t$. Is said that the frame satisfies φ, $\mathcal{M} \models \varphi$, if every $I \in \mathcal{I}$ satisfies φ. For a set Γ such that any sentencence of Γ is satisfied by $\mathcal{M}$, is said that the frame $\mathcal{M}$ satisfies Γ, $\mathcal{M} \models \Gamma$

3 Knowledge and Belief

Propositional language $\mathcal{L}$ is extended by adding modal operators of **Knowledge** K, **Belief** B, **Potential Knowledge** K_p, and **Aposteriori Knowledge** K_{aps}. The language extended with these modal operators is called $\mathcal{EL}$ by epistemic language. Arguments of $\mathcal{EL}$ are any kind of object or epistemic formulas. In the following definitions we take I' being a ***direct succesor*** of an element of I_ψ, *i. e.* $I' = I_{E_i}$ for some E_i. A direct succesor of $I \in I_\psi$ is a partial interpretation I' such that for any I'', if $I \sqsubset I'' \sqsubseteq I'$ then $I'' = I'$. In the satisfaction relation $I \models_{\mathcal{M}} \varphi$, whenever the model frame $\mathcal{M}$ is clear the subindex is ommited. (Easy reading of the following definitions could be done with example 2 in 3.2.)

Definition (Knowledge)

A formula φ is knowledge in the model set up frame $\mathcal{M}$, $I \models_{\mathcal{M}} K(\varphi)$ if and only if,
1) $I \models_{\mathcal{M}} \varphi$ or
2) $I \models_{\mathcal{M}} K_{aps}(\varphi)$

Definition (A posteriori knowledge) (Fig. 1)
$I \models_{\mathcal{M}} K\text{aps}(\varphi)$ if and only if, for all I' direct succesor of I, $I' \models_{\mathcal{M}} (\alpha)$, or $I' \models_{\mathcal{M}} K_{aps}\alpha$

Definition (Maximal interpretation for a set ψ)
An interpretation J is maximal for a set ψ, if and only if it assigns truth values *true* or *false* to every subformula of any formula in ψ.

Observation
$I \models K_{aps}\alpha$ *iff* for all maximal J, $J \models \alpha$

Observation
If J is maximal, $J \models K_{aps}\alpha$ *iff* $J \models \alpha$

Observation
$I \models K_{aps}(\alpha \vee \beta)$ *iff* for all maximal J, $J \models \alpha$ or $J \models \beta$

Proposition

$I \models K_{aps}(\alpha \vee \beta) \rightarrow K_{aps}\alpha \vee K_{aps}\beta$

Intuitively, *a posteriori* knowledge allows to know certain facts about ambiguous *a priori* knowledge: it is possible to know the possible alternatives about one fact altough not know which possibility is really *true*. For example, one can know that today will rain or not, but not know what effectively will occur. This is the case of tautological sentences; but not only that: I can know that my friend from Grenoble comes to Barcelone by plane, train or bus, altough I do not know which one of these transportations efectively he will use (see example 1).

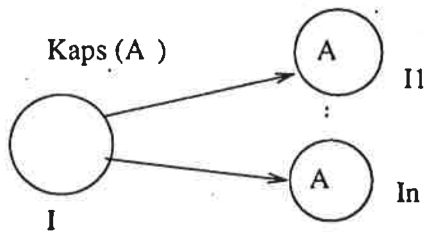

Figure 1: Aposteriori Knowledge

Definition (Belief B)

$I \models B(\varphi)$ iff $I \not\models \varphi$, and exists I', direct succesor of I, $I' \models \varphi$ (Fig. 2).

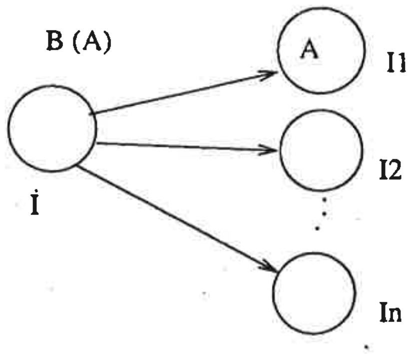

Figure 2: Belief

Some useful semantic charachteristics of defined operators are mentioned.

