Sub-nanosecond pulse filtering and amplification through first-order controlled circuit instability


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Abstract— In this paper we investigate how to use controlled circuit instability in the filtering and amplification of narrow pulses. The basic circuit consists of a first-order RC network coupled to a negative resistance controlled by an external quench generator, which yields alternating periods of stability and instability. The study shows that when the circuit acts as a signal generator it can produce narrow exponential or Gaussian pulses, depending on the quench waveform applied. Included as a receiver front end, the circuit behaves as a high-gain narrow-pulse filter. We also estimate the circuit parameters required to generate and receive sub-nanosecond pulses.

Index Terms—Radio communication, circuit stability, signal generators, RC circuits, ultra-wideband communication, impulse radio.

I. INTRODUCTION

Pulse modulation is of spread use in communication systems. A paradigm currently in great expansion is that of ultra-wideband (UWB) systems, which promise low complexity, low power, low costs and high data rate connectivity for a variety of applications such as public safety, business and consumer products. In particular, UWB impulse radio uses baseband pulse modulation, including pulse amplitude (PAM), pulse position (PPM) modulation and phase reversal keying (PRK), to achieve low complexity and low costs [1].

In this paper, we apply the principle of superregeneration to a first-order RC circuit and describe how it can be used to generate and receive narrow pulses. The proposed circuit can be understood as a lowpass version of the superregenerative receiver, which provides exceptional simplicity, low cost and low power consumption [2], [3], [4]. Ref. [5] describes an implementation of the circuit in which the principle is used to sample and amplify baseband signals. In this paper we give a detailed description of the characteristic parameters that determine the operation of the circuit with both generic inputs and quench waveforms and analyze its performance in generating and receiving sub-nanosecond impulses.

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II. BASIC OPERATING PRINCIPLE

Fig. 1 shows the basic schematic of the circuit considered in this paper. It consists of an RC network coupled to a negative conductance that varies periodically under the control of an external quench generator. Let \( G_0 \) be the quiescent circuit conductance in its passive state and let \(-G_a\) be the negative conductance generated by the active devices. The quench generator varies \( G_a \) so that the net conductance

\[
G(t) = G_0 - G_a(t)
\]

becomes alternatively negative, which yields a period of instability in which the capacitor voltage increases exponentially, and positive, which produces a period of stability during which the voltage decreases. As we show in the following section, the voltage generated in a given quench cycle of period \( T_q \) is controlled by the small signal \( i(t) \) present around the zero-crossing of \( G(t) \) with a negative slope. Therefore, the generated voltage \( v_o(t) \) is composed of a series of pulses that represent amplified samples of the input signal. If we assume that the circuit is linear, the output voltage is characterized by the linear, time-varying, first-order differential equation

\[
\dot{v}_o(t) + \frac{G(t)}{C} v_o(t) = \frac{1}{C} i(t),
\]

whose general solution is given by

\[
v_o(t) = v_o + \frac{1}{C} \int_{t_a}^{t} i(\xi) e^{-\frac{1}{C} \int_{\xi}^{t} G(\tau)d\tau} d\xi - e^{-\frac{1}{C} \int_{t_a}^{t} G(\tau)d\tau} \int_{t_a}^{t} G(\tau)d\tau
\]

where \( V_o \) is the initial voltage at the quench cycle start time \( t_a \).

III. PULSE CIRCUIT RESPONSE

The analysis of a specific implementation of the circuit shown in Fig. 1 (a) is presented in [5]. In this paper we focus
on the circuit response to a generic input pulse of the form

\[ i(t) = I p_s(t) \]  

(4)

where \( I \) is the peak current amplitude and \( p_s(t) \) a unit-normalized shaping function that is assumed to be zero beyond the limits defined by \( t_a \) and \( t_b \) in Fig. 1 (b). Let \( R_0 = 1/G_0 \) and \( \tau_\text{q}=R_0C \) be the quiescent values of the resistance and the time constant of the circuit, respectively. If we assume that there is no residual voltage from the previous quench cycles when the current cycle starts (i.e., \( V_o=0 \)), we can follow a procedure analogous to that described in [2] with appropriate parameter and function definition to express the response as

\[ v_o(t) = I R_0 K_s K_r p(t) \]  

(5)

with the following parameters and functions:

1) Regenerative gain:

\[ K_r = \frac{1}{\tau_\text{q}} \int_{t_a}^{t_b} p_s(\tau)s(\tau) d\tau \]  

(6)

2) Sensitivity function:

\[ s(t) = e^{-\frac{1}{C R_0} \int_{t_a}^{t_b} G(\lambda) d\lambda} \]  

