A DIRECT APPROXIMATION TECHNIQUE FOR DESIGNING DIGITAL EQUALIZERS WITH SIMULTANEOUS SPECIFICATION OF MAGNITUDE AND PHASE.

J.B. Mariño-Acebal, A. Mayer-Pujadas, A.R. Figueiras-Vidal.
E.T.S.I. Telecomunicación, Barcelona, Spain.

ABSTRACT.
A new direct method for designing nonrecursive and recursive digital equalizers approximating specified attenuation and phase responses is introduced. This method allows to control both magnitude and phase characteristics independently; to the authors' knowledge, this is the only direct method proposed in the literature which allows this independent control in nonrecursive designs without using sucessive equalization.

I. INTRODUCTION.

The previously proposed direct methods for the simultaneous approximation of both magnitude and phase of a given frequency response do not allow an independent control of both characteristics, or they require an unnecessary high number of coefficients in the transfer function of the equalizer.

Starting from the Taylor series for the natural logarithm of the transfer function of a nonrecursive digital equalizer having maximun phase, A.T. Johnson [1] has developped a direct method for approximating the magnitude of a given frequency response. His procedure is extended in this paper to the whole transfer function.

II. TRANSFER FUNCTION ANALYSIS.

If we assume that the transfer function of the equalizer has no zeros on the unit circle of the z-phane, we can write:

$$H(z) = N_1(z^{-1})\ N_2(z^{-1}) / D(z^{-1})\ ;$$

where $D(z)$ and $N_2(z)$ ($N_1(z)$) are polynomials having their roots outside (inside) the unit circle. If we denote by n_1 the degree of $N_1(z)$, the following series expansions are valid in the unit circle (see Appendix):

$$\ln z^{n_1} N_1(z^{-1}) = \sum_{n=0}^{\infty} a_n z^n \qquad (1.a)$$

$$\ln N_2(z) = \sum_{n=0}^{\infty} b_n z^n \qquad (1.b)$$

$$\ln D(z) = \sum_{n=0}^{\infty} c_n z^n \qquad (1.c)$$

If the frequency response corresponding to H(z) is considered as a function of its attenuation $\alpha(\omega)$ and its phase $\phi(\omega)$, we can easily obtain from (1):

$$\alpha(\omega) = -\operatorname{Re}\left[\ln H(\exp j\omega)\right] = -\sum_{n=0}^{\infty} (a_n + b_n - c_n) \cos n\omega \qquad (2.a)$$

$$\phi(\omega) = \operatorname{Im}\left[\ln (\exp j\omega)\right] = -n_1\omega + \sum_{n=1}^{\infty} (a_n - b_n + c_n) \operatorname{sen} n\omega \qquad (2.b)$$

III. DESIGN METHOD.

If the transfer function to be approximated has an attenuation $\hat{\alpha}(\omega)$ and a phase $\hat{\phi}(\omega)$, these can be approximated in the form:

$$\hat{\alpha}(\omega) \simeq \bar{\alpha}(\omega) = \sum_{n=0}^{P} \alpha_n \cos n\omega \qquad (3.a)$$

$$\hat{\phi}(\omega) \simeq \bar{\phi}(\omega) = -n_1\omega + \sum_{n=1}^{Q} \beta_n \operatorname{sen} n\omega \qquad (3.b)$$

where the cosine and sine sums do not have to be obtained necessarily from Fourier series truncation. Identifying (3) with the first terms in the series (2), we obtain:

$$-\alpha_n = a_n + b_n - c_n \qquad n = 0, 1, \ldots, P \qquad (4.a)$$

$$\beta_n = a_n - b_n + c_n \qquad N = 1, 2, \ldots, Q \qquad (4.b)$$

where we require $c_o = 0$ in order to insure the usual normalization for the constant term of $D(z^{-1})$. To obtain a_n, b_n and c_n, additional conditions must be imposed.

If the indetermination is resolved choosing as the order of $H(z)$ the minimum possible, we arrive to one of the two following situations:

a) Recursive design; if b_n is selected as zero for all n, we will obtain a maximum phase recursive filter.

b) Nonrecursive design: c_n is assumed to be zero for all n.

In general, P and Q are different, and we cannot obtain a_n, b_n, c_n from (4). Let's suppose, for example, that P >Q and that we want a nonrecursive design ($c_n = 0$ for all n). The problem is due to the fact that from n = Q to n = P we only have the α_n for obtaining a_n and b_n; this will be solved if we think that, given a polynomial, its series development (1) is known, and, then, there will be no difficulty in order to obtain the coefficients of the other polynomial. This consideration allows us to establish the following steps for the design method:

<u>a</u>) Obtain the cosine and sine series (3) for $\hat{\alpha}(\omega)$ and $\hat{\phi}(\omega)$;

<u>b</u>) Determine the first N + 1 (N = min (P,Q)) coefficients of

(1) by using (4);

c) Obtain one of the two polynomials constituting H(z) using the coefficients calculated in b), and (A.2);

d) With T = max (P,Q), calculate for the polynomial in c) the coefficients of its corresponding expansion (1) from the (N+1) th to the Tth. To that end, it suffices to impose: $P_i = 0$, when i = N + 1, N + 2, ... , T, in (A.2), from which:

$$d_{N+i} = -\frac{1}{N+i}\,\frac{1}{P_0}\sum_{j=0}^{N}(N+i-j)\,d_{N+i-j}\,P_j \qquad i > 0$$

e) Determine, using (4), for the other polynomial constituing H(z) the coefficients of its expansion (1) from the (N + 1)th to the Tth.

f) Obtain this polynomial by means of (A.2).