Semantical observations

1. $\neg B(\neg\alpha)\ iff\ I \not\models B(\neg\alpha)\ iff\ \forall I' > I,\ I' \not\models \neg\alpha$
2. $\neg K_{apr}(\alpha)\ iff\ I \not\models K_{apr}(\alpha)\ iff\ I \not\models \alpha$
3. $\neg K_{aps}(\alpha)\ iff\ I \not\models K_{aps}(\alpha)\ iff\ \exists I' > I,\ I' \not\models \alpha$

Definition (Potential Knowledge K_p)

$I \models K_p(\varphi)$ if and only if the following conditions are satisfied.

1) $I \not\models \varphi$,
2) $\forall I'$, I'direct succesor $I' \not\models \neg\varphi$ (Fig. 3).

Observation
$I \models K_p(\varphi)\ iff\ I \models \neg B(\neg\varphi)$

A particular case is $I \models B(\alpha) \wedge \neg B(\neg\alpha) \rightarrow K_p(\alpha)$. It means that while exist some expansions satisfing α and no one satisfing $\neg\alpha$, then α is potential knowledge. Thus, potential knowledge is belief, but belief not necesarily is potential knowledge, because *true* formula in a state and *false* in another is belief but not potential knowledge. In extreme case, a sentence can be potential knowledge altough not being belief. In fact, any not believed sentence in present state can be consider potential knowledge:

$I \models \neg B(\neg\alpha)$	then	$I \models K_p(\alpha)$	(1)
$I \models \neg B(\alpha)$	then	$I \models K_p(\neg\alpha)$	(2)
$I \models B(\alpha)$	then	$I \models \neg K_p(\neg\alpha)$	(3)

Thus potential knowledge definition can be rewriten as follows:

$$I \models \alpha^u \quad \text{and for all} \quad I' \ \text{succesor}, \ I' \not\models \neg\alpha$$

where u denotes *undefined* truth-value for α

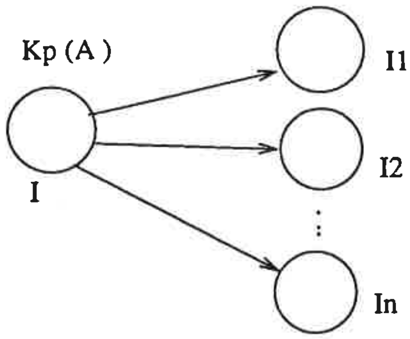

Figure 3: Potential Knowledge

Rationality of one agent is usually related with consistency in the set of beliefs. Intuitively we consider that given a context (*frame*), a statement α is consistent with beliefs of an agent whenever the agent does not believe the negation of the statement; in this case the affirmative statement can become to be knowledge and seems natural to consider it as potential knowledge for the agent. In our approach we will consider the potential knowledge conditions the ones that preserves the consistency in epistemic states. In this sense they could be compared with justifications that are satisfied in deduction rules of defaults theories (see example 3). Moreover, interpretation of default sentences in epistemic context is very intuitive: a default conclusion, by its own character of *provisional*, seems natural to be consider potential knowledge or belief, and conversely.

However, deppending on the certainty that some agent has about a statement, it can believe the affirmative, the negative or both. If some agent is compelled to believe a statement affirmed and negated, except in the case to be trivial and uninteresting, it must try with each one in local and different subframes. Our formalism provides this possibility. Consistency condition of AGM expansion prevents to include contradictory statements in the same expansion set. Thus any one statement can be consider **local knowledge** only, being *true* in a subframe (subcontext) of full frame (context) $\mathcal{M}$. The point is that in such case, any of the statements can not be considereded potential knowledge, and thus can not become to be knowledge (see example 3).

