Signal to noise ratio in non-linear optical amplification process

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ABSTRACT

In this communication, one looks at the influence of the saturation process over the statistical fluctuations (signal to noise ratio) of the optical power at the output of a travelling wave semiconductor optical amplifier (TWOA). The statistical behaviour of this device is described starting from the photon density matrix equation. The obtained theoretical results, both for the noise figure and for the signal to noise ratio, allow to take out some conclusions about noise behaviour in the non-linear light amplification process.

1.INTRODUCTION.

The numerous performances that semiconductor optical amplifiers can offer in optical fiber transmission systems have been proved throughout the present decade by many authors[1,2,3]. On these papers it is pointed out that in a light transmission system, information is transmitted by means of an intensity modulated, frequency modulated or phase modulated optical carrier. At the end of certain distance, signal must be amplified because of optical fiber losses during light propagation. Really, this function can be realized by a linear optical amplifier acting as a line repeater, both in long distance land links and in submarine links. Another application of this device is its use as an optical preamplifier before the photodetector in order to increase receiver sensibility[4,5]. In this context, signal to noise ratio concept constitutes a suitable figure of merit when estimating quality of transmitted information. In this communication one deals with some questions related to signal to noise ratio at semiconductor optical amplifier output but looking at saturation phenomenon. Due to this effect which unavoidably always exists the optical amplifier can show a certain non-linear behaviour.

This paper is organized as follows: in Section 2 the model used to characterize a non-linear optical amplifier is described, which is based on the photon density matrix equation[6,7]. The fact to bear in mind saturation phenomenon forces to solve non-linear differential equations which characterize the device, in order to caculate the mean and the variance of the amplified light. The method used to solve these equations is described in reference[8]. Signal to noise ratio and noise figure concepts used in this communication are defined in Section 3. In the next one (Section 4), numerically obtained results of the mean, the variance, the noise figure and the signal to noise ratio of the amplified optical power are shown. Signal to noise ratio can be optimized, under a theoretical point of view, as a function of different parameters taking part in the amplification process. Results related to gain and figure of merit which have been obtained by means of our simulations are in good agreement with some experimental results obtained at CNET (Lannion, France) within RACE 1027 Program[9]. The main conclusions are exposed in Section 5. Among them, the existence of optimum saturation parameter values that allows to obtain a maximum signal to noise ratio, and hence a minimum noise figure, is maybe the most outstanding.

2. TRAVELLING WAVE OPTICAL AMPLIFIER MODEL.

In a semiconductor optical amplifier the laser transition occurs between distributions of electronic states in the conduction and valence bands. We assume a model of the gain medium as an ensemble of two-level systems (particles), each consisting of their respectives conduction-band and valence-band electronic states. The statistical behaviour of the transitions that take place can be characterized by the Photon Density Matrix Equation (P.D.M.E.)[10].The rate equation for its diagonal elements describes the evolution during the amplification process of the probability density function, $P_m(t)$, of having m photons in the considered mode at an instant t when there is a fixed number of photons at the amplifier input (t=0). $P_m(t)$ evolution is given by

$$\frac{dP_m(t)}{dt} = \frac{(m+1)A}{1+(m+1)s} P_m(t) + \frac{mA}{1+ms} P_{m-1}(t) + \frac{(m+1)B}{1+(m+1)s} P_{m+1}(t) - \frac{mB}{1+ms} P_m(t) - $$

$$- (m+1)C\, P_{m+1}(t) - mC\, P_m(t) \qquad (1)$$

where A and B are coefficients associated to stimulated emission and stimulated absorption processes, respectively, s is the saturation parameter which is equal to the spontaneous emission coefficient[11], and C is a coefficient that represents losses mechanisms in the amplifier medium.

