Noise Subspace Communications: Waveform Optimization and Interference Management

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by

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“The secret of my happiness is not to strive for pleasure, but to find pleasure in effort”

André Gide
Abstract

Nowadays, the increasing demand for high data-rate services increases the interest of spectral efficiency. As it is well-known, the spectrum is a limited resource. Thus, those newest techniques should exploit it efficiently.

The proposed techniques to accomplish this objective are, mainly, cognitive radio, device-to-device communications (D2D) and heterogeneous networks (HetNet). Basically, the difference between these techniques is that cognitive radio increases the device intelligence to efficiently exploit the spectrum, whereas D2D and HetNet improves the design of wireless networks increasing its agility and flexibility.

All of these techniques are included in the area known as Opportunistic Communications, which is based on allowing new users (or devices) to transmit while the existing ones have to be protected.

This work consists on analyzing an opportunistic transmission strategy exploiting the second-order statistics of the existing users in a highly dense distributed network.

First of all, the waveform optimization problem is tackled, as well as a deep study of the properties that the optimal waveform presents. Pursuing the objective of avoiding interfere to the external network, i.e. the existing system, a study of the interference level is performed considering some negative effects in the monitoring of external-network observations. The following step is designing a strategy which allows mitigating this undesired interference. Moreover, this strategy also improves the transmission performance in the internal network by reducing the energy bit loss. In order to compare the proposed solution and a well-known modulation as the Orthogonal Frequency-Division Multiplexing (OFDM), an analysis of the Peak-to-Average Ratio (PAR) is included.

At the first part of the study, the channel is assumed to be ideal with Additive White Gaussian Noise (AWGN). But, as we know, the majority of the scenarios presents a frequency-selective channel. Therefore, the asymptotic case of the proposed solution is analyzed in this more demanding scenario and a new transmission strategy is studied to mitigate the multipath fading.
Resum

Avui dia, la gran demanda per a donar suport a aplicacions que requereixen una alta velocitat de les dades converteix l’eficiència espectral en un ítem a tenir en compte a les noves estratègies de comunicacions. Com és ben sabut, l’espectre de freqüències és un recurs limitat. Així, aquestes noves tècniques han d’exploitar-lo de manera eficient.

Les tècniques que es proposen per aconseguir aquests objectius són, pròpicament, la ràdio cognitiva, les comunicacions dispositiu-a-dispositiu (D2D) i les xarxes heterogènies (HetNet). Bàsicament, la diferència entre aquestes tecnologies és que la ràdio cognitiva dota els dispositius d’intèl·ligència per explotar eficientment l’espectre, mentre que D2D i HetNet milloren el disseny de les xarxes inalàmbriques incrementant la seva agilitat i flexibilitat.

El conjunt d’aquestes tècniques s’engloba a l’àrea de les comunicacions oportunistes, que es basa en permetre la transmissió de nous usuaris o dispositius amb el requeriment de no molestat i, per tant, protegir els usuaris existents.

Aquest treball consisteix en analitzar una estratègia de transmissió oportuista lliure d’interferències fent ús de l’estadística de segon ordre dels usuaris existents en xarxes distribuïdes molt denses.

Primerament, s’aborda el problema de la optimització de la forma d’ona, així com l’estudi de les seves propietats. Perseguint l’objectiu de no interferir els usuaris existents, s’estudia el nivell d’interferència a la xarxa externa, és a dir, als usuaris existents, considerant efectes negatius sobre el monitoratge de les observacions que obtenen el nous usuaris dels ja existents. El pas següent és presentar una estratègia que permeti mitigar aquesta interferència i millorar la transmissió de la xarxa interna, és a dir, dels usuaris nous, tot reduint la pèrdua de l’energia de bit. Per tal de poder comparar la solució proposada amb modulacions existents, com ara l’OFDM, s’analitzà la relació de potència pic a potència mitjana (PAR).

A la primera part de l’estudi, el canal s’ha considerat ideal amb soroll additiu Gaussià i blanc. A la pràctica, sabem que majoritàriament els canals seran selectius en freqüència. Per tant, el cas asimptòtic de la solució proposada s’analitzà en un escenari no ideal així com també es proporciona una estratègia de transmissió per a reduir les pèrdues per propagació multicamí.
Comunicación de Subespacio de Ruido: Optimización de Forma de Onda y Gestión de Interferencias

Resumen

Hoy en día, la gran demanda para dar soporte a aplicaciones que requieren una alta velocidad de los datos convierte la eficiencia espectral en un punto a tener en cuenta en las nuevas estrategias de comunicaciones. Como es bien sabido, el espectro de frecuencias es un recurso limitado. Así pues, dichas técnicas tienen que explotarlo de forma eficiente.

Las técnicas que se proponen para conseguir esta meta son, principalmente, la radio cognitiva, las comunicaciones dispositivo-a-dispositivo (D2D) y las redes heterogéneas (HetNet). Básicamente, la diferencia entre estas tecnologías es que la radio cognitiva dota a los dispositivos de inteligencia para explotar eficientemente el espectro, mientras que D2D y HetNet mejoran el diseño de las redes inalámbricas incrementando su agilidad y flexibilidad.

El conjunto de dichas técnicas se engloba en el área de las comunicaciones oportunistas, que se basa en permitir la transmisión de nuevos usuarios o dispositivos con el requisito de no molestar y, por tanto, proteger a los usuarios existentes.

Este trabajo consiste en analizar una estrategia de transmisión oportuna libre de interferencias haciendo uso de la estadística de segundo orden de los usuarios existentes en redes distribuidas muy densas.

Primeramente, se aborda el problema de la optimización de la forma de onda, así como el estudio de sus propiedades. Persiguiendo el objetivo de no interferir a los usuarios existentes, es decir, a la red externa, se estudian casos negativos de monitorización de las observaciones de la red externa que obtienen los usuarios nuevos, es decir, la red interna. El siguiente paso es presentar una estrategia que permita mitigar dicha interferencia y mejorar la transmisión de la red interna reduciendo la pérdida de la energía de bit. Para poder comparar la solución con modulaciones ya existentes, como OFDM, se analiza la relación de potencia de pico a potencia media (PAR).

En la primera parte del estudio, el canal se ha considerado ideal con ruido aditivo Gaussiano y blanco. En la práctica, sabemos que mayoritariamente los canales serán selectivos en frecuencia. Por tanto, el caso asintótico de la solución propuesta se analiza en un escenario no ideal así como también se presenta una estrategia de transmisión para reducir las pérdidas por propagación multicamino.
L'entrega d'aquest Treball Final de Grau representa el final d'una etapa molt important de la meva vida. En els darrers quatre anys, he conegut fantàstiques persones; persones que han significat, signifiquen i estic segur que significaran una part de mi.

Durant aquests anys, he descobert també el què realment m’agrada, el què realment m’ompli. Aquest fet marcarà la direcció del meu camí els propers anys, cosa que m’enorgulleix. Tot això, no hagués estat possible sense la gran ajuda, ànims i consells que he rebut d’aquelles persones que han estat, estan i estaran amb mi. Aquest treball va dedicat a tots ells.

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Finalment, però no per això menys important, voldria fer arribar el meu agraïment a la meva família. És molt difícil escriure en poques línies el què signifiqueu per a mi. Gràcies pels vostres consells, els ànims, la confiança que m’heu donat i tots els esforços que heu fet i el fet d’estar sempre al meu costat tot i la distància. Tot això sempre tindrà un racó especial en mi. Per tot això i molt més, moltes gràcies.
List of Acronyms

3GPP 3rd Generation Partnership Project.

AWGN Additive White Gaussian Noise.

BER Bit-Error Rate.

CSI Channel State Information.

DoF Degrees of Freedom.

D2D Device-to-Device communication.

EVD Eigenvalue Decomposition.

HetNet Heterogeneous Networks.

IA Interference Alignment.

ICI Inter-Carrier Interference.

i.i.d Independent and identically distributed.

ISI Inter-Symbol Interference.

MAP Maximum A Posteriori.

MIMO Multiple-Input Multiple-Output.

MISO Multiple-Input Single-Output.

ML Maximum Likelihood.

MNW Minimum-Norm Waveform.

MRC Maximal-Ratio Combining.

OFDM Orthogonal Frequency-Division Multiplexing.

PAM Pulse-Amplitude Modulation.

PAR Peak-to-Average Ratio.
PSD  Power Spectral Density.

QAM  Quadrature Amplitude Modulation.

RX   Receiver.

SNR  Signal-to-Noise Ratio.

s.t.  Subject to

SVD  Singular Value Decomposition.

TX   Transmitter.

VFDM Vandermonde-Subspace Frequency-Division Multiplexing.
Notation

\( x(t) \)  A random process.

\( X(\omega) \)  The Fourier transform of \( x(t) \).

\( \phi_x(\omega) \)  The PSD of \( x(t) \).

\( x \)  A column vector.

\( X \)  A matrix.

\( I \)  The identity matrix.

\( 0 \)  All-zeroes matrix or vector with appropriate dimensions.

\( F \)  A normalized Fourier matrix whose spectral information is organized in a decreasing order.

\( \mathcal{X} \)  A set or subspace.

\( \text{tr}(X) \)  The trace of \( X \).

\( \text{diag}(x) \)  A matrix whose diagonal contains the elements of \( x \).

\( \text{span}(X) \)  The subspace generated by all linear combinations of the columns of \( X \).

\( \|X\|_F \)  The Frobenius norm of \( X \).

\( \odot \)  The Hadamard product (also known as Schur product).

\( (\cdot)^H \)  The hermitian (transpose and conjugate) operator.

\( (\cdot)^* \)  The conjugate operator.

\( X^# \)  The Moore-Penrose pseudoinverse of \( X \).