IV. STABILITY AND CONVERGENCE.

For a correct design, it is necessary that the roots of the polynomials constituting H(z) as found above verify the conditions imposed in the Section II. Let z_k (p_k) be a root of $N_1(z)$ ($N_2(z)$ or D(z) respectively for nonrecursive or recursive desings). The conditions are:

$$|z_k| < 1 \qquad k = 1, \ldots, n \qquad (5.a)$$

$$|p_k| > 1 \qquad k = 1, \ldots, m \qquad (5.b)$$

where $m = n_2$ or $m = d$, depending of the case. In this way, when we consider a recursive equalizer, a correct design guarantees that $D(z^{-1})$ has all its roots in the unit circle.

However, when H(z) has low order and α_n and β_n are relatively high, (5) can be violated. Nevertheless, since the coefficients of the polynomials obtained with (A.2) go to zero as 1/k (k high enough), one can easly prove that a desing verifying (5) is finally obtained completing (3) with $\alpha_n = 0$, $\beta_n = 0$ for $P < n \leqslant M$, $Q < n \leqslant M$, respectively.

The approximation errors can be written as:

$$\bar{\alpha}(\omega) - \alpha(\omega) = -\sum_{n=P+1}^{\infty}\frac{1}{n}\Big(\sum_{k=1}^{n_1} z_k^{\,n} \pm \sum_{k=1}^{m} p_k^{-n}\Big) \qquad (6.a)$$

$$\bar{\phi}(\omega) - \phi(\omega) = \sum_{n=Q+1}^{\infty}\frac{1}{n}\Big(\sum_{k=1}^{n_1} z_k^{\,n} \mp \sum_{k=1}^{m} p_k^{-n}\Big) \qquad (6.b)$$

(upper (lower) signs refer to recursive (nonrecursive) cases). The convergence as the order of the approximating H(z) is increased according to the method given above, is seen clearly in (6), since we have the double effect of increasing P and Q and decreasing the moduli of z_k and p_k^{-1}.

V. AN EXAMPLE: WIDEBAND DIFFERENCIATOR.

Let us consider:

$$H(\exp j\omega) = (\omega/\pi)\ \exp(j\pi/2)\ \exp(-jk\omega) \qquad 0 \leqslant \omega \leqslant \pi$$

an ideal differenciator. It is appropiate to work with:

$$H(z) = (1-z^{-1})\ H_1(z)$$

The desing problem is reduced to that of $H_1(z)$.

To obtain recursive designs when the polynomials constituing H(z) have a common degree M, one has to impone the condition: $M = P = Q + 1$; and calculate $N_1(z^{-1})$ first.

Table I summarizes maximum errors and the angular frequency ω_{max} for which maximum magnitude errors occur. The $\bar{\alpha}(\omega)$ cosine series has been obtained by using the DAPMM subroutine [2], which enables us to have a constant relative maximum error for the modulus of H(z). Table I includes previous results [3] for comparison.

TABLE I: MAXIMUM ERRORS

M	MAGNITUDE	ω_{max}	PHASE	MAGNITUDE	PHASE
2	$3.41\ 10^{-2}$	0.860	5.34º	$1.10\ 10^{-2}$	10.5º
4	$2.01\ 10^{-2}$	0.900	2.16º	$0.63\ 10^{-2}$	10.5º
6	$1.42\ 10^{-2}$	0.943	1.29º	$0.60\ 10^{-2}$	10.5º
	Obtained values.			Previous values [3].	

VI. CONCLUSIONS.

A direct method for designing digital equalizers approximating both magnitude and phase of a given frequency response has been introduced. The obtained designs, provided that the approximation is good enough, will have guaranteed stability. To the authors' knowledge, this is the only direct method proposed in the literature which allows independent control of both magnitude and phase in nonrecursive equalization without using sucessive equalization.

APPENDIX [3]

Let us consider the polynomial of degree M

$$p(z) = \sum_{k=0}^{M} p_k\ z^k$$

and define: $a = \{\min |z_k|\ ,\ k = 1, \ldots, M\}$; being z_k the k-th root of p(z). If $|z| < a$, we can write

$$\ln p(z) = \sum_{n=0}^{\infty} d_n z^n \qquad (A.1)$$

where

$$d_o = \ln p_o; \; d_n = -\frac{1}{n} \sum_{k=1}^{M} z_k^{-n} \qquad n = 1, 2, \ldots$$

It had been proved that p(z) can be reconstructed from the d_n by using the following equations:

$$p_o = e^{d_o}; \; p_k = \frac{1}{k} \sum_{i=0}^{k-1} (k-i) \, d_{k-i} \, p_i \quad i = 1, \ldots, M \qquad (A.2)$$

Note that, although the reconstruction of p(z) requieres only the M + 1 first coefficients of the series (A.1), the validity of (A.2) holds for <u>any natural</u> number k.

The authors are with
E.T.S.I. Telecomunicación, Aptdo. 2549, Barcelona, Spain.

REFERENCES

[1] JOHNSON,A.T.: "Magnitude Equalization Using Digital Filters". I.E.E.E. CT - 20, pp. 308-311; May 1973.

[2] IBM Systems/360 Programming Manual, Sci. Subroutine Package (360 A - CM - 03 X), Fortran Prog. APMM, pp. 283-288.

[3] RABINER, L.R.; STEIGLITZ, K.: "The Design of Wide-band Recursive and Nonrecursive Digital Differenciators". I.E.E.E. AU - 18, pp. 204-209; June 1970.