Modelling and analysis of crack turning on aeronautical structures

Doctoral Thesis

Llorenç Llopart Prieto

Director:
Dr. Marc Anglada i Gomila

Departament de Ciència dels Materials i Enginyeria Metal·lúrgica
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Company director:
Elke Hombergsmeier

EADS
European Aeronautical Defence and Space company

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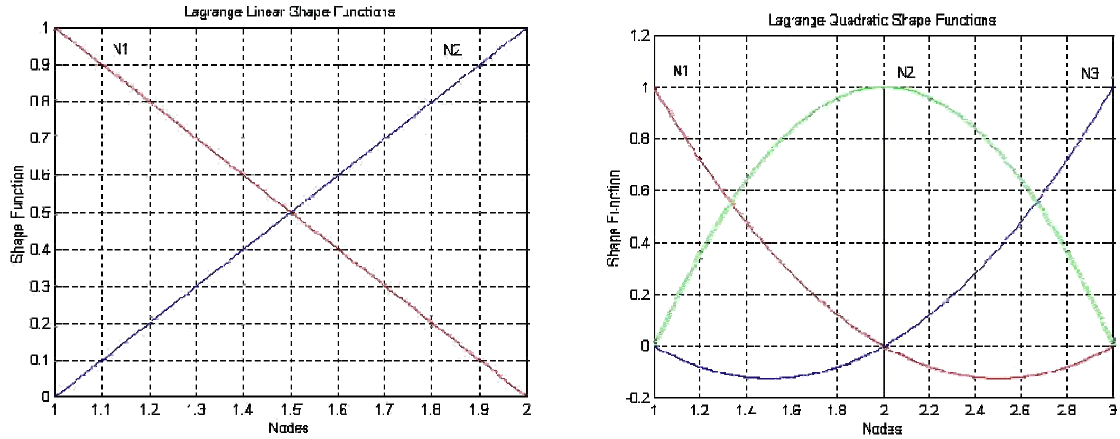
A Annexes

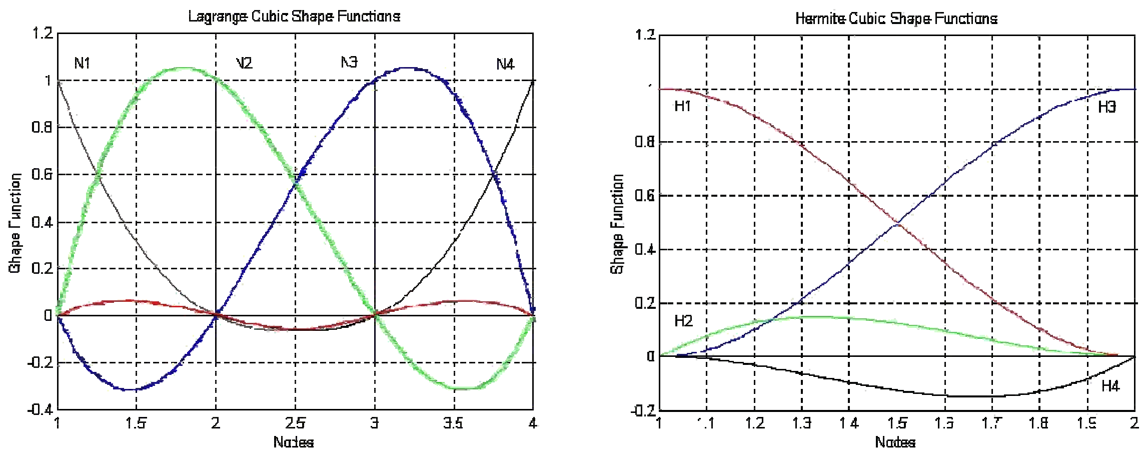
A.1 Numerical methods

The evaluation of a variable field, *U'*, inside the finite or boundary element is solved by the shape functions. The most used are as introduced in chapter 2.3, the Lagrange linear function, quadratic or cubic and the Hermite cubic function. A representation of these is shown in Figures A.1 and A.2.



a) ***b)***

Figure A.1. Lagrange shape function a) linear and b) quadratic



a) ***b)***

Figure A.2. Cubic shape function a) Lagrange and b) Hermite

The characteristic of these functions is that they have a value equal to *1* at its associated node and *0* at the other nodes.

