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Spatial and Performance Optimality in Power Distribution Networks

Lingen Luo, Giuliano Andrea Pagani, and Marti Rosas-Casals

Abstract—Complex network theory has been widely used in vulnerability analysis of power networks, especially for power transmission ones. With the development of the smart grid concept, power distribution networks are becoming increasingly relevant. In this paper, we model power distribution systems as spatial networks. Topological and spatial properties of 14 European power distribution networks are analyzed, together with the relationship between geographical constraints and performance optimization, taking into account economic and vulnerability issues. Supported by empirical reliability data, our results suggest that power distribution networks are influenced by spatial constraints which clearly affect their overall performance.

Index Terms—Complex networks, failure analysis, optimization, power distribution, reliability.

I. INTRODUCTION

THE need of modeling systems with many interacting components organized in nontrivial topologies has given birth to a new approach of analyzing interconnected systems known as complex networks [1]. An example of such a complex system is the power grid. Power grids, especially transmission networks, have been widely studied applying the complex network approach. Basic topological characteristics, statistical global graph properties, and vulnerability (or robustness) analysis have been performed on many power grids in different parts of the world [2], being the vulnerability characteristic of the power grid the main motivation for these studies. In fact, topological properties play an important role in shaping the performance of power grids [3]–[6]. As a result, there is an increasing interest in analyzing structural vulnerability of power grids by means of complex networks methodology.

However, up to now most of the scientific literature using complex networks approach to power grids has been focused on transmission (i.e., high voltage) networks, while little attention has been paid to distribution grids, with some minor exceptions such as in [7]. As the authors in this reference stress, with the development of the smart grid, the main role of high-voltage

transmission networks may change, with distribution networks gaining more and more importance and even requiring substantial upgrade to adapt to this new framework. Most of the research that focuses on modeling the power grid considering network science principles uses simple graph models with sometimes the use of basic properties such as direction and weight. However, as shown in [2], [8], and [9], scholars tend to miss an important characteristic of any power grid: its spatial features. In its most restricted framework, spatial properties are basically the coordinates of the generators, transformers, and substations, and the length of power cables. Here, we focus on understanding the spatial aspects of electricity distribution networks and their relationship with topological (i.e., graph theory) features.

One key aspect of many practical engineering problems concerns optimization. Optimization can be applied also in the networks context, with the objective of identifying optimal topologies, network models, and/or dynamics, such as flow or traffic [10]–[12]. Optimization in power systems is also an important topic in dispatching power generation [13], power distribution network reconfiguration [14] or placing phasor measurement units and optimal control strategy for power system facility and stability, which covers static and dynamic analysis of power systems [15]. Two key issues should be taken into consideration in the optimization of power grids once stability is achieved: performance (associated with vulnerability and measured in terms of connectivity) and cost. Furthermore, in order to assess the performance of a power system from an engineering point of view, empirical reliability data (i.e., total loss of power, energy not supplied or restoration time) must be considered [16].

The first question we address is how the performance of power distribution networks competes with its wiring cost defined as the sum of the Euclidean length of power cables [17]. We use two systematic ways to modify the structure of the networks and minimize the wiring cost function: edge exchange (EE) [18] and vertex swapping (VS) [19] shuffling methods. We adopt a Monte-Carlo (MC) scheme with simulated annealing [20], using the wiring cost as our Hamiltonian. The second question we address is to assess the robustness of power distribution networks when modeled as spatial network and under cascading failures schemes. Since cascading failures are an important vulnerability issue of complex systems [21]–[24], we carry out a cascading failure simulation based on the classical model by Motter and Lai [21] but incorporating the distributed flow property of power distribution networks, not found in other studies. The addition of this distributed flow property in our simplified model is essential to capture specific aspects of the power distribution grid and consequently, to obtain results that are valid also from an electrical point of view. In this sense, this approach based on the

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L. Luo is with the Department of Electrical Engineering, Shanghai Jiao Tong University, Shanghai, 200240, China (e-mail: llg523@sjtu.edu.cn).

G. A. Pagani is with the Distributed Systems Group, Computer Science Department, Rijksuniversiteit Groningen, Groningen AG 9747, The Netherlands, and also with Alliaander N.V., Arnhem, EA 6835, The Netherlands (e-mail: g.a.pagani@rug.nl).

M. Rosas-Casals is with the Sustainability Measurement and Modelling Laboratory, Universitat Politècnica de Catalunya—BarcelonaTech, 1, Terrassa 08222, Spain (e-mail: rosas@mmt.upc.edu).

