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PERFORMANCE OF OPTICAL AMPLIFIER-FIBER CHAINS
IN COHERENT OPTICAL FIBER TRANSMISSION LINKS.

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This paper deals with the performances of a coherent optical fiber transmission link with linear optical amplifiers chains in which identical sections of attenuating fiber alternate with identical amplifiers. The overall signal to noise ratio degradation is derived by using the optical noise factor concept.

1. INTRODUCTION

Despite the present day low attenuation fibers, in long distance links it is necessary to compensate in some way the losses introduced by them [1] and so amplification is required. An excellent option against the usual optoelectronic repeater is to use the semiconductor laser as an on line all optical amplifier. With the use of those amplifiers we can profit to the maximum the high bandwidth of single mode fibers working at wavelengths of 1.55 μm . Nevertheless, laser amplifiers introduce noise accumulated all along the chain.

Another problem of the optical amplifier is that it modifies the parameters which characterize the input light, mainly the mean and variance of the photon flux or of the electric field. In intensity modulation and direct detection (IM-DD) systems it is important to know the statistic of the photon number, but in coherent systems it is important to know the electric field properties. Coherent systems are of interest because the sensitivity of the heterodyne detection receiver is better than that of the IM-DD receivers [2].

In this paper, the degradation of the signal to noise ratio of the electric field introduced by the cascaded single mode fiber-laser amplifier sections is modeled through the definition of a noise factor. Finally, the performances of a coherent system with heterodyne detection and cascaded sections of fiber-optical amplifier (figure 1) are evaluated.

2. NOISE FACTOR OF CASCADED FIBER-OPTICAL AMPLIFIER SECTIONS

Due to the fact that this paper

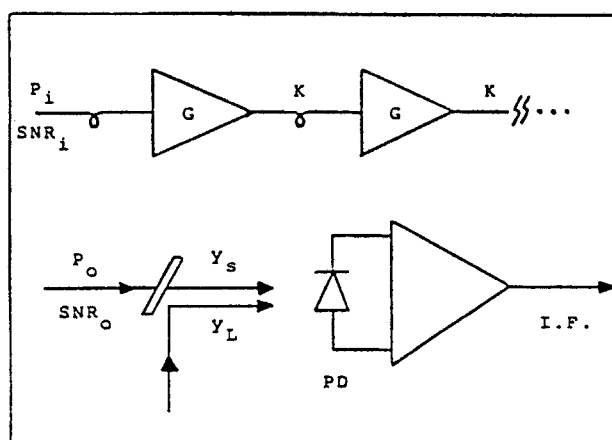


Figure 1

Cascaded sections of fiber-optical amplifiers with heterodyne detection.

deals with coherent optical systems, the definition of the signal to noise ratio, SNR, which we are going to use is

$$SNR = \frac{\langle E \rangle^2}{\sigma_E^2} \quad (1)$$

where $\langle E \rangle$ is the average value of the electric field module and σ_E^2 its variance. The bracket $\langle \rangle$ operator indicates average value.

The optical noise factor F measures the signal to noise ratio degradation

$$F = \frac{SNR_i}{SNR_o} \quad (2)$$

In this expression, subindex 'i' refers to input and 'o' to output.

In a single mode laser amplifier, the following expressions are verified [3]:

$$\langle E_o \rangle = \langle E_i \rangle G^{1/2} \quad (3)$$

$$Q_o^2 = G Q_i^2 + B(2P-1)(G-1) \quad (4)$$

where G is the amplifier gain, P the population inversion parameter and B a constant defined as

$$B = H / (2\epsilon_o V) \quad (5)$$

being H the photon energy, ϵ_o the vacuum dielectric constant and V the quantization volume.

Defining the Q parameter as

$$Q = Q_e^2 / 2B - 0.5 \quad (6)$$

we obtain

$$Q_o = G Q_i + P(G-1) \quad (7)$$

With the latter formulae set and for coherent light at the input ($Q_i = 0$, [3]), the noise factor of the amplifier can be expressed as

$$F_A = \frac{2Q_o + 1}{G(2Q_i + 1)} \Big|_{Q_i=0} = \frac{2P(G-1) + 1}{G} \quad (8)$$

The noise factor of a fiber, F_f , of length L and attenuation α dB/Km, is calculated knowing that [3]

$$\langle E_o \rangle = \langle E_i \rangle K^{1/2} \quad (9)$$

$$Q_o = K Q_i \quad (10)$$

and

$$K = 10^{-\alpha L / 10} \quad (11)$$

The expression obtained for F_f , with $Q_i = 0$, is the following one:

$$F_f = K^{-1} \quad (12)$$

The total optical noise factor, F_T , for the configuration shown in figure 1, with $KG=1$, is, for coherent light at the input ($Q_i=0$)

$$F_T = 2rP(G-1) + G \quad (13)$$

being r the number of optical amplifiers.