Definition (Local knowledge)
α is a local knowledge in $\mathcal{M}$ if and only if, in some partial interpretation I of $\mathcal{M}$, $B(K_{aps}(\alpha))$ is satisfied.

Recursivity and epistemical acceptance

Definitions of knowledge, potential knowledge and belief are recursive: they appeal the modalities argument satisfaction in possibles states to decide if it is knowledge, potential knowledge or belief. Each succesor repeats the process. Whenever the conditions are satisfied the process stops and sentence epistemic status is established. This is a constructive process because status of any sentence is determined when conditions are fulfilled.

Unsucces of $K_p(\alpha)$ to become to be $K_{aps}(\alpha)$ due to eventual satisfaction of $\neg\alpha$, could be interpreted as *refutation* in the model frame $\mathcal{M}$ of α and could suggests any of the following options: the change of frame or the suitable reception of refutation against a *virtual* or potential knowledge.

An epistemic sentence is knowledge —*a priori* or *a posteriori*— potential knowledge or beliefe, or no have any of these epistemic status. In fact, an object formula that is argument of one epistemic is declared not known when is not knowledge, potential knowledge nor belief. But unknown sentences can change to be believed or known, it depends of evolving of possible states. By the other hand, object sentences are *true*, *false* or *undefined* meanwhile epistemic are not concerned in the same semantic way: there are not *undefined* epistemic sentences.

Now we give some obvious epistemic observations and metarules to be satisfied by epistemic operators.

3.1 Epistemic metarules

Observations
$K_{apr}(\wedge_1^n \alpha_i) \implies K_{apr}(\alpha_i)$ for $i = 1, \ldots, n$

$B\,(\wedge_1^n \alpha_i) \implies B(\alpha_i)$ for $i = 1, \ldots, n$

Metarules
1.) $K_{apr}(\vee_1^n \alpha_i) \wedge (\forall i \ \neg K_{apr}(\alpha_i)) \implies K_{aps}\alpha_i$ for some $i \in \{1, \ldots, n\}$

2.) $K_{apr}(\vee_1^n \alpha_i) \wedge (\exists i \ \neg B(\neg\alpha_i)) \implies K_p\alpha_i$ for some i

Metarule 1.) says that at least a part of every ambiguous knowledge sentence must be unambiguous in the future. Seems natural to ask this condition for a rational agent or system. Metarule 2.) established the plausibility of a substatement of disjunctive knowledge.

Proposition

$K_{apr}((\wedge_1^n \alpha_i \rightarrow \beta) \equiv K_{apr}(\vee_1^n(\alpha_i) \rightarrow \beta))$ and exists expansions $E_1, \ldots E_m$ of ψ such that, respectively. $\alpha_i \rightarrow \beta. \alpha_i \in E_i$ then $I \models K_{aps}(B)$

3.2 Examples

Example 2

Table 1

ψ	E_1	E_2	Epistemic status in MES_ψ
$\{\alpha = a \vee b\}$	α, a	α, b	$K(\alpha)$, $B(a)$, $B(b)$, $K_p(a)$, $K_p(b)$
	α, a	$\alpha, \neg a$	$K(\alpha)$, $B(a)$, $B(\neg a)$, $\neg K_p(a)$, $K_p(b)$
	α, a	α, a, b	$K(\alpha)$, $K_{aps}(a)$, $B(b)$, $K_p(b)$

Each row represents possible expansion states from ψ. Epistemic sentences deppends of actual expansions. In model epistemic state of thirth row $E_1 \sqsubseteq E_2$ is satisfied.