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TABLE I
POWER GRID DATASETS

Acronym/Spatial scale	n	m	Type	Population density (hab/km ²)
UCTE/Europe	2777	3762	–	–
S1/Spain	519	557	Urban	1854
S2/Spain	240	263	Urban Cluster	430
N1/The Netherlands	451	492	Rural	98
N2/The Netherlands	473	505	Urban Cluster	323
N3/The Netherlands	241	254	Rural	191
N4/The Netherlands	287	305	Rural	144
N5/The Netherlands	221	231	Rural	93
N6/The Netherlands	193	209	Rural	288
N7/The Netherlands	957	1095	Urban	2497
N8/The Netherlands	371	391	Rural	191
N9/The Netherlands	223	237	Rural	188
N10/The Netherlands	204	207	Rural	280
N11/The Netherlands	271	279	Urban Cluster	367
N12/The Netherlands	480	509	Urban Cluster	351

spatial graph theory, mathematical optimization, and cascading failure models with flow redistribution, complements other very promising results aimed at reproducing realistic network topologies under performance (i.e., network cost, network losses, and reliability) and environmental constraints [25]–[27].

The remainder of this paper is organized as follows. Section II introduces the power networks dataset and their spatial model. In Section III, the mathematical methods used in the paper are described. Section IV gives the results of the analysis considering topological and spatial issues. An empirical reliability assessment, using real electric distribution quality parameters [e.g., system average interruption duration index (SAIDI)] is presented in Section V in order to validate our results. Section VI highlights the key findings from this paper and draws conclusions.

II. DISTRIBUTION POWER GRID DATASETS

We have analyzed 14 distribution and 1 transmission power grids (this last used for comparative purposes). Distribution power grids come from two samples in Spain (S) and the rest from The Netherlands (N). Basic information on these networks is presented in Table I, with the acronym and spatial scale, number of nodes (n), being them substations, transformers and alike, and number of power cables connecting them and considered as edges (m). Finally, we present population density (inhabitants per square kilometer) to characterize the urban, urban cluster, or rural character of the network (with density higher than 1500 hab/km² as the urban limit, 300 hab/km² as the limit for urban cluster and less than 300 hab/km² for rural population in EU, following [25]).¹

From a graph theory perspective, a network can be abstracted as an undirected graph $G = (n, m)$, consisting of two sets n and m , such that $n \neq \emptyset$ and m is a set of unordered pairs of elements of n . The elements of $n \equiv n_1, n_2, \dots, n_N$ are the nodes (or vertices)

¹The Dutch population density data are from the Dutch official statistical authority *Centraal Bureau voor de Statistiek* (<http://www.cbs.nl>). The Spanish data come from the Spanish official statistical authority *Instituto Nacional de Estadística*. [Online]. Available: <http://www.ines.es>



Fig. 1. Example of an urban power distribution network. Red lines are power cables (i.e., edges) and blue dots are substations (i.e., nodes). Inset: spatial graph representation of the same network.

of the graph G , while the elements of $m \equiv \{m_1, m_2, \dots, m_K\}$ are the links (or edges). We consider all the substations and transformers equal and they are abstracted as nodes while electric cables are abstracted as edges [29].

Fig. 1 shows an example of a power distribution network modeled as an undirected and unweighted network. The inset shows the same network as spatial graph, where nodes and edges are geographically located. When modeling power grids as spatial networks, important information such as the geographical position of nodes and the cable length must be kept. These data, not shown in Table I, complete the overall set of parameters considered in the analyses presented in this paper.

III. METHODS

In order to assess how spatial constraints are related with the overall performance of distribution networks, we use the following approach. First, we characterize the topology of these networks by means of several metrics commonly used in complex network analysis. Additionally in this first step, we present cumulative cable distance probability distributions to distinguish, at a minimum level, the spatial character of these networks. Second, we adopt a rerouting strategy and simulated annealing method to see how spatial constraints affect economy (i.e., cost) optimality. In this sense, we finally perform a cascading failure analysis, incorporating distributed flowing to those networks for which we have impedance values according to their length (i.e., S1 and S2).

A. Topology

Graph theory offers a well-established battery of metrics to characterize the topology of any networked system [30]. Among these, average degree $\langle k \rangle$, average path length L , average clustering coefficient C , and graph density $\langle \rho \rangle$ (as percentage) will be used to evaluate the 15 power networks from a pure topological approach.

The degree k_i of a node i is the number of edges incident to that node. Here, we use 1) the average degree $\langle k \rangle$ as a measure

TABLE II
BASIC TOPOLOGICAL RESULTS

Network name	$\langle k \rangle$	L	C	$\langle \rho \rangle$
UCTE	2.71	22.7	0.07	0.10
S1	2.14	24.6	0.01	0.41
S2	2.19	15.8	0.00	0.92
N1	2.21	11.0	0.00	0.48
N2	2.15	17.0	0.01	0.45
N3	2.11	11.6	0.00	0.88
N4	2.18	12.7	0.01	0.74
N5	2.12	10.2	0.00	0.95
N6	2.17	9.2	0.00	1.13
N7	2.34	9.8	0.00	0.24
N8	2.11	15.0	0.00	0.57
N9	2.16	10.8	0.00	0.96
N10	2.05	15.6	0.00	1.00
N11	2.08	14.7	0.00	0.76
N12	2.15	13.1	0.00	0.44

of the global connectivity of a network and 2) the cumulative degree probability distribution $P(k)$ over the whole network.