For the calculation of the total optical noise factor it can also be used a formula equivalent to the Friis formula which is used for electrical signals

$$F_T = F_f + \frac{F_A - 1}{K} + \frac{F_f - 1}{KG} + \dots + \frac{F_A - 1}{K^r G^{r-1}} + \frac{F_f - 1}{K^r G^r} \quad (14)$$

3. HETERODYNE DETECTION

For heterodyne detection (Fig.1), if both optical signals are unimodal and monochromatic of frequencies f_s and f_L with $|f_s - f_L| = f_i$ (intermediate frequency, I.F.), we can express

$$y_s(t) = \sqrt{2P_s} \cos(2\pi f_s t + \theta_s) \quad (15)$$

and

$$y_L(t) = \sqrt{2P_L} \cos(2\pi f_L t + \theta_L) \quad (16)$$

where P_s and P_L are, respectively, the optical powers of the received signal and of the local oscillator.

The photodetector (PD) current without multiplicative effect can be expressed as [4]

$$i_D = SP + i_R \quad (17)$$

where

$$P = P_s + P_L + 2\sqrt{P_s P_L} \cos(2\pi f_i t + \theta_i) \quad (18)$$

and being

$S = \eta q / H$ = photodetector's sensitivity.

η = photodetector's quantum efficiency.

q = electron's charge.

i_R = noise current.

If P_L is high enough, in the expression of i_R we will only take into account the shot noise produced by P_L whose current spectral density is qSP_L . From (17) we obtain:

$$i_D = i + i_N \quad (19)$$

$$i = S \langle P \rangle \quad (20)$$

$$i_N = S(P - \langle P \rangle) + i_R \quad (21)$$

4. CALCULATION OF THE SIGNAL TO NOISE RATIO

From (17)-(21) and being $P_s = AE_s^2$ and $P_L = AE_L^2$, the signal to noise ratio (neglecting the thermal noise due to the intermediate frequency amplifier) is

$$SNR = \frac{2S^2 A^2 \langle E_s \rangle^2 \langle E_L \rangle^2}{S^2 \sigma_p^2 + \sigma_r^2} \quad (22)$$

being [3]

$$\sigma_r^2 = 2qS \langle P_L \rangle \Delta f \quad (23)$$

$$\sigma_p^2 = \sigma_s^2 + \sigma_L^2 + 4A^2 \{ \langle E_L \rangle^2 \sigma_{ES}^2 + \langle E_S \rangle^2 \sigma_{EL}^2 + \sigma_{ES}^2 \sigma_{EL}^2 \} \cos^2(2\pi f_T t) + 4A \{ \langle E_L \rangle \langle E_S P_S \rangle - \langle E_S \rangle \langle P_S \rangle + \langle E_S \rangle \langle E_L P_L \rangle - \langle E_L \rangle \langle P_L \rangle \} \cos(2\pi f_T t) \quad (24)$$

where σ_s^2 , σ_L^2 , σ_{ES}^2 and σ_{EL}^2 are, respectively, the variances of the power of the received signal, of the power of the local oscillator, of the module of the received signal electric field and of the module of the local oscillator electric field.

If the variations of the field associated to the local oscillator are neglected, and $\sigma_L^2 \gg \sigma_s^2$, the maximum value of σ_p^2 can be approximated by

$$\sigma_p^2 = \sigma_L^2 + 4A^2 \langle E_L \rangle^2 \sigma_{ES}^2 \quad (25)$$

Taking into account that [4]

$$\sigma_L^2 = H \langle P_L \rangle \Delta f \approx HA \langle E_L^2 \rangle \Delta f \quad (26)$$

from (23)-(26), the signal to noise ratio is

$$SNR = \frac{2S^2 A^2 \langle E_L \rangle^2 \langle E_S \rangle^2}{S^2 HA \langle E_L^2 \rangle \Delta f + 4S^2 A^2 \langle E_L \rangle^2 \sigma_{ES}^2 + 2qSA \langle E_L^2 \rangle \Delta f} \quad (27)$$

If it is verified that $\langle E_L^2 \rangle \approx \langle E_L \rangle^2$, the relative error is approximately $\sigma_L^2 / 8 \langle P_L \rangle^2$ and then we have the following approximate signal to noise ratio:

$$SNR \approx \frac{1}{\begin{bmatrix} 1 & 1 \\ -+ & - \\ 2 & \gamma \end{bmatrix} \frac{H \Delta f}{\langle P_S \rangle} \left[\frac{\sigma_{ES}^2}{\langle E_S \rangle^2} + 1 \right] + 2 \frac{\sigma_{ES}^2}{\langle E_S \rangle^2}} \quad (28)$$