A.2 Crack turning criteria

A.2.1 Crack turning criteria for 2D-structures

In quasi-static loading one of the pioneers on assessing the direction of a crack was Griffith [67] who held that the crack will grow in the direction where the elastic energy release per unit of crack extension is maximal [62]. Moreover, the crack will start growing when this energy reaches a critical value [21].

On further investigations [59], it was found that this energy is independent on the mixed mode of loading, and so a material property.

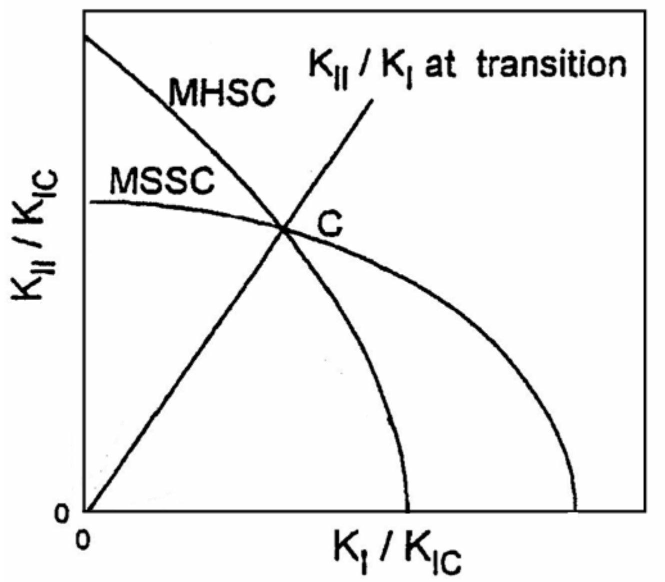
Some authors proposed that the crack would grow in the normal direction to the plane of the maximum circumferential stress [3, 79, 111].

Under pure *Mode I*, the maximum hoop stress criterion predicts no turning when $K_I = K_{Ic}$. Under pure *Mode II*, the maximum turning angle, φ_c, is $-70.5°$ which corresponds to a *SIF*-value equal to the fracture toughness under *Mode II*, i.e. $K_{II} = K_{IIc} = 0.866K_{Ic}$.

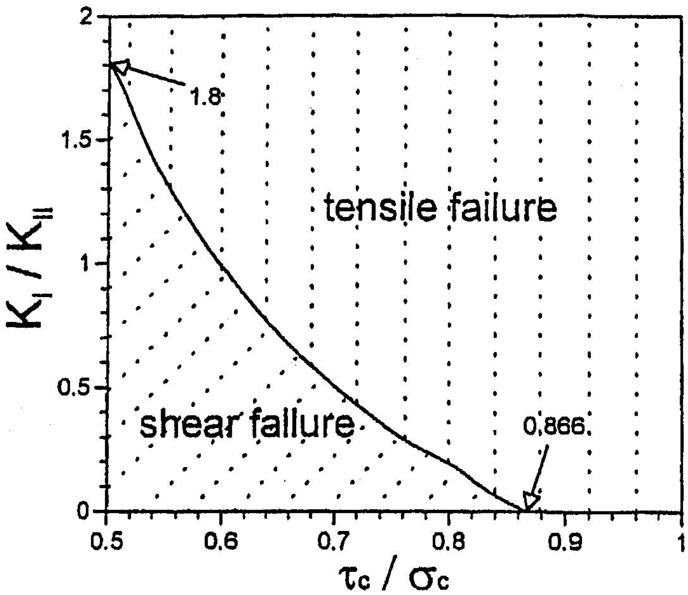
It was observed that under *Mode II* loading the crack does not follow the maximal tensile stress as Erdogan & Sih found out under *Mode I*. Instead the crack growth followed the maximal shear stress [27].

Under pure *Mode I*, the *MSS* criterion predicts a maximal turning angle of $\varphi_c = 70.5°$ and a corresponding value for the *SIF* under *Mode I* equal to $K_I = 2.6(\tau_c/\sigma_c)K_{Ic}$. Under pure *Mode II*, the criteria predicts a turning angle equal zero and $K_{II} = (\tau_c/\sigma_c)K_{Ic}$.