A measure of the typical separation between two nodes in the graph is given by the average path length L , defined as the mean of geodesic lengths d over all couples of nodes

$$L = \frac{1}{n(n-1)} \sum_{i,j \in n, i \neq j} d_{ij}. \quad (1)$$

In a network, clustering can be essentially seen as the level of transitivity (i.e., the presence of a high number of triangles). It can be quantified as the fraction of connected triples of nodes (triads) which also form triangles. The overall level of clustering in a network is measured by as the average of the local clustering coefficients C_i over all vertices n [31]

$$C = \frac{1}{n} \sum_{i \in n} C_i. \quad (2)$$

The fraction of possible edges that exist in a graph is known as graph density

$$\rho = \frac{2m}{n(n-1)}. \quad (3)$$

Finally, cumulative cable length probability distributions have been obtained for every network to define the spatial character of these networks.

B. Rerouting Strategy

Real-world spatial networks including electric circuits, the internet, power grids, and neuronal networks, face the challenge of balancing performance and cost [10]–[15]. If connectivity is not a constraint, redundancy can be increased and the performance of a network can be greatly enhanced using sufficiently many edges. However, more edges imply more resources in all practical situations, and, thus, we would expect a competition between these two antagonist measures. In this sense, low clustering coefficients (see Table II, Section IV) is the expected outcome of a distribution of electric power which avoids triangles in order to

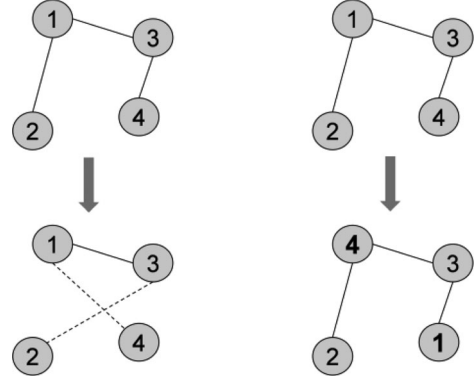


Fig. 2. EE shuffling (left) and VS shuffling (right). Dashed edges (left) and black over white nodes (right) denote changes after each kind of shuffling.

reach as much population as possible with minimum cost (total power cable length) and where energy production is centralized.

Here, we use the same procedure as in [17]–[19] to investigate the role, if any, of the wiring cost in the performance of a power distribution network. Two systematic methods to shuffle each sample are used: 1) EE and 2) VS. In the EE method presented in [18], vertices of two randomly selected edges exchange their partner vertices. The degree (1) of each vertex remains unchanged and the positions of all vertices remain the same. In the VS method presented in [19], two randomly chosen vertices simply exchange their positions while preserving all the connections. In this case, the connection structure of the network never changes while the distances are altered as we repeat the process (see Fig. 2).

In the traditional power engineering domain, one distinctive evolving feature of power distribution network is its adaptive capacity to meet load demand: any power distribution network will be extended where there is power demand. Although there is a general expansion planning at the initial stage, the changing development of urban/rural infrastructure, industry and human migration, to name a few, complicate the network evolution and nonoptimal connections may appear in terms of wiring cost. In order to assess the optimality of the wiring cost, defined here as the sum of all cable lengths in terms of Euclidean distances, a global optimization problem needs to be solved. There are generally two kinds of methods: analytic and iterative. Here, we adopt the simulated annealing iterative method because as a numerical method it allows including the shuffling procedure in a more convenient way. Inspired by the method used in [18], we apply an MC scheme controlled by a given “temperature” T (introduced only as an updating control parameter for the algorithm). The fully random shuffling of the network using either EE or VS method corresponds to the MC simulation at $T = \infty$, and it has been denoted as EE(inf) and VS(inf), respectively. Simulated annealing technique, starting from $T = \infty$ and slowly decreasing it until $T = 0$ is reached, has been used to get the optimal value denoted as EE(0).

C. Flow Redistribution Algorithm

Flow redistribution mechanism is a key feature in cascading failures studies of networks [21], [22]. Although these studies

are not realistic representations of complex cascading phenomena, cascading models offer the possibility to both simplify assumptions and computational load, and to devise useful approaches to improve the realism of the model in further developments. In this sense, in Motter's work [21], the network flow is quantified by the shortest path length, which simply counts the number of edges connecting a consecutive pair of nodes. On the other hand, in Crucitti's work [22], a weighted network is used, and the shortest path is the path with the minimal sum of weights of edges between two nodes. However, a power network is a flow-based network constrained by Kirchhoff's law, where power is transmitted from power plant to consumer not just along the shortest path but also using remaining paths following not only geodesic ones but also those where the aforementioned law applies, so almost all power cables are involved. The inclusion of electrical laws together with topological analysis is a novel and additional point we consider in our analysis in comparison with the traditional network analysis performed in the literature.