\( X_{a:b} \)  Given an \( N \)-column matrix, the subset of columns from \( a \) to \( b \).

\( \text{rank}(X) \)  The rank of \( X \).

\( x \propto y \)  The two vectors are proportional, i.e. there exists a constant \( \lambda \) such that \( x = \lambda y \).

It also applies to matrices.
\( \mathbf{X} \succeq 0 \) \( \mathbf{X} \) is positive semidefinite.

\( \mathbb{C} \) The field of complex numbers.

\( \mathcal{CN}(\mathbf{m}, \mathbf{C}) \) Complex Gaussian distribution with mean \( \mathbf{m} \) and covariance matrix \( \mathbf{C} \) of appropriate dimensions.

\( \mathbb{E} \) The mathematical expectation operator.
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1.1 Project Outline

1.1.1 Scope

The design of an opportunistic communication strategy implies studying how the information has to be modulated and how the interference that may be caused to existing users can be mitigated. Hence, denoting the existing users as *external network* and the new users, *internal network*, the objectives of this work can be summarized as follows:

1. Optimization of the waveform used to modulate the information.
2. Minimization of the interference level caused to the external network.
3. Characterization and study the proposed solution in different scenarios: AWGN and frequency-selective channels.

1.1.2 Requirements and Assumptions

The main requirement, as explained above, is allowing the internal-network users to transmit without interfering the communication in the external network. In case that it is not possible, a strategy to mitigate this effect has to be proposed. With the idea of improving the strategy under study, another requirement is that the proposed algorithms should optimize the computational cost.

Concerning to the former assumptions, it is important to remark that the proposed study shows many similarities to some classical signal processing, information theory and wireless communications problems. As the objective of this work is not to solve these problems, the following ideas are further on assumed:

1. The complete knowledge of the total number of degrees of freedom and the ones used by the external network. This problem is known as Model Order Selection and it is not the scope of this work.

2. The estimation errors and the effects of the possible shadowing between the external and the internal networks are modeled as complex Gaussian random variables, because this kind of variables degrades maximally the information. Therefore, it is the worst-case in statistical terms.

3. The channels between the external network and internal network are assumed to be unknown and only shadowing effects are considered.
1.1.3 Time Planning

The realization of this work lasted one year. On July 2015, the objectives of this work were proposed and some lessons to strengthen concepts in signal processing and communications were performed.

Taking into account the organization of this Degree thesis, the first half of chapter 2, i.e. the minimum norm solution, the study of their properties and the interference level was conducted during the last quarter of 2015. At the beginning of 2016, new scenarios were thought, as the frequency-selective channel and the use of spatial diversity. From February and until May, both scenarios were analyzed, but, finally, only the frequency-selective channel scenario has been included in this Degree thesis. During this period, the second half of chapter 2 has been developed.

In Appendix A, the included calendar shows the evolution through these ten months of the performed work.

1.2 Organization

This Degree thesis is organized in four chapters. The first one is the introduction, where the project outline and the organization of this work are explained. The starting point and contributions are also included, as well as a state of the art in opportunistic communication, with some relevant references.

In Chapter 2 the solution in a single AWGN channel is studied. This chapter includes the optimal waveform and their properties as well as a study of the effects caused by shadowing at transmitter side and how to mitigate them, whereas Chapter 3 includes the analysis of the optimal solution in a frequency-selective channel. In order to reduce the mathematical complexity described in Chapter 2, the asymptotic case is considered. In addition, a transmission strategy which mitigates the multipath fading effects is proposed.

The Chapter 4 includes the conclusions and future tasks that will have to be studied in order to perform a complete analysis in some actual scenarios. Notice that before the list of contents, there are some notation and acronyms to facilitate the comprehension of the following chapters, whereas at the end there is a list of references used along this work or interesting to delve into some of the topics exposed on the state of the art.

![Diagram](Figure 1.1: Degree Thesis structure.)
1.3 Contributions

The idea of exploiting the second-order statistics of the external network comes from the PhD Dissertation \cite{FS14} by Josep Font-Segura. In this dissertation, the waveform optimization using the minimum norm as the design criterion was established to transmit in a wideband cognitive radio scenario. Moreover, some of the properties studied in this work were introduced as well as the concept of interference to the external network.

In this work, apart from the properties proposed in \cite{FS14}, the Peak-to-Average Ratio and the equal distribution of power among all available degrees of freedom were proposed to be shown and also the study of the effects of shadowing at transmitter side, from a theoretical and simulation perspective. This last study lead us the idea of designing a strategy to mitigate these effects, which is introduced in this work.

Although the behavior in the frequency domain was studied in \cite{FS14}, we now propose to study the use of this approach to consider the transmission through frequency-selective channels as well as the use of Maximal-Ratio Combining (MRC) to mitigate the effects of multipath fading and exploit any potential diversity gain provided by the channel memory.

1.4 State of the art

The aim of Opportunistic Communications is allowing the coexistence of new users (in this work, internal-network users) and the existing users (in this work, external-network users), also known as secondary and primary users, respectively. The idea is that secondary users use the bands assigned to the primary users opportunistically and with the constraint of not providing interference to primary users. In order to deal with this problem, it is important to take into account that a careful and dynamic planning of the wireless networks is needed to avoid the undesired interference. However, the wireless network performance is undermined.

In this sense, some literature proposed the use of interference alignment (IA) in order to mitigate the possible interference that the secondary users may provide to primary users. This idea implies projecting the signals sent by the secondary users to the orthogonal subspace such that the interfered users, i.e. the primary users, decode the interference as noise. As it is stated in \cite{LLMY14} by Lu Lu et al. a possible way to achieve interference-free transmissions is putting the signals from secondary users into the null space of the primary users in the frequency domain and/or time domain.

Concerning to the frequency domain, the idea is exploiting Orthogonal Frequency-Division Multiplexing (OFDM) modulation by computing the null space of the channel matrix whose dimension is related to the length of the cyclic prefix. As it is suggested, the interference-free transmission may be achieved using Vandermonde precoders, also known as Vandermonde-Subspace Frequency-Division Multiplexing (VFDM). This technique consists on projecting the signal to the null space of the interference channel from the secondary transmitter to the primary receiver. This implies that the primary link does not suffer any interference. Looking for further information about this technique, in \cite{CCKD10} a study of the performance of VFDM is carried out. Moreover, it is shown that the only cost to obtain a good performance of this technique, exploiting the non-used degrees of freedom of the primary system, is perfect channel state information (CSI). In this paper, it is shown that the performance, in terms of Bit-Error Rate (BER), is similar to OFDM. However, assuming that the primary users do not know the existence of the secondary ones, the transmission in the primary link can interfere the communication in the secondary link.

The use of the null space in the spatial domain consists on implementing Multiple-Input Multiple-Output (MIMO) systems. With this approach, the dimension of the null space is the difference between the number of antennas at the secondary transmitter and at the primary receiver. In this sense, some work has been developed to perform IA in MIMO systems. S.M. Perlaza et al. propose a strategy to re-use the non-occupied degrees of freedom of a MIMO primary system in \cite{PFLD10}. This paper also provides a technique to build signals which allows exploiting the spatial resources and maximizes the opportunistic transmission rates, which can achieve the same order as that of the
primary link, thanks to a power allocation policy. As it is pursued in some strategies, the obsession to maximize transmission rates can cause interferences. In this last strategy, in order to maximize transmission rates, more degrees of freedom are needed. Hence, it could be interesting to study the maximum number of degrees of freedom that secondary link can exploit to achieve the maximum transmission rate as possible with the constraint of not providing interference to primary link. In this sense, an outer bound on the degrees of freedom that a secondary system can use without interfere the primary system is studied by M. Amir et al. in [AEKN11]. Implementing IA, the number of available degrees of freedom tends to the half of the product between the number of secondary users and the number of antennas of each secondary user (assuming a symmetric scenario, i.e. both transmitter and receiver have the same number of antennas).

As we can read, opportunistic communication requires certain conditions to achieve its goal, as perfect CSI and the fact that the primary users do not know the existence of the secondary system. It is well-known that the knowledge of CSI consumes some radio resources which affects negatively on the performance of the whole system. Hence, some current works propose to relax this conditions. For example, A. Mukherjee et al. propose in [MPSW12] a MIMO precoding with completely unknown primary CSI. Also, in [LLMY14] there is the proposal that the tolerance to certain level of interference will facilitate the design of opportunistic transmission strategies and, moreover, this proposal fulfills with the assumption of imperfect CSI. Notice that with the last assumption, interference-free transmission is not guaranteed. Another option can be a cooperation between primary and secondary users to assist in the monitoring of the available degrees of freedom. This may suppose the use of pilots to communicate to secondary users where the unused spaces are. This option, initially, implies the need of a feedback algorithm, because a constant communication between primary and secondary users allows to use the complete space, which suppose an increase of the Quality of Service in each system. The use of a feedback strategy may decrease the performance of the system. In this sense, K. Ntougias et al. propose in [NTP15] a cooperation scheme for dynamic spectrum sharing minimizing the feedback needed.

The presented strategies are based on cognitive radio. Generally, they increase the device intelligence to efficiently exploit the spectrum. These techniques need to know where are the spectral holes. Until now, two techniques have been frequently used: spectrum sensing and features detection. The first one consists on a full-time sensing to detect spectral holes to transmit. While the secondary is transmitting, sensing is required to avoid the occupancy of these holes if the primary users wants to use them. The second one is based on looking at the main features of the primary users, as transmit power. Then, knowing these features, we can determine if there is a spectral hole and, in case of occupying it, wether the primary users request these holes.