The maximal hoop stress criterion leaves reliable predictions for structures under a *MM* loading ratio near *Mode I*, $K_{II}/K_I \sim 0$. On the other hand, the maximum shear stress criterion predicts accurately fracture of components charged with a *MM* ratio close to *Mode II*, $K_I/K_{II} \sim 0$. Therefore, in order to provide a prediction for the whole *MM* domain, it seems logical to mix the two previous criteria, as represented in Figure A.3 [31]. However, the mixed mode ratio characterizing the transition from tensile controlled growth to shear is not defined, but it is accepted to be dependent on the material [112].



a)



b)

Figure A.3. Schemas showing a) the competition of MHS and MSS criterion b) the type of failure determined by K_I/K_{II} versus the material ductility τ_c/σ_c

Shih [30] extended the Hutchinson, Rice and Rosengreen theory for small scale yielding to include mixed mode under plane strain conditions, in order to extend the maximum circumferential stress theory for elastic-plastic crack initiation [26]. For this purpose, the elastic mixed mode parameter, M_e, and the plastic mixed mode parameter, M_p, were defined.

$$M_e = \frac{2}{\pi} arctg \lim_{r\to\infty} \left| \frac{\sigma_{\varphi\varphi}(r,\varphi=0)}{\sigma_{r\varphi}(r,\varphi=0)} \right|_{r>r_{pl}} = \frac{2}{\pi} arctg \lim_{r\to\infty} \left(\left| \frac{J_I}{J_{II}} \right| \right)^{1/2}_{r>r_{pl}} = \frac{2}{\pi} arctg \left(\frac{K_I}{K_{II}} \right) \quad \textbf{(A.2.1)}$$

$$M_p = \frac{2}{\pi} arctg \lim_{r\to 0} \left| \frac{\sigma_{\varphi\varphi}(r,\varphi=0)}{\sigma_{r\varphi}(r,\varphi=0)} \right| \quad \textbf{(A.2.2)}$$

Shih developed a curve, which represents the crack turning angle, φ_c, in terms of the elastic mixed mode parameter for different hardening exponents [27]. By means of this curve and the value of M_e, equation A.2.1, it is possible to compute the turning angle, φ_c.

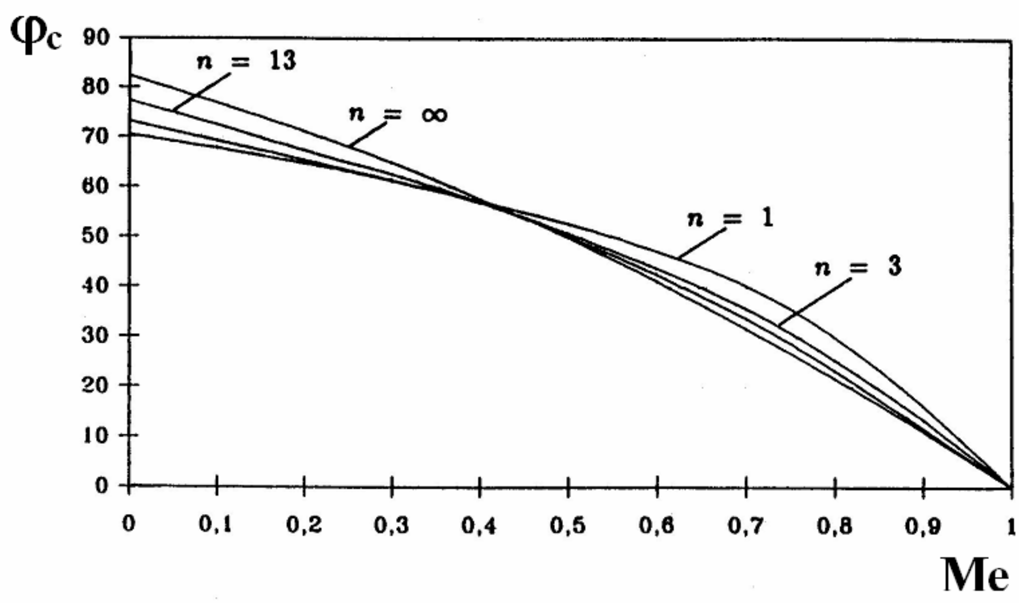


Figure A.4. Crack turning angle as a function of the elastic mixed mode parameters for different hardening exponents n [26]

Sumi et al. [64] performed a similar analysis as Cotterell and Rice, including one additional higher order term in the stress field expansion, i.e. the 3rd term. Therewith, they obtained additional information about whether the crack was approaching a region of greater stability or instability [80].