Assuming unit current flowing through one source–target pair of nodes, the load on an arbitrary line $l \equiv (i, j)$ is the current along that line: $p_{ij} = I_{ij}^{(st)}$. Accordingly, the load on an arbitrary node (i) is the current passing through that node: $p_i = I_i^{(st)}$. The relationship between line load and node load is

$$I_i^{(st)} = \frac{1}{2} \sum_j I_{ij}^{(st)}. \quad (4)$$

Based on our previous works [32], [33] on equivalent impedance which takes into account the magnitude of voltage drop between source node i and target node j , for I units of current and for a given source–target pair, the current flowing through line l_{ij} is

$$I_{ij}^{(st)} = A_{ij} (V_i - V_j) = A_{ij} I Z_{eq} \quad (5)$$

where A is the adjacency matrix of a network (i.e., a square $n \times n$ matrix such that A_{ij} is one when there is an edge from node i to node j , and zero when there is no edge [32]), V is the unknown column voltage vector which will be replaced by I and Z_{eq} , and Z_{eq} is the equivalent impedance defined as

$$Z_{eq} = z_{ii} - 2z_{ij} + z_{jj} \quad (6)$$

where z_{ij} is the i th, j th element of the impedance matrix [35].

Equation (5) illustrates that current flowing through the power network involves almost all the cables (edges) uniquely determined by the network topology. Furthermore, the impedance also implies a spatial factor as the cable length is one of the most important parameters when calculating impedance.

In order to incorporate this flow-based consideration we apply a cascading failure model on power distribution networks carrying distributed flow according to the classical model by Motter and Lai [21]. Each node is characterized by its capacity defined as the maximum load that the node can handle. The capacity Cap_i of node i is proportional to its initial load

$$\text{Cap}_i = (1 + \alpha) L_i(0) \quad (7)$$

where $\alpha \geq 0$ plays the role of a tolerance parameter, and $L_i(0)$ is the load of each node for the intact network.

From the power engineering point of view, the existence of redundant lines in distribution networks would lead to a possibility that some nodes may form particular motifs such as triangles. When this happens, node capacity according to (7) cannot capture this feature. Based on this consideration, a condensation operation, as addressed in [36], will be performed before the node capacity calculation. In summarized form, this operation includes three steps: first, a directed graph model is built with flow directions obtained from the power flow direction according to (5); second, a condensed graph, characterized by the absence of cycles, is generated following a condensation operation given in [36]. Finally, the capacity of each node in the condensed graph is calculated using (7). The cascading failure is modeled in the following way: when a node on the network fails (i.e., understood in this paper as a random failure), this node is removed from the network. Then the flow undergoes redistribution, and consequently, the loads on each surviving node change. If the node has a relatively small load, its removal will not cause a significant unbalance of global loads, and subsequent cascading failures are unlikely to occur. On the contrary, when a failing node has a large load, its removal will probably affect the loads of the rest of the nodes. If the new load of any surviving node exceeds its capacity (i.e., $L_i \geq \text{Cap}_i$), then that node will also fail which leads to a further redistribution and possible further failures until a certain time when all the loads of the remaining nodes are lower than or equal to their capacity. The essential difference here with respect to previous models [21]–[23] is the application of a flow redistribution algorithm, which includes distributed flow involving all nodes and branches of the network. The distributed flow is a simplified dc power flow calculation, where generation and load data for each network is not fully available. In this sense, the fundamental electrical features of power networks are not ignored. The flow computation has been performed on the S1 and S2 samples where information of generator attachment point to the network and loads were available.

Following the notation introduced in [21], the resilience of a network is also quantified in terms of the fractional size of the surviving largest connected component after the cascade ends

$$\text{Res} = N'/N \quad (8)$$

where N' is the number of nodes belonging to the largest network component after the cascade occurs and N is the original number of nodes. For power systems, the largest connected network measures the surviving power consumption.

IV. RESULTS

A. Topology

Topological metrics defined previously for each network under consideration are shown in Table II. Both transmission and distribution networks are very sparse graphs (i.e., low density $\langle \rho \rangle$) with very similar topological values. The slightly higher value of the average degree $\langle k \rangle$ and clustering coefficient C for the UCTE transmission network give us a hint of the more

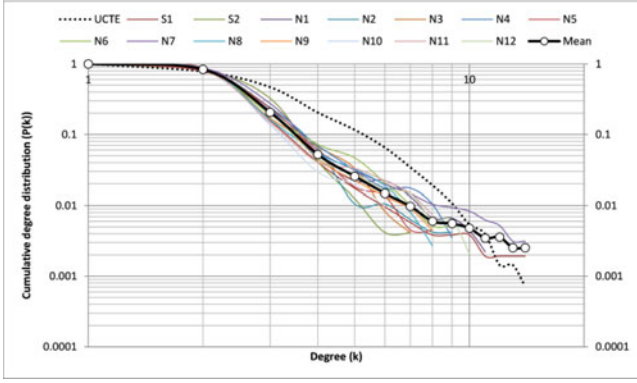


Fig. 3. Cumulative probability degree distribution of transmission (dashed line) and distribution (colors and solid line, average) networks.