(7)

3) Superregenerative gain:

\[ K_s = e^{-\frac{1}{C R_0} \int_{t_a}^{t_b} G(\lambda) d\lambda} \]  

(8)

4) Normalized envelope of the output pulse:

\[ p(t) = e^{-\frac{1}{C R_0} \int_{t_a}^{t_b} G(\lambda) d\lambda} \]  

(9)

Eq. (5) shows that the response of the circuit to the input pulse shaped by \( p_s(t) \) is an amplified pulse shaped by \( p(t) \). \( R_0 \) represents the amplification factor of the circuit in its passive state (i.e., when the quench is disabled, \( G_0=0 \)), whilst \( K_r \) and \( K_s \) are additional factors introduced by the quench operation. The product \( R_0 K_s K_r \) is the total peak gain.

The regenerative gain \( K_r \) is obtained by integrating the input pulse \( p_s(t) \) weighted by the sensitivity function \( s(t) \). The latter is also a normalized function that takes its maximum value of unity at \( t=0 \) and usually decreases rapidly towards zero due to its exponential dependence on time. This function describes the impact of each portion of the input signal, depending on its temporal location, and its effective duration defines the sensitivity period (Fig. 1 (b)). It can be concluded from (6) that the maximum output signal-to-noise ratio for a given input pulse energy is achieved when \( p_s(t)=s(t) \). The circuit behaves as a matched filter under this condition. Since the factor \( K_r \) depends on the cross-correlation of \( p_s(t) \) and \( s(t) \), it also determines how the output signal changes when the input pulse is advanced or delayed with regard to \( t=0 \). The resulting phase discrimination characteristic may eventually be used to synchronize the receiver.

The superregenerative gain \( K_s \) is an amplification factor associated with the exponential build-up of the output voltage during the negative conductance period and is the most relevant amplification factor. The value of \( K_s \) depends on the area of the negative portion of the conductance. Eq. (5) holds after the end of the sensitivity period and assumes that \( K_r >> 1 \) and \( s(t_a), s(t_b) << 1 \), as is usually the case. Fig. 2 shows the characteristic functions with two different types of quench waveform.

IV. INFLUENCE OF THE QUENCH WAVESHAPES

It is well known from the literature on superregenerative reception that there are two possible modes of operation depending on the quench waveform generated: the step-controlled state and the slope-controlled state [3].

A. Step-controlled state

The step-controlled state is characterized by a discontinuous transition of the conductance at \( t=0 \). A rectangular quench is a common example that is easy to generate, in which the conductance alternates between two constant values \( G_- \) and \( -G_+ \), as shown in Fig. 2 (b). The negative conductance period is equal to \( t_b \). A two-sided
Fig. 2. (a) Input pulse. Conductance, sensitivity function and generated pulse: (b) in the step-controlled state; (c) in the slope-controlled state.

exponential pulse shape is obtained and each side has a time constant that is related to the corresponding conductance $G_+$ or $G_-$. Table I shows the characteristic functions and parameters of this mode of operation. For the sake of simplicity, we assumed that $G_+ = G_- = G_0$, which also ensures that the circuit generates and receives symmetrical pulses. In this case, the pulse width $t_w$ at 60.7% of the peak amplitude is equal to the time constant $\tau_0$. The regenerative gain was calculated for two input pulse shapes: (a) matched to the sensitivity function and (b) constant during the entire sensitivity period.

**TABLE I.**

<table>
<thead>
<tr>
<th></th>
<th>Step controlled</th>
<th>Slope controlled $(t_r=t_f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s(t)$ ($t_e&lt;t_b$)</td>
<td>$\frac{1}{\tau_0} e^{-t/\tau_0}$</td>
<td>$-\frac{1}{\tau_0} e^{-t/\tau_0}$</td>
</tr>
<tr>
<td>$p(t)$ ($0&lt;t&lt;T_q$)</td>
<td>$\frac{1}{\tau_0} e^{-t/\tau_0}$</td>
<td>$e^{-(t-t_b)^2/\tau_0}$</td>
</tr>
<tr>
<td>$t_w$ (60.7%)</td>
<td>$\tau_0$</td>
<td>$\sqrt{2t_f \tau_0}$</td>
</tr>
<tr>
<td>$K_r$</td>
<td>matched pulse</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>constant pulse</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>$e^{-t_0}$</td>
<td>$e^{-t_f/2}$</td>
</tr>
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The sensitivity function (7) is determined by the slope of the conductance at $t=0$. The pulse waveform generated is defined by a Gaussian function of standard deviation