Cognitive radio does not consider the design of wireless networks. In fact, there are also techniques which aim raising the agility and flexibility of wireless networks. In this sense, heterogeneous networks (HetNet) have been studied. As a particular case of HetNets, device-to-device communications (D2D) have been proposed recently to take advantage of the physical proximity of communicating devices, increasing resource utilization and improving cellular coverage. These features may suppose power savings, increased throughput and higher spectral efficiency. For example, G. Fodor et al. studied in [FDM12] the design of D2D systems using the 3GPP Long Term Evolution as a baseline. In this paper, some techniques to manage the interference between D2D pairs and between the D2D and the cellular layers are proposed.
2

Single AWGN channel scenario

2.1 Introduction

The scope of this chapter is analyzing an opportunistic communication strategy based on the use of the second-order statistics of the observations from the external network. The optimal waveform is obtained in section 2.2 and their properties are analyzed in section 2.3. In section 2.4, the effects of shadowing in the External-Network to Internal-Network Transmitter channel are studied whereas in section 2.5 a strategy is proposed to mitigate the interference level caused. In section 2.6, the PAR is studied to observe the influence of the number of degrees of freedom used in the opportunistic transmission. Finally, in section 2.7 an iterative algorithm is proposed to compute orthogonal waveforms to avoid Inter-Carrier Interference (ICI) at receiver.

Problem statement

A suitable way to understand the problem is observing the figure 2.1. In that figure, \( x \) is a single time-domain observation of size \( N \) from the external network. From this observation, and making use the assumption established in Chapter 1 whereby we consider known the number of degrees of freedom used by the external network (\( M \)), the observations autocorrelation matrix is computed using \( k = 1, \ldots, M \) observations:

\[
\hat{R}_{ext} = \sum_{k=0}^{M-1} x_k x_k^H
\]  

(2.1)

This matrix corresponds to an estimation of the external-network observations correlation matrix. In order to tune the estimation into a real scenario, the effect of shadowing should be taken into account. If this effect was considered on the observations got by the receiver, it would only suppose a bit energy loss, whereas if it was considered on the observations got by the transmitter, an interference, apart from the bit energy loss, between the internal link and the external network would be produced. This effect will be modeled with the matrix \( \Delta \) and from now, we will assume that:

\[
R_T = \hat{R}_{ext}
\]

(2.2)

and

\[
R_R = \hat{R}_{ext} + \Delta
\]

(2.3)

where \( \hat{R}_{ext} = \hat{R}_S + \hat{R}_N \) is an estimation of the autocorrelation matrix of the external-network observations. The model of the matrix error \( \Delta \) is not studied until section 2.4.

Initially, we are going to assume ideal channel.
Once the second-order statistics of the external network are studied, the opportunistic communication problem has to be formulated.

Let \( s(t) \) be a Pulse-Amplitude Modulation (PAM) transmitted signal in the internal network, we can write it as:
\[
s(t) = \sum_{k} a_k \varphi_k(t-nT)
\]
for \( k = 1, 2, \ldots, P \) where \( \varphi_k(t) \) is the transmission waveform. Moreover, we know that its power spectral density (PSD) will be proportional to \( |\Phi_k(\omega)|^2 \). As it is described in Chapter 1, the objective implies to build a transmission waveform such that:
\[
\min \frac{1}{2\pi} \int_{-\pi}^{\pi} \phi_x(\omega) |\Phi_k(\omega)|^2 d\omega \tag{2.4}
\]
where \( \phi_x(\omega) \) is the PSD of the external-network observation. That is, the overlapped spectrum between the transmitted signal and the observation from the external network must be minimum.

At this point, we may see that proceeding with the minimization stated in (2.4) is exactly the same as minimizing \( \varphi_k^H R_S \varphi_k \), where \( \varphi_k \) is the vector representation of the transmission waveform and \( R_S \) is the autocorrelation matrix of the external-network observations. Since the last expression is dimensionally a scalar number, minimizing it is the same as minimizing its trace, and making use of the properties of the trace operator we have that:
\[
\min \text{tr} (\varphi_k^H R_S \varphi_k) = \min \text{tr} (R_S \varphi_k \varphi_k^H) \tag{2.5}
\]
That is, operating (2.4) and equivalently the left side of (2.5) implies a minimization of the norm of the transmission waveform, as it is seen in the right side of (2.5). This lead us to establish the minimum norm as a design criterion.

2.2 Minimum-norm solution

The main objective is to design a waveform such that the interference to the external network caused by the internal network will be null. This condition is given by the orthogonality principle between the useful signal and the noise. Hence, we will have to estimate the external-network noise subspace from its autocorrelation matrix.

The spectral theorem of the correlation matrix states that it admits the following decomposition (SVD):
\[
R_{ext} = R_s + R_N = U \Sigma V^H \tag{2.6}
\]
where $\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_R]$ and $\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_R]$ are unitary by the left $P$-by-$R$ and $Q$-by-$R$ matrices, respectively and $\mathbf{\Sigma} = \text{diag} ([\sigma_1, \ldots, \sigma_R])$ is a diagonal $R$-by-$R$ matrix that contains the singular values. Since this decomposition is too generic, and $\mathbf{R}_{\text{ext}}$ is a square matrix, we can use the Eigenvalue Decomposition (EVD):

$$\mathbf{R}_{\text{ext}} = \mathbf{R}_s + \mathbf{R}_N = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H,$$  \tag{2.7}$$

The matrix $\mathbf{U}$ is a square matrix which contains the eigenvectors ($\mathbf{u}_k$) of $\mathbf{R}_{\text{ext}}$ and $\mathbf{\Lambda}$, a diagonal matrix that contains its eigenvalues ($\lambda_k$). The mentioned theorem states also that each eigenvalue can be written as:

$$\lambda_k = \lambda_S + \lambda_N,$$  \tag{2.8}$$

for $k = 1, \ldots, N$, where $\lambda_S$ and $\lambda_N$ denote the signal and noise eigenvalues, respectively.

In our case, only the dimensions used by the external network will have $\lambda_S \neq 0$, so the first $M$ eigenvalues will accomplish this statement.

The desire of obtaining a waveform orthogonal to the external network, will lead us to use a linear combination of the eigenvectors associated to the noise eigenvalues to compute a waveform that satisfies our objectives, that is $\{\mathbf{u}_k^N\}_{M+1 \leq k \leq N}$. These eigenvectors are the last $N - M + 1$ columns of matrix $\mathbf{U}$. Thus, the noise eigenvectors matrix, $\mathbf{U}_N$ is:

$$\mathbf{U}_N = \mathbf{U}_{M+1:N} \tag{2.8}$$

Finally, the condition to fulfill is:

$$\varphi_k \in \text{span} \{\mathbf{U}_N\} \Rightarrow \varphi_k \perp \mathbf{R}_s \tag{2.9}$$

Before proceeding with the computation of the optimal waveform, an important issue is avoiding a trivial solution. As the minimization of the norm is the criterion to design the desired waveform, clearly the trivial solution will be that the norm of this waveform should be zero. At this point, we need to establish the non-trivial solution constraint:

$$\varphi_k^H \mathbf{e}_k = 1,$$  \tag{2.10}$$

where $\mathbf{e}_k = [0, \ldots, 0, 1, 0, \ldots, 0]^T_{N-k, k-1}$.

The question is where the non-null component of (2.10) should be to accomplish the objectives stated in our problem. In order to proceed, we have to define the Lagrangian function to be minimized, where (2.10) will be the constraint:

$$\mathcal{L} (\varphi, \mu) = \|\varphi\|^2 - \mu (\varphi^H \mathbf{e}_k - 1) \tag{2.11}$$

Since the optimal waveform is a linear combination of the noise eigenvectors, we can write it as:

$$\varphi_k = \mathbf{U}_N \lambda \tag{2.12}$$

and its squared norm is:

$$\|\varphi_k\|^2 = \lambda^H \mathbf{U}_N^H \mathbf{U}_N \lambda \tag{2.13}$$

With this, (2.11) may be rewritten as:

$$\mathcal{L} (\lambda, \mu) = \lambda^H \mathbf{U}_N^H \mathbf{U}_N \lambda - \mu (\lambda^H \mathbf{U}_N^H \mathbf{e}_k - 1) \tag{2.14}$$

Computing the gradient of (2.14), we obtain that the optimal linear combination is $\lambda = \mu \mathbf{U}_N^H \mathbf{e}_k$. With this result, (2.12) may be written as: $\varphi_k = \mu \mathbf{U}_N^H \mathbf{e}_k$. Using the non-trivial constraint we may find the expression $\mu^* = (\mathbf{e}_k^H \mathbf{U}_N \mathbf{U}_N^H \mathbf{e}_k)^{-1}$. Thus, the optimal waveform is given by the following expression:

$$\varphi_k = \frac{\mathbf{U}_N \mathbf{U}_N^H \mathbf{e}_k}{\mathbf{e}_k^H \mathbf{U}_N \mathbf{U}_N^H \mathbf{e}_k} \tag{2.15}$$
As (2.15) is the equation of the Minimum-Norm Waveform (MNW), computing its squared norm we can see how to obtain the waveform.

\[
\| \varphi_k \|^2 = e_k^H U_N U_N^H U_N^H e_k \frac{1}{(e_k^H U_N U_N^H e_k)^2} = e_k^H U_N I U_N^H e_k \frac{1}{(e_k^H U_N U_N^H e_k)^2} = \frac{1}{e_k^H P e_k}
\]

(2.16)

We may notice that the position of the non-null component of vector \( e \) correspond to a value of the diagonal of the noise subspace projector, \( P = U_N U_N^H \). So that, the optimal waveform will be the column of the projector which contains the inverse of the minimum value of its diagonal, that is, the maximum value of the diagonal.