5. RESULTS AND DISCUSSION

From figure 1, if $KG=1$, we have

$$\langle P_i \rangle = G \langle P_s \rangle \quad (29)$$

From (1) and (2):

$$\frac{\sigma_{ES}^2}{\langle E_S \rangle^2} = F_T \frac{\sigma_L^2}{\langle E_L \rangle^2} \quad (30)$$

If we have coherent light at the input, it is verified [3], [4]

$$\frac{\sigma_L^2}{\langle E_L \rangle^2} = \frac{H \Delta f}{4 \langle P_i \rangle} \quad (31)$$

From (28)-(31) we can obtain

$$SNR \approx \frac{4 G N_s}{\begin{bmatrix} 1 & 1 \\ -+ & - \\ 2 & \gamma \end{bmatrix} \left[\frac{F_T}{N_s} + 4G \right] + 2F_T} \quad (32)$$

where $N_s = \langle P_s \rangle T / H$ is the photon mean number in the time interval T and we have assumed $\Delta f = 1/T$ (bit rate).

From (13) and for r large enough

$$F_T = 2rP(G-1) + G \approx 2rPG \quad (33)$$

and so

$$SNR \approx \frac{N_s}{\begin{bmatrix} 1 & 1 \\ -+ & - \\ 2 & \gamma \end{bmatrix} \left[1 + \frac{rP}{2N_s} \right] + rP} \quad (34)$$

With coherent systems and neglecting the phase noise of the transmitter and local oscillator lasers, we have [5]

- Homodyne detection

$$\begin{cases} P_e = 0.5 \operatorname{erfc}(\sqrt{2K} \eta N_s) \\ \text{FSK is impossible} \end{cases}$$

- Heterodyne detection

$$\begin{cases} \text{Coherent} & P_e = 0.5 \operatorname{erfc}(\sqrt{K} \eta N_s) \\ \text{Noncoherent} & P_e = 0.5 \exp(-K \eta N_s) \end{cases}$$

The K parameter is 0.25 for ASK, 0.5 for FSK and 1 for PSK and erfc is the complementary error function.

If there is only shot noise, the ideal signal to noise ratio is

$$SNR' = \eta N_s \quad (35)$$

For instance, an error probability of 10^{-9} for heterodyne PSK with coherent detection is achieved with $SNR=18$. In table I, the values of N_s for several values of the r and η parameters are shown. In figure 2 we can see the degradation of the signal to noise ratio defined as

$$\Delta SNR = 10 \log_{10} \frac{SNR'}{SNR} \quad (36)$$

If we take into account the phase noise of the transmitter and local oscillator lasers, from [6] we have calculated the signal to noise ratio necessary to obtain a determined error probability (figure 3). In that figure $\sigma_{\Delta\gamma}^2$ is the variance of the phase noise of the

transmitter and local oscillator lasers which we have supposed to be the same. The signal to noise ratio of this figure is that given by (34).

r	N _s		
	η = 1	η = 0.7	η = 0.5
1	118	126	136
2	209	215	226
4	388	396	407
6	567	577	588
8	749	755	766
10	928	936	947
50	4528	4537	4548
100	9029	9035	9048

Table I ($P_e = 10^{-9}$, $P = 5$)

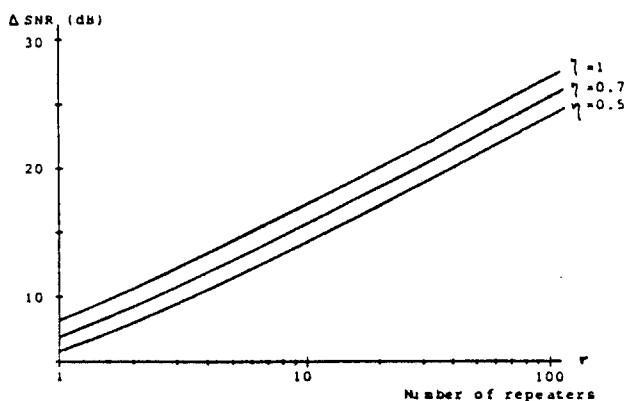


Figure 2

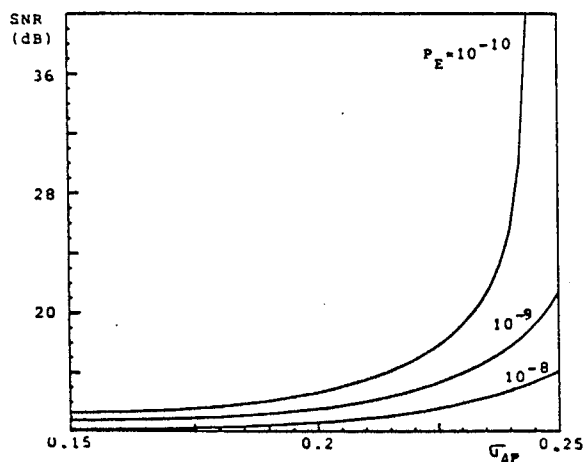


Figure 3

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