Design of a Flight Management System to support four-dimensional trajectories

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Abstract. This paper presents a design and simulation of the functions of a Flight Management System (FMS) in order to follow automatically four-dimensional trajectories. This is achieved by controlling the aircraft airspeed, altitude, heading and vertical speed in order to arrive to the merging point in the specified time. The system receives data from the aircraft and computes new control parameters based on mathematical equations and prediction trajectories algorithms. Additional features has been added to the FMS-4D, such as the capability of predicting the arrival time taking into account previous flight parameters and speed/altitude constrains. Finally, a testing phase is performed using a flight simulator in order to obtain the performance and results of the designed system.

Keywords: FMS, 4D-trajectories, four dimensional navigation, control system.

1. Introduction

The procedures and tasks related to the air traffic management have been improved since several years ago in order to increase the safety and reliability of the air transportation. The concept of four dimension airspace management has become more prominent in the recent years due to labours performed by the international organizations with the objective of taking the airspace into a new dimension of performance and reliability.

According to Eurocontrol, the most-likely scenario C (Regulated Growth) has 14.4 million flights in 2035. This is equivalent to 50% more than 2012 and a growth average of 1.8% per year. This forecast pretends to alert to the international organizations to perform important improvements in the airspace management. In this scope, a possible solution in order to improve the air traffic transportation aims to the concept of 4D navigation. [1]

The 4D trajectory management consists in establishing far in advance a sequence for all aircraft converging to a specific point in a congested area. This is achieved using trajectory predictions computed by air traffic management ground and airborne systems. The main idea is providing each aircraft with a time constraint to get a specific merging point while allowing this aircraft to perform an autonomous flight in order to achieve this merging point in the given time. [2]
If the merging point is not achieved in the specified time, the aircraft has to perform holding patterns in order to wait a new time to achieve the merging point and to avoid disrupting the flight sequence predicted previously by the ground systems. Several problems are related to this fact; in one hand, the aircraft consumes more quantity of fuel which is traduced to economic losses for the airline companies and even worst the CO2 emissions produced by this consumption. In the other hand, a new merging point sequence has to be computed by the ground systems and this produces delays to the airlines as well as conflicts in the airports.

According to the problems described above, it is important to provide the aircraft with more accurate avionics systems capable to follow precise trajectories to achieve the specific merging point in the assigned time with a very low errors tolerance.

In this paper, it is described the design of a Flight Management System (FMS) in order to support navigation procedures using four dimensional trajectories as part of the improvements for the future aircraft avionics and navigation systems necessary to evolution the air traffic transportation.

2. State of the Art

2.1 Next Generation Flight Management System (NG-FMS)

The Next Generation Flight Management System or NG-FMS is an avionic system developed and launched in 2010 by Honeywell© Inc. The system is based on performance that meets both the Single European Sky ATM Research (SESAR) and the NextGen Air Traffic Management (ATM) objectives [8].

According to a research performed by R. Sabatini et al. [9], the Next Generation Flight Management System (FMS) generates optimal trajectories (operationally and environmentally) thanks to mathematical models which were developed for this purpose. In the performance field, several cost functions have been considered in order to optimize the fuel consumption, the flight time and the CO2 emissions; in addition, the models include aircraft dynamics, engine, atmospheric, noise and weather.

Other important features of the NG-FMS are enabling the Required Navigation Performance (RNP) 0.1, support Wide Area Augmentation System – Localizer Performance with Vertical guidance (WAAS-LPV), Future Air Navigation System 1 (FANS-1) and FANS-2 [8].

This system is currently used by few modern aircraft like the Boeing 747-8 as a stand-alone system and integrated in other avionics systems of the Gulfstream G650.

2.2 Flight Management Computer (FMC) and 10.7 GE Aviation Systems Update

The Flight Management Systems used in the new generation of Boeing B737-NG (-600 / 700 / 800) are provided with trajectory tracking function, which is a feature enabled by the 10.7 GE Aviation Systems Update [7].

These FMS use ARINC 702A-1 Trajectory Bus which is a technology capable to send each minute or when the Flight Plan (FP) changes, information about the state of the aircraft. This is latitude, longitude, altitude, turn radius, turn direction, fly-by waypoint type, and of course time. With this information, it is possible to obtain the full aircraft trajectory including vertical waypoints and turns.