The mathematical approach described above lead us to perform a simulation. The PSD of the computed waveforms are shown in the next figure:

![Waveforms contained in the noise subspace of the external network.](image)

**Figure 2.2:** Waveforms contained in the noise subspace of the external network.

In this simulation, it is considered \( M = 100 \) degrees of freedom used by the external network and the size of the observation vectors is \( N = 1024 \). With these parameters, the number of waveforms computed is 924. It is observed that all of the waveforms present a white shape, but the optimal is flatter than the other ones.

### 2.3 Properties of the optimal waveform

The optimal waveform is obtained using the minimum norm as the design criterion. This fact will have certain properties, which will be discussed below.

#### 2.3.1 Interference to external network

The first property, and the most trivial one, is that the interference caused to external network by the internal one must be null. This property is trivially shown by (2.9). As the optimal waveform is contained in the span of the noise eigenvectors matrix, \( U_N \), it will be orthogonal to the signal subspace as it is stated in the orthogonality principle.
2.3.2 Invariance to rotations

The second property which is going to be discussed is that the optimal waveform relies on the noise subspace projector, which exhibits invariance to rotations within the external-network noise subspace.

As it is said in (2.16), the optimal waveform depends on the noise subspace projector, \( P = U_N U_N^H \). The properties of the projectors lead us to find the orthogonal projector or the signal subspace projector: \( P^\perp = I - P = U_S U_S^H \), where \( U_S \) denotes the signal subspace eigenvectors. Let \( \Xi \) be a rotation matrix, which satisfies \( \Xi \Xi^H = I \) and \( \det (\Xi) = \pm 1 \). Thus, the rotated version of \( U_S \) is given by:

\[
U_S^{(r)} = U_S \Xi
\]  

(2.17)

It is easy to prove that the orthogonal projector, \( P^\perp \), given by \( U_S U_S^H \) is the same as the given by \( U_S^{(r)} (U_S^{(r)})^H \):

\[
P^\perp = U_S^{(r)} (U_S^{(r)})^H = U_S \Xi \Xi^H U_S^H = U_S U_S^H
\]  

(2.18)

In (2.18) is shown that the optimal waveform is rotationally invariant by the right, which means that if the internal-network users get a rotated version of the external-network observations auto-correlation matrix, the optimal waveform is not affected by this rotation. However, if the rotation is not within this subspace, i.e. rotation by the left, a mismatching between subspaces occurs.

2.3.3 Whiteness

The third property that is directly derived by the use of minimum norm is the whiteness as it is stated by Kumaresan and Tufts in [KT82] and it is going to be shown:

Our problem may be expressed as:

\[
\min_{\varphi_k} \|\varphi_k\|^2 \text{ s.t. } U_S^H \varphi_k = 0 \text{ and } \varphi_k^H e_k = 1
\]  

(2.19)

The expression (2.19) is able to be written as:

\[
\min_{\varphi_k} \frac{\|\varphi_k\|^2}{\varphi_k^H e_k} \text{ s.t. } U_S^H \varphi_k = 0
\]  

(2.20)

The fraction in (2.20) is equivalent to:

\[
\frac{\|\varphi_k\|^2}{\varphi_k^H e_k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} |\Phi_k(\omega)|^2 d\omega \quad \frac{1}{2\pi} \int_{-\pi}^{\pi} |\Phi_k(\omega)|^2 d\omega \quad \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| e^{-j2\pi \omega(k-1)} \right|^2 d\omega
\]  

(2.21)

Using the Cauchy-Scharwz inequality, the denominator in (2.21) can be expressed as:

\[
\left| \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_k(\omega) e^{-j2\pi \omega(k-1)} d\omega \right|^2 \leq \frac{1}{2\pi} \int_{-\pi}^{\pi} |\Phi_k(\omega)|^2 d\omega \times \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| e^{-j2\pi \omega(k-1)} \right|^2 d\omega
\]  

(2.22)

Operating (2.22) we obtain the following:

\[
\frac{1}{2\pi} \int_{-\pi}^{\pi} |\Phi_k(\omega)|^2 d\omega \geq \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| e^{-j2\pi \omega(k-1)} \right|^2 d\omega
\]  

(2.23)

In (2.23) the right term is 1, which means that numerator and denominator of the left term are equal. According to Cauchy-Schwarz inequality, the product of squared norms will only be equal to the square of the scalar product when both functions are directly proportional. Hence, the PSD of \( \varphi_k(t) \) is proportional to 1, which implies that this spectrum is as white as possible.
As it is observed in figure 2.2, the optimal waveform is the whitest one. Given so, minimizing the norm, i.e. selecting the column of the projector which contains the largest value of its diagonal, means maximizing the whiteness.

**2.3.4 Equal distribution of power among all available degrees of freedom**

The last exposed property is closer to the whiteness, and it is that the minimum norm solution provides us an equal distribution of power among all available degrees of freedom. Demonstrating it requires using only the relevant dimensions of the optimal waveform, that is:

$$
\varphi_k' = U_N^H \varphi_k
$$

(2.24)

Assuming zero-mean solution, the autocorrelation matrix can be used instead of the covariance matrix:

$$
R_{\varphi'} = \mathbb{E} \left[ \varphi_k' (\varphi_k')^H \right]
$$

$$
= \mathbb{E} \left[ U_N^H \varphi_k \varphi_k^H U_N \right]
$$

$$
= \mathbb{E} \left[ U_N^H U_N \left( U_N^H e_k e_k^H U_N \right) U_N^H U_N \right]
$$

$$
= \mathbb{E} \left[ U_N^H U_N \left( U_N^H e_k e_k^H U_N \right) U_N^H e_k \right]
$$

(2.25)

In (2.25), the product $U_N^H U_N$ is the identity matrix, $(U_N^H e_k e_k^H U_N)^\# = (U_N^H e_k e_k^H U_N) = I$ due to the existence of the Moore-Penrose pseudoinverse matrix and $e_k^H U_N e_k$ is dimensionally a scalar value, concretely the inverse of the squared norm of the $k$-th noise eigenvector, which is assumed to be a unitary norm vector. Thus,

$$
R_{\varphi'} = I
$$

(2.26)

The fact that the covariance matrix, and equivalently the autocorrelation matrix, is diagonal means that power is distributed equally among all available degrees of freedom.

**2.4 Study of the effects of shadowing at TX side**

Since we consider that the observations got at transmitter and receiver side are different, two possible situations must be studied. On the one hand, the number of available degrees of freedom estimated at transmitter side is larger than the estimated one at receiver side. This situation may suppose a certain interference on the degrees of freedom estimated as available when they are really occupied by the external network and also a loss of bit energy, given that the transmitter will send information using some degrees of freedom which are not contemplated by the receiver. On the other hand, when the number of degrees of freedom estimated at receiver side is larger than the estimated one at transmitter side, the transmission through the opportunistic link will not interfere the external network, this will only suppose a loss of bit energy ($E_b$). The following study is focused solely on analyzing the first situation, because the main objective pursued in this work is not to interfere the communication in the external network. In addition, the error matrix will introduce a rotation which will result in a mismatch between subspaces. This fact will suppose a phase ambiguity between the transmission and reception pulses. It is important to remember that phase ambiguity is also an inherent problem in any communication system which can be compensated using an algorithm, e.g. a training-based algorithm that estimates the phase ambiguity using a known sequence of pilots.

Observing (2.2) and (2.3), it is seen that these equations model the situation under study. The question is how to model the error matrix, $\Delta$. First of all, we have to take into account that, as the mathematical model consists on adding the error matrix to the external-network observations correlation matrix, $\Delta$ may follow the mathematical model of a correlation matrix, thus:
1. $\Delta$ has to be hermitian and it must have a Toeplitz structure.

2. $\Delta$ has to be positive semidefinite, i.e. $\Delta \succeq 0$, with positive and real values on its diagonal.

The way to obtain a matrix that accomplishes these points is using the estimation of a correlation matrix. Thus,

$$\Delta = \sum_{k=1}^{D} \delta_k \delta_k^H$$

(2.27)

where $D$ is the rank of the error matrix.

The question now is which statistics have to follow the vectors $\delta_k$ in order to increase the reliability of the error model. The desire is that the statistics of $\delta_k$ are as bad as possible in terms of information degradation. From information theory, it is a well-known result that a Gaussian interference will cause the maximum information degradation as possible if the input noiseless signal follows a Gaussian distribution and subject to power constraints. So, from now it is assumed that:

$$\delta_k \sim \mathcal{CN} (0; \sigma_\delta^2 I)$$

(2.28)

### 2.4.1 Interference level

As a first step, the interference level caused to the external network is going to be studied. Hence, let $\xi_\varepsilon (D)$ be the interference level provided by the opportunistic transmission as a function of the rank of error matrix, $D$, and for a given uncertainty, $\varepsilon$, such that:

$$\xi_\varepsilon (D) = \mathbb{E} \left[ \varphi^H \Delta \varphi \right]$$

(2.29)

#### Rank-one error matrix

In that particular case, where $D = 1$, the error matrix is able to be written as:

$$\Delta = \delta \delta^H = [\Delta_{mn}]_{1 \leq m,n \leq N} = [\delta_m \delta_n^*]_{1 \leq m,n \leq N}$$

(2.30)