Example 3

Application of epistemic definitions is shown by modeling the well known Nixon Diamond problem. Let r be taken for republican, q for quacker and p for pacifist. It is guessed that republicans are not pacifist, quackers are pacifists and that Nixon is republican and quacker. In our approach this can be write as follows:

$r \wedge \neg K_p(p) \rightarrow B(\neg p)$
$q \wedge \neg K_p(\neg p) \rightarrow B(p)$
$r \wedge q$

As has been mentioned, a sentence is consistent with present knowledge —satisging potential knowledge conditions,— whenever the negation of the sentence is not believed, and the converse. In the example, there is no problem dealing with contradictory information about Nixon. Contradictory statements are treated in diferent expansion sets belonging to diferent *branches* of model frame $\mathcal{M}$. So, divided opinion about Nixon pacificity, due to belong to both identified groups, can not be established as knowledge. But is possible to deal with it adequatetly.

The proposed belief definition resembles with preferred modalities in Cumulative Default Logic (CDL) of Meyer and Hoeck [22]. When a belief sentence is true in a given state ES, it is true in every possible state from ES. It means that a *branch* of model frame satisfies it; this branch can be associated with a preferred modality in CDL (see example). Potential knowledge condition corresponds with consistency condition in CDL.

4 Related works

4.1 Awareness, implicit and explicit beliefs

In the attempt to avoid logical omniscience and ideal reasoning Levesque introduced distinction between explicit and implicit beliefs [18]. Relevance logic [AnBe 75] is used to reason with explicit beliefs, while implicit beliefs satisfy a $S5$ modal sytem. Because relevance logic is a weakening of the propositional one, every explicit belief is implicit, but not conversely.

However is not so clear how, eventually, an implicit belief can become to be explicit. It means: how to extend the *a priori* belief (knowledge) that an agent has?.

Our model-based approach provides dynamical evolution of initial explicit beliefs. We can treat explicit belief as *a priori* knowledge and implicit belief as undefined or ambiguous information. Deppendig of actual cappabilities of the agent is possible to extend explicit beliefs. An approach having intuitions close to ours is the logic of awareness [10] defined over Kripke possible worlds. Explicit agent knowledge is the true and aware sentences in possible worlds for the agent. Our approach can model this characterization of awareness in a dynamical way, and in this sense it constitutes an overcome to awareness operator that does not seem to provide a dynamical process.

4.2 Defaults Theories

In the proposed frame, we avoided to take a definitive position about any statement whenever there are not enough information to do it. At this sense, our epistemic logic is similar with default epistemic logics [4], [7], [21], [22], and is essentially different with theories that use closed world assumption —in strong or weak versions *e. g.* [20], [19]— in which the negation of information not explicitely given or deduced is taken.

Our model-based approach leads with nonmonotone conclusions in the line of preferred default theories [24], [4], [7], [22], [1]. Belief and potential knowledge —as default conclusions— are satisfied by some preferred models, and whenever consistency condition between belief and knoweledge is preserved, belief being potential knowledge, can become to be knowledge. (This graduality is as preferential entailment leads with non-monotonicity [25].)

Forthcoming work

A more accurate way to do possible sets by expansion is taking the minimal set $\mathcal{P}$ in $\mathcal{Q}$ that results in $\psi^{+}_{\mathcal{P}}$ [2]. It means to apply preferential criterion over set $\mathcal{Q}$ and to expand ψ using the minimal information. By the other hand, for the present paper porpouse, restriction to values t, f and u of four valued logic of Driankov is enough, because we define expansion sets of actual state in a cumulative a logic. A suitable extension for states obtained by contraction or revision AGM operation, could be done using *contradictory* or *superinformed* truth-value k [AlGo 95] in the four-valued Driankov logic. Following this way, seems that we could allows to generalize revision operator in the line suggested by Katsuno and Mendelzon [14] and Del Val [6].

Conclusions

A epistemic logic that captures —we hope in a suitable way— dynamical process to extend knowledge from ambiguous or undefined information is proposed. Modal operators of *a posteriori* knowledge, knowledge and belief have been defined. *A posteriori* knowledge is obtained from undefined information or ambiguous knowledge, in such way that intermediate steps correspond with potential knowledge and/or belief. We take a frame model of possible states defined on a partial logic, that is syntactically based on AGM expansion operation.

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