From their studies, four possible stability situations were identified dependent on two defined stability coefficients, β^* which represents the normalised *T*-stress and γ^*. The exactly mathematical definition of this criterion as well as its parameters is described in reference [80].

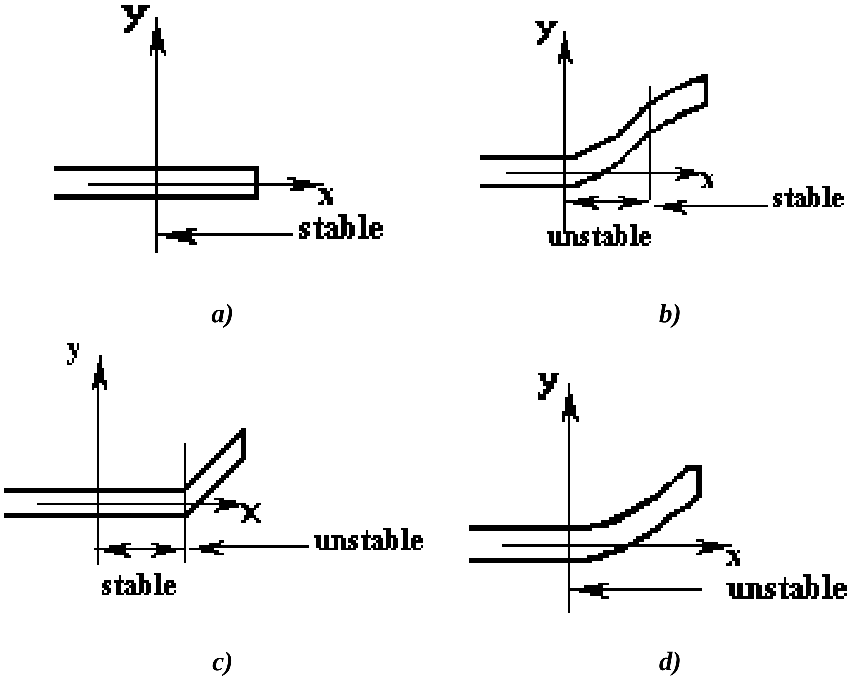


Figure A.5. a) stable crack $\beta^* < 0$; $\gamma^* < 0$; b) re-stabilized crack $\beta^* > 0$; $\gamma^* < 0$; c) prediction of instability $\beta^* < 0$; $\gamma^* > 0$ d) unstable crack $\beta^* > 0$; $\gamma^* > 0$

The stability coefficient γ^* was used to explain the stability of the crack path. $\gamma^* < 0$ cause the crack to remain stable despite $T > 0$.

This criteria shows that the criteria which only uses the second order term on the William's expansion series, would predict that all crack path with positive β^* are unstable, which clearly misjudge a large group of experimental stable paths.

A.2.2 Crack turning criteria for 3D-structures

For the determination of crack turning and twisting angles φ_c and ψ_c, Pook [104, 105] proposed that the crack turning angle is principally due to *Mode I* and *II*, so that φ_c should be calculated according to the *MTS*-criterion, i.e. equation 2.4.1. The twist angle, ψ_c, was defined using equation A.2.3 and the comparative stress intensity factor $K_{vI,II}$, that is,

$$\tan^2\psi_c = \frac{2K_{III}}{K_{vI,II}(1-2\nu)} \quad \textbf{(A.2.3)}$$

$$\mathrm{Kv}_{\mathrm{I,II}} = \frac{0.83K_I + \sqrt{0.4498K_I^2 + 3K_{II}^2}}{1.5} \quad \textbf{(A.2.4)}$$

Schöllman [22] made the assumption that crack growth develops perpendicular to a special maximum principal stress, σ'_I. Thus, the turning angle φ_c is found maximizing σ'_I. The twist angle, ψ_c, was defined as:

$$\psi_c = \arctan\left(\frac{2\tau_{\varphi z}(\varphi_c)}{\sigma_\varphi(\varphi_c) - \sigma_z(\varphi_c)}\right) \quad \textbf{(A.2.5)}$$

A.3 Experimental tests

As already introduced in chapter 4.3.2, the *2SP*-specimens were reinforced around the clamping area by means of dopplers. Its dimensions are plotted in Figure A.6.