From equation (1) one can obtain the system of equations which describes the temporary evolution of the statistical moments of the photon distribution. For the r^{th}-order moment we have

$$\frac{dm_r}{dt} = \sum_{j=1}^{r} \binom{r}{j-1} \left[\left< \frac{(m+1)m^{j-1}}{1+(m+1)s} \right> A + (-1)^{r+j-1} \left< \frac{m^j}{1+ms} \right> B + (-1)^{r+j-1} \left< m^j \right> C \right] \qquad (2)$$

where $m_r = \left< m^r \right>$ and < > denotes the statistical mean value. For different values of r equation (2) provides a non-linear differential equations system, coupled and without analytic solution due to the dependence of each moment with regard to all the others. In particular, for r=1 and r=2 one obtains

$$\frac{dm_1}{dt} = \left< \frac{m+1}{1+(m+1)s} \right> A - \left< \frac{m}{1\ ms} \right> B - Cm_1 \qquad (3a)$$

$$\frac{dm_2}{dt} = 2\left(<\frac{(m+1)m^2}{1+(m+1)s}>A - <\frac{m^2}{1+ms}>B - Cm_2\right) + <\frac{m+1}{1+(m+1)s}>A + <\frac{m}{1+ms}>B + Cm_1 \qquad (3b)$$

The solution of equations (3) gives the first two statistical moments of the light when it passes through a medium with saturable gain and linear losses, starting with the moments corresponding to the incident optical power. The knowledge of the mean and the variance is fundamental in order to calculate any figure of merit upon the statistical fluctuations of the amplified optical power. In the next Section we will see the figures of merit we will use to describe the non-linear optical amplifier statistical behaviour.

3.SIGNAL TO NOISE RATIO AND NOISE FIGURE.

Starting with the definition given by

$$\sigma_m^2(t) = m_2(t) - m_1^2(t) \qquad (4)$$

we define

$$F(t) = \frac{m_1^2(t)}{\sigma_m^2(t)} \qquad (5)$$

where t represents the amplification time for a given amplifier length. F(t) will be the criterion used to calculate the signal to noise ratio of the amplified optical power. It is about a useful expression when the photon number involved in the amplification process is high.

The optical amplifier noise figure can be defined starting from (5) by

$$N.F. = \frac{F_{IN}}{F_{OUT}} = \frac{F(0)}{F(t)} \qquad (6)$$

If we suppose a linear situation then the product of the saturation parameter by the photon number is much smaller than unity (sm<<1). Assuming high gain, coherent light at the amplifier input and noises depending on the signal (i.e., shot noise and beating noise between spontaneous emission and signal), expression (5) takes the value

$$F \simeq \frac{A-B-C}{2A}\, n_1 \qquad (7)$$

being n_1 the incident photon mean number. Accordingly the noise figure will be

$$N.F. \simeq 2\frac{A}{A-B-C} \qquad (8)$$

Nevertheless, values predicted by (7) and (8) will only be strictly true if one despises the saturation phenomenon that unavoidably always occurs. Now our aim is to take precisely into account this phenomenon.

4.RESULTS AND DISCUSSION.

Using the solving method developed in reference[8] we have calculated the mean, the variance, the signal to noise ratio and the noise figure at the optical amplifier output, all of them as a function of the saturation parameter. Particularly, the mean and the variance have been respectively represented on curves a) and b) in figure 1, in a case of having input coherent light with a photon mean value of 10^5. We can observe that for small saturation parameter values both magnitudes are approximately constants and with clearly different values. This situation corresponds to the linear amplification case, i.e., that one which verifies $sm \ll 1$. On the other hand, both the mean and the variance decreases when the saturation parameter increases because of the device gain diminution, so that for large saturation parameter values ($>10^3$) both the mean value and the variance tend to the same asymptotic value. This characteristic points out the possibility to reduce the relative statistical fluctuations of the amplified light after a non-linear amplification process.

In order to see the influence of the saturation on the signal to noise ratio, figure 2 shows the F ratio defined in (5) as a function of the saturation parameter. It is clear there exists a saturation parameter value (around 10^{-6}) for which the F ratio is maximum and, therefore, the relative statistical fluctuations of the amplified light are minimum. On the other hand, with the aid of figure 1 we can observe that when F is maximum the gain of the device has suffered a drastic reduction. Really, the gain is approximately 23 dB in the case of a negligible saturation and it decreases down to 13 dB for $s \simeq 10^{-6}$. In consequence, we can say that when the non-linearity of an optical amplifier represents a good quality, its gain is considerably reduced. However in this paper, performances of a non-linear optical amplifier are calculated with regard to the possible reduction of the relative statistical fluctuations of light and not with regard to its gain. To calculate magnitudes represented in figures 1 and 2 one has assumed $A=2.8\cdot10^{12}$ s^{-1}, $B=1.82\cdot10^{12}$ s^{-1}, $C=0.2\cdot10^{12}$ s^{-1}, an amplification length of 500 μm and a light speed in the medium $v=7\cdot10^{7}$ $m\cdot s^{-1}$. These are typical values for semiconductor laser materials operating at a wavelength around 1.5 μm.