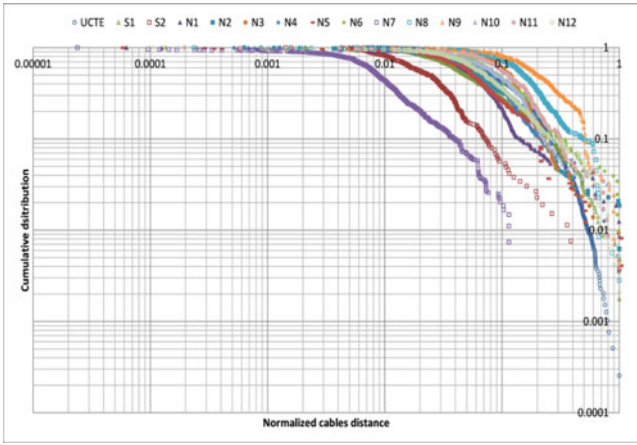


Fig. 4. Cumulative normalized power cable distance distribution of each network.

radial-like structure of the distribution grids, while transmission networks present a much more meshed topology.

Degree cumulative probability distributions give us an insight of the general properties of the network and allow us to classify them. Fig. 3 shows a log-linear cumulative node degree distribution of each network. According to the literature [2], we observe that the degree distribution of UCTE-ENTSO network follows an exponential distribution (dashed line) while the rest of distribution networks do not present this trend. For these networks, the probability of having intermediate degrees is lower than in transmission networks (see Fig. 3, solid black line as the average). This feature implies a remarkably different topology, much more uniform in terms of low degrees, but with the presence of much more highly connected nodes (see Fig. 1) that act as hubs for the circular topology of these networks.

A distinct spatial feature of our power distribution dataset is cable length. Similarly to the previous statistical analysis for the degree, the cumulative probability distribution of cable length is reported in Fig. 4, where cable length has been normalized to the maximum length present in each network sample. It is shown that while some distributions follow exponential functions, especially having consistent exponential effects in the tails [37],

TABLE III
KS TEST OF FITTING FUNCTIONS

Network name	Power law	Power law + exp. cutoff	Log- norm	Stretched exp.	Exp.	%
S2	0.0546	0.1224	0.0957	0.1109	0.1483	0.8212
N7	0.0363	0.0588	0.0539	0.0613	0.1891	0.7665
S1	0.0809	0.0883	0.0863	0.0867	0.1667	0.4930
N1	0.0906	0.0673	0.0756	0.0694	0.1146	0.0455
N2	0.0348	0.1233	0.1047	0.1186	0.3843	0.3271
N3	0.0715	0.4477	0.3452	0.341	0.5105	0.0741
N4	0.0826	0.6163	0.3449	0.3408	0.6163	0.1004
N5	0.0685	0.401	0.3847	0.3847	0.4236	0.1020
N6	0.0707	0.0791	0.0791	0.0791	0.2303	0.4631
N8	0.128	0.0937	0.0987	0.0937	0.2218	-
N9	0.0752	0.0887	0.0677	0.0782	0.1316	-
N10	0.075	0.0687	0.0507	0.0634	0.1897	-
N11	0.0508	0.043	0.0428	0.0428	0.0885	-
N12	0.0735	0.0397	0.0538	0.045	0.1496	-

some other distributions deviate from this behavior, especially S2 and N7.

In order to analytically detect these differences, we follow the methodology described in [38], which offers the possibility to statistically fit a heavy-tailed function to the tail of a probability distribution. Given an observed dataset and a hypothesized distribution from which the data are drawn, one fundamental step is detecting whether the hypothesis is a plausible one. The Kolmogorov–Smirnov (KS) statistic is used as a goodness-of-fit test between real data and synthetically generated data. Results for five fitting functions (i.e., power law, power law plus exponential cutoff, log-normal, stretched exponential, and exponential) are shown in Table III. The last column (%) represents, for each network, the ratio of occurrences in the tail of the distribution over the total data points.

Bold values denote those networks that more likely follow the distribution functions of their respective column. We observe that the KS test value for networks S1, S2, and N1–N7 are small enough so to be considered well fitted by a power-law distribution. Complementarily, the value in the percentage column (%) for S2 and N7 suggests a much higher ratio of occurrences in their power-law tail than that of S1 and N1–N6. An interesting pattern arises for S2 and N7 power distribution networks: from the cumulative degree distribution in Fig. 3 and modeling these networks just as undirected graphs, we cannot distinguish S2 and N7 from the rest of the networks since they all generally follow similar exponential distributions. But when the spatial properties of these networks are considered, we notice that cable lengths in S2 and N7 follow power-law distributions and with a much more significant percentage of occurrences well fitted by this function than the rest, which also follow this same distribution.