Taking into account the statistics of $\delta$ and (2.30), the following results may be obtained:

$$\mathbb{E} [\Delta] = \sigma_\delta^2 I$$

(2.31)

$$\mathbb{E} \left[ \|\Delta\|_F^2 \right] = \text{tr} (\Delta \Delta^H) = \sum_{m=1}^{N} \sum_{n=1}^{N} |\Delta_{mn}|^2 = \sum_{m=1}^{N} \sum_{n=1}^{N} \mathbb{E} [\delta_m \delta_n^* \delta_n \delta_m^*]$$

$$= \sum_{m=1}^{N} \sum_{n=1}^{N} (\mathbb{E} [\delta_m \delta_m^*] \times \mathbb{E} [\delta_n \delta_n^*] + \mathbb{E} [\delta_m \delta_n^*] \times \mathbb{E} [\delta_n \delta_m^*])$$

$$= N (N + 1) \sigma_\delta^4 \triangleq \varepsilon^2$$

(2.32)

and according to the Law of Large Numbers, $\|\Delta\|_F^2 \approx (N^2 + N) \sigma_\delta^4 \triangleq \varepsilon^2$ for large $N$. The total interference level provided by the minimum-norm solution will be given by:

$$\xi_\varepsilon (1) = \mathbb{E} \left[ \varphi^H \Delta \varphi \right] = \text{tr} (\mathbb{E} [\Delta \varphi \varphi^H]) = \sigma_\delta^2 \|\varphi\|^2 = \sigma_\delta^2 = \frac{\varepsilon}{\sqrt{N (N + 1)}}$$

(2.33)
Rank-two error matrix

For the case $D = 2$, we consider the following model:

$$\Delta = \delta_1 \delta_1^H + \delta_2 \delta_2^H = [\Delta_{mn}]_{1 \leq m, n \leq N} = \left[\delta_{1,m} \delta_{1,n}^* + \delta_{2,m} \delta_{2,n}^*\right]_{1 \leq m, n \leq N} \quad (2.34)$$

According to the current model, now we have:

$$\mathbb{E}[\Delta] = 2\sigma_\delta^2 \mathbf{I} \quad (2.35)$$

\begin{align*}
\mathbb{E}\left[\|\Delta\|_F^2\right] &= \text{tr}(\Delta \Delta^H) = \sum_{m=1}^N \sum_{n=1}^N |\Delta_{mn}|^2 = \sum_{m=1}^N \sum_{n=1}^N \left(\mathbb{E}[\delta_{1,m} \delta_{1,n}^* \delta_{1,n} \delta_{1,n}^*] + \mathbb{E}[\delta_{2,m} \delta_{2,n}^* \delta_{2,n} \delta_{2,n}^*]\right) \\
&+ \sum_{m=1}^N \sum_{n=1}^N \left(\mathbb{E}[\delta_{2,m} \delta_{2,n}^* \delta_{2,n} \delta_{2,n}^*]\right) = 2N (N + 2) \sigma_\delta^4 \triangleq \varepsilon^2 \quad (2.36)
\end{align*}

In (2.36) the null elements have not been written. Finally, for a total uncertainty $\varepsilon$ the interference level is:

$$\xi_\varepsilon(2) = \mathbb{E}[\varphi^H \Delta \varphi] = \text{tr}(\mathbb{E}[\Delta \varphi \varphi^H]) = 2\sigma_\delta^2 \|\varphi\|^2 = 2\sigma_\delta^2 = \sqrt{\frac{\varepsilon}{\sqrt{N (N + 2)}}} \quad (2.37)$$

Rank-D error matrix

The extension to the general case of rank $D$ is given by the following model:

$$\Delta = \sum_{k=0}^{D-1} \delta_k \delta_k^H = [\Delta_{mn}]_{1 \leq m, n \leq N} = \sum_{k=0}^{D-1} \left[\delta_{k,m} \delta_{k,n}^*\right]_{1 \leq m, n \leq N} \quad (2.38)$$

The results for this case may be obtained generalizing the previous results:

$$\mathbb{E}[\Delta] = D\sigma_\delta^2 \mathbf{I} \quad (2.39)$$

\begin{align*}
\mathbb{E}\left[\|\Delta\|_F^2\right] &= \text{tr}(\Delta \Delta^H) = \sum_{m=1}^N \sum_{n=1}^N |\Delta_{mn}|^2 = D \sum_{m=1}^N \sum_{n=1}^N \left(\mathbb{E}[\delta_{m} \delta_{m}^*] \times \mathbb{E}[\delta_{n} \delta_{n}^*]\right) \\
&= DN (N + D) \sigma_\delta^4 \triangleq \varepsilon^2 \quad (2.40)
\end{align*}

As in the previous case, in (2.40) only the non-null elements have been considered. Thus, in the general case, the interference level provided by the minimum norm solution for a total uncertainty $\varepsilon$ is given by:

$$\xi_\varepsilon(D) = \mathbb{E}[\varphi^H \Delta \varphi] = \text{tr}(\mathbb{E}[\Delta \varphi \varphi^H]) = D\sigma_\delta^2 \|\varphi\|^2 = D\sigma_\delta^2 = \sqrt{D \frac{\varepsilon}{\sqrt{N (N + D)}}} \quad (2.41)$$

It is interesting to see the evolution of the interference level as a function of the rank of error matrix and setting the total uncertainty $\varepsilon$.
As it is expected, the uncertainty does not affect on the shape of the interference level, it only introduces an offset of 10 dB per magnitude order.

### 2.4.2 Robustness Analysis: Worst-case error matrix

Once the interference is studied assuming that the error matrix is a random matrix computed as a typical estimation of an autocorrelation matrix, where each vector follows a complex normal distribution, it is necessary to study which error matrix provides the worst case. In order to carry out this computation, the following equation has to be considered:

\[
\min_{\varphi_k} \max_{\Delta} \varphi_k^H (R_S + \Delta) \varphi_k \text{ s.t. } \|\Delta\|_F^2 \leq \varepsilon^2
\]  

(2.42)

The Lagrangian associated to this problem, taking into account the maximum squared norm of the error matrix, is given by:

\[
\mathcal{L} (\Delta, \mu) = \varphi_k^H (R_S + \Delta) \varphi_k - \mu (\varepsilon^2 - \text{tr} (\Delta \Delta^H))
\]  

(2.43)

As the first term in (2.43) is dimensionally a scalar value, its trace may be considered. Then, using the properties of the trace and taking the gradient with respect to $\Delta$, the following result is obtained:

\[
\Delta = \varepsilon^2 \varphi_k \varphi_k^H
\]  

(2.44)

From (2.44) we can conclude that the external network is using the degrees of freedom estimated as available by the internal-network transmitter but the shadowing does not allow the transmitter to discard these degrees of freedom. In the next figure, selecting a certain value of uncertainty, e.g. $\varepsilon = 1$, there is a comparison between the interference level assuming a random error matrix as it is described in (2.27) and the worst-case error matrix.
Notice that the worst-case error matrix does not depend on $D$. As we can observe in figure 2.4, the worst-case error matrix provides a large interference value which corrupts completely the communication in the external network. It is interesting to observe that the difference between the interference level provided by the worst-case error matrix and the provided by a random error matrix, is given by the following equation:

$$|\Delta \xi| = |\xi_{\text{worst-case}} - \xi(D)| = \frac{\sqrt{N}}{\sqrt{N(N+D)}} \to \frac{1}{\sqrt{N}},$$

independent from $\varepsilon$ as the uncertainty only supposes a certain offset.

### 2.4.3 Bit energy loss

The system thought for this scenario includes a typical receiver, composed by a scalar product computation between the received waveform and the optimal waveform obtained at receiver (this operation is equivalent to implement a match filter and a sampler) and an optimal MAP/ML decider.

In an ideal case, it is expected a complete matching between the waveforms computed at transmitter and receiver sides, thus:

$$\varphi_{\text{Tx}}^H \varphi_{\text{Rx}} = 1$$

That means that the pulses will not introduce a bit energy loss. In that case, the performance decrease will be caused by the noise introduced by the channel. In fact, when the error matrix is considered, this situation will not occur because the waveforms used by transmitter and receiver come from different subspaces. It could be seen clearly by observing equations (2.2) and (2.3). In the receiver waveform, the matrix $\Delta$ is going to reduce the number of available degrees of freedom at RX side. At the beginning of this section, it it stated that the error matrix will depend on two parameters: its rank and its squared Frobenius norm (which in turn, it is the squared uncertainty).

In communication systems, it is typical to normalize the pulse energy. As it is seen in figure 2.3, the uncertainty will only introduce an offset, which does not affect the bit energy loss after normalizing the waveform. However, we realize that the rank of the error matrix plays an important role in the level of degradation of the bit energy. Thus, it is expected that the bit energy loss is proportional to the rank of $\Delta$. Hence, the theoretical loss is given by:

$$\vartheta(D) = 10 \times \log_{10} (D) \ [\text{dB}]$$

In the next figure, it may be observed the evolution of the bit energy loss:
2.5 Improvements in system performance

In this section, there is a description of the technique used to improve the system performance. This strategy is based on a first-order feedback algorithm that establishes a consensus between the noise subspaces estimated at transmitter and receiver sides in order that both users use the same degrees of freedom.

It is important to notice that this technique does not exclude the need of compensate the phase ambiguity. Assuming a perfect compensation of this phenomenon, what we may expect is to obtain a remarkable performance in terms of BER. To sum up, the first-order feedback provides us a completely match between the noise subspaces estimated at each node of the link, whereas the phase ambiguity compensation corrects the possible rotations observed in the received constellation.

2.5.1 Noise subspaces consensus

The way studied to reach consensus between noise subspaces is using a first-order feedback algorithm. As it is mentioned before, the number of degrees of freedom used by the receiver will be lower than the ones used by the transmitter.

The general idea to perform the feedback is computing in which degrees of freedom exist a complete matching between the transmission waveform and the noise subspace estimated at receiver side. A scheme of the scenario is shown in figure 2.6.