The main idea of the use of this bus is to track this information and sending it through an ACARS download data-link or output on a dedicated ARINC 429 in order to perform four-dimensional trajectory intent. The first trial flights utilizing this technology were performed in Sweden with the collaboration of SAS Airlines [7].

3. Theoretical Model

3.1 Four Dimensional Navigation

In order to perform navigation, it is necessary to know the initial waypoint and the final waypoint (and they are provided in the flight plan). In the mathematical scope, a point in the space is composed by three dimensional components (x, y and z). However, when these points are used in order to represent a waypoint, it is taken its position with respect to the Earth; this position is given by its geographical coordinates.

Hence, a three dimension waypoint is composed by latitude, longitude and altitude.

The four dimensional navigation consists in achieving a three dimensions waypoint at a required time of arrival by changing the aircraft flight profile. In this context there are several variables and parameters involved in the concept of changing a flight profile. According to S. Mohleji [10], the airspeed is considered the most important control variable in order to achieve a waypoint at a given time of arrival.

3.2 Distance Estimation

The distance estimation is considered one of the most important parameters of the mathematical model for navigation and trajectory prediction. The Haversine equation for orthodromic distances provides simplicity for computational calculation and it is used for navigation purposes.

The equation provides the orthodromic distance between two points in a sphere. As mentioned above, this equation is often used for navigation purposes due to its great precision for small distances [12], because
of this, it result to be used for estimation of four-dimensional precision trajectories.

The Haversine equation [12] can be divided in three main elements.

The square of half the chord length between two points denoted by:

\[ a = \sin^2\left(\frac{\Delta \varphi}{2}\right) + \cos(\varphi_1) \cdot \cos(\varphi_2) \cdot \sin^2\left(\frac{\Delta \lambda}{2}\right) \]  

(3.1)

The angular distance expressed (in radians) by:

\[ c = 2 \cdot \text{atan}^2\left(\sqrt{a}, \sqrt{1-a}\right) \]  

(3.2)

The orthodromic distance expressed (in kilometers) by:

\[ d = R \cdot c \]  

(3.3)

3.3 Bearing Estimation

The bearing refers to the direction that the aircraft have to move in order to reach final point of a path. The bearing could change depending of the latitude of the initial and final point as well as the total distance of the path. Because of this, it is possible to compute an Initial Bearing and a Final Bearing.

However, when the distance between two points is really small, the Initial Bearing value is closer to the Final Bearing value. [13]

The equation of the initial bearing from a given initial point to a final point in a straight line along a great-circle arc is denoted by the following equation:

\[ \theta = \text{atan}^2\left(\sin(\Delta \lambda) \cdot \cos(\varphi_2) \cdot \cos(\varphi_1) \cdot \sin(\varphi_2) - \sin(\Delta \varphi) \right) \]  

(3.4)

3.4 Time Estimation

The time that an aircraft spend to move from an initial point \((\text{lat}_1, \text{lon}_1)\) to a final point \((\text{lat}_2, \text{lon}_2)\) can be represented as following:

\[ t = \frac{1}{\beta} \cdot \int_{x_1}^{x_2} \frac{\partial x}{V_a(z_c)} \]  

(3.5)

where \(V_a\) is the true airspeed of the aircraft at a given altitude and \(\beta\) is a conversion factor from knots to feet / seconds.

3.5 Wind Effect

An important element that has to be taken into account for time computation is the along-track wind effect. The wind produces an important change of the airspeed of the aircraft and this could affect the estimation of the time of arrival. The effect of the along-track wind over the aircraft airspeed in cruise level is associated with two elements [10]:

1. The direction of the wind with respect to the aircraft. Depending of the relationship between the heading of the along-track wind and the aircraft, it is called tailwind or headwind.

2. The magnitude of the along-track wind which represents the constant velocity of the wind at a given altitude (cruise altitude).