$$\sigma_s = \sqrt{\frac{t_f}{2}}. \quad (10)$$

Table I summarizes the main parameters and functions in the slope-controlled state. For the sake of simplicity and symmetry, we also assumed that $G_+ = G_0$ and that rise and fall times were equal, $t_r = t_f$. The transition time must meet the condition $t_f > 4 \sigma_s$ to prevent the premature truncation of the Gaussian function due to the slope change out of the transition period. The 60.7% pulse width $t_w$ is equal to $2 \sigma_s$, which is controlled in this case by both the time constant $\tau_0$ and the fall time $t_f$. The transition time $t_f$ can therefore be increased if necessary, provided that the time constant $\tau_0$ is decreased accordingly.

**V. PULSE GENERATION**

As mentioned above, the proposed circuit can generate exponential and Gaussian pulses with the quench waveforms considered. The slope-controlled state can be of particular interest because Gaussian pulses and their derivatives are widely used in UWB communications [1]. By controlling the initial startup voltage of the RC circuit by means of an external circuit, it is possible to modulate the amplitude and/or the polarity of the generated pulses.

**VI. PULSE FILTERING AND AMPLIFICATION**

Eq. (6) and (8) show that the circuit uses the sensitivity function $s(t)$ to sample the input signal and that it can achieve high gain, particularly as a result of the superregenerative factor $K_r$. Therefore, when the parameters
are correctly selected to generate a narrow sensitivity function, the circuit is ideal for sampling and amplifying low energy narrow pulses, e.g., as a receiver front end. The optimum results will be achieved when the input pulses are matched to the sensitivity function, which cause the circuit to operate as a matched filter.

VII. EXPERIMENTAL RESULTS

We calculated the circuit parameters required to generate/receive pulses having \( t_w = 0.5 \text{ ns} \). Considering a total capacitance, including the contribution of the active devices, of 2.5 pF, and operation in the slope-controlled state with \( t_b = t_s = t_f = 1.2 \text{ ns} \), we obtained \( R_0 = 44 \Omega \left( \tau_0 = 110 \text{ ps} \right) \) and a theoretical gain \( K_s K_b = 60 \text{ dB} \). Fig. 3 shows the schematic of the implemented circuit using bipolar transistors in a cross-coupled differential configuration. The lower transistor acts as a current source controlled by the quench signal. Fig. 4 shows the applied quench voltage and the generated pulse for an input peak amplitude of approximately 0.3 mV. The pulse characteristics are in good agreement with the theoretical predictions. Fig. 5 shows the results for a practical application under PRK modulation at 20 Mbit/s. In this case, wider quench pulses having \( t_w = 10 \text{ ns} \) are applied to increase gain. As a consequence, the output voltage reaches positive or negative saturation depending on the input signal polarity. The amplitude and the width of the generated pulses are consequently wider, which facilitates data detection, whereas the sensitivity function is kept narrow provided that the transition times of the quench voltage remain unchanged. Table II summarizes the main circuit features.

VIII. CONCLUSION

In this paper we show how the controlled instability of a first-order RC circuit can be applied to the generation, filtering and amplification of sub-nanosecond pulses. Exponential pulses can be processed optimally in the step-controlled state whereas Gaussian pulses are best-suited to the slope-controlled state. The circuit is very simple, achieves high gain and low power consumption. Further optimization of the circuit will allow the reception of signals operating under the UWB spectrum mask.

![Schematic of the implemented circuit](image1)

Fig. 3. Schematic of the implemented circuit.

![Generation of a 0.5-ns pulse](image2)

Fig. 4. Generation of a 0.5-ns pulse: quench voltage (upper trace, 0.5 V/div) and differential voltage measured at the cross-coupled pair (lower trace, 100 mV/div). The time scale is 1 ns/div.

![Reception of PRK modulation](image3)

Fig. 5. Reception of PRK modulation: quench voltage (upper trace, 2 V/div) and differential voltage measured at the cross-coupled pair (lower trace, 500 mV/div). The time scale is 50 ns/div.

<table>
<thead>
<tr>
<th>TABLE II. SUMMARY OF CIRCUIT FEATURES.</th>
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<tbody>
<tr>
<td>Pulse width</td>
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<tr>
<td>Pulse repetition frequency = quench frequency</td>
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<tr>
<td>Operation mode</td>
</tr>
<tr>
<td>Peak-to-peak output voltage</td>
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<tr>
<td>Total supply current</td>
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<tr>
<td>Total power consumption</td>
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REFERENCES