As the optimal waveform is a concrete column of the noise subspace projector, a way to achieve
the objective is projecting the transmission waveform with the receiver noise subspace projector:

$$\varphi_{opt} = P_{Rx} \varphi_{Tx} = U_{N} (R_{Rx}) (U_{N} (R_{Rx}))^{H} \varphi_{Tx},$$

(2.48)

where $\varphi_{opt}$ is the optimal waveform that the receiver may communicate to the transmitter in order to achieve a consensus. What this idea is looking for is exploiting the statement that the noise subspace estimated at receiver side is included in the noise subspace estimated at transmitter side, as the error matrix $\Delta$ will only introduce an increase in the rank of correlation matrix, an scale factor (compensated by normalization) and a rotation. But taking into account the rotational invariance, this phenomenon will not affect. If $\mathcal{N}_{Tx}$ and $\mathcal{N}_{Rx}$ denote the noise subspaces estimated at transmitter and receiver sides, respectively, thus $\mathcal{N}_{Rx} \subseteq \mathcal{N}_{Tx}$ and the equality is only given when the error matrix is the all-zeroes matrix.

The studied solution is sensitive to the additive noise introduced by the channel. The effect of the noise is that detection gets worse. In order to compensate this effect, we can use a de-noising strategy as it is proposed by Josep Font-Segura et al. in [FSVR11]. This strategy implies a complete knowledge of the noise at the receiver.

Using the normalized version of these waveforms and implementing a perfect de-noising strategy, the interference level and the bit energy loss are expected to be zero. In the next figure it is shown how the bit energy loss is mitigated using the proposed strategy:

![Figure 2.7: $E_b$ loss reduction given by the feedback strategy proposed.](image)

Notice that a perfect de-noising strategy used at the receiver antenna guarantees the correct operation of feedback algorithm since the bit energy expected tends to be null.

### 2.6 Study of the PAR

In this section, an analysis of the Peak-to-Average Ratio is performed. Since the born of the OFDM, this property has taken a relevant role in order to compare the energy efficiency between modulations. As it is well-known, for a given signal $x(t) = \sum_{m} a_{m} \varphi(t - mT)$, where $a_{m}$ are, e.g., QAM symbols and $T$ denotes the symbol time, the PAR is defined as:

$$PAR = \frac{\text{max} \left| x(t) \right|^2}{\text{E} \left[ \left| x(t) \right|^2 \right]},$$

(2.49)

that is, the quotient between the peak and the average powers. Intuitively, this ratio will be the product of the PAR caused by the influence of the pulse shaping and the PAR caused by the symbols. Only the terms that come from the pulse shaping are the ones where the influence of the strategy
exposed in this work may affect. Assuming $\varphi(t)$ as the minimum-norm waveform, the peak and the average powers may be computed:

$$\max |x(t)|^2 = \max \left| \sum_{m=0}^{K-1} a_m \varphi(t-mT) \right|^2 \leq \max \left( \sum_{m=0}^{K-1} |a_m| |\varphi(t-mT)| \right)^2$$

$$\max |x(t)|^2 = \max_{0 \leq m \leq K-1} a_m^2 K^2 |\max \varphi(t)|^2$$ (2.50)

$$\mathbb{E} \left[ |x(t)|^2 \right] = \mathbb{E} \left[ \sum_{m=0}^{K-1} a_m \varphi(t-mT) \right]^2$$

$$\mathbb{E} \left[ |x(t)|^2 \right] = \sum_{m=0}^{K-1} \sum_{m'=0}^{K-1} \mathbb{E} [a_m a_m^*] \varphi(t-mT) \varphi^*(t-m'T)$$

$$\mathbb{E} \left[ |x(t)|^2 \right] = \alpha K R_{\varphi\varphi^*} \left( (m-m')T \right)$$

Neglecting the terms $|\max_{0 \leq m \leq K-1} a_m|^2$ from (2.49) and $\alpha$ from (2.50), as they correspond to the influence of the symbols, the PAR caused by the pulse shaping is given by:

$$\text{PAR} \bigg|_{\text{pulse shaping}} = K |\max \varphi(t)|^2$$ (2.52)

assuming unit energy transmission waveform. As it is seen in (2.52), the PAR depends on the number of degrees of freedom available for the noise subspace, $K$. In the next figure, some waveforms are plotted varying the number of degrees of freedom available. The observed ratio between the peaks of two concrete waveforms $\varphi_i$ and $\varphi_j$ is:

$$\kappa = \frac{\max \varphi_j(t)}{\max \varphi_i(t)} = \sqrt{\frac{K_j}{K_i}}$$ (2.53)

where $K_i$ and $K_j$ denote the available degrees of freedom in $\varphi_i$ and $\varphi_j$, respectively.

**Figure 2.8:** Optimal waveforms in time domain for different number of available degrees of freedom.

We can observe that the designed waveform presents a peaky shape in the time domain. As the available number of degrees of freedom is a parameter which cannot be controlled, there exists the possibility of forcing the maximum value to be zero in order to reduce the PAR. This modification may suppose some changes in the behavior of the transmission waveform.
In figure 2.9, the PSD of the MNW will be compared for 32 and 256 degrees of freedom.

![Figure 2.9: Comparison between the MNW PSD and the Non-peaky MNW PSD.](image)

Observing figure 2.9, we can say that as the number of available degrees of freedom increases, the non-peaky MNW tends to the optimal MNW. Another aspect to take into account is the interference level provided by each possibility. As the uncertainty parameter \( \varepsilon \) of (2.41) only supposes an offset, the next figure shows only the interference level for \( \varepsilon = 1 \).

![Figure 2.10: Interference level provided by MNW and non-peaky MNW for \( \varepsilon = 1 \) as a function of error-matrix rank, with \( N = 1024 \).](image)

As the interference does not present important variations and the ones observed comes, probably, from a lack of numerical resolution of the simulations, the use of the non-peaky MNW may be a solution to decrease the PAR. Therefore, a comparison between the PAR provided by each of the proposed MNW solutions and the provided by OFDM can be observed in figure 2.11.

Notice that the PAR considered for OFDM is the maximum as possible and the available degrees of freedom are, in this case, the number of subcarriers. As it is shown in Appendix B, the maximum PAR is equal to the number of subcarriers.
At this point, we may think on using the non-peaky MNW as the PAR improves. Another option could be not to use all available degrees of freedom. However, the maximum whiteness is only guaranteed when all available degrees of freedom are used.

### 2.7 Iterative computation of transmission waveforms

Until now, the study performed only applies for the first iteration of the transmission. As it is said previously, if $N$ is the total number of degrees of freedom and $M$, the ones occupied by the external network, then we will have $N - M$ degrees of freedom available. The idea is to design the waveform in iteration $k$ under the constraint that it has to be orthogonal to the $k - 1$ waveforms computed previously, in order to avoid ICI at receiver side.

To accomplish this constraint, in each iteration we will have a degree of freedom less than in the previous one. Hence, the noise subspace projector has to be updated in each iteration such that the degree of freedom used cannot be used anymore. Mathematically, it could be modeled as:

$$U_{S}(k+1) = [U_{S}(k) \ \varphi_{k}]$$ (2.54)

By computing the noise subspace projector, we may see the following:

$$P(k+1) = I - (U_{S}(k+1)U_{S}^{H}(k+1))$$
$$= I - [U_{S}(k) \ \varphi_{k}][U_{S}(k) \ \varphi_{k}]^{H}$$
$$= I - (P_{\perp}(k) + \varphi_{k}\varphi_{k}^{H})$$ (2.55)

Thus, the following algorithm is proposed to compute the $N - M$ possible waveforms:
Algorithm 2.1 Iterative Computation of Transmission Waveforms

\begin{algorithm}
\begin{algorithmic}
\State \textbf{begin initialize} \texttt{usedIndex}, $N$, $M$, $U$
\State $U_S = U_{1:M}$
\State $P^\perp = U_S U_S^H$
\State $P = I - P^\perp$
\State $indMax = \arg \max (\text{diag}(P))$
\State $\texttt{usedIndex}(1) = indMax$
\State $\varphi_1 = P_{indMax}$
\State $\varphi_1 = \frac{\varphi_1}{\sqrt{\varphi_1^H \varphi_1}}$
\State $U_S = [U_S \varphi_1]$
\State $P^\perp = U_S U_S^H$
\State $P = I - P^\perp$
\For{$j = 2$ until $N - M$}
\If{$indMax \in \texttt{usedIndex}$}
\State $indMax = \arg \max (\text{diag}(P))$
\EndIf
\Until{$indMax \notin \texttt{usedIndex}$}
\State $\texttt{usedIndex}(j) = indMax$
\State $\varphi_j = P_{indMax}$
\State $\varphi_j = \frac{\varphi_j}{\sqrt{\varphi_j^H \varphi_j}}$
\State $U_S = [U_S \varphi_j]$
\State $P^\perp = U_S U_S^H$
\State $P = I - P^\perp$
\EndFor
\State \Return $\varphi_1, \ldots, \varphi_{N-M}$
\State \textbf{end}
\end{algorithmic}
\end{algorithm}

In figure 2.12, we can observe the evolution of the MNW PSD in the first four iterations. Notice that the properties studied previously will be fulfilled by all computed waveforms except the whiteness, because the whitest waveform needs all available degrees of freedom. As we are losing degrees of freedom in each iteration, the whiteness will be degraded.

\begin{figure}[h]
\centering
\includegraphics[width=\columnwidth]{Figure2.12.png}
\caption{PSD of the first four MNW}
\end{figure}
3

Single frequency-selective channel scenario

3.1 Introduction

In the previous chapter, the channel between the opportunistic users has been assumed ideal. The objective of this chapter is analyzing the solution proposed in Chapter 2 but now considering the effects of a frequency-selective channel. Moreover, perfect CSI is assumed at both transmitter and receiver sides. The mathematical complexity induced by the SVD may be simplified by using a circular structure as it is done in OFDM or studying the asymptotic case. As we can find in some literature on statistics, when the number of samples is large enough, the limiting case may be considered. Thus, from now it is assumed that the length of observations is large enough to consider the case $N \to \infty$. The following statement is said about the decomposition of an autocorrelation matrix under this condition:

Let consider the spectral theorem which states that an autocorrelation matrix decomposes as $R = U^H A U$. Then, at the limiting case, the matrix $U$, that contains the eigenvectors of $R$, tends to the normalized Fourier matrix such that the spectral information is organized in a decreasing order, $F$.