Because calculus presented in this paper correspond to a non-linear amplification process, any presented result will necessarily depend on the incident optical power. In order to analyze the influence of this incident power, figure 3 shows the signal to noise ratio as a function of the saturation parameter, taking the incident photon mean number as a parameter. The optical amplifier parameters values (length, v, A, B and C) are those of the previous figures. In figure 3 one can observe as the incident optical power level increases the maximum value of the signal to noise ratio is obtained with a lower saturation parameter. The relative statistical fluctuations of the amplified light are minimum for a concrete value of the saturation parameter which decreases when the incident photon number increases.

For several reasons, in a general situation, the ligth incident to the amplifier could not be totally coherent (e.g., owing to the possible quantum noise accumulated before the considered amplification process). Then, in a more general way, we can assume that the variance at the amplifier input is such

$$\sigma_n^2=\alpha n_1 \tag{9}$$

where n1 is the incident photon mean number. The α parameter is larger or equal to unity ($\alpha=1$ for the coherent light case). Figure 4 shows F ratio as a function of s taking α coefficient as a parameter and $n1=10^5$. It can be observed that the position of the maximum F ratio shifts to the right if α increases. This implies larger values of the optimum saturation parameter as the incident optical power is less coherent. As a concrete example we now consider the case of curve 4c) which corresponds to an input signal to noise ratio around 22 dB ($\alpha=640$). The maximum value of the signal to noise ratio is obtained for $s\simeq4\cdot10^{-6}$ and its value is approximately 35 dB. This represents a signal to noise ratio improvement of 13 dB.

The simulation processes realized about the non-linear optical amplifier, some of them are presented in this communication, take part of the tasks realized within the RACE 1027 Program field of activity. Hence, now we present the obtained results in a numerical simulation of a semiconductor optical amplifier used at the CNET Laboratories[9], whose characteristics are presented in Table I. According to them, the new values of the emission, absorption and losses coefficients are $A=1.01146\cdot10^{12}$ s^{-1}, $B=3.375\cdot10^{11}$ s^{-1} and $C=2.125\cdot10^{11}$ s^{-1}, respectively. In figure 5 we have shown the noise figure as it was defined in (6) as a function of the product of the saturation parameter by the incident photon mean number (sn1). This representation has been chosen because this product is in fact that which determines the saturation degree of the device. In this figure we can observe that for a lower values of the sn1 product the noise figure is constant and it corresponds to the value obtained by[9] for the operating wavelength shown in Table I. As the sn1 product increases the noise figure decreases down to a minimum value. Note that when this minimum is lower than unity it means that the relative fluctuations of the amplified optical power are lower than the input ones. Therefore, under given conditions it is possible to achieve a signal to noise ratio larger than the input one.

5.CONCLUSION.

Our aim here has been to analyze the signal to noise ratio at the output of a semiconductor optical amplifier taking into account the saturation phenomenon and, in consequence, its unavoidable non-linear behaviour. Actually, the effect we must consider is the product of the saturation parameter by the number of photons present inside the amplifier medium. When this product is much lower than the unity the amplifier is said to be linear (i.e., the saturation effect is negligible). On the contrary, when this product is appreciable the amplifier shows a non-linear behaviour. The signal to noise ratio improvement is maximum when this product is of the order of unity. Thus, the noise figure and the signal to noise ratio results presented in this paper show that under certain conditions a non-linear optical amplifier tends to reduce the relative fluctuations of the amplified light. These results are quite attractive for future all optical transmission systems.

