AN ADAPTATIVE SPECTRAL ESTIMATOR BASED ON THE PREDICTIVE INNOVATION PROCESS AND KALMAN FILTERING

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A new algorithm is proposed for the simultaneous estimation of poles and zeros in power spectral estimation. The method is based in the Kalman filtering and uses the innovation sequence to generate the numerator of the rational estimator.

The algorithm is adaptative and does not require any preliminary processing or analysis of the sampled signal. The procedure achieves promising levels of insensibility with respect to initial values and to the estimator order.

The results obtained when applying the proposed algorithm to analyze simulated systems and natural electroencephalogram show that the procedure gives a very accurate fit of the spectra of the systems analyced.

The signal model employed in this work is based on the Gauss signal model |1| and is show in (1)

$$x(k)=\sum_{i=1}^{Q} a(i,k)x(k-i)+\sum_{j=1}^{P} b(j,k)\delta(k-j)$$

$$a(i,k+1)=a(i,k)+\omega(k) \qquad (1)$$
$$b(j,k+1)=b(j,k)+v(k)$$

Where x(k) denotes the kth signal sample, a(i,k) are the denominator coefficients, b(j,k) are the numerator coefficients and ω(k), v(k) y δ(k) are white noise terms of zero mean and unknow variances.

In order to extract the coefficients a(i,k) and b(j,k) of the observed signal x(k), it is proposed the filtering showed in (2)

$$\hat{x}(k)=\sum_{i=1}^{Q} \hat{a}(i,k)x(k-i)+\sum_{j=1}^{P} \hat{b}(j,k|e(k-j)$$

$$e(k)=x(k)-\hat{x}(k) \qquad (2)$$

The estimated spectrum $\hat{S}_{xx}(\omega)$ can be expressed in terms of the filter coefficients defined in (3) as follows:

$$\hat{S}_{xx}(\omega) = B(\omega)/A(\omega)$$

$$B(\omega) = 1 + \sum_{j=1}^{P} \hat{b}(j,k_o)\exp(-j\omega) \qquad (3)$$

$$A(\omega) = 1 - \sum_{i=1}^{Q} \hat{a}(i,k_o)\exp(-j\omega)$$

The value k_o is the required for achieve the properties of an innovation sequence on e(k). In the conventional approaches to solve the problem, global cost functions over e(k) were minimized. We propose in this work the minimization of a quadratic cost functions over the coefficients of the model. In other words, we select the set a(i,k) and b(j,k) which produce minimum mean squere-error respect the optimum set that is expresed in the model given in (1).

The variances of v(k) and ω(k) were estimated using an smoothed version of the innovation sequence.

The up dated values of the coefficients a(i,k+1) and b(j,k+1) were obtained from (4) by the application of the orthogonality principle |2|

$$a(i,k+1) = a(i,k) + A(k,i)e(k)$$
$$b(j,k+1) = b(j,k) + B(k,j)e(k) \qquad (4)$$

Where,

$$A(k,i) = \alpha(k)x(k-i)/E(k)$$
$$B(k,j) = \alpha(k)e(k-j)/E(k)$$
$$E(k) = \alpha(k)(Qx(k)^2 + Pe(k)^2) + 1$$
$$\alpha(k+1) = \alpha(k) - (\alpha(k)^2 x(k)^2/E(k)) + gE(k)$$

The parameter g is related with the estimation of the covariances of v(k) and ω(k) by maximum likehood.

References

|1| Brian D.O., Anderson, John B. Moore; "Optimal filtering". Prentice Hall. Chapter 3, 1979.

|2| A. Papoulis; "Probability, Random variables and stochastic Processes". Mc. Graw-Hill. Chapter II. 1965.