This way, in this chapter the optimal solution for the asymptotic case is discussed in section 3.2. The obtained result will avoid us to study their properties. In section 3.3, an iterative algorithm to compute orthogonal waveforms is presented and the PAR of the four first iterations is studied. In section 3.4, the transmission through a frequency-selective channel using the asymptotic solution is discussed. Also, we show that the transmission strategy of the first iteration and the following ones cannot be the same if we want to avoid ICI.

3.2 Waveform optimization in the asymptotic case

The first step of the study is analyzing the waveform optimization problem when the observations autocorrelation matrix is diagonalized by the normalized Fourier matrix. In this scenario, we also assume the knowledge of the number of degrees of freedom occupied by the external network, $M$.

Following the conditions established in Chapter 2, the optimal waveform may be written as a linear combination of the columns of $F$ that corresponds spectrally to the noise. Hence, let $F_N \triangleq F_{M+1:N}$ be the columns of $F$, i.e. the eigenvectors of $R$, contained in the noise subspace of the external network. Thus,

$$\varphi_k = F_N \lambda$$  \hspace{1cm} (3.1)
and its squared norm is:

$$\|\varphi_k\|^2 = \lambda^H F_N^H F_N \lambda$$  \hspace{1cm} (3.2)$$

Considering the non-trivial constraint (2.10), as it is done in Chapter 2, the Lagrangian associated to this problem is given by:

$$\mathcal{L}(\lambda, \mu) = \lambda^H \lambda - \mu (\lambda^H F_N^H e_k - 1)$$  \hspace{1cm} (3.3)$$

where \(\mu\) is the Lagrange multiplier for the non-trivial constraint. By taking the gradient with respect to \(\lambda\), and using the non-trivial constraint to find the scaling parameter, the optimal waveform is:

$$\varphi_k = \frac{P}{e_k^H P e_k} F_N F_N^H e_k$$  \hspace{1cm} (3.4)$$

being \(P\) the noise subspace projector. Thus, its squared norm may be obtained by replacing (3.4) in (3.2):

$$\|\varphi_k\|^2 = \frac{1}{e_k^H P e_k}$$  \hspace{1cm} (3.5)$$

Notice that the result in (3.5) is the same as the obtained one in the AWGN channel and finite-size observations scenario. The optimal waveform is the column of the noise subspace projector that contains the largest value of its diagonal.

The properties of the Fourier matrix lead us to establish an additional criterion to select the transmission waveform in the first iteration. As all columns of a Fourier matrix will be orthogonal between them and their squared norms are the unit, then all columns of the noise subspace projector will be a possible MNW. Therefore, all them maximize the whiteness. As it is seen in section 2.6, the large PAR could lead a poor spectral efficiency. Given so, in the first iteration, the transmission waveform selected will be the one with minimum PAR.

![Figure 3.1: Some of the MNW including the one with minimum PAR.](image)

In this simulation, the total number of degrees of freedom is \(N = 1024\). Notice that in the available bands, the PSD of all waveforms presents a white shape. Taking into account the mathematical expression of the MNW shown above, the properties stated in section 2.3 are still true in this case.
3.3 Iterative computation of transmission waveforms

The idea of this section is the same as the pursued one in section 2.7. The difference is that in this case we do not have the noise eigenvectors matrix, but we have at our disposal the matrix $F$. Thus, the mathematical model in iteration $(k+1)$ is:

$$FS(k+1) = [FS(k) \varphi_k],$$

(3.6)

Hence, the updated noise subspace projector is computed as:

$$P(k+1) = I - (FS(k+1)F^H_S(k+1))$$

$$= I - [FS(k) \varphi_k][FS(k) \varphi_k]^H$$

$$= I - (P^\perp(k) + \varphi_k\varphi_k^H)$$

(3.7)

The following algorithm is just a modified version of algorithm 2.1 such that the eigenvectors matrix is replaced by Fourier matrix and the first MNW corresponds to the one that has the minimum PAR:

**Algorithm 3.1 Iterative Computation of Transmission Waveforms in the asymptotic case**

1: begin initialize usedIndex, $N$, $M$, $F$
2: $FS = F_{1:M}$
3: $P^\perp = FSF^H_S$
4: $P = I - P^\perp$
5: $ind\text{MinPAR} = \arg \min (PAR(P))$
6: usedIndex(1) = $ind\text{MinPAR}$
7: $\varphi_1 = P_\text{ind\text{MinPAR}}$
8: $\varphi_1 = \frac{\varphi_1}{\sqrt{\varphi_1^H\varphi_1}}$
9: $FS = [FS \varphi_1]$
10: $P^\perp = FSF^H_S$
11: $P = I - P^\perp$
12: for $j = 2$ until $N - M$
13: do
14: if $ind\text{Max} \in \text{usedIndex}$
15: $ind\text{Max} = \arg \max (\text{diag}(P))$
16: end
17: until $ind\text{Max} \notin \text{usedIndex}$
18: usedIndex($j$) = $ind\text{Max}$
19: $\varphi_j = P_{\text{ind\text{Max}}}$
20: $\varphi_j = \frac{\varphi_j}{\sqrt{\varphi_j^H\varphi_j}}$
21: $FS = [FS \varphi_j]$
22: $P^\perp = FSF^H_S$
23: $P = I - P^\perp$
24: end
25: return $\varphi_1, \ldots, \varphi_{N-M}$
26: end

In figure 3.2, we can observe the evolution of the MNW PSD in the first four iterations.
Notice that the properties studied previously will be fulfilled by all computed waveforms except the whiteness, because the whitest waveform needs all available degrees of freedom. As we are losing degrees of freedom in each iteration, the whiteness will be degraded.

As we have commented, the first iteration will be the waveform with minimum PAR. However, can be interesting to study the PAR for the MNW in the asymptotic case. In order to perform a study similar to section 2.6, we will consider the MNW for different number of available degrees of freedom:

We can observe that in the asymptotic case, the waveforms also present a peaky shape in time domain. Following the strategy proposed in section 2.6, the peak is going to be forced to be zero. The PSD of the MNW can be seen in figure 3.4 for 64 and 256 degrees of freedom.

From figure 3.4 we can conclude that the proposed technique in section 2.6 cannot be used in the asymptotic case because the reserved bands are occupied by the non-peaky waveforms.
3.4 Transmission in a frequency-selective channel

In this section it is studied the behavior of the optimal waveform in a frequency-selective channel scenario. After some considerations, a space diversity strategy is proposed to mitigate the multipath propagation and exploit any potential diversity gain provided by the channel memory.

First of all, let \( h \) be the impulse response of a Rayleigh channel. Hence, each tap \( h_l \) follows a complex Gaussian distribution, i.e. \( h_l \sim \mathcal{CN}(0, \sigma^2) \), for \( 1 \leq l \leq L \), being \( L \) the length of the impulse response. It is important to remark that the random variables \( h_l \) cannot be assumed to be i.i.d., because in this scenario the signal bandwidth will be higher than the coherence bandwidth. This fact will cause a certain correlation between the channel coefficients. Also, in order to reduce the complexity of the problem, we assume \( \sigma = 1 \).

At this point, the channel output may be obtained as the convolution between the input and the impulse response. Thus,

\[
y(n) = \varphi(n) * h(n)
\]  

(3.8)

In order to reduce the mathematical difficulty and to avoid the possible Inter-Symbol Interference (ISI), it is possible to use a cyclic prefix to induce a circular convolution instead of (3.8). The use of the cyclic prefix requires to copy a certain amount of the last samples of input signal, e.g. the last 25%, at the beginning of the input signal. Hence, the new definition of \( \varphi(n) \) is:

\[
\varphi' = \begin{bmatrix} c_p \\ \varphi \end{bmatrix},
\]  

(3.9)

where vector \( c_p \) contains the samples which correspond to cyclic prefix. Now, the channel output may be found as:

\[
y(n) = \varphi'(n) \odot h(n)
\]  

(3.10)

Thanks to (3.10), in the frequency domain the channel output is just the product between the input and the channel response. This principle allows to study mathematically this channel as a set of \( L \) frequency flat fading channels. Hence, the signal at receiver antenna is:

\[
y = F^H \left( h' \odot F \varphi \right) = F^H HF \varphi,
\]  

(3.11)

where \( h' \) is an \( N \)-th size vector and \( F \) is the normalized Fourier matrix. It is not necessary that \( L = N \), being \( N \) the length of the transmitted waveform. Then, the vector \( h' \) contains the channel coefficients repeated to achieve the \( N \)-th size required. Thus, the channel matrix \( H \) is a diagonal...
matrix which contains the elements of $h'$, i.e. $H = \text{diag}(h')$. Notice that the cyclic prefix is neglected at receiver side.

As an example, in figure 3.5, we can see the multiplicative effect in the frequency domain when the cyclic prefix is used.

![Figure 3.5: Comparison between the asymptotic MNW PSD and the Non-peaky MNW PSD.](image)

As it is well-known, in a scenario such the one under study, the effects of the multipath propagation may cause a degradation of the received signal. Hence, it will be a smart idea to think about a transmission strategy that mitigates its effects. In this case, we have proposed an MRC at TX side.