Table 1: Constants and variables of the wind effect equations

<table>
<thead>
<tr>
<th>Constant</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_w)</td>
<td>Speed of the wind at the given altitude.</td>
</tr>
<tr>
<td>(H_w)</td>
<td>Heading of the wind at the given altitude.</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Factor to correct the magnetic north to true north.</td>
</tr>
<tr>
<td>(B)</td>
<td>Bearing of the aircraft track.</td>
</tr>
</tbody>
</table>

The along-track component of wind in the horizontal plane can be represented as following [10]:

\[ W(z_c) = V_w(z_c) \cdot \cos \left( |B(z_c) - H_w(z_c) + \delta| \right) \]  

(3.6)

By using the equation 3.5 into 3.6, the resultant equation to compute the time an aircraft flight between an initial point and a final point with constant velocity, is the following:

\[ t = \frac{1}{\beta} \cdot \int_{x_1}^{x_2} \frac{\partial x}{V_a(z_c) + \sqrt{V_w(z_c)^2 \cdot \cos \left( |B(z_c) - H_w(z_c) + \delta| \right)}} \]  

(3.7)

By integrating the equation 3.7, the result is the time estimation equation with wind effect.

\[ ^1 \text{The along-track component of the headwinds is negative and for tailwinds it is positive.} \]
Note that this equation can be used in order to estimate the minimum time at which an aircraft can fly from the initial point to the final point at cruise airspeed. In addition, it is possible to isolate the airspeed in order to obtain the velocity that an aircraft should fly in order to arrive to the final point in a specific time or even it is possible to isolate the factor \( \frac{2 - \dot{x}_1}{\beta} \) in order to estimate the distance flew in a specific time at a given velocity.

This is an important statement that should be remembered in order to understand the computation of the flight control parameters explained in the next section.

### 3.2.4 Vertical Speed Estimation

The vertical speed is the rate of climb or descent of an aircraft. It can be estimated using the following equation:

\[
VS = \frac{|Z_1 - Z_2|}{t} \cdot \frac{1}{\hat{\beta}}
\]

Where \( \hat{\beta} \) is a conversion factor from seconds to minutes and \( Z_1 - Z_2 \) is the difference of altitude between the initial point and the final point of the vertical trajectory.

The vertical speed is represented as a negative value when the aircraft is descending and positive value for the contrary case.

### 4. Design

#### 4.1 Heading Control

The heading control allows the aircraft to perform a correct navigation in the horizontal plane. The basic idea is computing the heading necessary to achieve a target waypoint (composed by latitude and longitude) from the current aircraft position (composed by latitude and longitude as well).

To perform this control, the position of the aircraft is received and by using the equation 3.4 from the previous chapter, it is computed a new heading which is sent to the aircraft autopilot panel.

The Figure 1 (a) shows the Heading Control performed by the FMS-4D over the aircraft.

#### 4.2 Vertical Speed Control

In the same way that the aircraft uses the Heading Control in order to achieve a point in the horizontal plane, in the vertical plane, the situation is similar.

When the aircraft is flying in a constrained situation (e.g. below a constraint flight level), the vertical speed remains constant. This has been designed in order to obtain a correct prediction of the time when the aircraft crosses the constraint altitude.

In the other hand, when the aircraft is flying in a non-constrained situation (e.g. over a constraint flight level) the vertical speed is computed using the equation 3.9 and the vertical speed control is performed as shown in the following figure.

![Image](image.png)

**Figure 1 a)** Heading Control, **b)** Altitude Control, **c)** Airspeed Control loops.

#### 4.3 Airspeed Control

The airspeed control is considered the core of the flight control regarding to four-dimensional navigation. The airspeed is the most important parameter to validate the aircraft arrives to the target waypoint at the expected arrival time.

The airspeed of the aircraft is sensitive to different factors. When an aircraft is ascending, the airspeed tends to decrease (also in the other way around). In addition, the aircraft airspeed is modified by the flaps position, airbrakes and landing gear. However, most of these changes can be controlled with a change in the aircraft thrust, which is traduced directly as a change of the aircraft airspeed.
The equation 3.8 describes the time spent by an aircraft while flying a known distance at a given airspeed. The main idea of the airspeed control is that the time and the distance are known parameters; therefore it is possible to obtain the required true airspeed of the aircraft in order to arrive to the target waypoint in the defined arrival time.

The airspeed provided by the equation 3.8 refers to true airspeed (TAS). However, the airspeed sent to the aircraft is indicated airspeed (IAS). For this reason, the true airspeed (TAS) is converted to indicated airspeed (IAS) before sending to the aircraft using the density information provided by the variables of the simulator.

Additionally to the routines shown in the (c), it has been added a proportional controller feature. The main idea is to improve the airspeed computation with respect to the delays performed by the aircraft (the airspeed changes slower than other parameters such as vertical speed and heading).