B. Rerouting Strategy

With this strategy, the first aspect that we would like to address is the tradeoff between optimality in terms of cost and performance. As we have said, if there is no spatial constraint, redundancy can be increased and the performance of a network can

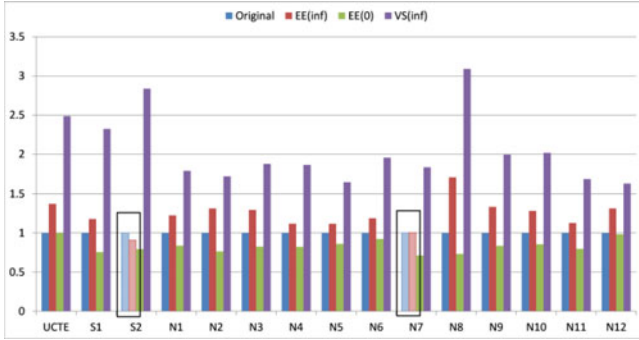


Fig. 5. Wiring costs (normalized) of networks obtained by fully random vertex and edge shuffling. All networks present optimal or almost optimal configurations for EE(0) except S2 and N7, highlighted in black boxes, which is not optimal under EE(inf) as well. UCTE transmission network has been used with comparative purposes.

be greatly enhanced using sufficiently many edges. However, more edges imply more resources in all practical situations, and thus we would expect a competition between performance and cost (simply measured, in this case, as the sum of the Euclidean lengths of edges). Therefore, we have simulated a rerouting process by means of simulated annealing to find out the optimality under spatial constraints for our sample networks.

The simulation was performed 50 000 times for each network and the average results for the 15 networks are shown in Fig. 5, where EE(inf), EE(0), and VS(inf) have been all normalized using the original wiring length cost. We observe that all networks can be spatially optimized by method EE(0), but VS(inf) and EE(inf) methods are not able to optimize the current spatial topology of the distribution networks except for S2 and N7. The rerouting simulation shows S2 and N7 having different optimality characteristics compared with other power networks (both transmission and distribution ones) which is also discriminated in the spatial statistical analysis shown in Fig. 4. This fact suggests that a correlation between connectivity for power delivery and economy optimality (keeping the connectivity with as less connections as possible) exists when modeling power distribution networks as spatial networks. In other words, power cables' routing strategy of power distribution networks, which is largely dictated by the geographical constraints, plays an important role in the behavior (or performance) of the networks.

C. Cascading Failures Model

Vulnerability is one important property to consider in network performance. Here, we focus on the analysis of cascading failures spreading characteristic on power distribution networks in which the connectivity of nodes and wiring of edges are constrained by geographical factors. Unfortunately, the reliability data are available only for the S1 and S2 samples (see Section V). Therefore, the analysis described here is only performed for these two networks. The simulation is carried out as follows.

- Step 1) Obtain the condensed graph model as addressed in Section III-C.
- Step 2) Generate the load of each bus by normal distribution.

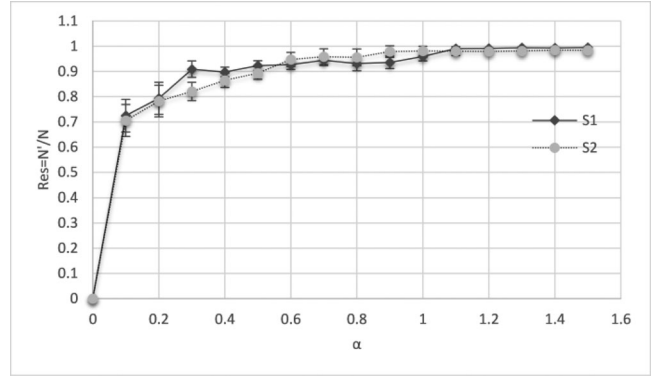


Fig. 6. Cascades triggered by random removals in S1 and S2 networks.

- Step 3) Calculate the capacity according to (7) and setting the initial value of α to 0.0 for each bus.
- Step 4) Random removal of a bus from the network.
- Step 5) Find the largest island among the fractions caused by the node removal with a generator (only an island with a generator has a source for energy dispatching).
- Step 6) Flow redistribution following (5) in the island found in step 5. Compare the updated load with the capacity for each bus. Remove the buses whose load exceeds their capacity.
- Step 7) Calculate the fractional size of the surviving largest connected component according to (8). If the simulation time is less than a run-step control parameter, go to step 5. Otherwise, go to step 4.

This process being an MC-based simulation, a run-step control parameter is used in order to balance accuracy and computational time. For the above procedure, this balance has been found to perform adequately using a 1000 runs for both S1 and S2. Average values and corresponding standard deviations of Res against α are reported in Fig. 6. It shows that the power distribution networks under study are more vulnerable than we thought when geographical features are taken into consideration. Even one node removal will cause a cascading failure when the capacity control parameter α is small (less than 0.5), which is consistent with the recent research published in [39]. Another important observation is that S2 is more fragile than S1 when α is less than 0.6. (Such a high tolerance is not meaningful in the current practice and the distribution network operators are already trying to find ways to mitigate the overloads caused by additional electricity demand [40] to achieve a balance between function and cost.) Together with the cost optimality analysis of the previous section, we observe how spatial properties (i.e., geographical constraints) of power networks play an important role in the analysis of their overall performance, especially their vulnerability.