### 3.4.1 Transmission of the first iteration

The use of MRC supposes the use of a diversity scheme at TX side. First of all, complete CSI at both transmitter and receiver sides is assumed. Thus, a perfect knowledge of the random channel let assume it as a deterministic channel, which also avoids the estimation of the water-pouring coefficients. At this point, it is interesting to study the diversity gain that the used strategy will provide at the output. Remark that this diversity gain will suppose a decrease in the BER for a given SNR.

The channel output of a system such the one under study will be the sum of all branches. Thus,

$$y = F^H \sum_{l=1}^{L} \beta_l h_l \varphi + w$$

$$= F^H F \|h\| \varphi + w, \quad (3.12)$$
where $\beta_l$ are the water-pouring coefficients, defined as:

$$\beta_l = \frac{h_l}{\|h\|}$$  \hspace{1cm} (3.13)

From (3.12), the diversity gain, $G_D$, is $\|h\|^2$. At this point, from (3.11) it may be said that matrix $F^H HF$ is a circular matrix, let it be named $H_c$. Thus, taking into account the properties of the Fourier matrix and the squared Frobenius norm of $H_c$, the diversity gain is $\|H_c\|^2$. Notice that in a complete CSI scenario, the diversity gain is a known variable. However, with imperfect CSI, this variable will follow a Chi-squared distribution with $2L'$ degrees of freedom, where $L'$ are the number of channels that can be assumed i.i.d., fact that will depend on the coherence bandwidth.

Notice that the MRC at transmitter side can be used only for the first iteration. The receiver for this strategy is as simple as a sum of all branches. If we want to avoid ICI at receiver side, for the second iteration we cannot sum all branches because the received signal will corrupt the received signal in the previous iteration.

With this technique, the mathematical complexity of the transmission can be minimized using the proposed circular structure. Moreover, the use of the cyclic prefix, apart from reducing the possible ISI, allows us to study this problem as a set of $L$ frequency flat fading channels in the frequency domain. The proposed diversity strategy reduces the effects of multipath fading and exploits the diversity gain, $G_D$, provided by the channel memory.

### 3.4.2 Computation of the second iteration

In this case, we will have to take into account the effects of the channel and the last consideration of the previous subsection. In order to avoid the ICI at receiver, an idea is to implement two independent MRC at receiver side, one for the first iteration and the other one for the second iteration. Notice that the diversity gain will not change from the computed one in subsection 3.4.1.

We know that the received signal may be written as:

$$y_k = H_c \varphi_k$$  \hspace{1cm} (3.14)

Then, the condition to fulfill at receiver side between the first and the second iterations is:

$$y_1^H y_2 = \varphi_1^H H_c^H H_c \varphi_2 = 0$$  \hspace{1cm} (3.15)

Developing (3.15), we can see the following:

$$\text{tr} \left( \varphi_1^H H_c^H H_c \varphi_2 \right) = \text{tr} \left( F \varphi_2 \varphi_1^H H_c^H F \varphi_2 \right) = 0$$  \hspace{1cm} (3.16)

Thus, from (3.16) we can formulate the conditions for the second iteration:

$$\text{tr} \left( H_c^H H_c \varphi_k \right) = 0 \hspace{0.5cm} \text{s.t.} \hspace{0.5cm} H_c^H H_c \neq 0$$  \hspace{1cm} (3.17)

That is, the waveforms must be orthogonal at the receiver side without taking into account the influence of the channel. The extension to the general case, can be written by forcing the orthogonality of the waveform in the iteration $k$ with the $k-1$ waveforms built previously at receiver side. Let $\Phi = [H_c \varphi]_{1 \leq i \leq k-1}$ be the matrix that contains on its columns all the received waveforms. Hence,

$$\Phi^H H_c \varphi_k = 0$$  \hspace{1cm} (3.18)

With this condition, ICI will be avoided at receiver side although the orthogonality is not guaranteed at transmitter side, which is not a problem. As it is said, the diversity strategy should be as much independent MRCs as waveforms computed. If they are orthogonal, the scalar product of each branch of the receiver will cancel the influence of other waveforms keeping only the one of interest.
Conclusions and Future Work

4.1 Conclusions

This work has tackled two problems in the area of Opportunistic Communications: the transmission between new users avoiding the interference to the existing ones and, in case of providing interference, how this undesired situation can be mitigated.

In Chapter 2, assuming ideal channel, the optimal waveform has been computed and their properties studied. As we have seen, when the estimation is perfect and there is not shadowing, the proposed solution accomplishes the basic objective of opportunistic transmission. If these effects have been considered, then a bit energy loss and interference to the external network is caused. However, using a first-order feedback the performance achieves the expected goals. In terms of PAR, the obtained waveform presents a peaky shape in the time domain which suppose that for a large number of available degrees of freedom available, the PAR is similar to the one for OFDM. The idea of sending this peak to zeros is proposed and the performance and accomplishment of the properties have been studied. As it is seen, the use of the non-peaky MNW is more or less the same as using the optimal MNW for a large number of available degrees of freedom. Finally, an iterative algorithm is proposed in order to avoid the ICI at receiver side. To achieve this, the orthogonality between transmission waveforms has to be forced, with the price of losing whiteness in each iteration.

In Chapter 3, the asymptotic case is firstly studied. Considering that the correlation matrix can be diagonalized by the Fourier matrix such that spectral information is organized in a decreasing order, the mathematic load is reduced. Consequently, in this scenario, we do not need the SVD to get the noise subspace eigenvectors. The particularities of the Fourier matrix lead us to select, in the first iteration, the waveform that presents the minimum PAR, as all possible waveforms fulfills the minimum norm criterion. In further iterations, as the whiteness will be degraded, the waveform that maximizes it will be the used one. As we have seen, the proposed strategy for the finite-size observations case cannot be used in the asymptotic case because the bands occupied by the external network will be encroached. The transmission in a frequency-selective channel is also studied in the asymptotic case. We have used the cyclic prefix to force a circular structure, which reduces the mathematical difficulty and avoids the possible ISI. Finally, an MRC at TX is proposed to transmit the first iteration to mitigate the multipath fading effects and to exploit the diversity gain. The mathematical condition that the second iteration must fulfill is that it has to be orthogonal to the first one at receiver side. In the general case, the iteration \( k \) has to be orthogonal at receiver side to the \( k - 1 \) waveforms computed previously.
4.2 Future Work

Once this Degree thesis is finished, the next topics are identified as future research.

4.2.1 Multichannel scenario
In the present study, only single channel is considered. It could be interesting to take into account the advantages of spatial diversity to see the behavior of the proposed strategy and what the use of multiantenna setting will suppose in terms of interference provided to the external network and how the performance of the proposed strategy will vary.

4.2.2 Frequency-selective channel with imperfect CSI
In the performed study in Chapter 3, perfect CSI is assumed at transmitter and receiver sides. The next step is assuming an incomplete knowledge of CSI. This issue will complicate the problem because the effects of the channel will have to be included in the optimization problem.

4.2.3 Interference level in frequency-selective channels
In Chapter 3, the estimation of the noise subspace at both transmitter and receiver sides is assumed to be ideal and the shadowing effects have been neglected. The following step will be to consider the effects of the error matrix $\Delta$ when the channel is not ideal.

4.2.4 Verification of the mathematical models
The mathematical models described on this work have not been verified using real data. Capturing real data and modulate it with the proposed strategy will give us an actual idea of its performance. In this sense, the mathematical model of the error matrix should be verified.

4.2.5 Iterative algorithm to compensate the interference and the channel effects
In this work, two iterative algorithms have been proposed. They do not take into account the effects of the channel. The idea is to design an iterative computation such that the effects caused by the shadowing at TX side and the ones introduced by the channel should be compensated in each iteration. Thus, the waveforms computed keep the orthogonality at RX side and the transmission of the internal-network users does not interfere the communication in the external network.
References


Further Information of Chapter 1

A.1 Calendar

The calendar associated to subsection 1.1.3 is attached.

![Figure A.1: Time planning of the Degree Thesis.]

Notice that (*) does not included as a part of this work and it corresponds to an item of Future Work.
An OFDM signal is the sum of multiple sinusoids separated $\frac{1}{T}$ in the frequency domain. Each sinusoid gets modulated by independent information $a_n$. Thus, mathematically, the transmitted signal may be written as:

$$x(t) = \sum_{n=0}^{N-1} a_n \exp \left\{ j \frac{2\pi nt}{T} \right\}$$  \hspace{1cm} (B.1)

being $N$ the total number of subcarriers. For simplicity, let us assume $a_n = 1$ for all subcarrier. In that scenario, the peak power of the transmitted signal is:

$$\max |x(t)|^2 = \max [x(t)x^*(t)] = \max \left[ \sum_{n=0}^{N-1} a_n \exp \left\{ j \frac{2\pi nt}{T} \right\} \sum_{n'=0}^{N-1} a_{n'}^* \exp \left\{ -j \frac{2\pi n't}{T} \right\} \right]$$

$$= N^2$$  \hspace{1cm} (B.2)

The average power is:

$$\mathbb{E} [|x(t)|^2] = \mathbb{E} \left[ a_n a_n^* \sum_{n=0}^{N-1} \exp \left\{ j \frac{2\pi nt}{T} \right\} \sum_{n'=0}^{N-1} \exp \left\{ -j \frac{2\pi n't}{T} \right\} \right] = N$$  \hspace{1cm} (B.3)

Given so, the PAR for an OFDM system with $N$ subcarriers and all of them are given the same modulation is,

$$\text{PAR} = N$$  \hspace{1cm} (B.4)

It is reasonably intuitive that (B.4) corresponds to the maximum PAR, when all the subcarriers are equally modulated, aligned in phase and the peak value hits in the maximum.