6. REFERENCES.

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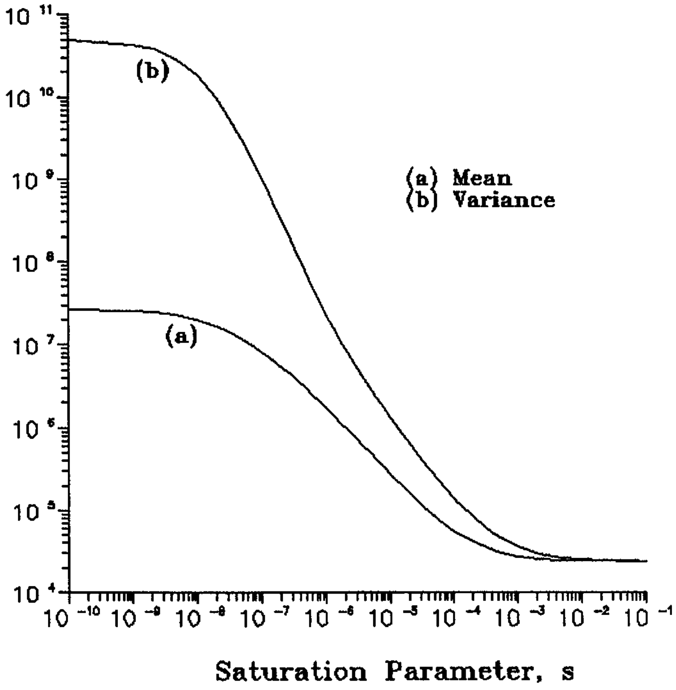


Figure 1. Plot of m1 and σ_m^2 as a function of saturation parameter.

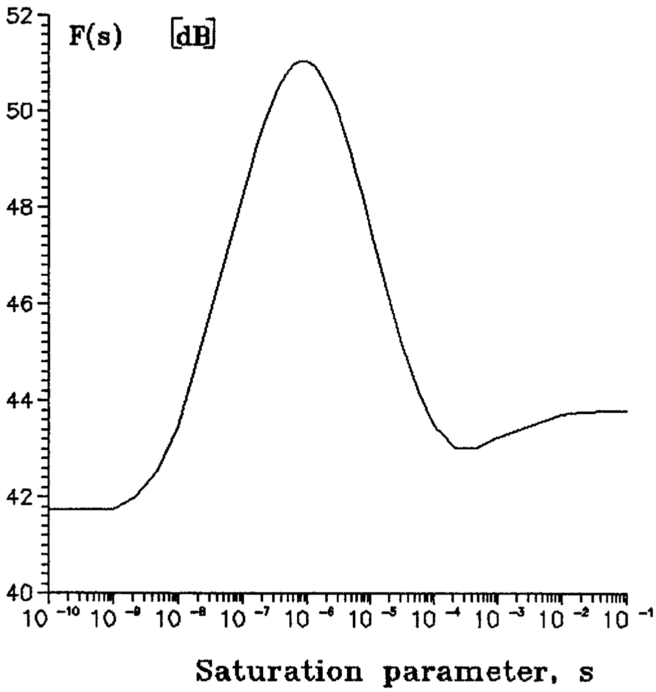
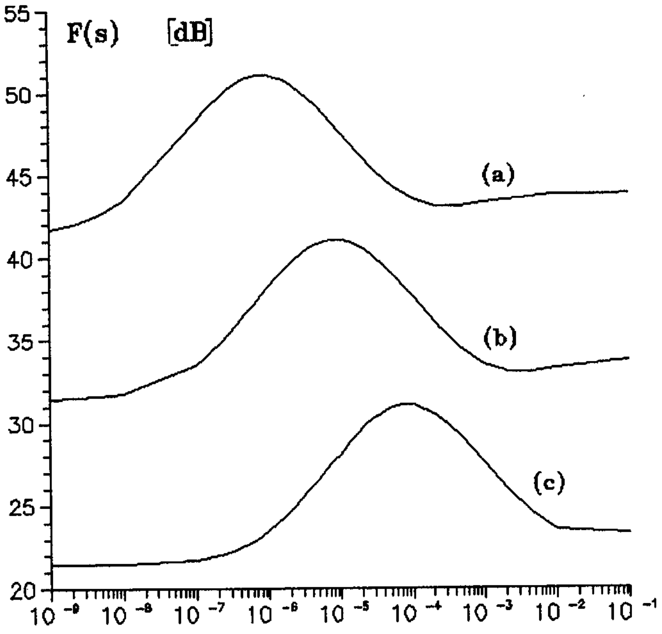


Figure 2. Plot of the F ratio as a function of the saturation parameter.