Finally, we would like to investigate the cascading failure propagation of power distribution networks with different spatial constraints. Again we use the rerouting strategy to generate different spatial models for S1 and S2. The cascading failure simulation described in steps (1)–(6) is performed after each VS shuffling. Here, only VS method is taken into consideration

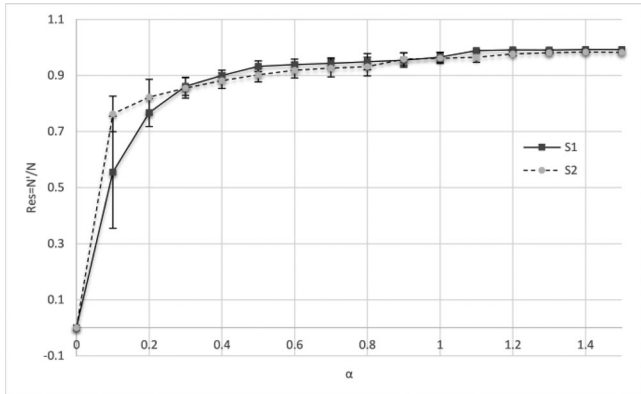


Fig. 7. Cascades triggered by random removals in S1 and S2 networks under VS shuffling scenarios.

because it keeps the connectivity (i.e., impedances) unchanged. On the contrary, for the EE method, connections will be changed after shuffling, and only Euclidean distance can be calculated and used. Fig. 7 shows the simulation results of cascading failure propagation triggered by random removals in S1 and S2 in different scenarios after VS shuffling. The lines (dash and solid) are the average value and the error bars denote the standard deviation.

Using VS shuffling method (recall it as the most optimizing procedure from Fig. 5), S2 increases its robustness (i.e., higher Res) to random removal compared to S1, (i.e., average lines in the low load capacity region, $0.1 < \alpha < 0.3$). Since S2 has not achieved its optimal topology under its original geographical constraints from both economy and performance points of view, a new more optimal topology, stemming from the vertex shuffling procedure, would allow a remarkable increase in Res. This is not the case, though, with S1, which nearly approaches its maximum resilience capacity of cascading failure propagation especially for $0.1 < \alpha < 0.3$ (i.e., it already has optimal or almost optimal configuration, as shown in Figs. 5 and 6).

V. RELIABILITY VALIDATION

The empirical validation of the consequences of these results on the performance of distribution networks has never been an easy task. A common approach has been the correlation of major events (i.e., equivalent time of interruption, energy not supplied, restoration time, power loss, etc.) with some topological characteristic of the network (e.g., average degree $\langle k \rangle$ in Table II). This approach has been shown useful in order to, for example, segregate European power transmission networks into fragile and robust ones [5]. Although a similar procedure can be applied to distribution networks, access to data is highly restricted. Whereas transmission system operators are forced to inform UCTE/ENTSO about major events on their grids, distribution systems operators are not. This fact poses a difficult drawback on this kind of validation processes.

For this study, we have had access to electric distribution quality data for only two distribution networks: S1 and S2. An appropriate reliability measure would be the average consequence

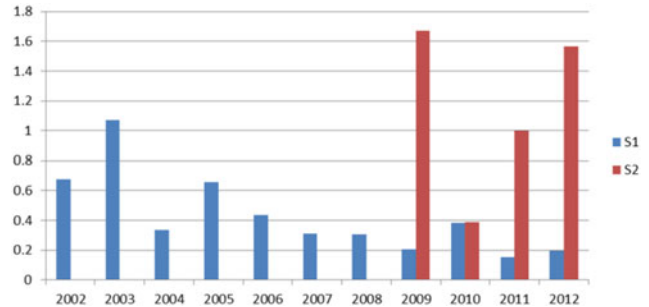


Fig. 8. Evolution of TIEPI values for S1 and S2 distribution networks.

of a fault (i.e., sum of all customer interruption durations over the number of faults), since fault rates from the same equipment depend on maintenance routines, the skill of cable joiners, etc. Due to the lack of this type of data, here, we use TIEPI values to measure the quality of electricity supply instead. TIEPI index, Spanish acronym for equivalent time of interruption of the installed capacity in medium voltage (i.e., $1 \text{ kV} < V < 36 \text{ kV}$) and similar to the English SAIDI [41], is a numerical index that measures the effect of the number and/or duration of interruptions affecting customers longer than three minutes. It is defined as

$$\text{TIEPI (SAIDI)} = \frac{\sum U_i N_i}{N_T} \quad (9)$$

where N_i is the number of customers, U_i is the annual outage time for location i , and N_T is the total number of customers served.

Fig. 8 shows TIEPI values for distribution networks S1 and S2 and for several years. As we can observe, distribution network S2, which was previously noted as nonoptimal from two of the three shuffling methods used, accumulates higher values of TIEPI for all years of data available, suggesting a lower performance for this distribution system.