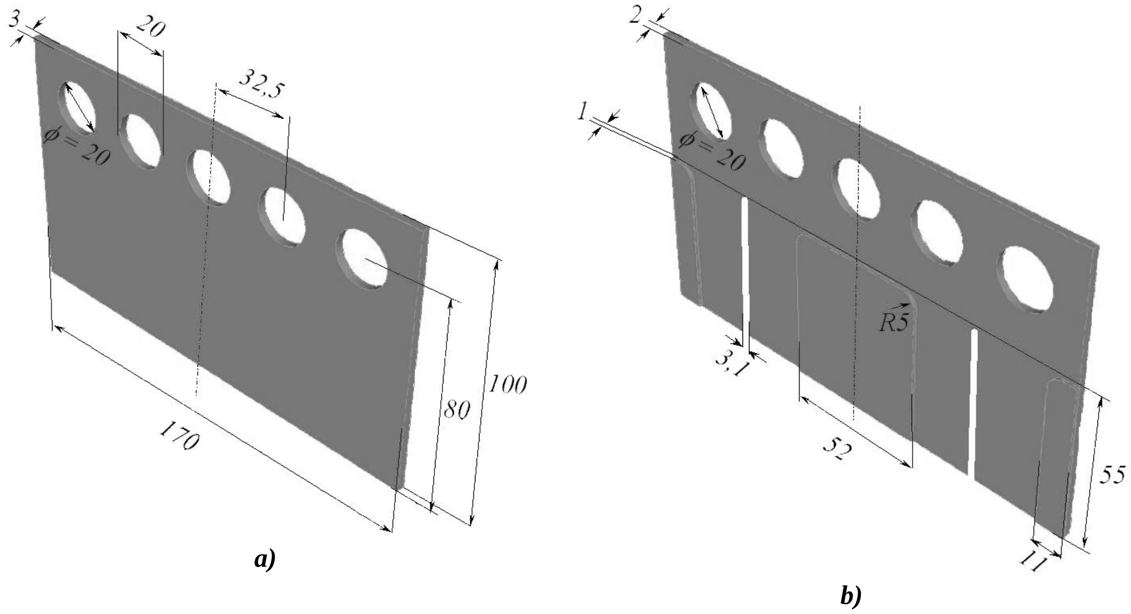


Figure A.6. Doppler dimensions a) rear and b) front

A.4 Other modelling tools

SAMCEF®

SAMCEF® is a tool developed by the SAMTECH Group S.A. It computes the *SIF* in a linear elastic analysis for both two and three-dimensional problems using nodal displacements values. Crack direction is computed by means of the Sih criterion. The transition zone between plane strain and plane stress conditions in *3D*-models can be imposed or evaluated automatically and the *J*-integral extraction capability is available for *2D*-problems. One of the advantages of this code compared with most commercial *FE*-programs is the capability to compute the *SIF*s in all three modes, i.e. K_I, K_{II} and K_{III} [W7].

ABAQUS®

ABAQUS® is a multi-purpose finite element analysis program, which can analyse stress/displacement distributions, manufacturing processes and also explore concepts for a new design in different fields. The release Version 6.5 of ABAQUS/CAE® includes modelling and post processing capabilities for fracture mechanics analyses. It can calculate stress intensity factors and contour integrals. It is implemented with different material damage and failure models and crack growth can be simulated by means of cohesive elements [W8].

BEASY®

BEASY® is a tool developed by the Computational Mechanics Group. The fracture mechanics part includes fatigue crack growth models, which support a wide range of crack growth and retardation models. The calculated *SIF* data can be exported to other software and user defined fatigue models can be specified [W9].