### 4.4 Speed and Altitude Constraints

The main idea of the speed constraints is that the FMS-4D performs a check before computing a required airspeed. If the flight level is below the constraint level, then the airspeed is not computed using the airspeed control loop but it remains constant at the constraint airspeed (except that the airspeed control loop determines that the airspeed have to be lower than the constraint airspeed).

The Figure 2 shows the typical flow performed by the airspeed control in order to follow the airspeed constraint.

![Figure 2: Speed / Altitude Check](image)

### 4.2.5 Minimum Arrival Time Prediction

What could happen if the ATM Ground Systems assigns an arrival time that cannot be achieved by the aircraft due to technical limitations or due to speed / altitude constraints?

This section pretends to get a response to the question above by explaining one of the key features of the FMS-4D: How to compute the Minimum Predicted Arrival Time (PAT).

The Predicted Arrival Time (PAT) is a parameter designed for this FMS-4D in order to give to the pilots an idea of the best performance of the aircraft. This parameter pretends to inform to the crew the minimum time that the aircraft requires in order to achieve a target waypoint. By using this information, the crew could discuss with the ATC if is possible to be assigned a specific arrival time or if this time has to be delayed instead.

When a new waypoint is added to the FMS-4D, a new PAT for that waypoint is computed taking into account the following parameters and / or situations:

- The maximum aircraft cruise speed.
- The existence of speed / altitude constraints.

If there is not existence of speed / altitude constraints in the trajectory, the PAT is computed using the equation 3.8. For this case, the distance is computed using the equation 3.3 and the speed is a constant parameter equivalent to the maximum aircraft cruise speed. 

If the case is the trajectory involves a speed altitude constraint, it is necessary to compute the PAT by using the following equation:

\[
PAT = PAT_{CS} + PAT_{NCS}
\]  

(4.1)

Where \( PAT_{CS} \) is the predicted arrival time from the initial position until the point where the constraint altitude (CAP) is achieved, and \( PAT_{NCS} \) is the predicted arrival time from the point where the constraint altitude (CAP) is achieved and the target waypoint. Hence, the trajectory of the aircraft is divided into two sections as shown in the Figure 3.

Initially, with the conditions of a normal flight controlled by the FMS-4D and the equations explained in the previous sections, it is not possible to know the exact components of the point CAP (latitude and longitude) or the distance from the CAP point and the Target Point.

In order to figure out this problem, it has been decided that since the aircraft airspeed is constant in most of the cases while flying before the CAP point, therefore the vertical speed could be defined as a constant parameter according to the aircraft specifications.

Hence, by knowing the vertical speed and the airspeed at which the aircraft would be flying from Initial Point to CAP, it is possible to predict the time when the

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2 The time is computed continuously and the distance is computed using the Haversine equation (3.3)

3 The maximum cruise speed is a parameter defined in the FMS-4D and depends of the aircraft model, and other technical specifications.

4 This equation is used for any situation where the aircraft trajectory involves speed / altitude constraints. Despite of the Figure 4 it shows an ascending situation, this procedure is applied for descending situations as well.
aircraft crosses the constraint flight level and the
distance at that point.

This means that the $\text{PAT}_{CS}$ is calculated using a
constant vertical speed as following:

$$\text{PAT}_{CS} = \frac{|Z_{IP} - Z_{CAP}|}{V_{CS}} \cdot \frac{1}{\partial} \quad (4.2)$$

Where $Z_{IP}$ is the altitude at the initial point,
$Z_{CAP}$ is the constraint altitude and $V_{CS}$ is the
constant vertical speed defined by the aircraft
specifications. Also $\partial$ is a conversion factor from
minutes to seconds for the vertical speed.

Once the $\text{PAT}_{CS}$ is computed, it is necessary to
compute the other component of the equation 4.1.

Then, in order to calculate the value of $\text{PAT}_{NCS}$ it
is necessary computing the distance from $\text{CAP}$ to the
Target Point as follows:

$$D_{\text{CAP} IP} = D_{\text{total}} - D_{\text{CAP} TP} \quad (4.3)$$

Where $D_{\text{total}}$ is a well-known parameter which is
calculated using the Haversine equation 3.3 and
$D_{\text{CAP} TP}$ is calculated using the equation 3.8,
isolating the distance $x_{1}$ and replacing the
time with the $\text{PAT}_{CS}$.

Finally, the value of $\text{PAT}_{CS}$ is calculated by using
again the equation 3.8 using the Maximum Cruise
Speed of the aircraft.