VI. CONCLUSION

In this paper, we investigate the effect of spatial constraints to performance optimality of power distribution networks modeled as spatial networks. First, the analysis of the cable length probability distributions offers two differentiated patterns for two particular networks (i.e., S2 and N7), which follow power-law functions in their tails, given by cables with much longer lengths than the rest of the networks. Based on the analysis above, we would like to know whether power distribution networks are more prone to failures because of these particular geographical features (for example, shorter cable lengths due to their urban placement). In Table I, we can see that S2, N2, N11, and N12 are also partially urban networks. Power-law distribution properties and more short cable lengths found in the spatial modeling can help us to further investigate the spatial topological characteristics and even provide a hint of the vulnerability properties of power distribution networks. Thus, we suggest that topology, constrained by geographical conditions of each network, will affect its performance. In this paper, we have assessed the optimality

of the network in terms of connectivity (i.e., wiring cost) and robustness (i.e., cascading effect) to measure its performance. Second, we have examined the wiring cost in the spatial organization of these networks by means of shuffling nodes and edges in three systematic ways. We show that again S2 and N7 have different optimality characteristics compared with their other counterparts. Furthermore, it is remarkable that with two particular complete datasets (i.e., S1 and S2) in terms of spatially defined topology and electric quality indexes, two distinguishable behaviors appear, with one more prone to failures linked to a nonoptimal topology (i.e., S2). Therefore, we suggest that suboptimal networks (i.e., those with topologies not organized to reduce the wiring cost) seem more prone to accumulate failures. Third, cascading failures which incorporate distributed flow affected by both topology and geographical constraints in power distribution networks have been analyzed. We have found that when a power distribution network is modeled as a spatial network, even one node removal triggers a global cascading process. In this sense S2 is again more vulnerable than S1 to the random node removal, a feature consistent with empirical reliability data. Even more remarkable is the fact that given only the spatial constraints, S2 should be more robust than S1. However, due to its pretty low efficient wiring implementation, it exhibits a lower robustness. Spatial features, often neglected in the literature, play an important role in vulnerability analyses. For power networks planning, the comprehensive routing strategy under geographical constraints would lead to more robust and efficient networks.

In order to better predict how these systems respond to failures or dynamical variations, and how performance is linked with topology and optimal design of networks, more and better data processing is needed. Although topologies are statically similar, distribution networks have different objectives from transmission ones. The distribution grid has to deal with the last miles of the connectivity of the users and efficiency and costs are its first imperatives. It is also more difficult to trace failure data for specific distribution networks compared to power transmission grids. In fact, the operations tend to aggregate failure data at larger regional or national level thus posing even more difficulties for data-driven optimization. Our research goal in the near term is: 1) to have access to more data sources and distribution networks to provide a more sound statistical analysis to the promising results of this work; 2) developing more specific topological (i.e. spatial) and extended metrics, involving electrical engineering characteristics of the network and the more economic-related aspects (e.g. different kinds of costs such as maintenance, environmental, etc.) to be able to more precisely characterize the kind of tradeoff presented in this paper; 3) adding to the simplified topological features used here some other additional features, such as losses, and objects typically characterizing power grid operations such as fuses, circuit breakers and protective devices; and 4) improving the models used to analyze cascading failures in interdependent and spatially embedded power distribution networks, in order to include associated phenomena such as load shedding, islanding, and reclosing, to better define risk assessment in this kind of particularly critical infrastructures.

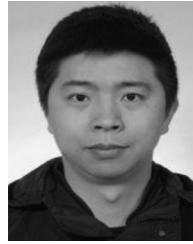
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Lingen Luo received the M.S. degree in electrical engineering from Shanghai Jiao Tong University, Shanghai, China, in 2008, and the Ph.D. degree in electrical engineering from the Politecnico di Torino, Turin, Italy, in 2013.

He is currently a Postdoctoral Researcher with Shanghai Jiao Tong University. His research interests include vulnerability analysis of power systems, smart grids, and complex networks.



Giuliano Andrea Pagani received the B.Sc. and M.Sc. degrees in computer engineering from the University of Parma, Parma, Italy, in 2004 and 2006, respectively, the M.S. degree in corporate management from the MIP Polytechnic University of Milan, Milan, Italy, in 2009, and the Ph.D. degree in computer science from the University of Groningen, Groningen, The Netherlands, in 2014.

He is currently a Postdoctoral Researcher with the University of Groningen. His research focuses on distributed energy generation with a particular interest

in the fields of complex network analysis, graph theory, smart grid technology, and renewable energy.



Marti Rosas-Casals received the Ph.D. degree in applied sciences from the Universitat Politècnica de Catalunya—BarcelonaTech, Barcelona, Spain, in 2009.

He is currently an Associate Professor with the Heat Engines Department, Terrassa School of Engineering, Universitat Politècnica de Catalunya—BarcelonaTech, and Head of the Sustainability Measurement and Modelling Laboratory. His research interests include energy, infrastructure networks, and complex systems.