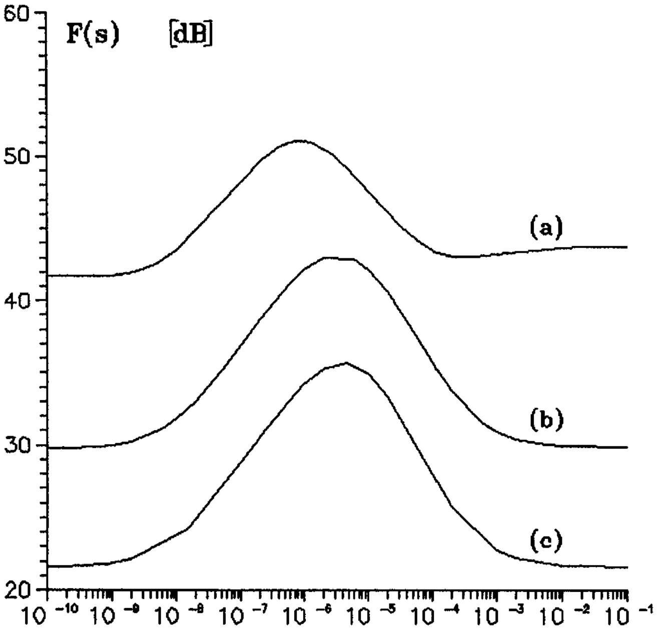


Figure 3

Figure 4

Figure 3. Plot of F ratio as a function of the saturation parameter for different values of input photon mean number:

curve a) $n1=\sigma_n^2=10^5$
curve b) $n1=\sigma_n^2=10^4$
curve c) $n1=\sigma_n^2=10^3$

Figure 4. Plot of F ratio as a function of saturation parameter for different values of α.

curve a) $\alpha=1$
curve b) $\alpha=100$
curve c) $\alpha=640$

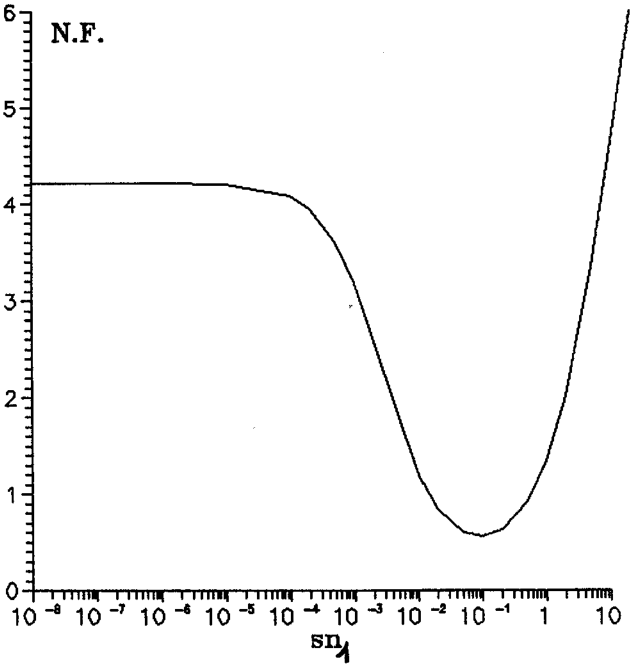


Figure 5. Plot of noise figure (N.F.) as a function of saturation parameter.

Length		L (μm)	785
Width		w (μm)	2.5
Thickness		d (μm)	0.1
Bias current		I (mA)	168
Maximum Linear Power		P_{max} (mW)	>20
		I_{max} (mA)	>277.2
Operating wavelength		λ (μm)	1.53

Table I- Characteristics of the laser used in the simulation.