The control functions use the information provided by
the input parameters in order to produce new output
parameters as shown in the following figure.

5. Test and Results

It has been designed and performed specific tests in
order to know the general behavior of the FMS-4D
algorithms. Taking into account the requirements of
the test, the route chosen was from Frankfurt Main Airport
(EDDF) to Amsterdam Schiphol Airport (EHAM).

The automatic flight performed by the FMS-4D has
been taken into account between the waypoints
MARUN (begin of ascending phase) to seconds before
REKKEN (begin of approach phase).

5.1 Horizontal Plane Navigation (Heading control)
The Latitude [deg] vs. Longitude [deg] plot represents
the horizontal navigation behavior of the aircraft. It can
be pointed out the constant changes of the aircraft
heading in order to achieve the flight plan waypoints in
an efficient way.

In the test simulation, the aircraft maintained the
heading of the aircraft with a very accurate precision.
When a new target waypoint had been followed, the
heading of the aircraft changes instantly to the new
target. It has been detected slowly changes in the
output of the heading control system few seconds after
turning and intercepting the predicted course.

The following figure shows the changes of heading
performed by the aircraft in the whole simulation as
well as a briefly overview of the path followed during
the simulation.

**Figure 5**: Horizontal Plane Navigation (Heading Control)

\[5\] The bank angle of the aircraft has been set to 30 degrees showing a normal performance
during the whole simulation.
The Minimum Distance recorded to the Target Waypoint is shown in the Table 2. According to this information, the performance of the FMS in the horizontal plane has been achieved with a very low error in the route path. The maximum error is below 0.5 nautical miles and the minimum is around 0.3 nautical miles.

The heading control of the FMS-4D allows the aircraft passing through all the Flight Plan waypoints with an acceptable low error.

Table 2: Minimum Distance to Target Waypoint Results

<table>
<thead>
<tr>
<th>Target Waypoint</th>
<th>Minimum Distance (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MARUN</td>
<td>0.36793</td>
</tr>
<tr>
<td>ARPEG</td>
<td>0.32781</td>
</tr>
<tr>
<td>BADGO</td>
<td>0.34483</td>
</tr>
<tr>
<td>ABILU</td>
<td>0.49713</td>
</tr>
<tr>
<td>ADEMI</td>
<td>0.49901</td>
</tr>
<tr>
<td>HMM (HAMM)</td>
<td>0.38090</td>
</tr>
<tr>
<td>REGBU</td>
<td>0.39680</td>
</tr>
<tr>
<td>RELBI</td>
<td>0.41239</td>
</tr>
<tr>
<td>REKKEN (RKN)</td>
<td>-</td>
</tr>
</tbody>
</table>

5.2 Vertical Plane Navigation (Vertical Speed control)

The Altitude [ft] vs. Absolute Time [sec] plot represents the aircraft behavior for the vertical navigation. The FMS-4D controlled the aircraft in order to achieve the requested altitude in a smooth and regular way in the time required for all the waypoints.

Figure 6: Vertical Plane Navigation (Vertical Speed Control)

The Predicted Arrival Time and Arrival Time shown in the Table 5.5 are measured with respect to the simulation starting point (Simulation Time = 0).
As shown in the previous table, the maximum error obtained was 3 seconds and the minimum one was 2 seconds. In all the waypoints, the aircraft arrives before than the required arrival time.

As result the true airspeed varies in a band of no more than ± 15 knots out of the defined cruise airspeed (350 knots). The continuous change of airspeed is the main responsible factor that allows the aircraft matching the Prediction Arrival Time (PAT) with very low errors.

5. Conclusion

In order to provide a possible solution to the problems faced by the aircraft in order to follow a four-dimensional trajectory, it has been designed a software-based Flight Managed System (FMS) that performs control of the flight in order to achieve three dimensional waypoints in the required arrival time estimated.

The system designed is composed by control functions that takes as input parameters that define the flight status and uses mathematical equations and estimation algorithms in order to calculate new values for heading, vertical speed and airspeed. By this way the aircraft is able to follow accurately a four dimensional trajectory defined by the Flight Plan (FP).

Also, the FMS-4D calculates a parameter defined as Prediction Arrival Time (PAT), which is an estimation of the time of arrival the aircraft takes to fly a specific path. The PAT is used to inform to the crew about the estimated performance of the aircraft in order to use this information to discuss the availability of the time constraints with air traffic management services.

6. References


